

Tarefa 1 de Métodos Computacionais para Equações Diferenciais

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Sumário

- 1 Introdução ao problema
- 2 Função escolhida para demonstração
- 3 Resultados (gráficos)
- 4 Resultados (tabulados)
- 5 Extra: Interatividade

Aproximação via diferenças finitas

Dada uma determinada função y , é possível aproximar, numericamente, a sua derivada (dada por y') a partir das seguintes fórmulas:

- Diferença **avançada**: $y'_x = \frac{y_{x+1} - y_x}{h} + O(h)$
- Diferença **atrasada**: $y'_x = \frac{y_x - y_{x-1}}{h} + O(h)$
- Diferença **centrada**: $y'_x = \frac{y_{x+1} - y_{x-1}}{2h} + O(h^2)$

Comportamento esperado do erro

Observamos que as diferenças avançada e atrasada possuem ordem de erro dominada assintoticamente por h , enquanto a diferença centrada é dominada assintoticamente por h^2 . Dessa maneira, é esperado que o erro do valor da derivada aproximado pela fórmula da diferença centrada seja muito menor para um mesmo valor de h .

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Para fazer a demonstração desse comportamento, é necessário escolher uma função com solução analítica conhecida, calcular o erro de cada aproximação e observar a evolução desses valores. No caso demonstrado, foi escolhida a função $y(x) = \cos(x^2)$, cuja derivada analítica é $y'(x) = -2x \sin(x^2)$.

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$h = 0.1$

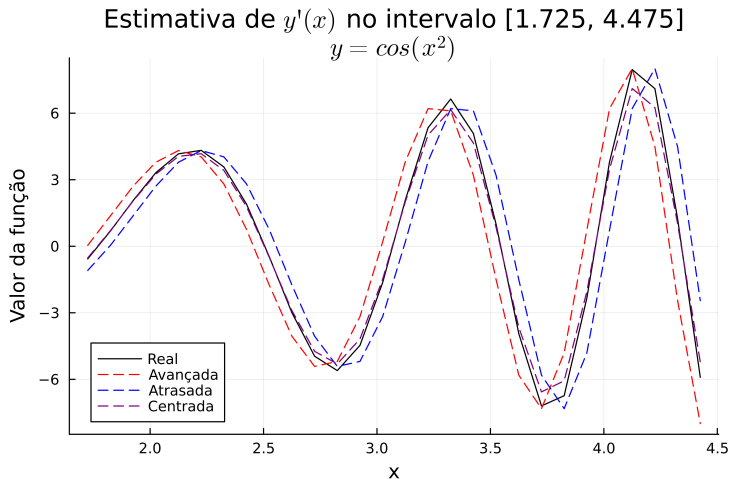


Figura: Resultados para $h = 0.1$

$h = 0.01$

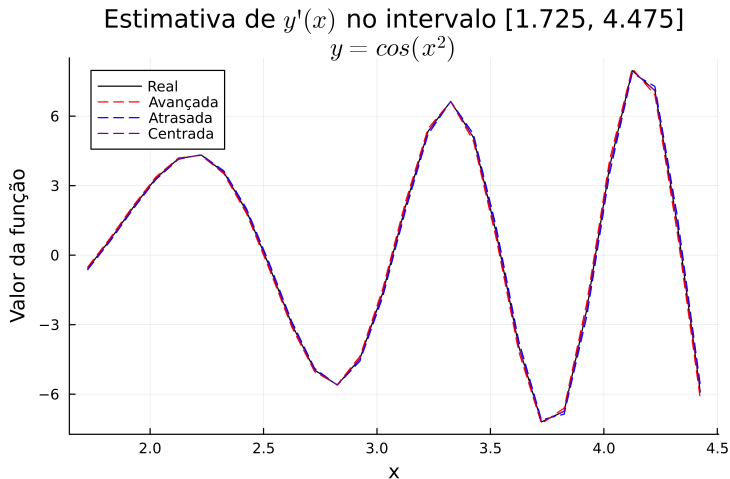


Figura: Resultados para $h = 0.01$

$h = 0.0001$

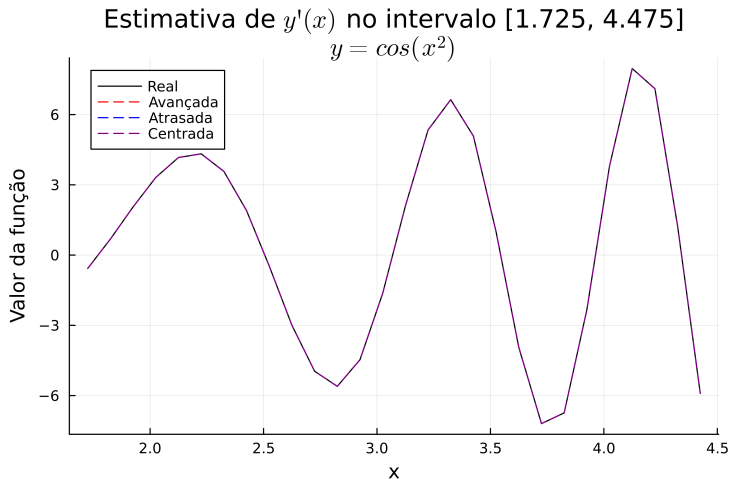


Figura: Resultados para $h = 0.0001$

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$$h = 0.1$$

x	ValorReal	ValorAvancada	ErroAvancada	ErroAvancadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
1.725	-0.5699633399572341	0.040724288704098166	0.6106876286613323	-1.071450715948071	-1.0914072093975213	0.5214438694402872	-0.9148726468607832	-0.5253414603467116	0.04462187961052955	-0.0782890345436439
1.825	0.6858662687548717	1.3708022336274606	0.6849359648725889	0.9986436074718011	0.040724288704094835	0.6454149800507768	0.9406235762127397	0.7057632611657777	0.019899692410906034	0.02901001742953073
1.925	2.0582034406719172	2.707938908603412	0.6497354679314999	0.315680887735748324	1.3708022336274606	0.6874012070444515	0.3339811767198511	2.039370571151436	0.018832869556475895	0.0091501496681183994
2.025	3.3154766755608374	3.7881562965586135	0.47267962099777616	0.14256762066281733	2.7079389086034165	0.6075377669574209	0.1832426214650684	3.248047602581015	0.06742907297982237	0.020337670741844758
2.125	4.16799345789127	4.314856178063338	0.14668272017206825	0.03523583269882845	3.7881562965586135	0.3798371613326563	0.09113189959871695	4.051506237310976	0.11648722058029382	0.02794803344994204
2.225	4.3243131595220925	4.030407758890633	0.2930540063122916	0.06796579937418559	4.314856178063338	0.0094566981458754399	0.0021869326086919084	4.172631968477101	0.15168119104499134	0.035076365990413865
2.325	3.5766969212971116	2.8103133340970055	0.7663835872001061	0.21427132465061433	4.030407758890633	0.45371083759375175	0.12685191045743138	3.4203605464939346	0.15633637480317697	0.0437097070965914
2.425	1.9001100229138226	0.7666954751501409	1.1344145477636817	0.601762842756511	2.810313334097012	0.910203511181895	0.47902635408928	1.7835044462435764	0.11660561829026118	0.06136782443336162
2.525	-0.466155895989333	1.7461844662579852	1.2886285721720468	-2.7459238114666915	0.7666954751501409	1.2228513703260793	-2.62226988703322	-0.494744460530221	0.28588690997983824	-0.06132841236168481
2.625	2.99652692885069	4.05735569694881	1.0608287680441908	-0.35401943428251786	1.7461844662579852	1.2503424615927048	-0.4172638829153691	-2.9017700820764327	0.09475684677425722	-0.0162222433642569
2.725	4.957590283378782	5.416815849955967	0.459225565771843	-0.09283080251299755	-0.45735569694881	0.9002345864819016	-0.18158712903595	-4.737085773425424	0.22050450995335866	-0.044478152320520175
2.825	5.6047516430439	-5.191686241970069	0.4130654014343209	-0.0736991445371956	-5.416815849955967	0.18793579344842293	-0.03353151136849814	-5.304251045963017	0.30050059744177236	-0.05361532795034695
2.925	4.468172446661291	-3.1718113081733623	1.2963161384879287	-0.29012514838997533	-5.191686241970082	0.723558795308791	-0.16193781487800984	-4.181748775071722	0.28637867158956887	-0.06409366675598274
3.025	-1.6379263478302744	0.2018301254055247	1.8397564732357998	-1.123222955460046	-3.1718113081733623	1.533884960343088	-0.9364798132599454	-1.4849905913839183	0.1529575644635604	-0.0933715711020503
3.125	2.089284516234264	3.8200509436002736	1.7307664273660097	0.8284015000913093	0.20183012540552547	1.8874543908287384	0.9033974912002699	2.0109405340029994	0.07834398171336448	0.03749799566435884
3.225	5.341741150375691	6.204813839335391	0.8630726889569998	0.16157141738307534	3.8200509436002736	1.5216902067754177	0.28486782940959154	5.012432391467832	0.3293087589078594	0.06164820601325818
3.325	6.63801335990995	6.1061014727369586	0.5310118875403639	0.08013118665184182	6.204813839335391	0.4331995205745587	0.06526041709871391	6.155457655852488	0.48255570405746173	0.07269580187527794
3.425	5.08130770558184	3.1948698080015707	1.883643897556133	0.3712506320949374	6.1061014727369595	1.024793766878755	0.65048524019265	4.30822246538918945	0.0847857461802458	0.01397083202356437
3.525	0.989893851196544	-1.505141154801285	2.4941249899209392	2.5219067302744964	3.1948698080015707	2.205885128960265	2.230456246667741	0.8448639266072111	0.14411990851244327	0.1457252418033775
3.625	-3.9382647116212866	-5.810271979766486	1.8720072681451994	-0.475381007175975	-1.505141154801285	2.4331235656820002	-0.6178161538101243	-3.6577065672838858	0.2805581443374008	-0.07123902654662633
3.725	-7.196643576478753	-7.322445892590608	0.12580231644185513	-0.01486992923715554	-5.810271979766486	1.3863715967122667	-0.19264141428971158	-6.56358936343547	0.630284601352508	-0.087580360680328
3.825	-6.73736027392146	-4.909694455159911	1.927669571071548	-0.285116625575718	-7.322445892590608	0.585889895346622	-0.0884236209221186	-6.06566174220209	0.617229831718468	-0.0963696614175301
3.925	-2.373625811138496	0.7358181823694	3.078210329371834	-1.3147385445410138	-4.909694455159911	2.4732236504085207	-1.0577406040133443	-2.070141368481582	0.3003487142553114	-0.13848999128334658
4.025	3.807430652788205	6.20968596146113	2.402254643357826	0.303818513472438	0.73581818233694	1.0717724709645466	0.8067835637964946	3.472671789849258	0.374591380135594	0.08792252420426456
4.125	7.606029954217385	8.013011721281266	0.0470714289456083	0.005908907237656731	6.20968596146113	1.756344600711917	0.22047909721261347	7.1113932406999	0.83463670453815	0.1078251482189732
4.225	7.105544823236437	4.484582186161313	2.620926363424284	0.3688616005705565	8.013011721281266	0.9075563799754285	0.12772509393052484	6.248841699017139	0.8264103128224698	0.12056825332001581
4.325	1.2047083202375673	-2.451987343420804	3.629660546674447	2.706280560419023	4.484582186161313	2.343873866575979	2.61533282066654	1.0162972618097412	0.2947104046060231	0.18087336917622962
4.425	5.90843247418909	-7.99705652891814	2.088213281472904	-0.3534047518327845	-2.451987343420804	3.456855512968629	-0.58503083738037	-5.224522131661947	0.684321157569629	-0.11581304277379278

Tabela: $h = 0.1$

$$h = 0.01$$

x	ValorReal	ValorAvancada	ErroAvancada	ErroAvancadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
1.725	-0.5096833399572341	0.51247117627810083	0.08749157717625888	-0.10086890356917628	0.626548408249783	0.059685868202548835	-0.09927843481441204	-0.5096510885513956	0.00045325444183853426	-0.000795234374821257
1.825	0.6858652687548717	0.753369454326855	0.0675030977781388	0.0984202365234727	0.6187754415992610	0.06709032715566977	0.09781910878549917	0.6860724036659737	0.002081343111020575	0.00300325449234440494
1.925	0.058203440671912	0.1259879199964913	0.06778444929300079	0.032931381886042915	1.99005264328048915	0.06815079741702057	0.033111788694110994	0.2058020160999902	0.00018314906201011283	8.898491684103044e-5
2.025	0.3157467556083174	0.370077192851284	0.05400517129044668	0.016488376234681427	3.25952880228039183	0.05594802775691904	0.01687482019382779	3.314802897827601	0.0006737777332364026	0.00020322197957324737
2.125	0.416799345789127	0.194273302885201	0.0262798844993931434	0.003518255669689675	4.193704440706641	0.028623017184628452	0.0008673373588042565	4.166821871795921	0.001175860953485909	0.0002810911595069598
2.225	0.4324313159520925	0.3091269260089025	0.01518623351319004	0.003518255669689675	4.336435264618027	0.0121221050599594366	0.0028032444011909286	4.322781095313465	0.0015320642086278369	0.00035429076297451944
2.325	0.3597699229711116	0.5137195955260103	0.062977325717110127	0.017607677462439884	3.6365025023056496	0.05980581008383056	0.016720897052370093	3.5751110489158298	0.0015858723812818276	0.00044339025036495754
2.425	1.9001100229138226	1.7946423522618327	0.10546767065198992	0.055500686163502805	2.003195895910348	0.10308587299652539	0.05425258103656702	1.8989191240869093	0.0011908988277322674	0.00062652766478889
2.525	-0.461558950859383	-0.594323917939685	0.1281680223080302	-0.2749466941405981	-0.3389596076793671	0.12755898740657118	-0.27364018936852663	-0.4664604125366678	0.003045174507295134	-0.00065325286035736
2.625	-2.9952602855059	-3.114415191039599	-0.11788826218890092	-0.3934163282626158	-2.876747128818978	0.1197702023171196	-0.03997267605989877	-2.995581458292885	0.000454660214014683	-0.00013152516168085963
2.725	-4.951950283378782	-5.028119209941049	-0.0865289256226628	-0.01382303067509664	-4.88466148923084	0.07398413448898834	-0.04721695478424435	-4.955362677430567	0.00222765948213583	-0.0004483324016072959
2.825	-5.6047516430439	-5.5914481598084	-0.0013302749634966	-0.002375801945845127	-5.611949295067125	-0.0071978516627338991	-5.601688805487583	-5.60305283791680649	-0.0005446874564725781	-0.0005446874564725781
2.925	-4.468127446661291	-4.362409273276646	0.10571817338464484	-0.023660509812511695	-4.5679946575075565	0.09896721084626549	-0.022351021101891137	-4.465201965392102	0.002925481269189234	-0.0006544743653101782
3.025	-1.6370263478302744	-1.4629230900279055	0.17500333880236885	-0.106844494052672	-1.8097675273980984	0.17184117956782408	-0.10491386245508437	-1.636345268213002	0.001581079617272385	-0.0009652934757211806
3.125	2.089284518234264	2.275866640694557	0.1865821244602932	0.08930431590839034	1.9011598026062004	0.188124736280635	0.0900426496085184	2.088513221650379	0.0007712945838851404	0.0003961668501307449
3.225	5.341741150375691	5.463207820966898	0.12146667059120642	0.022739153240823644	5.21359080809381	0.12815034187188168	0.023960369107059543	5.338399314735353	0.003341835640380756	0.0006256079331180321
3.325	6.63801333990995	6.629776822634109	0.00823653727584038	0.0012408136032966005	6.636387248711392	0.0016261111985578225	0.0002446965579672473	6.630382035672751	0.004931324237198886	0.000742891580631857
3.425	5.0813077058184	4.927005959207649	0.15430711267419106	0.030367598581893394	5.226751005580998	0.14544329999915817	0.02862320261364767	5.076875799244323	0.00431906337516445	0.0008721979841228618
3.525	0.989893831196554	0.7429129373049514	0.24607089781470293	0.24881185018047489	1.23912217299345737	0.24303789481491933	0.245745062946873	0.9874673336197626	0.001516501499891798	0.001533393680909668
3.625	-3.933647116212654	-4.16140226141414	0.223137145202764	-0.0566587901992938	-3.70944535245464	0.2388193683756426	-0.05810156887129187	-3.935423686693529	0.00284210272757481	-0.0007271361256712435
3.725	-1.196643576479753	-2.737133732308438	0.0489365582968313	-0.01040874796601248	-1.0882406638087	0.08719195100978829	-0.012023847718668208	-1.190138049344653	0.0064450271714099292	-0.000868041413298183
3.825	-6.737356027392146	-6.60067049617002	0.1366859717514346	-0.020287777166055134	-6.860178807093116	0.12828284353897044	-0.018230125146957977	-6.730424400274059	0.00691516711809512	-0.001028820097736797
3.925	-2.3373628511348996	-2.043196842576092	0.294169668588084	-0.258421502516952	-2.62532885154094	0.2787803040400044	-0.123168110961164	-2.33421484664076382174	-0.00134167016345626477	-0.0005713610345626477
4.025	3.8074306527882853	4.09406890724741	0.286638279366553	0.0752830251076680	3.593795618523	0.2934403666030554	0.0770711520085494	3.8404282835449854	0.0034023693332998484	0.0008936129488814722
4.125	7.966026994217306	8.05530193713546	0.08987198549624116	0.017582409362604	7.8591198005209195	0.10615395638546	0.013420577179024419	7.9571208710717233	0.008819084100071706	0.0011079614884308516
4.225	7.105544823236437	6.911913001879976	0.19363182135646095	0.02750805696002117	7.181357278872686	0.17581245563624837	0.02474299438133588	7.09663514037331	0.008090682860102644	0.0012539058584331147
4.325	1.2407083202357643	0.86977218865108754	0.3709665174288887	0.29091181903446294	1.660891509848002	0.36618318964003596	0.2951404320230982	1.238306690787878	0.00240161307926555	0.0019356935066239532
4.425	-5.908843247418909	-6.199791884532657	0.2909486371374774	-0.049239525391173544	-5.603790055499879	0.30505319191903	-0.051626550095449364	-5.901790970102628	0.007052277402641138	-0.0011935123621379081

Tabela: $h = 0.01$

$$h = 0.0001$$

x	ValorReal	ValorAvancada	ErroAvancada	ErroAvancadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
1.725	-0.5699633399572341	-0.59039286794339664	0.000570420132377883	-0.0010006924666638952	-0.5705337213546668	0.0005703813474327024	-0.0010007313936710345	-0.56996329466243316	4.532962542938314e-8	-7.953652343033109e-8
1.825	0.685862665948717	0.688539332230246	0.000673040681530312	0.0008813435924977085	0.6885193219657814	0.0006730292388002762	0.000881283462117078	0.685863893733583	2.0820531131583715e-8	1.0005005142037474e-8
1.925	2.058203440671912	2.0588832115864264	0.000679770914534216	0.000330273918059553184	2.057523631366022	0.0006798075353038303	0.00033029711066097497	2.0582034223615153	1.8310396754372996e-8	8.896300271516299e-8
2.025	3.315476655608374	3.3160294797629497	0.000552804202123447	0.0001067344566732124	3.314923373660214557	0.0005529389303816864	0.00010677595955564372	3.3154766081922027	6.7368834670827996e-8	2.0319441595658966e-8
2.125	4.16799345789127	4.168267960180512	0.0002745112952418083	6.586173851162735e-5	4.167718712280721	0.000274456105485213	6.5917956283819236e-5	4.1679933407336105	1.171576533565144e-7	2.81088680959046142e-8
2.225	4.3243131595220925	4.32417652343281	0.0001366360892829377	3.159717722618364e-5	4.3244494891855885	0.0001363296634959708	3.1526316079993995e-5	4.3243130063001995	1.5321289303926733e-7	3.543057299212804e-8
2.325	3.576669921297116	3.5760828100217523	0.000614112753592637	0.000171697879041021	3.577310715368043	0.0006137940709312062	0.00017160919262586272	3.5766697626948976	1.5860221402874686e-7	4.434320757913946e-8
2.425	1.9001100229138226	1.899066974666574	0.0010430482472481017	0.0005489409742961016	1.901152832940456	0.0010428100266333118	0.0005488156022805511	1.9001099038035152	1.19110370739467619e-7	6.28600552525916e-8
2.525	-0.4661558958958383	-0.4674388731846925	0.001278942086760949	-0.0027435931031724254	-0.4648770139314973	0.001278881154441025	-0.00274346230510697	-0.466155955880826	3.047215996199171e-8	-5.36903727504196e-8
2.625	-2.99652693853669	-2.997715560272347	0.001188977176571014	-0.00039605159028386143	-2.995381626205762	0.0011887662449137062	-0.000396714687750802	-2.9965168143140616	9.453662830338726e-8	-1.3548733099370806e-8
2.725	-4.957990233718782	-4.9582979113547165	0.000762801510141058	-0.0014278331150737796	-4.956882209681823	0.000760776969991328	-0.001432621829819032	-4.9579900609581	2.7277297251349837e-7	-4.4935736878115607e-8
2.825	-5.60475164304039	-5.606468918841171	0.00010270402321851066	-1.8324616281503337e-5	-5.604853778979315	0.0001020420354190919	-1.821566502713108e-5	-5.604751330884551	3.051983874482194e-7	-5.44757716581035e-8
2.925	-4.468127446661291	-4.467099046088219	0.0010284005730722967	-0.00023016366147367897	-4.46915526240017	0.0010278153788787492	-0.0002300326906849486	-4.468127154004194	2.92970967737812e-7	-6.548539634705314e-8
3.025	-1.6379263478302744	-1.636191484152829	0.0017348636779914717	-0.0010591829603874485	-1.6396608951929625	0.001734547302688188	-0.0010589898410181297	-1.6379261896726227	1.581576516418437e-7	-9.6559684605941813e-8
3.125	2.089284516234264	2.091158632944179	0.001874116709914997	0.0008970136404844054	2.0874102453050103	0.0018742709292536297	0.0008970874549110355	2.0892844391245946	7.710966931639973e-8	3.6902713315007264e-8
3.225	5.341741150375691	5.342989474669245	0.0012483243235532626	0.00023369240261023624	5.340492157608345	0.0012489927673460244	0.00023381753854886456	5.341740816153795	3.3422189638088184e-7	6.25679693141581e-8
3.325	6.63801335900995	6.63797986581846	3.37332810371407e-5	5.027607854071389e-6	6.63804574609745	3.238687702467047e-5	4.87900148262278e-6	6.6380128668441	4.93255395218021e-7	7.430318572430429e-8
3.425	5.08130770558184	5.08130807244599	0.001499633127705285	0.0026951274010867664	5.0826945209469	0.001496746512850424	0.00269495291363757485	5.0813072626963	4.333122100522996e-7	8.724372459589592e-8
3.525	0.989838351196544	0.988537160353321	0.003444674787302318	0.002437927965874281	0.9914302064362168	0.0034463173165634457	0.00243782134535399	0.989838613942844	1.512536993451423e-7	1.531415149087325e-7
3.625	-3.9382047118212866	-3.940525315613286	0.002286036915992327	-0.00057400101688260051	-3.9360035393742887	0.00228611722469978440	-0.00057415445955327559	-3.9382044273497872	2.8412749930959526e-7	-7.71453537544248e-8
3.725	-7.196643576478753	-7.197457094291759	0.00081350178130066478	-0.00011304128158053289	-7.1958287657925	0.0008128106862524429	-0.00011322093102269272	-7.196642930042129	6.464366233416285e-7	-8.982473794511908e-8
3.825	-6.737356027392146	-6.736057374071014	0.0012986532211316958	-0.0001927541480443827	-6.7386532939733845	0.001297260581238905	-0.000192548193452531	-6.73735354022199	6.933699464373886e-7	-0.02914259545481303e-7
3.925	-2.7373628511134986	-2.7344509375644584	0.00209113135490311976	-0.00124581151259991502	-2.7402741346778555	0.0020912813564595874	-0.0012455419846252632	-2.737362536171157	3.149023326617454e-7	-1.3476985819548257e-7
4.025	3.8074306578827853	3.8103324093766687	0.00029017659913833643	0.0007621296602663528	3.804528215594072	0.00290243711942134589	0.0007623086167276415	3.8074303124868072	8.43014150471506e-7	8.937823064416126e-8
4.125	7.966209954217765	7.967038111707476	0.00090876009470628	0.00012313002785676837	7.96504733232276	0.0009082621885029298	0.0001233515171766802	7.96620972019875	4.820192497745783e-7	1.07409446076275e-8
4.225	7.10544823239437	7.10399863934778	0.00184899301659292	0.000260213586329689	7.107391999866541	0.0018471766301040304	0.0002609670181129707	7.1054393190006	8.91333777630862e-7	1.2544284839514097e-7
4.325	1.240783202357643	1.237019652735971	0.003688377286051555	0.00297276787761796	1.2443961773323294	0.00368785706556106	0.0029723801785360223	1.240788079904898	2.4033902444471383e-7	1.937008115792023e-7
4.425	-5.908843247418909	-5.911824710629183	0.0029814634102778324	-0.0005045794941515736	-5.9058603730832715	0.0029828743356379306	-0.0005048152761440021	-5.908842541956227	7.05462620409739e-7	-1.19390996259248e-7

Tabela: $h = 0.0001$

Sumário

- 1 Introdução ao problema
- 2 Função escolhida para demonstração
- 3 Resultados (gráficos)
- 4 Resultados (tabulados)
- 5 Extra: Interatividade**