Homework 3 (score 7)

Deadline: Nov. 4 (Monday), 2024

I. Problem Description

Problem 1 (score 1): Suppose we have two independent cues x_A and x_V about a stimulus s, which have normal distributions around s with variances σ_A^2 and σ_V^2, respectively. Suppose the stimulus has a flat prior.

1) Show that the posterior $p(s|x_A, x_V)$ is a normal distribution. Show the mean and variance.
2) Show that the distribution of the MAP (maximum a posteriori) estimates is also a normal distribution. Show the mean and variance.

Hint: Directly using the following fact to solve the second problem: If random variables X and Y are independent and have normal distributions with means μ_X and μ_Y, and variances σ_X^2 and σ_Y^2, respectively, then the random variable $aX + bY$ has mean $a\mu_X + b\mu_Y$, and variance $a^2\sigma_X^2 + b^2\sigma_Y^2$.

Problem 2 (score 2): Complete the proof in page 51 of Lecture 6. I.e., show that the log posterior ratio in the visual search example is

$$d = \log \frac{1}{N} \sum_{j=1}^{N} \frac{p(x_j|T_j = 1)}{p(x_j|T_j = 0)}.$$

Problem 3 (score 4): An observer is trying to detect a signal of strength $s = 3$ in noise. The noise has a normal distribution with zero mean and standard deviation $\sigma = 2$. On each trial, an experimenter presents noise, or noise plus signal, each with 50% probability. The task of the observer is to respond whether the signal is present or absent.

1) We will simulate such an observer in Matlab. Start by simulating the measurement on each of 10,000 trials (use the `randn` function). Plot two histograms of the measurements: one for the trials when the signal was present and one for the trials when the signal was absent. Plot them in the same plot.
2) Calculate the decision variable (log posterior ratio) on each trial. Plot the histograms of the decision variables in the same way as you did in 1). How do these histograms compare to the ones in 1), and why?
3) Assume now that on each trial, the observer also provides a confidence rating by reporting "high confidence" when the absolute value of the log likelihood ratio exceeds 2, "medium confidence" when it lies between 1 and 2, and "low confidence" when it lies between 0 and 1. Create a 2-by-6 table of the two possible stimuli (signal present or absent) and the six possible responses. In each cell, put the numerical frequency of the response conditioned on the stimulus. Calculate the empirical ROC by cumulatively summing the response frequences. Plot the resulting points to form an empirical ROC.
4) What happens when you reduce the signal strength to $s = 2$? Interpret.

II. Files to be Submitted

1) A brief report about the results in either Chinese or English.
2) For problems 1 and 2, if you write the answers in a paper by hand, please scan or take a photo to change it to e-version. Please make sure every detail is clear in the e-version.
3) Complete source codes.