UNIVERSIDADE ESTADUAL PAULISTA
"JÚLIO DE MESQUITA FILHO"

FCT - Faculdade de Ciências e Tecnologia
DMC - Departamento de Matemática e Computação
Pós-Graduação em Matemática Aplicada e Computacional

2ª Tarefa de Métodos Computacionais para Equações Diferenciais

Guilherme Cesar Tomiasi

Presidente Prudente

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Conteúdo

1 Descrição da Tarefa

O objetivo dessa Tarefa é explicitar a diferença no comportamento de métodos numéricos para a solução de PVIs (isto é, Problemas de Valor Inicial) na solução de Equações Diferenciais Ordinárias. São realizados 3 exercícios, testando os seguintes métodos:

- Euler Explícito, método de **Primeira Ordem**;
- Euler Implícito, método de **Primeira Ordem**;
- Método dos Trapézios, método de **Segunda Ordem**;
- Método Runge-Kutta Clássico de **Terceira Ordem**;
- Método Runge-Kutta Clássico de **Quarta Ordem**;
- Método de Euler Modificado de **Segunda Ordem**;
- Método de Euler Aperfeiçoado de **Segunda Ordem**;

Para cada um dos métodos, são aplicadas quatro tamanhos diferentes de malha computacional, sendo esses tamanhos:

1. $h = 0.1$;
2. $h = 0.01$;
3. $h = 0.005$;
4. $h = 0.001$;

Foi realizada uma função que aplica todos os métodos explícitos, e um método específico foi preparado para a resolução via Euler Implícito. Todos os problemas de Valor Inicial foram descritos em um arquivo **.JSON**, cujo processo de leitura e de *parsing* foram feitos de forma automática por um *script* na Linguagem Julia. Os resultados para cada malha são salvos em arquivos tabulados (CSV), e as representações via gráficos são salvas em arquivos de imagem (PNG).

2 Resultados

Como as tabelas possuem números muito longos em precisão, as mesmas estão muito pequenas. No entanto, o *zoom* pode ser utilizado para permitir a leitura.

2.1 Exercício 1, item a

2.1.1 $h = 0.1$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.0000	1.0000	1.0000	1.0000
0.1000	0.9097	0.9000	0.9182	0.9100
0.2000	0.8375	0.8200	0.8529	0.8381
0.3000	0.7816	0.7580	0.8026	0.7824
0.4000	0.7406	0.7122	0.7660	0.7416
0.5000	0.7131	0.6810	0.7418	0.7142
0.6000	0.6976	0.6629	0.7289	0.6988
0.7000	0.6932	0.6566	0.7263	0.6944
0.8000	0.6987	0.6609	0.7330	0.7000
0.9000	0.7131	0.6748	0.7482	0.7145
1.0000	0.7358	0.6974	0.7711	0.7371

Tabela 1: Valores obtidos para o item a com $h = 0.1$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	9.67e-03	1.06e-02	8.51e-03	9.35e-03	3.25e-04	3.57e-04
2.00e-01	1.75e-02	2.09e-02	1.54e-02	1.84e-02	5.88e-04	7.03e-04
3.00e-01	2.36e-02	3.02e-02	2.10e-02	2.69e-02	7.99e-04	1.02e-03
4.00e-01	2.84e-02	3.84e-02	2.54e-02	3.43e-02	9.64e-04	1.30e-03
5.00e-01	3.21e-02	4.50e-02	2.88e-02	4.04e-02	1.09e-03	1.53e-03
6.00e-01	3.47e-02	4.98e-02	3.13e-02	4.49e-02	1.18e-03	1.70e-03
7.00e-01	3.66e-02	5.28e-02	3.31e-02	4.78e-02	1.25e-03	1.80e-03
8.00e-01	3.77e-02	5.40e-02	3.44e-02	4.92e-02	1.29e-03	1.85e-03
9.00e-01	3.83e-02	5.37e-02	3.51e-02	4.92e-02	1.32e-03	1.85e-03
1.00e+00	3.84e-02	5.22e-02	3.53e-02	4.80e-02	1.32e-03	1.80e-03

Tabela 2: Erros obtidos para o item a com $h = 0.1$

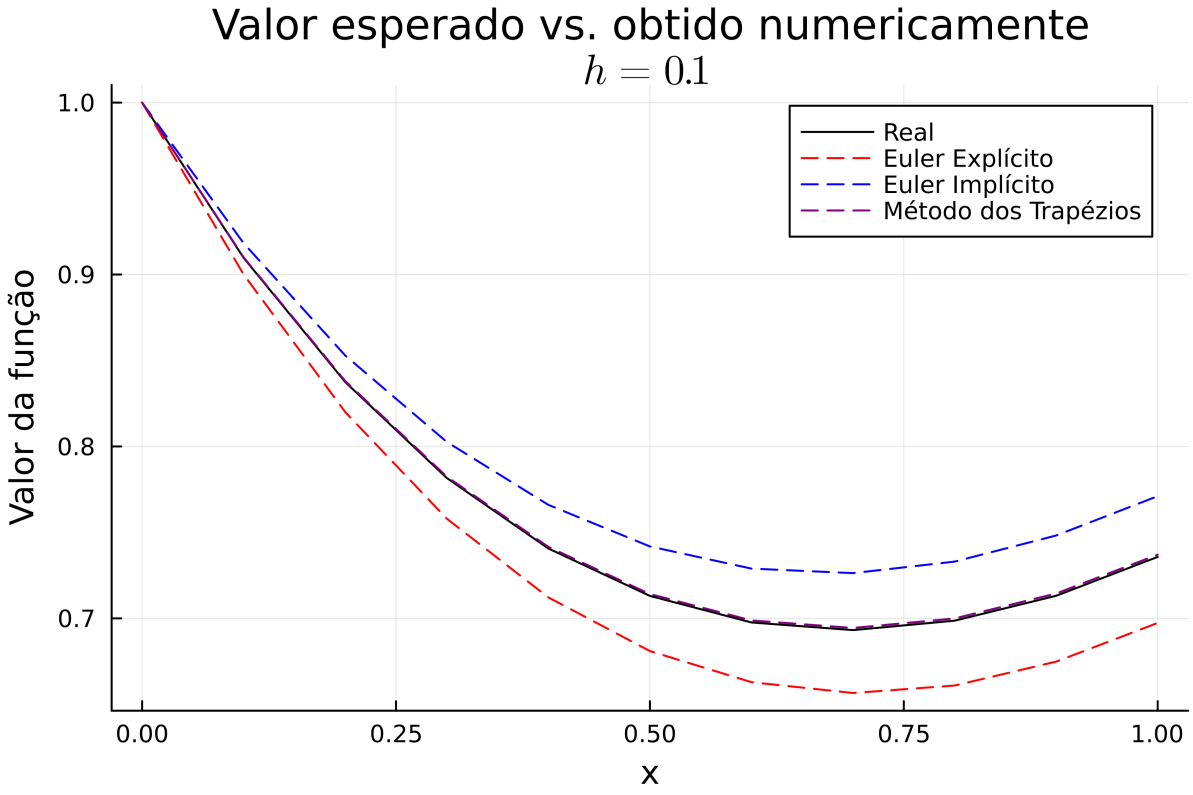

Figura 1: Gráfico plotado para o item a quando $h = 0.1$

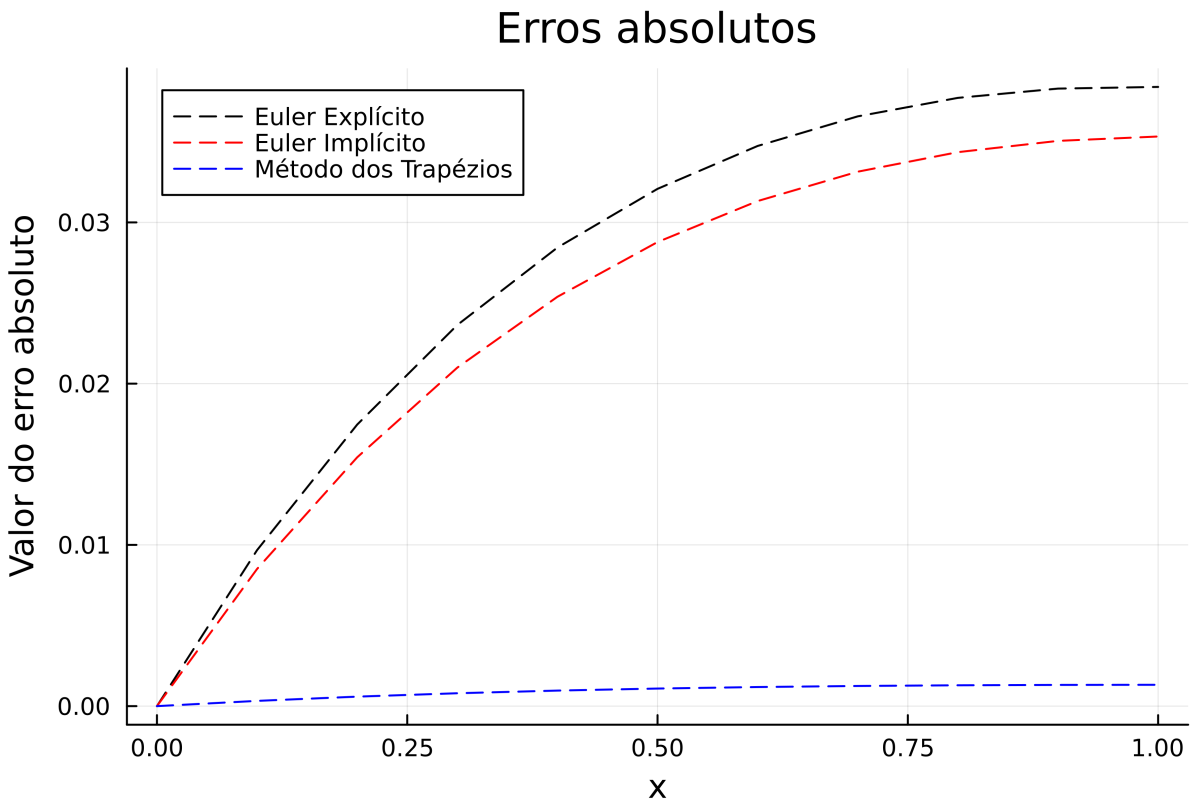

Figura 2: Gráfico plotado para o erro absoluto do item a quando $h = 0.1$

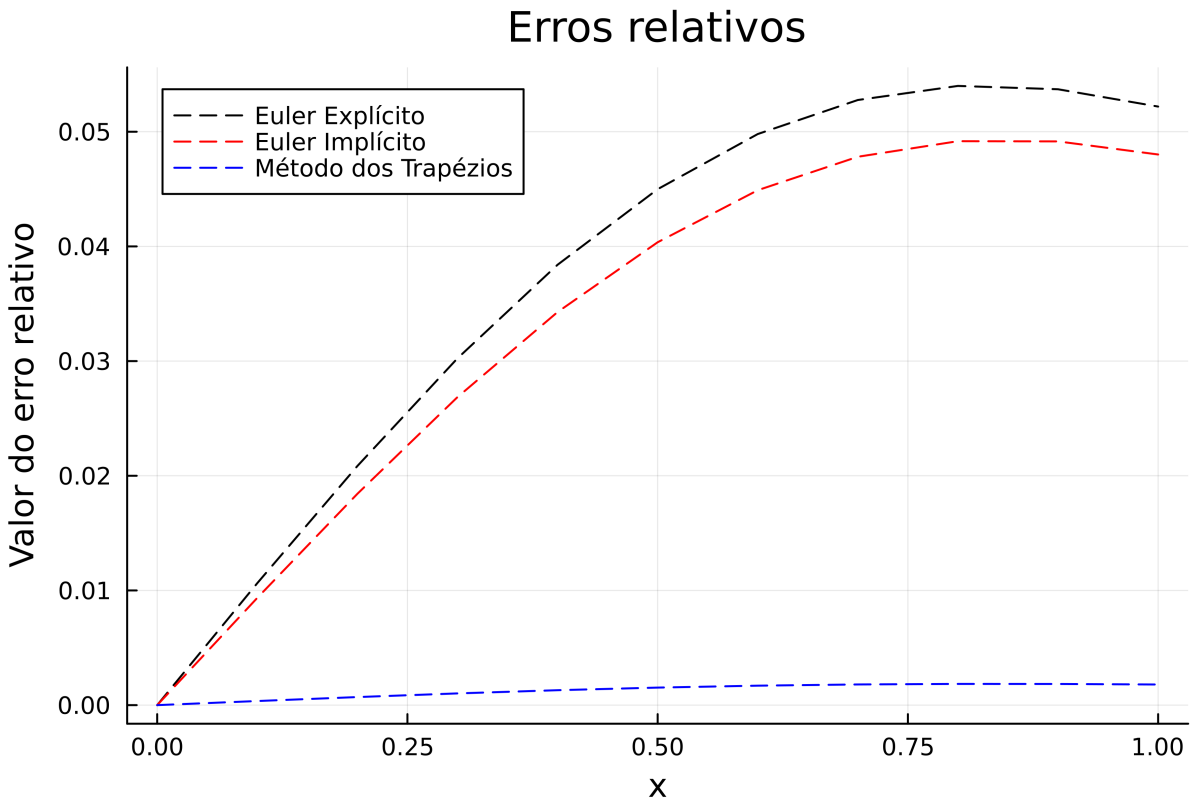

Figura 3: Gráfico plotado para o erro relativo do item a quando $h = 0.1$

2.1.2 $h = 0.01$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.0000	1.0000	1.0000	1.0000
0.1000	0.9097	0.9088	0.9106	0.9097
0.2000	0.8375	0.8358	0.8391	0.8375
0.3000	0.7816	0.7794	0.7838	0.7816
0.4000	0.7406	0.7379	0.7433	0.7406
0.5000	0.7131	0.7100	0.7161	0.7131
0.6000	0.6976	0.6943	0.7009	0.6976
0.7000	0.6932	0.6897	0.6966	0.6932
0.8000	0.6987	0.6950	0.7022	0.6987
0.9000	0.7131	0.7095	0.7168	0.7132
1.0000	0.7358	0.7321	0.7394	0.7358

Tabela 3: Valores obtidos para o item a com $h = 0.01$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	9.11e-04	1.00e-03	8.99e-04	9.88e-04	3.04e-06	3.34e-06
2.00e-01	1.65e-03	1.97e-03	1.63e-03	1.94e-03	5.50e-06	6.57e-06
3.00e-01	2.24e-03	2.86e-03	2.21e-03	2.83e-03	7.46e-06	9.55e-06
4.00e-01	2.70e-03	3.64e-03	2.67e-03	3.60e-03	9.00e-06	1.22e-05
5.00e-01	3.05e-03	4.28e-03	3.02e-03	4.23e-03	1.02e-05	1.43e-05
6.00e-01	3.31e-03	4.74e-03	3.28e-03	4.70e-03	1.11e-05	1.59e-05
7.00e-01	3.49e-03	5.04e-03	3.46e-03	4.99e-03	1.17e-05	1.68e-05
8.00e-01	3.61e-03	5.17e-03	3.58e-03	5.12e-03	1.21e-05	1.73e-05
9.00e-01	3.68e-03	5.15e-03	3.64e-03	5.11e-03	1.23e-05	1.72e-05
1.00e+00	3.69e-03	5.02e-03	3.66e-03	4.98e-03	1.24e-05	1.68e-05

Tabela 4: Erros obtidos para o item a com $h = 0.01$

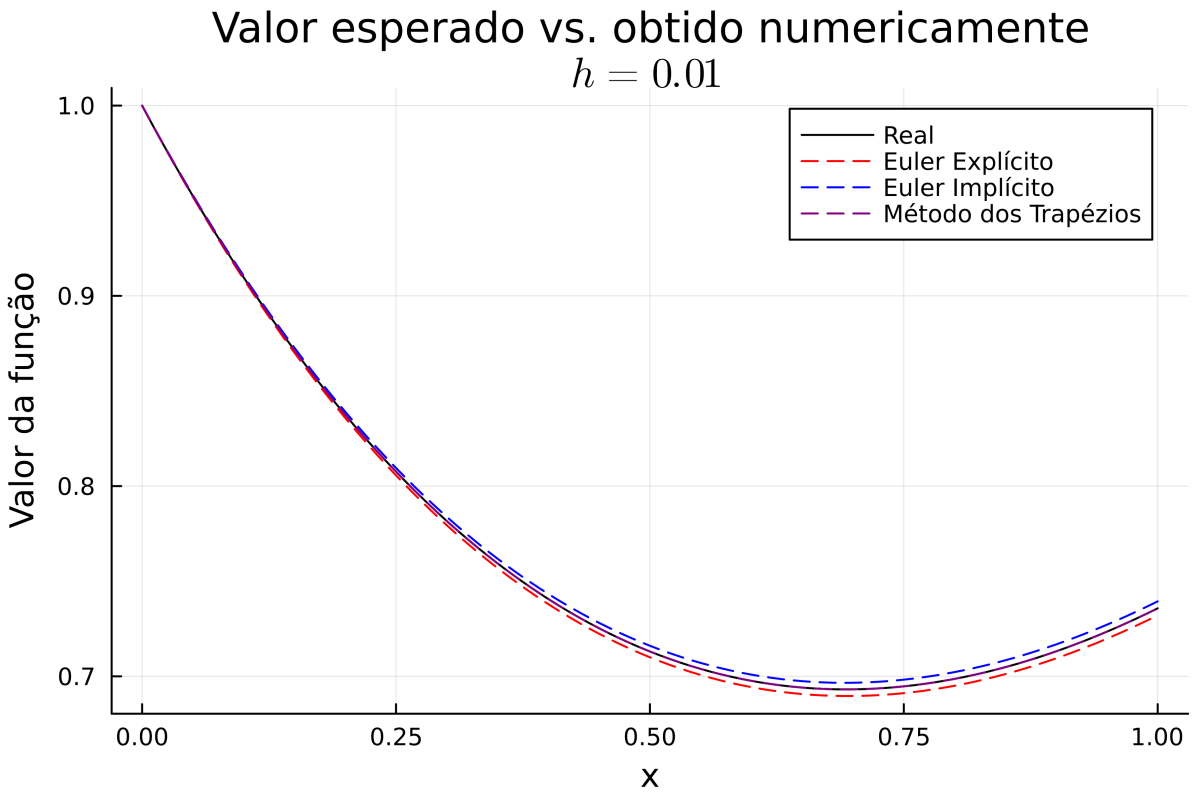

Figura 4: Gráfico plotado para o item a quando $h = 0.01$

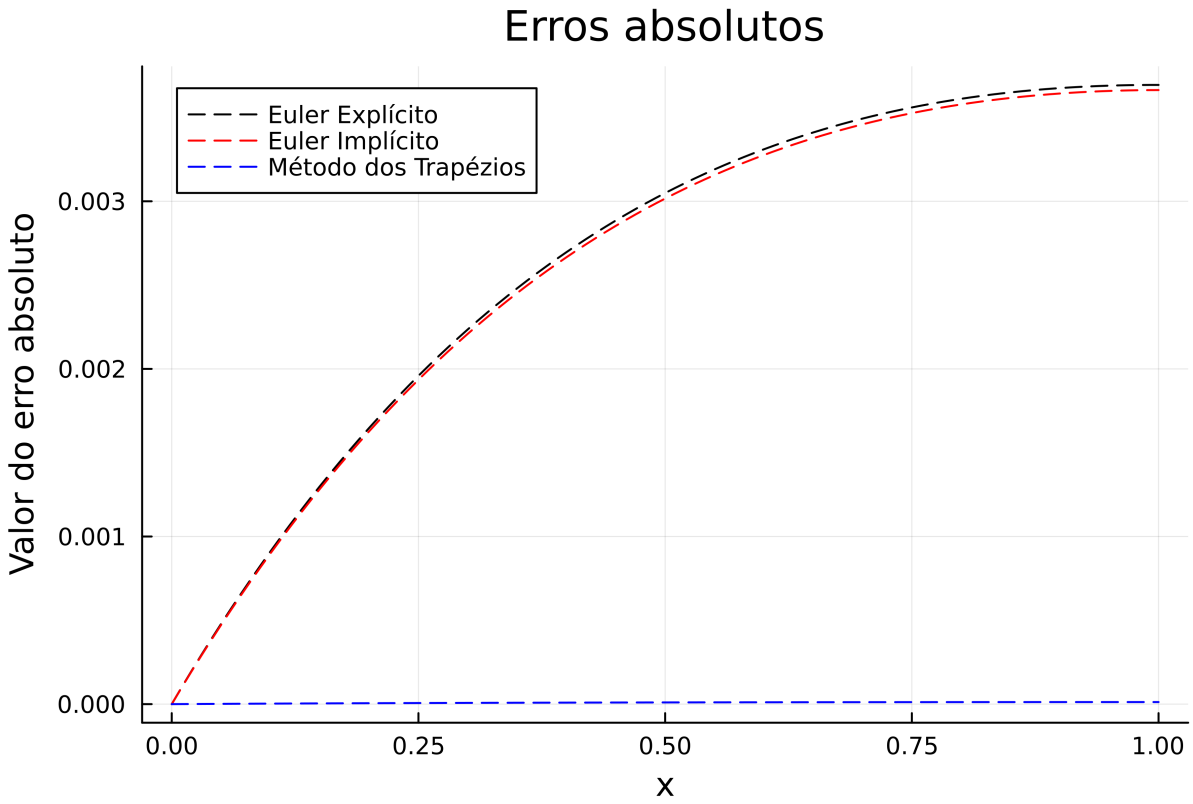

Figura 5: Gráfico plotado para o erro absoluto do item a quando $h = 0.01$

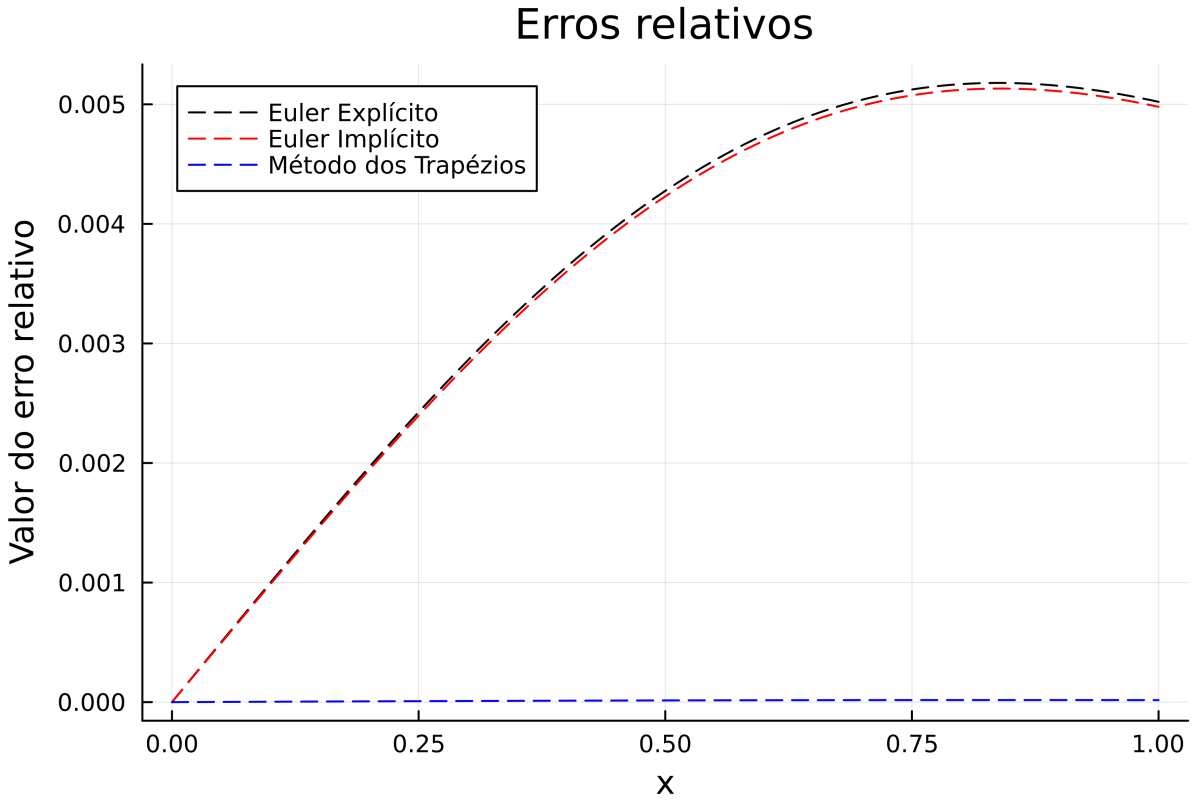

Figura 6: Gráfico plotado para o erro relativo do item a quando $h = 0.01$

2.1.3 $h = 0.005$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.0000	1.0000	1.0000	1.0000
0.1000	0.9097	0.9092	0.9101	0.9097
0.2000	0.8375	0.8366	0.8383	0.8375
0.3000	0.7816	0.7805	0.7827	0.7816
0.4000	0.7406	0.7393	0.7420	0.7406
0.5000	0.7131	0.7115	0.7146	0.7131
0.6000	0.6976	0.6960	0.6993	0.6976
0.7000	0.6932	0.6914	0.6949	0.6932
0.8000	0.6987	0.6969	0.7005	0.6987
0.9000	0.7131	0.7113	0.7150	0.7131
1.0000	0.7358	0.7339	0.7376	0.7358

Tabela 5: Valores obtidos para o item a com $h = 0.005$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	4.54e-04	4.99e-04	4.51e-04	4.96e-04	7.57e-07	8.32e-07
2.00e-01	8.21e-04	9.81e-04	8.16e-04	9.75e-04	1.37e-06	1.64e-06
3.00e-01	1.11e-03	1.43e-03	1.11e-03	1.42e-03	1.86e-06	2.38e-06
4.00e-01	1.34e-03	1.82e-03	1.34e-03	1.80e-03	2.24e-06	3.03e-06
5.00e-01	1.52e-03	2.13e-03	1.51e-03	2.12e-03	2.54e-06	3.56e-06
6.00e-01	1.65e-03	2.37e-03	1.64e-03	2.35e-03	2.75e-06	3.95e-06
7.00e-01	1.74e-03	2.51e-03	1.73e-03	2.50e-03	2.91e-06	4.19e-06
8.00e-01	1.80e-03	2.58e-03	1.79e-03	2.57e-03	3.01e-06	4.30e-06
9.00e-01	1.83e-03	2.57e-03	1.83e-03	2.56e-03	3.06e-06	4.29e-06
1.00e+00	1.84e-03	2.51e-03	1.84e-03	2.49e-03	3.08e-06	4.18e-06

Tabela 6: Erros obtidos para o item a com $h = 0.005$

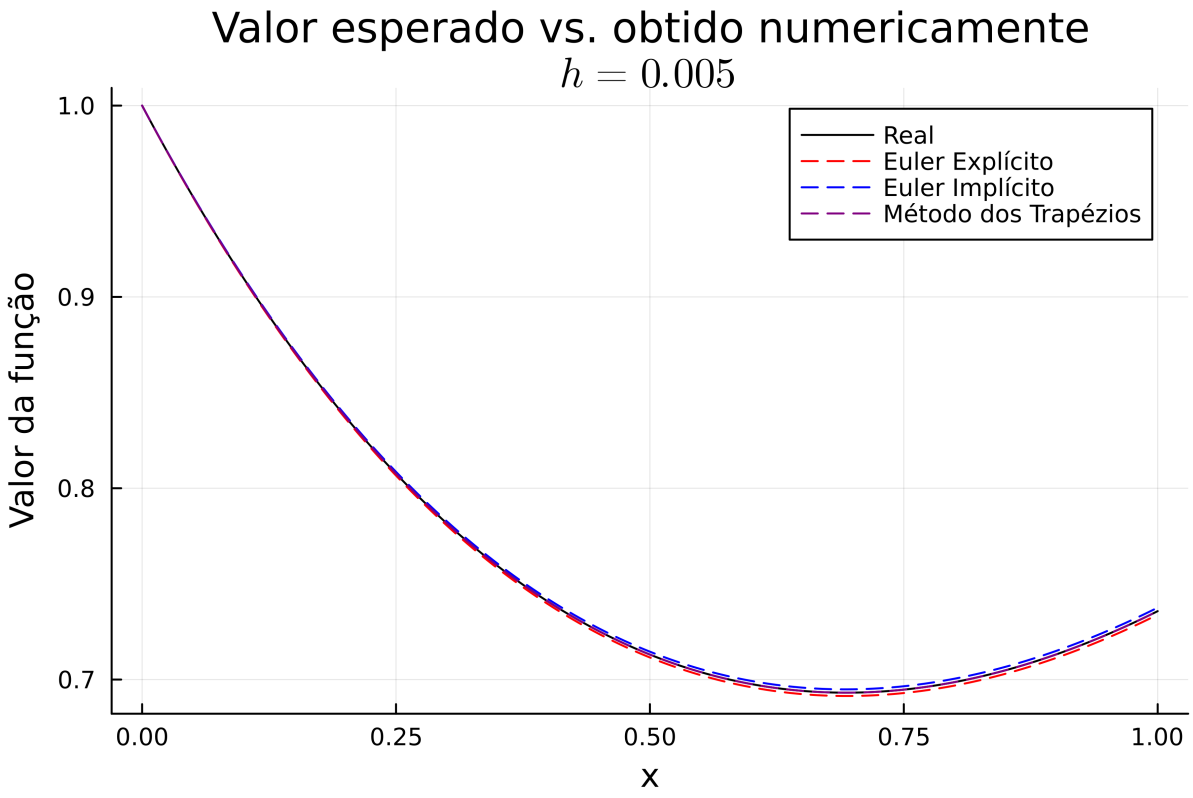

Figura 7: Gráfico plotado para o item a quando $h = 0.005$

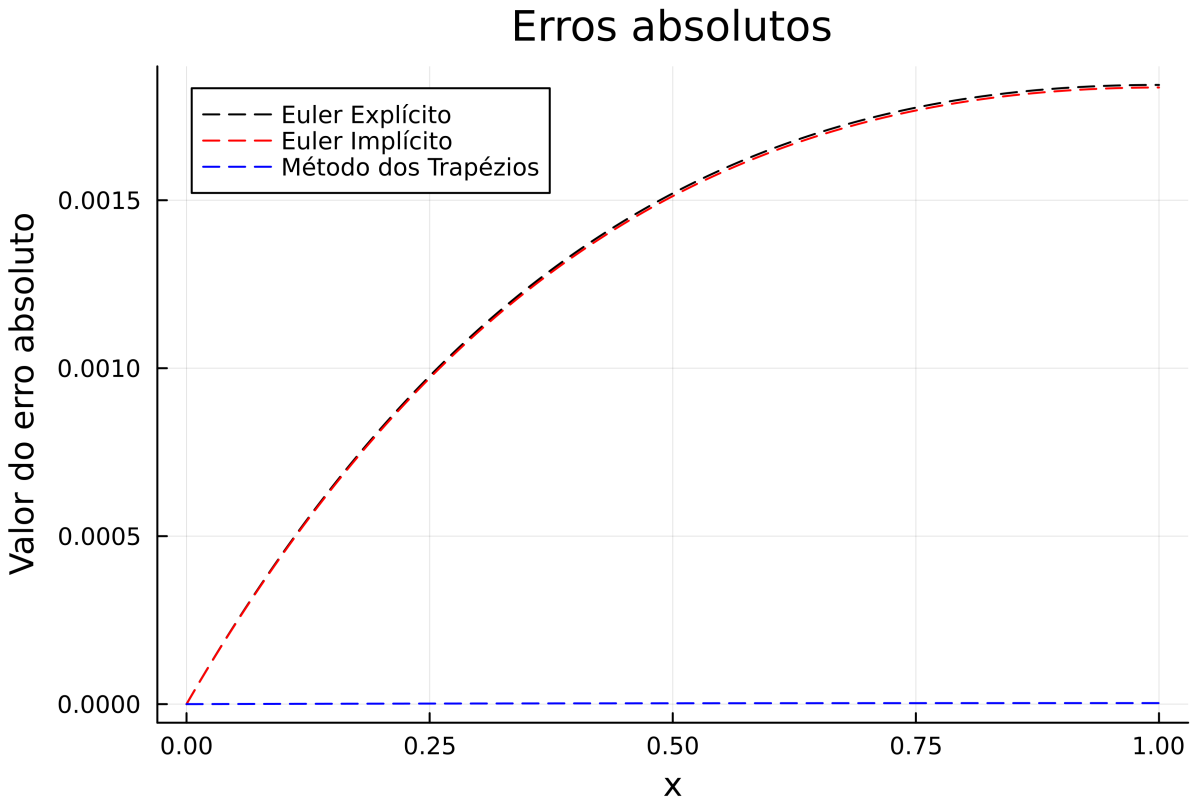

Figura 8: Gráfico plotado para o erro absoluto do item a quando $h = 0.005$

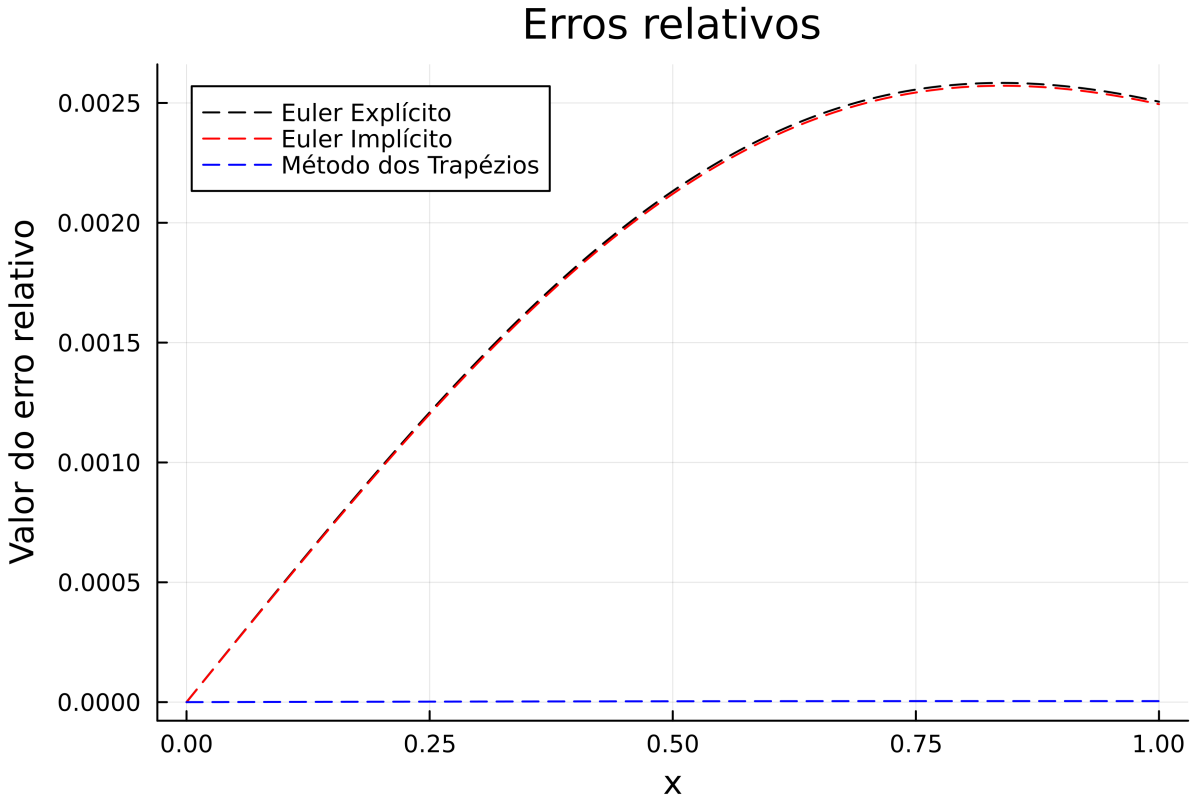

Figura 9: Gráfico plotado para o erro relativo do item a quando $h = 0.005$

2.1.4 $h = 0.001$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.0000	1.0000	1.0000	1.0000
0.1000	0.9097	0.9096	0.9098	0.9097
0.2000	0.8375	0.8373	0.8376	0.8375
0.3000	0.7816	0.7814	0.7819	0.7816
0.4000	0.7406	0.7404	0.7409	0.7406
0.5000	0.7131	0.7128	0.7134	0.7131
0.6000	0.6976	0.6973	0.6980	0.6976
0.7000	0.6932	0.6928	0.6935	0.6932
0.8000	0.6987	0.6983	0.6990	0.6987
0.9000	0.7131	0.7128	0.7135	0.7131
1.0000	0.7358	0.7354	0.7361	0.7358

Tabela 7: Valores obtidos para o item a com $h = 0.001$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	9.05e-05	9.95e-05	9.04e-05	9.94e-05	3.02e-08	3.32e-08
2.00e-01	1.64e-04	1.96e-04	1.64e-04	1.95e-04	5.46e-08	6.52e-08
3.00e-01	2.22e-04	2.85e-04	2.22e-04	2.84e-04	7.41e-08	9.48e-08
4.00e-01	2.68e-04	3.62e-04	2.68e-04	3.62e-04	8.94e-08	1.21e-07
5.00e-01	3.03e-04	4.26e-04	3.03e-04	4.25e-04	1.01e-07	1.42e-07
6.00e-01	3.29e-04	4.72e-04	3.29e-04	4.72e-04	1.10e-07	1.57e-07
7.00e-01	3.48e-04	5.02e-04	3.47e-04	5.01e-04	1.16e-07	1.67e-07
8.00e-01	3.60e-04	5.15e-04	3.59e-04	5.14e-04	1.20e-07	1.72e-07
9.00e-01	3.66e-04	5.13e-04	3.66e-04	5.13e-04	1.22e-07	1.71e-07
1.00e+00	3.68e-04	5.00e-04	3.68e-04	5.00e-04	1.23e-07	1.67e-07

Tabela 8: Erros obtidos para o item a com $h = 0.001$

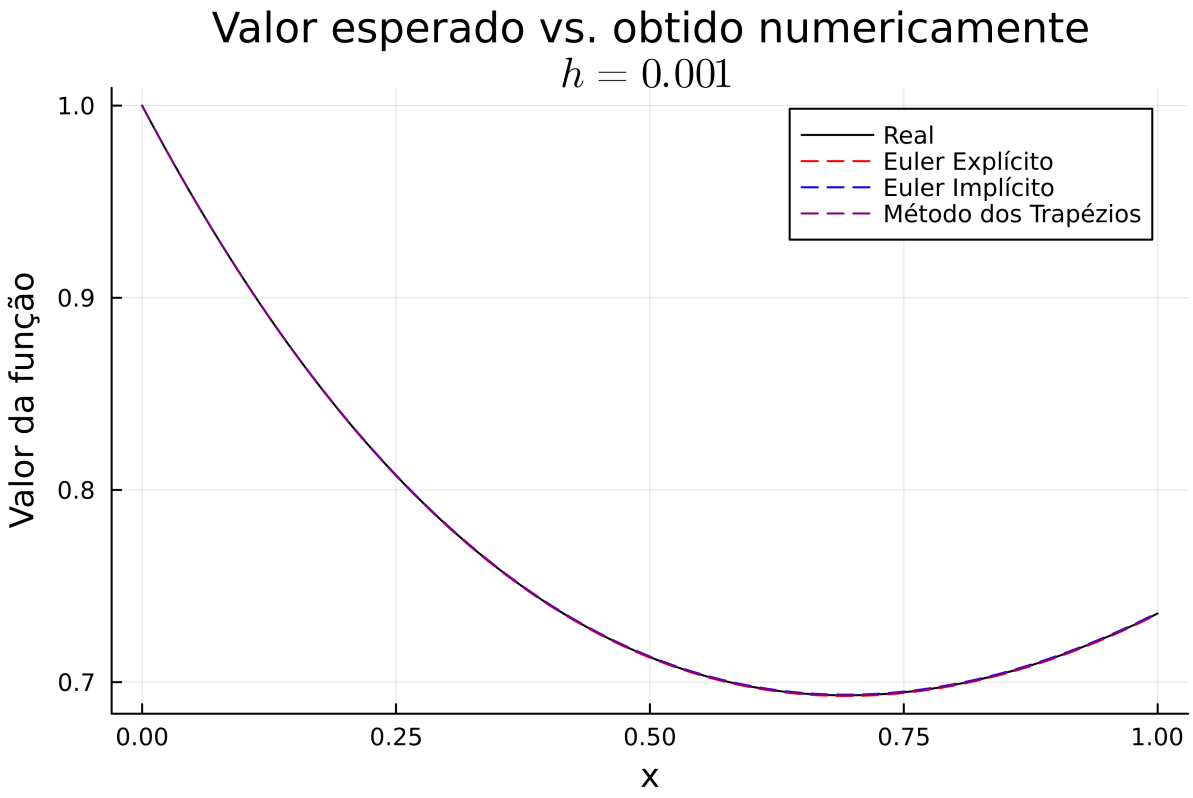

Figura 10: Gráfico plotado para o item a quando $h = 0.001$

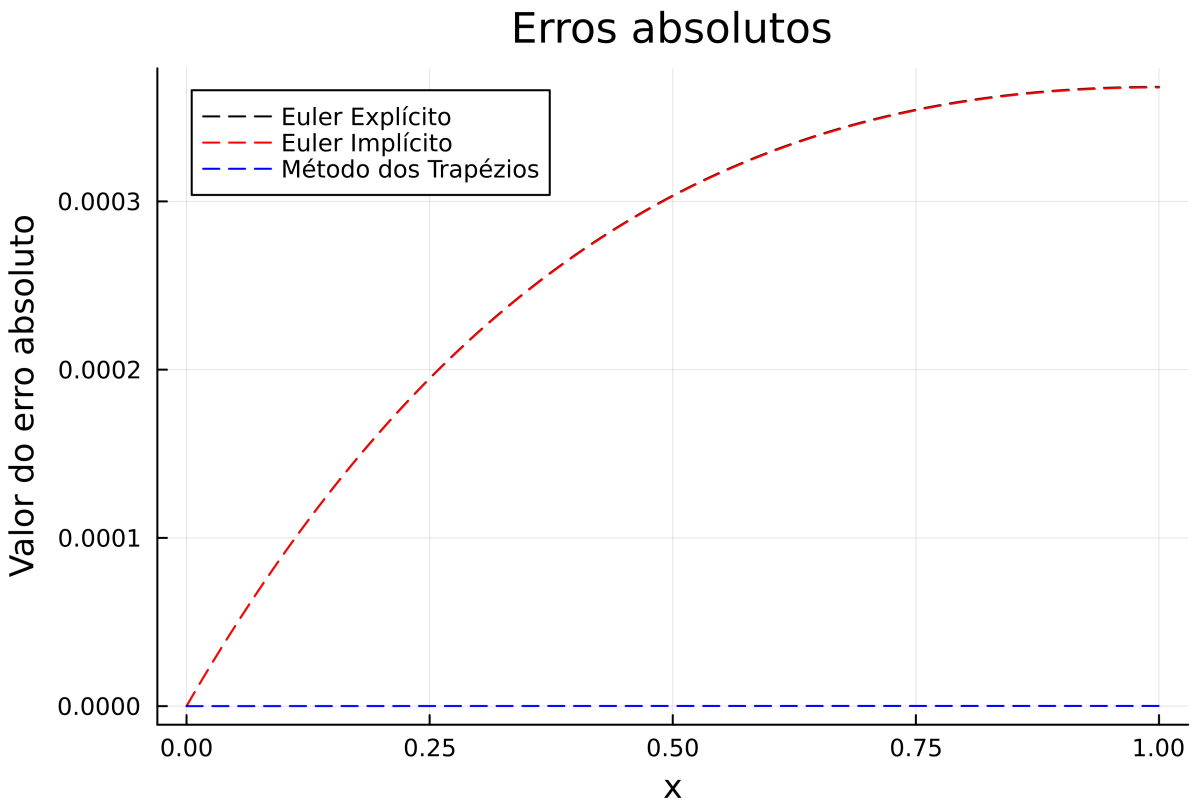

Figura 11: Gráfico plotado para o erro absoluto do item a quando $h = 0.001$

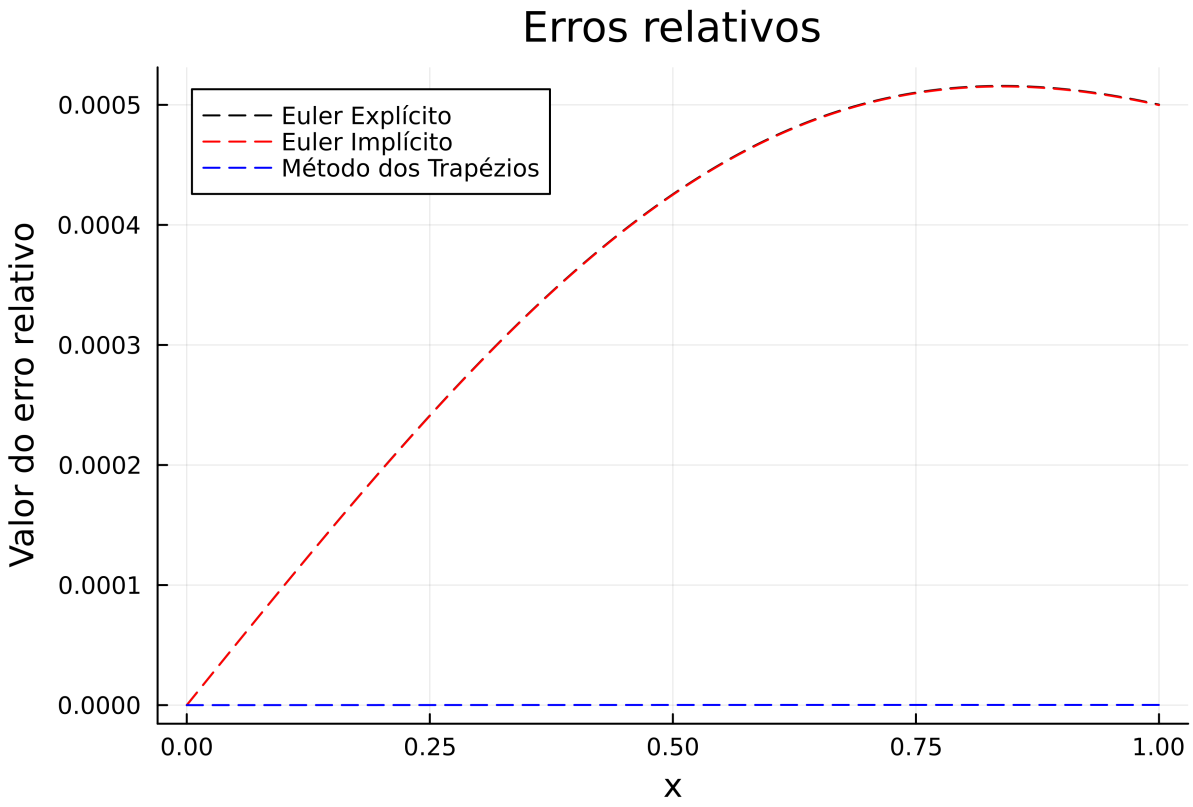

Figura 12: Gráfico plotado para o erro relativo do item a quando $h = 0.001$

2.2 Exercício 1, item b

2.2.1 $h = 0.1$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
-2.5000	0.0019	0.0019	0.0019	0.0019
-2.0000	0.0183	0.0128	0.0353	0.0173
-1.5000	0.1054	0.0595	0.2830	0.0970
-1.0000	0.3679	0.1886	1.1200	0.3346
-0.5000	0.7788	0.3955	2.3872	0.7062
0.0000	1.0000	0.5284	2.9341	0.9064
0.5000	0.7788	0.4299	2.1965	0.7059
1.0000	0.3679	0.2017	1.0473	0.3345
1.5000	0.1054	0.0510	0.3302	0.0974
2.0000	0.0183	0.0064	0.0710	0.0177
2.5000	0.0019	0.0003	0.0107	0.0021

Tabela 9: Valores obtidos para o item b com $h = 0.1$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
-2.50e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
-2.00e+00	5.52e-03	3.01e-01	1.70e-02	9.26e-01	9.86e-04	5.38e-02
-1.50e+00	4.59e-02	4.36e-01	1.78e-01	1.68e+00	8.45e-03	8.01e-02
-1.00e+00	1.79e-01	4.87e-01	7.52e-01	2.04e+00	3.33e-02	9.05e-02
-5.00e-01	3.83e-01	4.92e-01	1.61e+00	2.07e+00	7.26e-02	9.32e-02
0.00e+00	4.72e-01	4.72e-01	1.93e+00	1.93e+00	9.36e-02	9.36e-02
5.00e-01	3.49e-01	4.48e-01	1.42e+00	1.82e+00	7.29e-02	9.36e-02
1.00e+00	1.66e-01	4.52e-01	6.79e-01	1.85e+00	3.34e-02	9.07e-02
1.50e+00	5.44e-02	5.16e-01	2.25e-01	2.13e+00	8.01e-03	7.60e-02
2.00e+00	1.20e-02	6.53e-01	5.27e-02	2.88e+00	5.74e-04	3.14e-02
2.50e+00	1.58e-03	8.20e-01	8.79e-03	4.55e+00	1.51e-04	7.80e-02

Tabela 10: Erros obtidos para o item b com $h = 0.1$

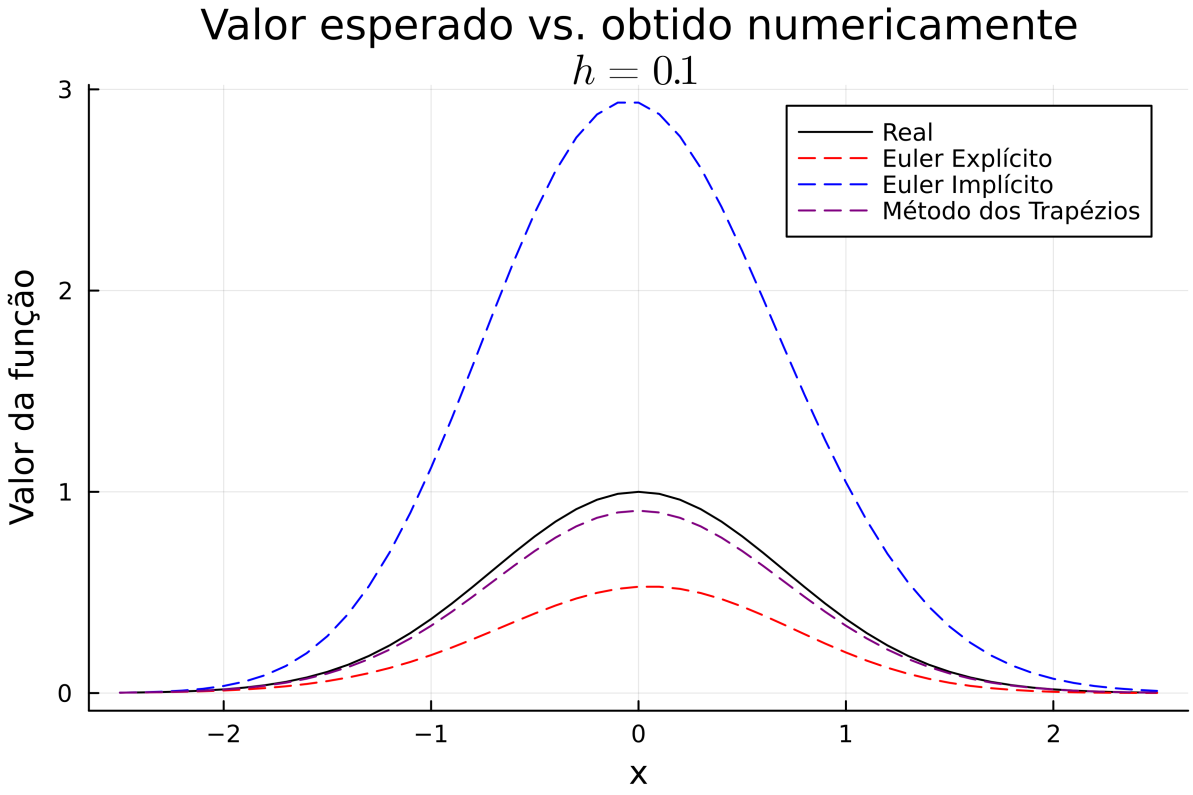

Figura 13: Gráfico plotado para o item b quando $h = 0.1$

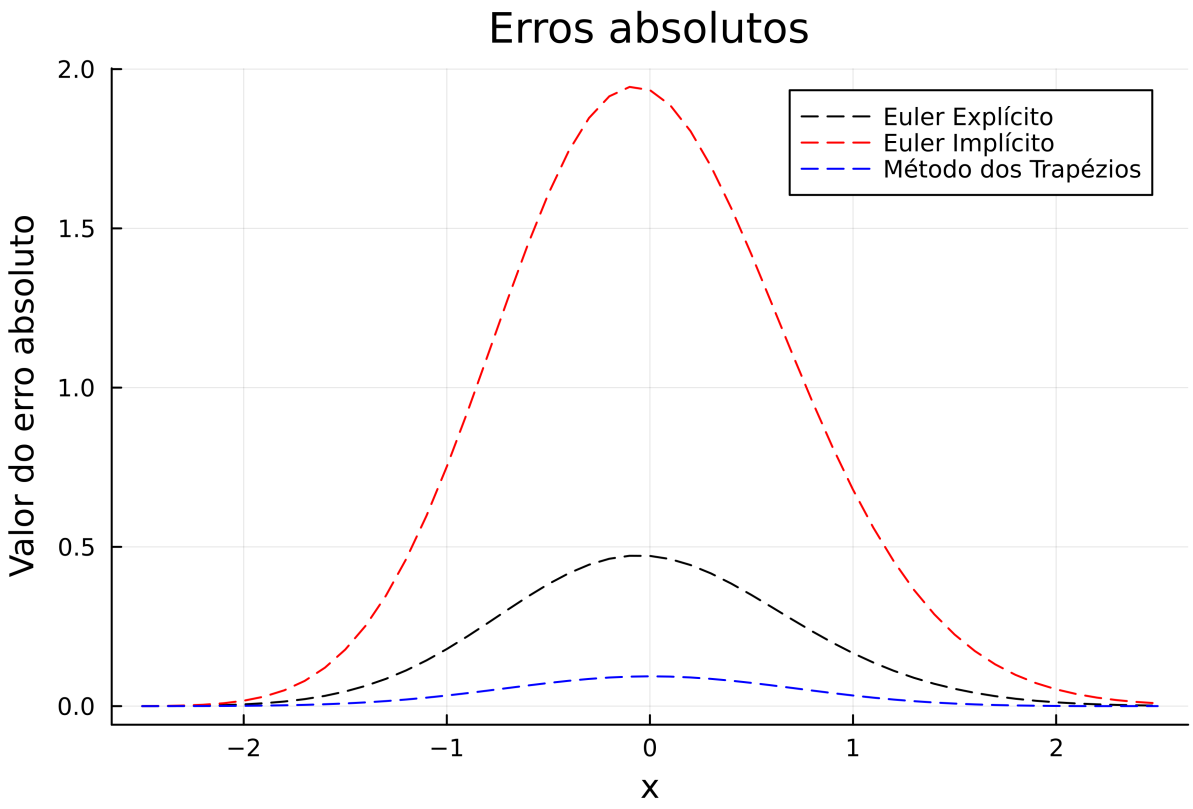

Figura 14: Gráfico plotado para o erro absoluto do item b quando $h = 0.1$

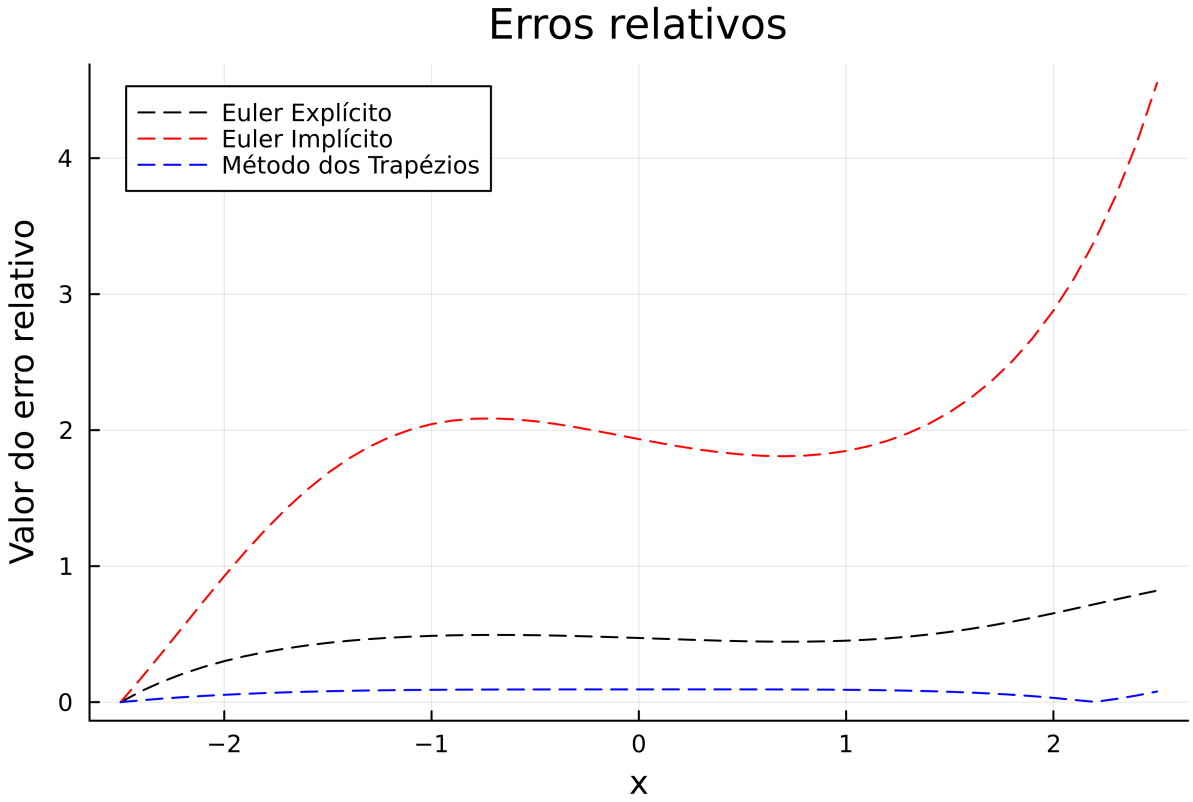

Figura 15: Gráfico plotado para o erro relativo do item b quando $h = 0.1$

2.2.2 $h = 0.01$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
-2.5000	0.0019	0.0019	0.0019	0.0019
-2.0000	0.0183	0.0175	0.0192	0.0183
-1.5000	0.1054	0.0983	0.1134	0.1053
-1.0000	0.3679	0.3394	0.4003	0.3674
-0.5000	0.7788	0.7179	0.8482	0.7778
0.0000	1.0000	0.9257	1.0846	0.9987
0.5000	0.7788	0.7239	0.8412	0.7778
1.0000	0.3679	0.3417	0.3977	0.3674
1.5000	0.1054	0.0968	0.1152	0.1053
2.0000	0.0183	0.0164	0.0205	0.0183
2.5000	0.0019	0.0016	0.0023	0.0019

Tabela 11: Valores obtidos para o item b com $h = 0.01$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
-2.50e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
-2.00e+00	7.98e-04	4.36e-02	8.85e-04	4.83e-02	1.36e-05	7.43e-04
-1.50e+00	7.11e-03	6.75e-02	8.05e-03	7.64e-02	1.16e-04	1.10e-03
-1.00e+00	2.85e-02	7.74e-02	3.25e-02	8.83e-02	4.53e-04	1.23e-03
-5.00e-01	6.09e-02	7.82e-02	6.94e-02	8.91e-02	9.83e-04	1.26e-03
0.00e+00	7.43e-02	7.43e-02	8.46e-02	8.46e-02	1.26e-03	1.26e-03
5.00e-01	5.49e-02	7.05e-02	6.24e-02	8.01e-02	9.83e-04	1.26e-03
1.00e+00	2.62e-02	7.12e-02	2.98e-02	8.10e-02	4.53e-04	1.23e-03
1.50e+00	8.58e-03	8.14e-02	9.76e-03	9.26e-02	1.15e-04	1.09e-03
2.00e+00	1.93e-03	1.05e-01	2.21e-03	1.21e-01	1.32e-05	7.19e-04
2.50e+00	2.83e-04	1.47e-01	3.31e-04	1.72e-01	1.46e-07	7.56e-05

Tabela 12: Erros obtidos para o item b com $h = 0.01$

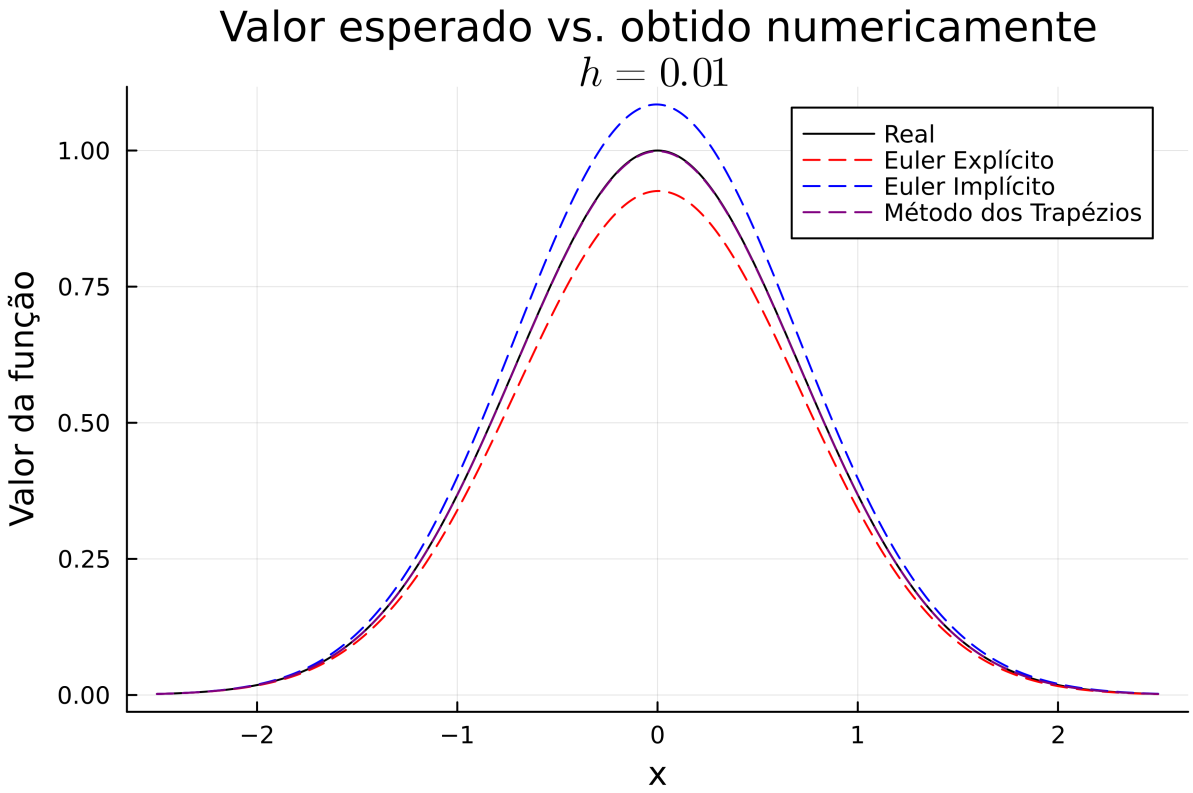

Figura 16: Gráfico plotado para o item b quando $h = 0.01$

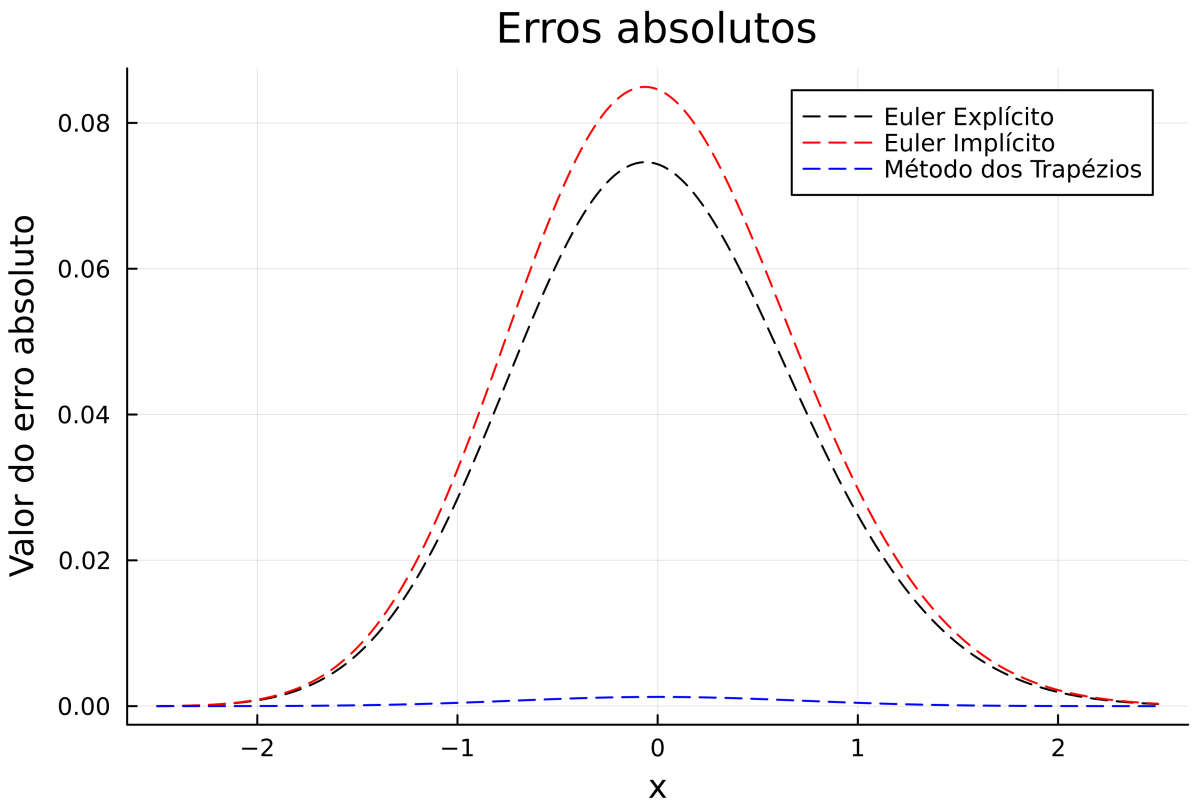

Figura 17: Gráfico plotado para o erro absoluto do item b quando $h = 0.01$

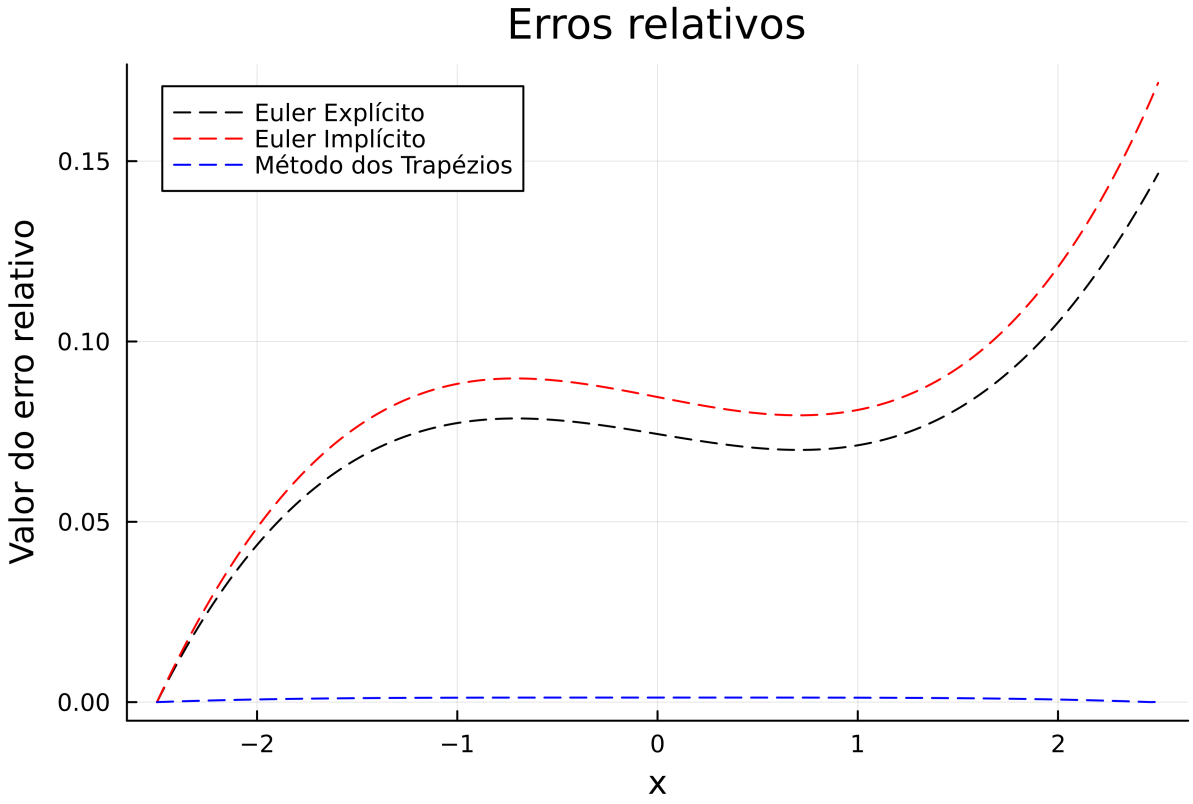

Figura 18: Gráfico plotado para o erro relativo do item b quando $h = 0.01$

2.2.3 $h = 0.005$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
-2.5000	0.0019	0.0019	0.0019	0.0019
-2.0000	0.0183	0.0179	0.0187	0.0183
-1.5000	0.1054	0.1017	0.1093	0.1054
-1.0000	0.3679	0.3532	0.3836	0.3678
-0.5000	0.7788	0.7474	0.8123	0.7786
0.0000	1.0000	0.9617	1.0409	0.9997
0.5000	0.7788	0.7505	0.8090	0.7786
1.0000	0.3679	0.3544	0.3823	0.3678
1.5000	0.1054	0.1010	0.1101	0.1054
2.0000	0.0183	0.0173	0.0194	0.0183
2.5000	0.0019	0.0018	0.0021	0.0019

Tabela 13: Valores obtidos para o item b com $h = 0.005$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
-2.50e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
-2.00e+00	4.09e-04	2.23e-02	4.31e-04	2.35e-02	3.46e-06	1.89e-04
-1.50e+00	3.66e-03	3.48e-02	3.90e-03	3.70e-02	2.94e-05	2.79e-04
-1.00e+00	1.47e-02	3.99e-02	1.57e-02	4.26e-02	1.15e-04	3.12e-04
-5.00e-01	3.14e-02	4.03e-02	3.35e-02	4.31e-02	2.49e-04	3.20e-04
0.00e+00	3.83e-02	3.83e-02	4.09e-02	4.09e-02	3.21e-04	3.21e-04
5.00e-01	2.83e-02	3.63e-02	3.02e-02	3.87e-02	2.49e-04	3.20e-04
1.00e+00	1.35e-02	3.67e-02	1.44e-02	3.92e-02	1.15e-04	3.12e-04
1.50e+00	4.42e-03	4.20e-02	4.72e-03	4.48e-02	2.93e-05	2.78e-04
2.00e+00	9.96e-04	5.44e-02	1.07e-03	5.82e-02	3.41e-06	1.86e-04
2.50e+00	1.47e-04	7.61e-02	1.59e-04	8.24e-02	1.82e-08	9.45e-06

Tabela 14: Erros obtidos para o item b com $h = 0.005$

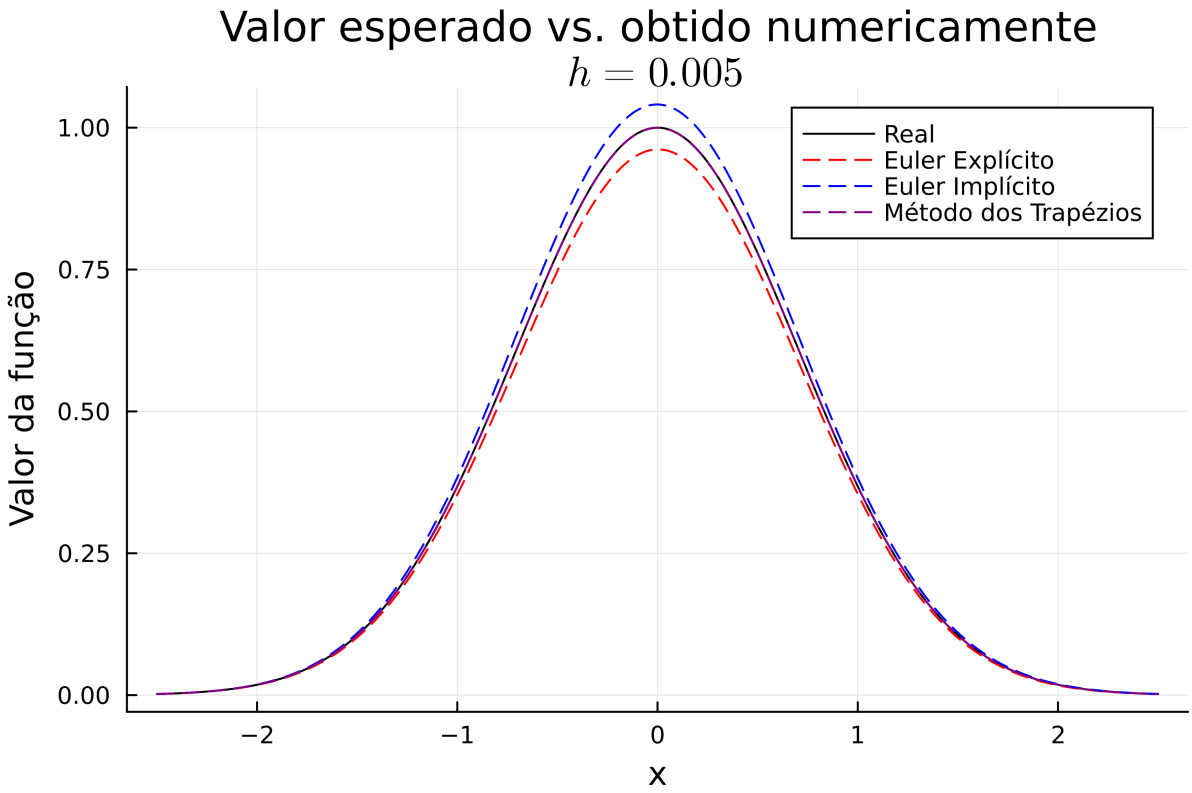

Figura 19: Gráfico plotado para o item b quando $h = 0.005$

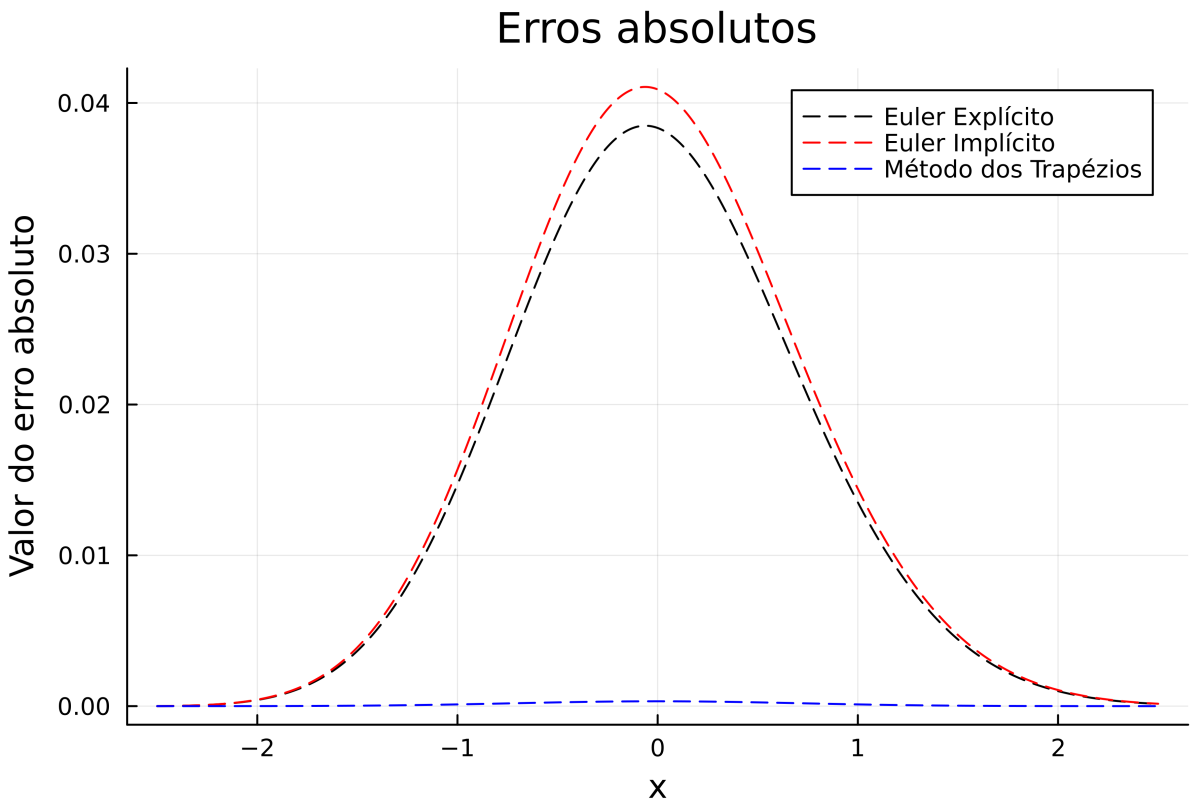

Figura 20: Gráfico plotado para o erro absoluto do item b quando $h = 0.005$

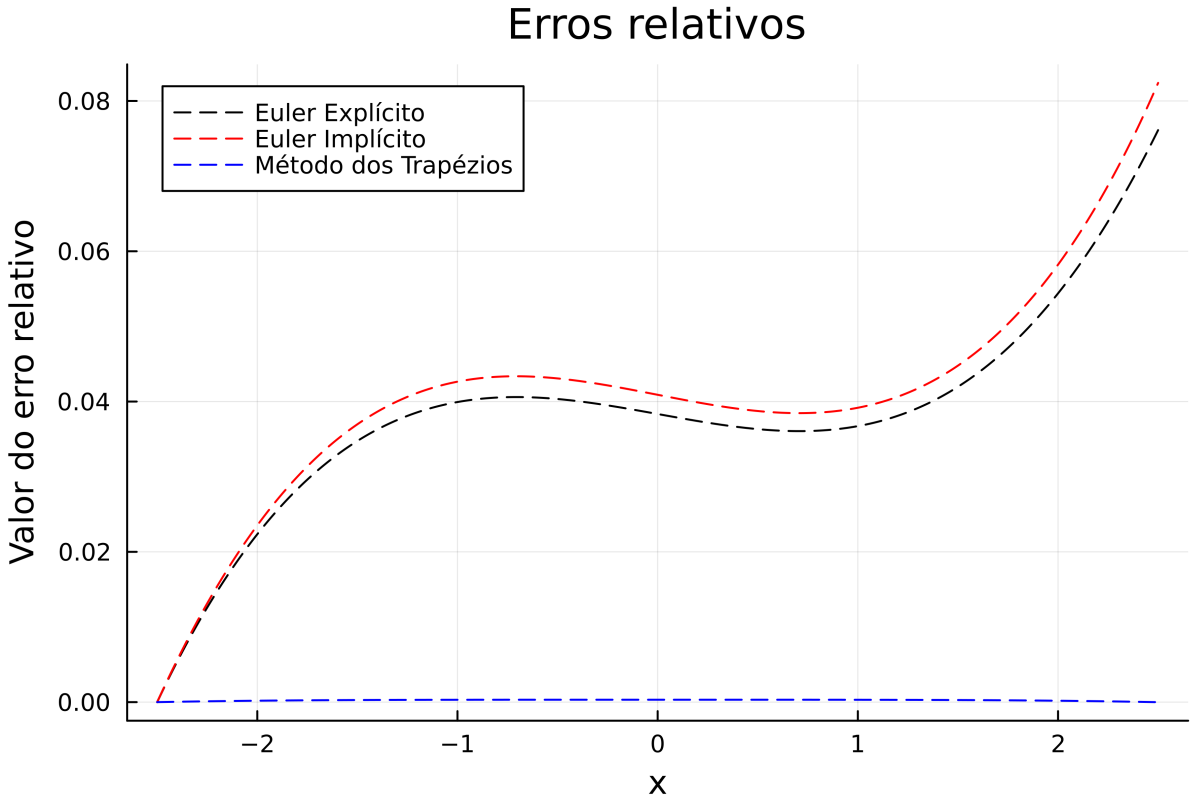

Figura 21: Gráfico plotado para o erro relativo do item b quando $h = 0.005$

2.2.4 $h = 0.001$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
-2.5000	0.0019	0.0019	0.0019	0.0019
-2.0000	0.0183	0.0182	0.0184	0.0183
-1.5000	0.1054	0.1046	0.1062	0.1054
-1.0000	0.3679	0.3649	0.3709	0.3679
-0.5000	0.7788	0.7724	0.7853	0.7788
0.0000	1.0000	0.9921	1.0080	1.0000
0.5000	0.7788	0.7730	0.7847	0.7788
1.0000	0.3679	0.3651	0.3707	0.3679
1.5000	0.1054	0.1045	0.1063	0.1054
2.0000	0.0183	0.0181	0.0185	0.0183
2.5000	0.0019	0.0019	0.0020	0.0019

Tabela 15: Valores obtidos para o item b com $h = 0.001$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
-2.50e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
-2.00e+00	8.35e-05	4.56e-03	8.44e-05	4.61e-03	1.40e-07	7.66e-06
-1.50e+00	7.51e-04	7.12e-03	7.60e-04	7.21e-03	1.19e-06	1.13e-05
-1.00e+00	3.02e-03	8.20e-03	3.06e-03	8.30e-03	4.65e-06	1.26e-05
-5.00e-01	6.45e-03	8.28e-03	6.53e-03	8.39e-03	1.01e-05	1.30e-05
0.00e+00	7.87e-03	7.87e-03	7.97e-03	7.97e-03	1.30e-05	1.30e-05
5.00e-01	5.80e-03	7.45e-03	5.88e-03	7.55e-03	1.01e-05	1.30e-05
1.00e+00	2.77e-03	7.53e-03	2.81e-03	7.63e-03	4.65e-06	1.26e-05
1.50e+00	9.08e-04	8.61e-03	9.19e-04	8.72e-03	1.19e-06	1.13e-05
2.00e+00	2.05e-04	1.12e-02	2.07e-04	1.13e-02	1.40e-07	7.64e-06
2.50e+00	3.03e-05	1.57e-02	3.08e-05	1.60e-02	1.46e-10	7.56e-08

Tabela 16: Erros obtidos para o item b com $h = 0.001$

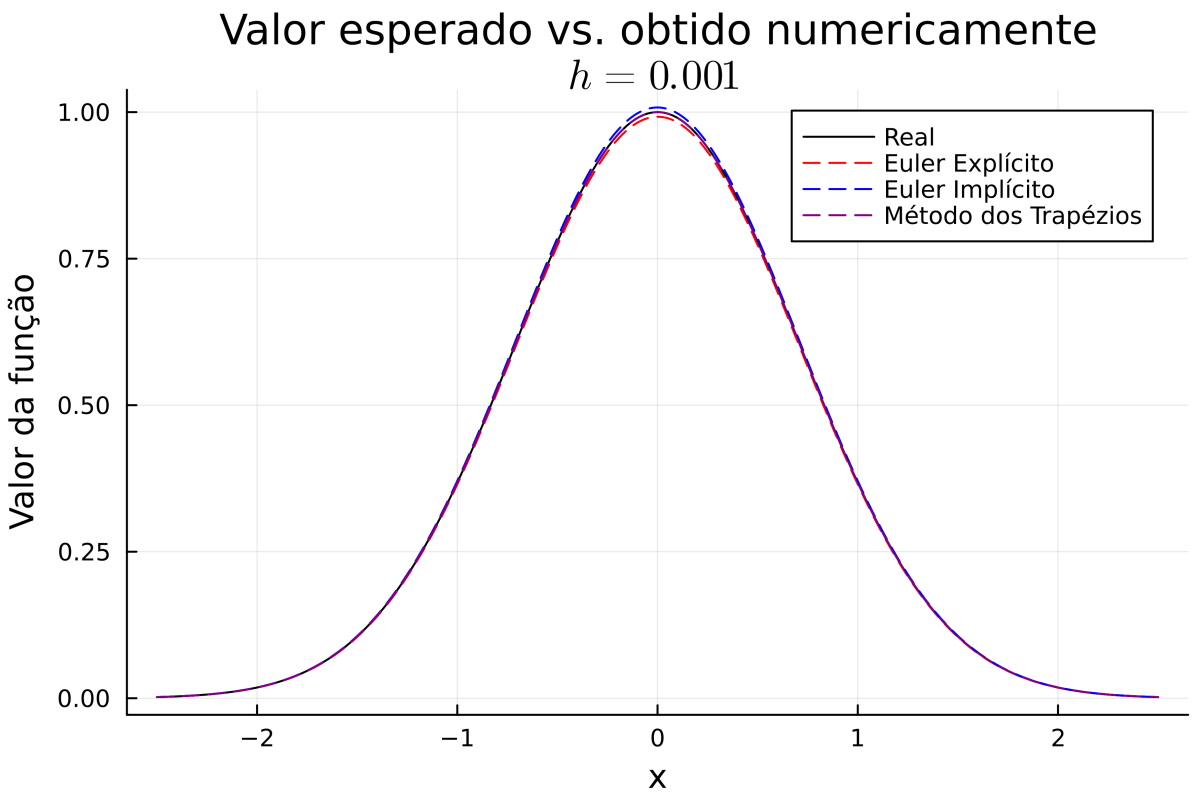

Figura 22: Gráfico plotado para o item b quando $h = 0.001$

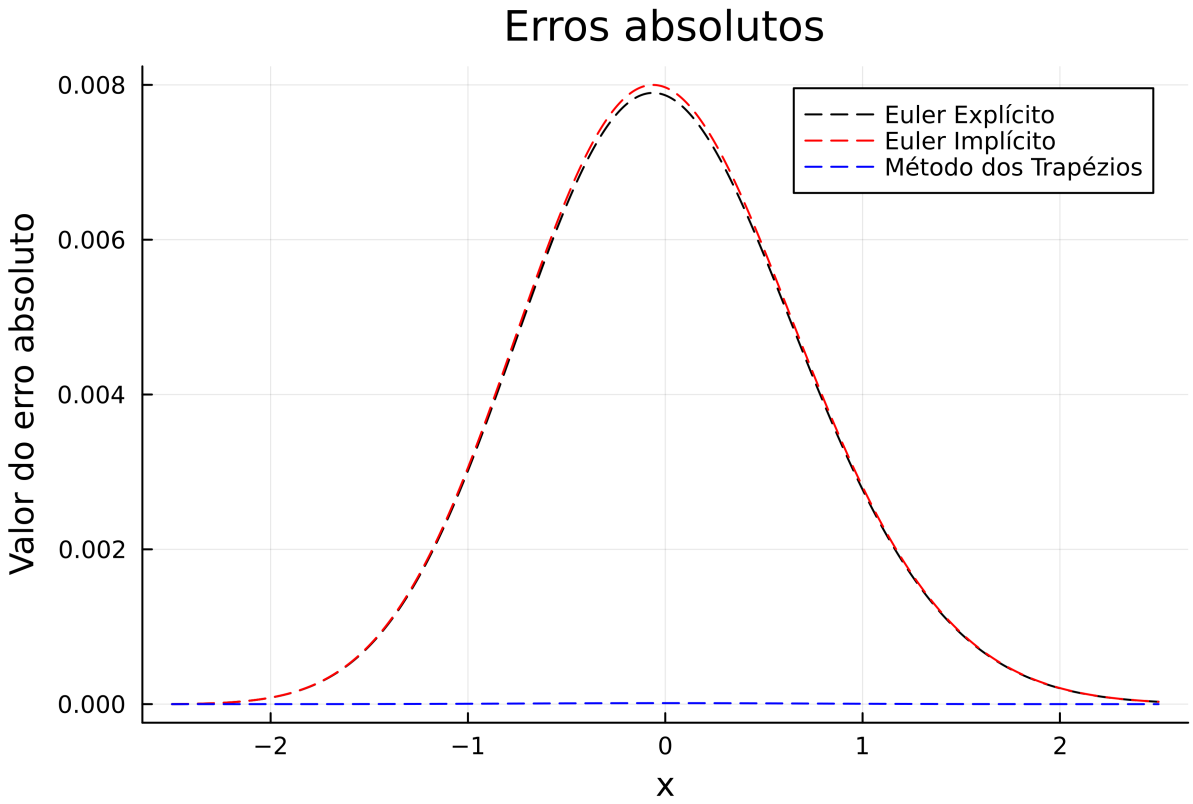

Figura 23: Gráfico plotado para o erro absoluto do item b quando $h = 0.001$

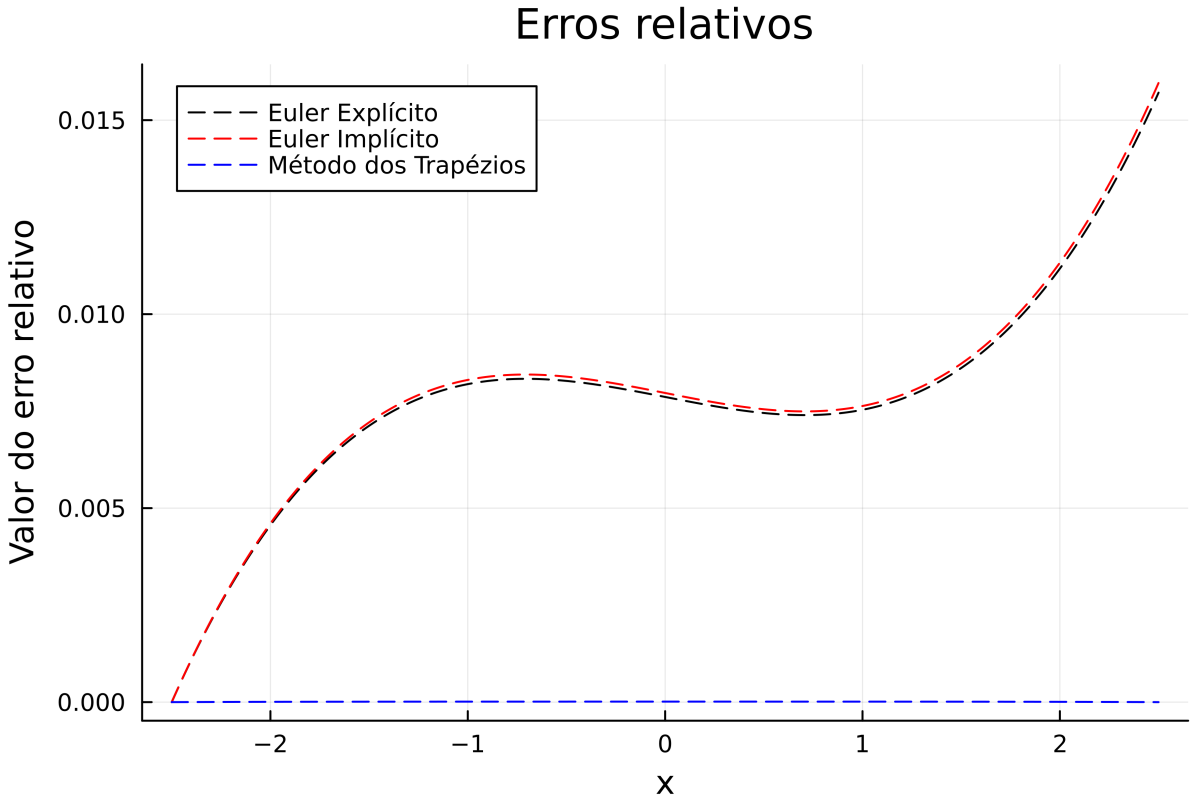

Figura 24: Gráfico plotado para o erro relativo do item b quando $h = 0.001$

2.3 Exercício 1, item c

2.3.1 $h = 0.1$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.2000	1.2000	1.2000	1.2000
0.2000	2.9183	2.4500	4.2000	2.8406
0.4000	7.5891	5.2625	16.2000	7.1729
0.6000	20.2855	11.5906	64.2000	18.6128
0.8000	54.7982	25.8289	256.2000	48.8213
1.0000	148.6132	57.8650	1024.2000	128.5907
1.2000	403.6288	129.9463	4096.2000	339.2318
1.4000	1096.8332	292.1293	16384.2000	895.4557
1.6000	2981.1580	657.0408	65536.2000	2364.2347
1.8000	8103.2839	1478.0919	262144.2000	6242.7291
2.0000	22026.6658	3325.4567	1048576.2000	16484.3784

Tabela 17: Valores obtidos para o item c com $h = 0.1$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	4.68e-01	1.60e-01	1.28e+00	4.39e-01	7.77e-02	2.66e-02
4.00e-01	2.33e+00	3.07e-01	8.61e+00	1.13e+00	4.16e-01	5.48e-02
6.00e-01	8.69e+00	4.29e-01	4.39e+01	2.16e+00	1.67e+00	8.25e-02
8.00e-01	2.90e+01	5.29e-01	2.01e+02	3.68e+00	5.98e+00	1.09e-01
1.00e+00	9.07e+01	6.11e-01	8.76e+02	5.89e+00	2.00e+01	1.35e-01
1.20e+00	2.74e+02	6.78e-01	3.69e+03	9.15e+00	6.44e+01	1.60e-01
1.40e+00	8.05e+02	7.34e-01	1.53e+04	1.39e+01	2.01e+02	1.84e-01
1.60e+00	2.32e+03	7.80e-01	6.26e+04	2.10e+01	6.17e+02	2.07e-01
1.80e+00	6.63e+03	8.18e-01	2.54e+05	3.14e+01	1.86e+03	2.30e-01
2.00e+00	1.87e+04	8.49e-01	1.03e+06	4.66e+01	5.54e+03	2.52e-01

Tabela 18: Erros obtidos para o item c com $h = 0.1$

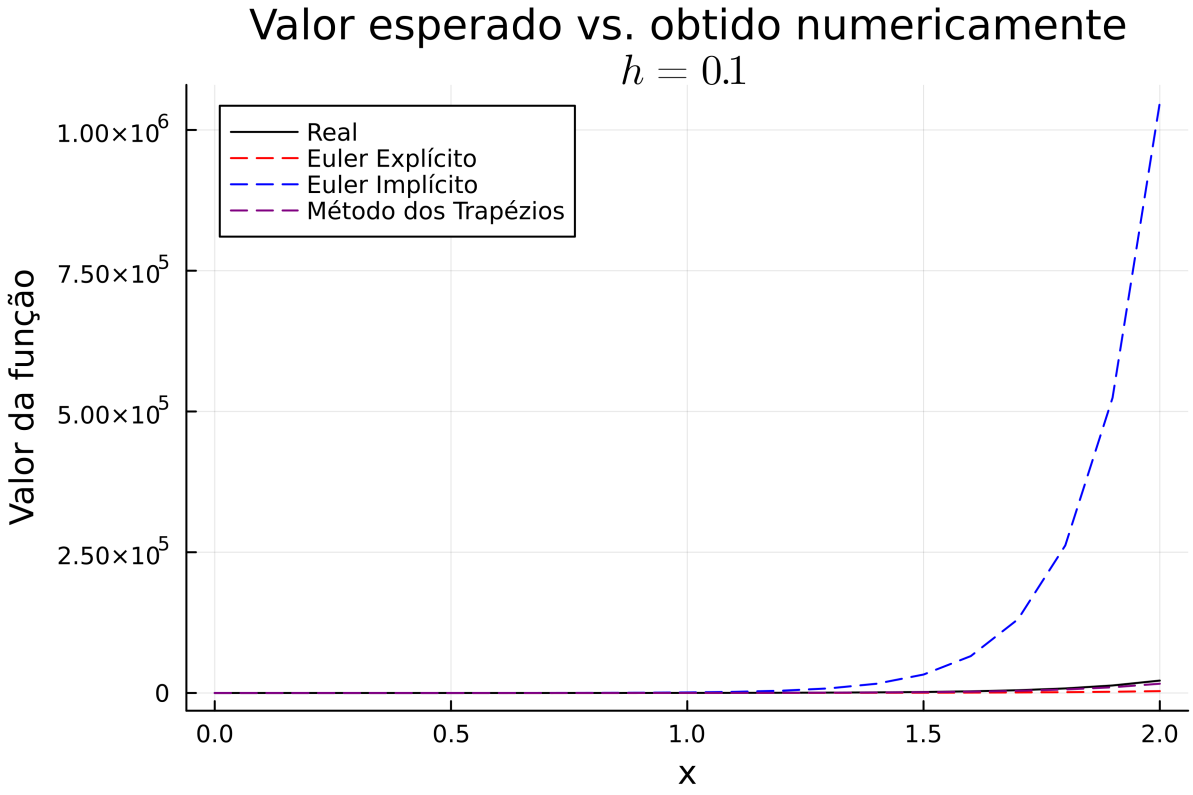

Figura 25: Gráfico plotado para o item c quando $h = 0.1$

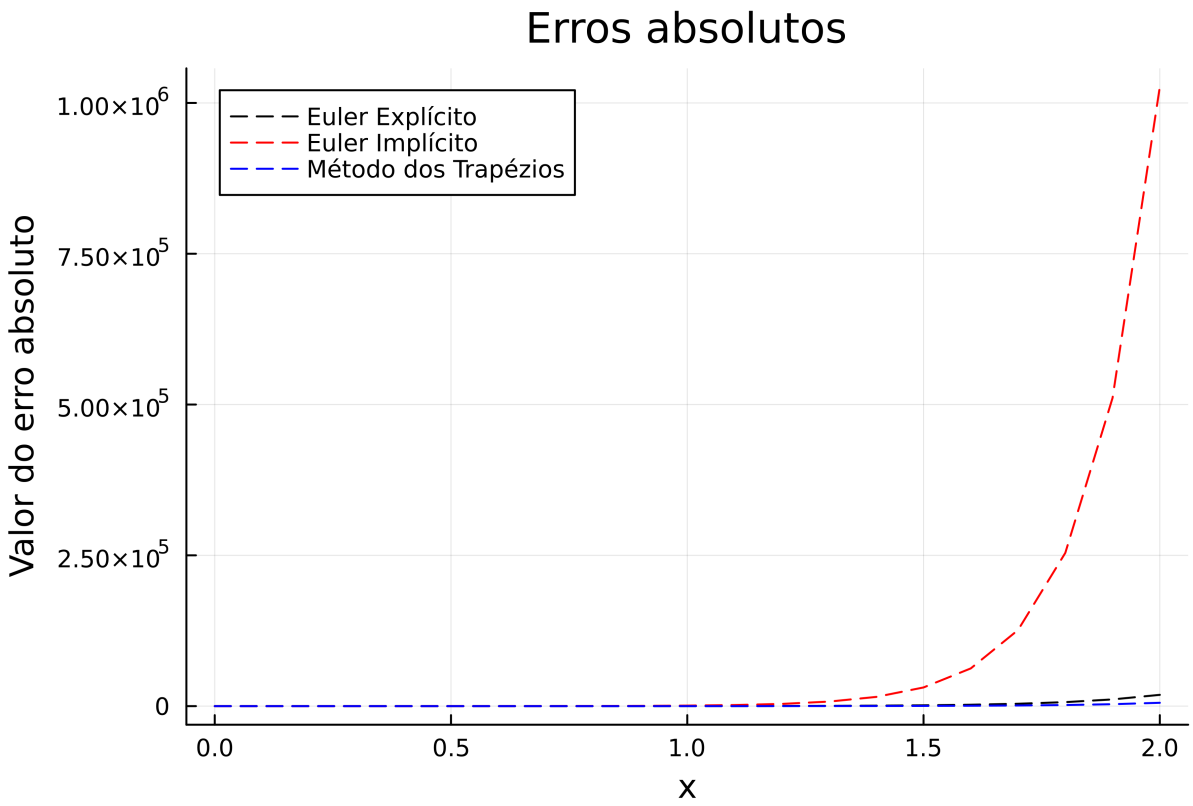

Figura 26: Gráfico plotado para o erro absoluto do item c quando $h = 0.1$

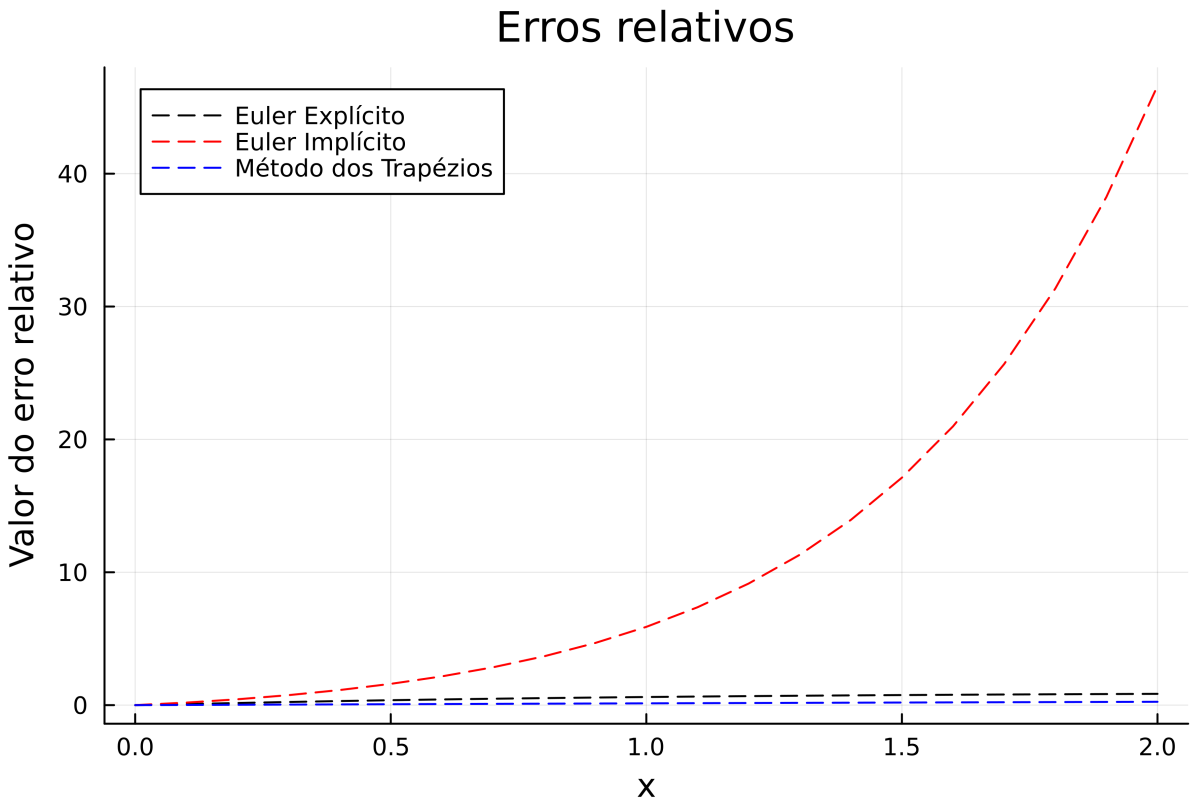

Figura 27: Gráfico plotado para o erro relativo do item c quando $h = 0.1$

2.3.2 $h = 0.01$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.2000	1.2000	1.2000	1.2000
0.2000	2.9183	2.8533	2.9895	2.9172
0.4000	7.5891	7.2400	7.9814	7.5831
0.6000	20.2855	18.8792	21.9062	20.2614
0.8000	54.7982	49.7614	60.7496	54.7106
1.0000	148.6132	131.7013	169.1038	148.3156
1.2000	403.6288	349.1120	471.3589	402.6585
1.4000	1096.8332	925.9674	1314.5023	1093.7565
1.6000	2981.1580	2456.5364	3666.4591	2971.6020
1.8000	8103.2839	6517.5918	10227.2657	8074.0669
2.0000	22026.6658	17292.7808	28528.7003	21938.4389

Tabela 19: Valores obtidos para o item c com $h = 0.01$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	6.50e-02	2.23e-02	7.12e-02	2.44e-02	1.09e-03	3.74e-04
4.00e-01	3.49e-01	4.60e-02	3.92e-01	5.17e-02	5.93e-03	7.81e-04
6.00e-01	1.41e+00	6.93e-02	1.62e+00	7.99e-02	2.42e-02	1.19e-03
8.00e-01	5.04e+00	9.19e-02	5.95e+00	1.09e-01	8.76e-02	1.60e-03
1.00e+00	1.69e+01	1.14e-01	2.05e+01	1.38e-01	2.98e-01	2.00e-03
1.20e+00	5.45e+01	1.35e-01	6.77e+01	1.68e-01	9.70e-01	2.40e-03
1.40e+00	1.71e+02	1.56e-01	2.18e+02	1.98e-01	3.08e+00	2.81e-03
1.60e+00	5.25e+02	1.76e-01	6.85e+02	2.30e-01	9.56e+00	3.21e-03
1.80e+00	1.59e+03	1.96e-01	2.12e+03	2.62e-01	2.92e+01	3.61e-03
2.00e+00	4.73e+03	2.15e-01	6.50e+03	2.95e-01	8.82e+01	4.01e-03

Tabela 20: Erros obtidos para o item c com $h = 0.01$

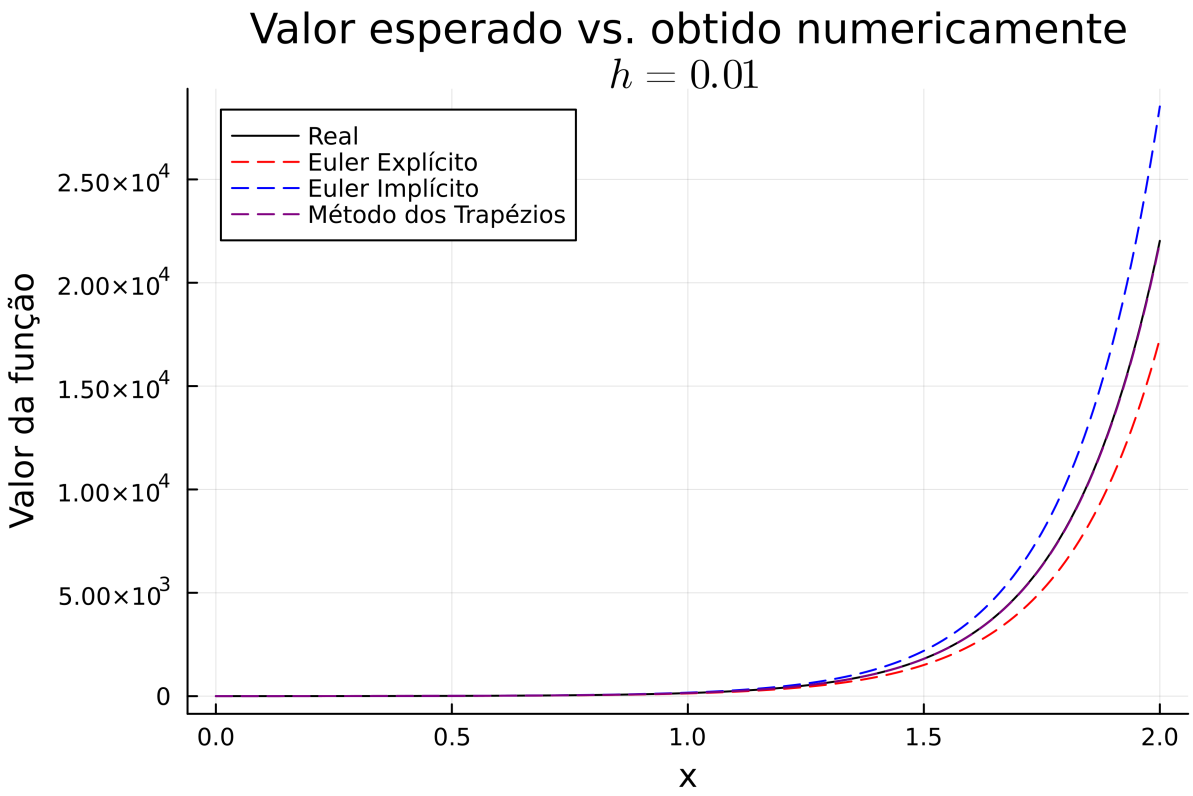

Figura 28: Gráfico plotado para o item c quando $h = 0.01$

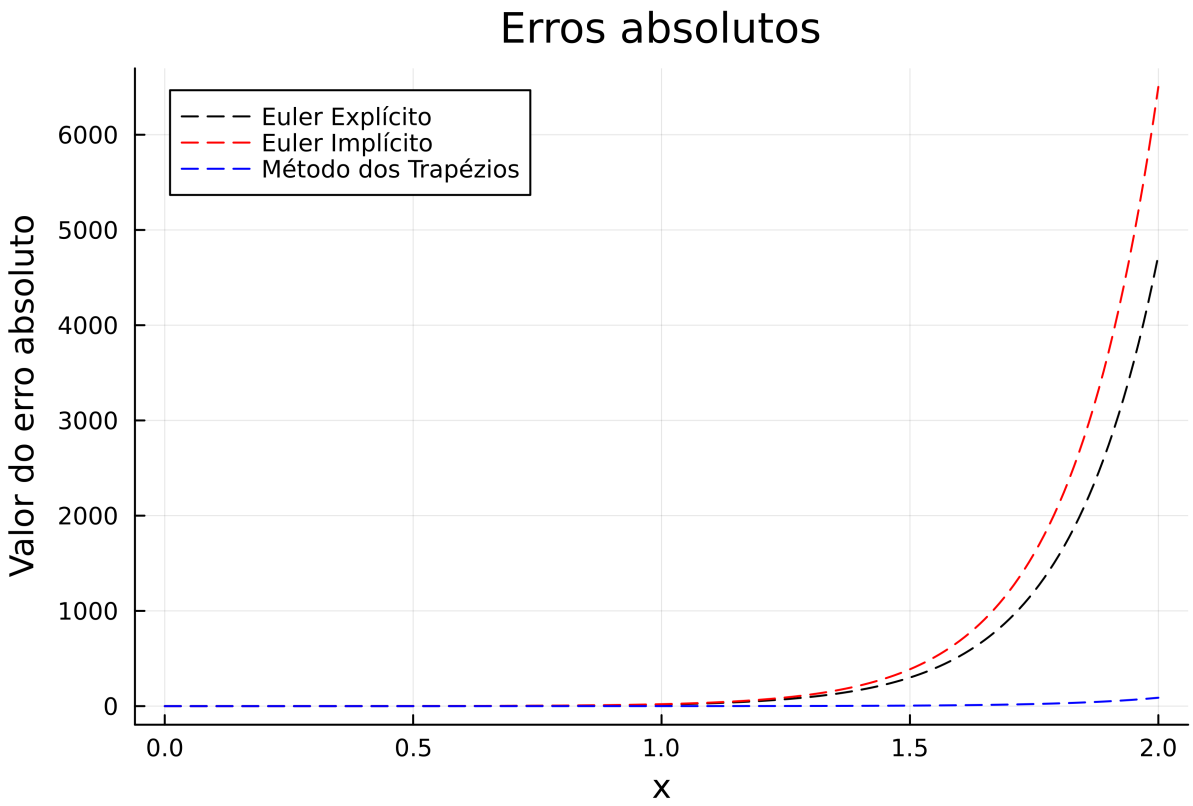

Figura 29: Gráfico plotado para o erro absoluto do item c quando $h = 0.01$

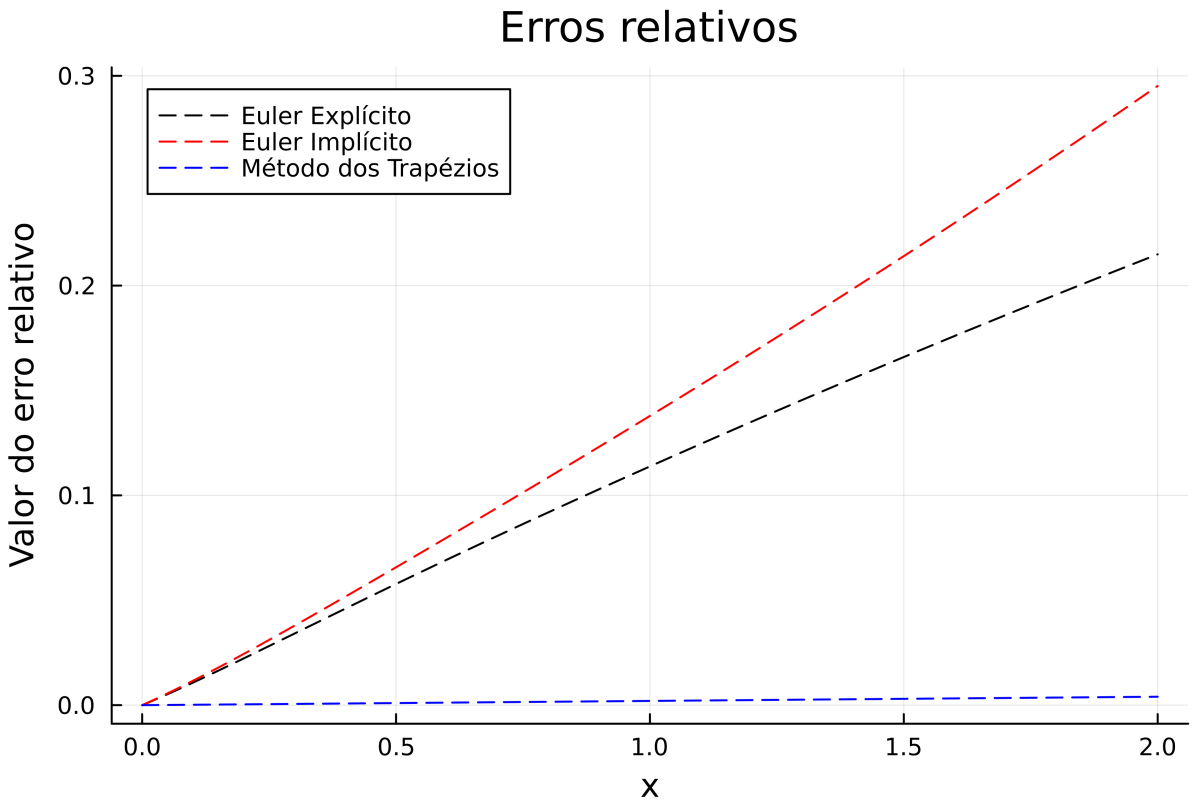

Figura 30: Gráfico plotado para o erro relativo do item c quando $h = 0.01$

2.3.3 $h = 0.005$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.2000	1.2000	1.2000	1.2000
0.2000	2.9183	2.8851	2.9531	2.9180
0.4000	7.5891	7.4096	7.7793	7.5875
0.6000	20.2855	19.5581	21.0663	20.2794
0.8000	54.7982	52.1779	57.6462	54.7758
1.0000	148.6132	139.7639	158.3528	148.5373
1.2000	403.6288	374.9380	435.6038	403.3814
1.4000	1096.8332	1006.3954	1198.8920	1096.0487
1.6000	2981.1580	2701.8988	3300.2687	2978.7210
1.8000	8103.2839	7254.4337	9085.4807	8095.8317
2.0000	22026.6658	19478.2805	25012.5053	22004.1590

Tabela 21: Valores obtidos para o item c com $h = 0.005$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	3.32e-02	1.14e-02	3.48e-02	1.19e-02	2.78e-04	9.52e-05
4.00e-01	1.79e-01	2.37e-02	1.90e-01	2.51e-02	1.51e-03	1.99e-04
6.00e-01	7.27e-01	3.59e-02	7.81e-01	3.85e-02	6.16e-03	3.04e-04
8.00e-01	2.62e+00	4.78e-02	2.85e+00	5.20e-02	2.23e-02	4.07e-04
1.00e+00	8.85e+00	5.95e-02	9.74e+00	6.55e-02	7.58e-02	5.10e-04
1.20e+00	2.87e+01	7.11e-02	3.20e+01	7.92e-02	2.47e-01	6.13e-04
1.40e+00	9.04e+01	8.25e-02	1.02e+02	9.30e-02	7.85e-01	7.15e-04
1.60e+00	2.79e+02	9.37e-02	3.19e+02	1.07e-01	2.44e+00	8.17e-04
1.80e+00	8.49e+02	1.05e-01	9.82e+02	1.21e-01	7.45e+00	9.20e-04
2.00e+00	2.55e+03	1.16e-01	2.99e+03	1.36e-01	2.25e+01	1.02e-03

Tabela 22: Erros obtidos para o item c com $h = 0.005$

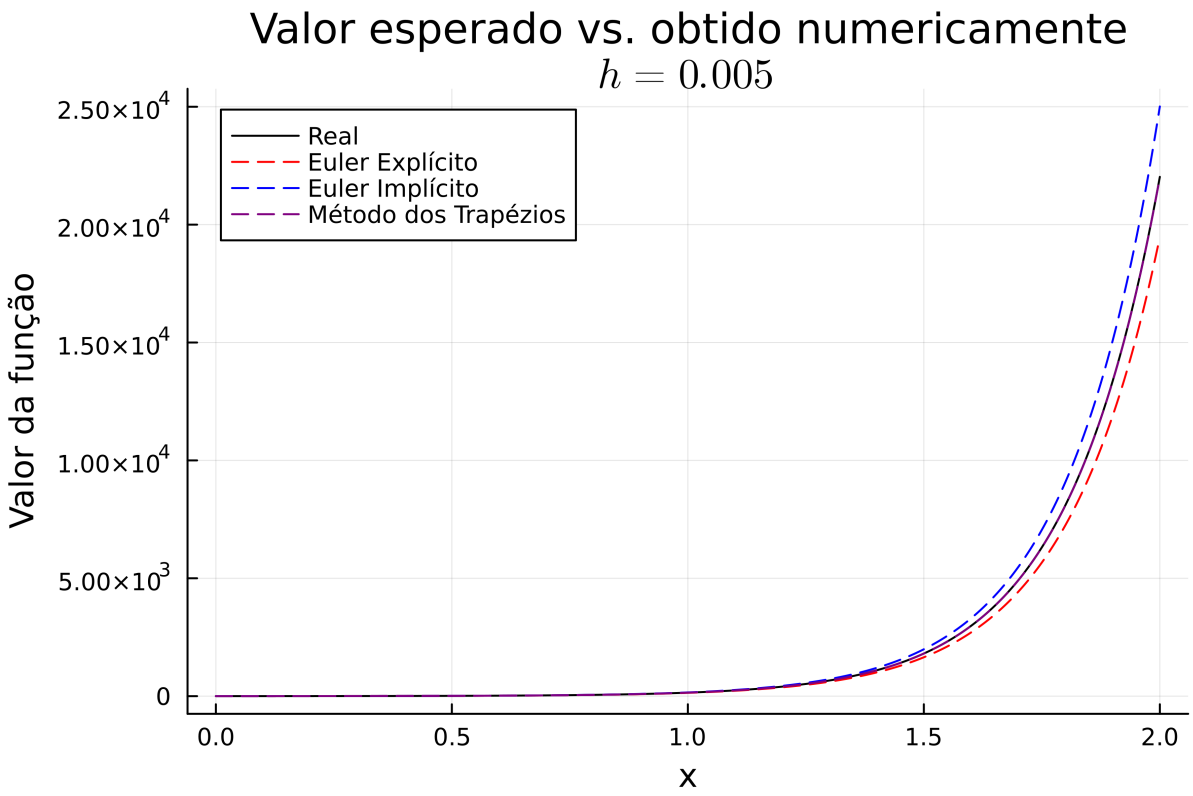

Figura 31: Gráfico plotado para o item c quando $h = 0.005$

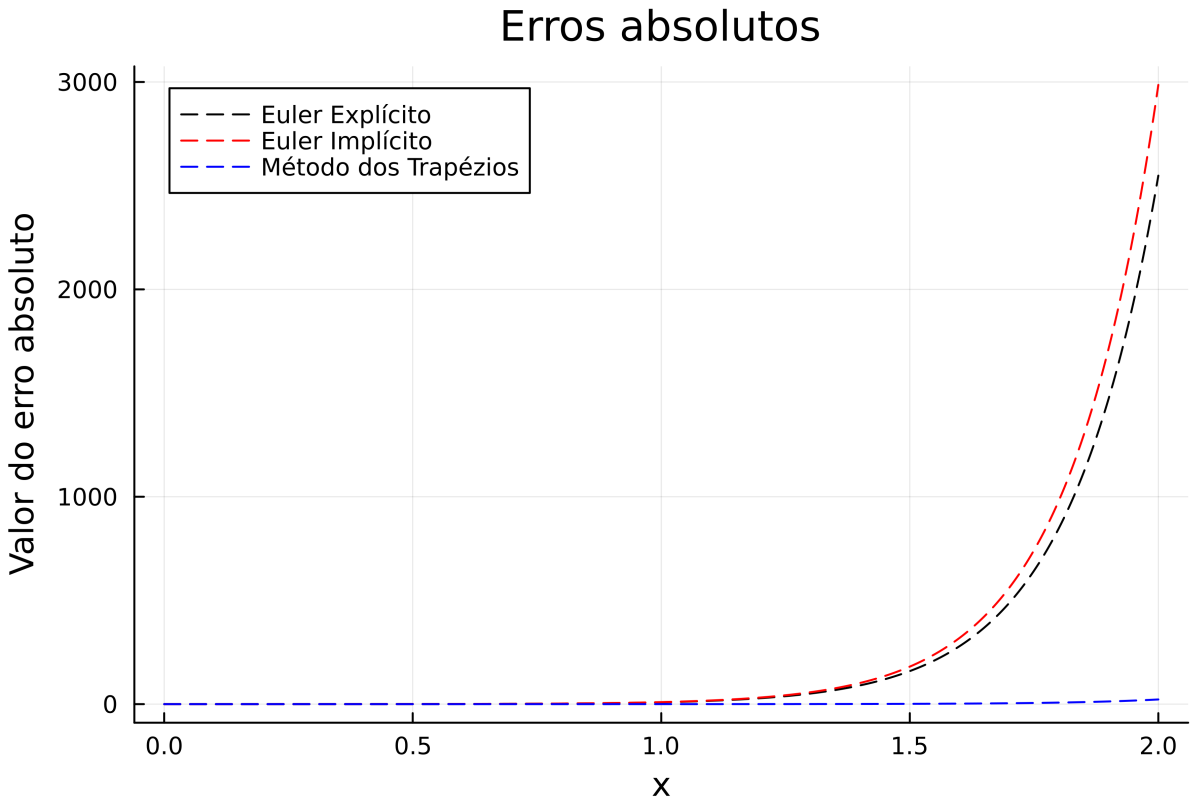

Figura 32: Gráfico plotado para o erro absoluto do item c quando $h = 0.005$

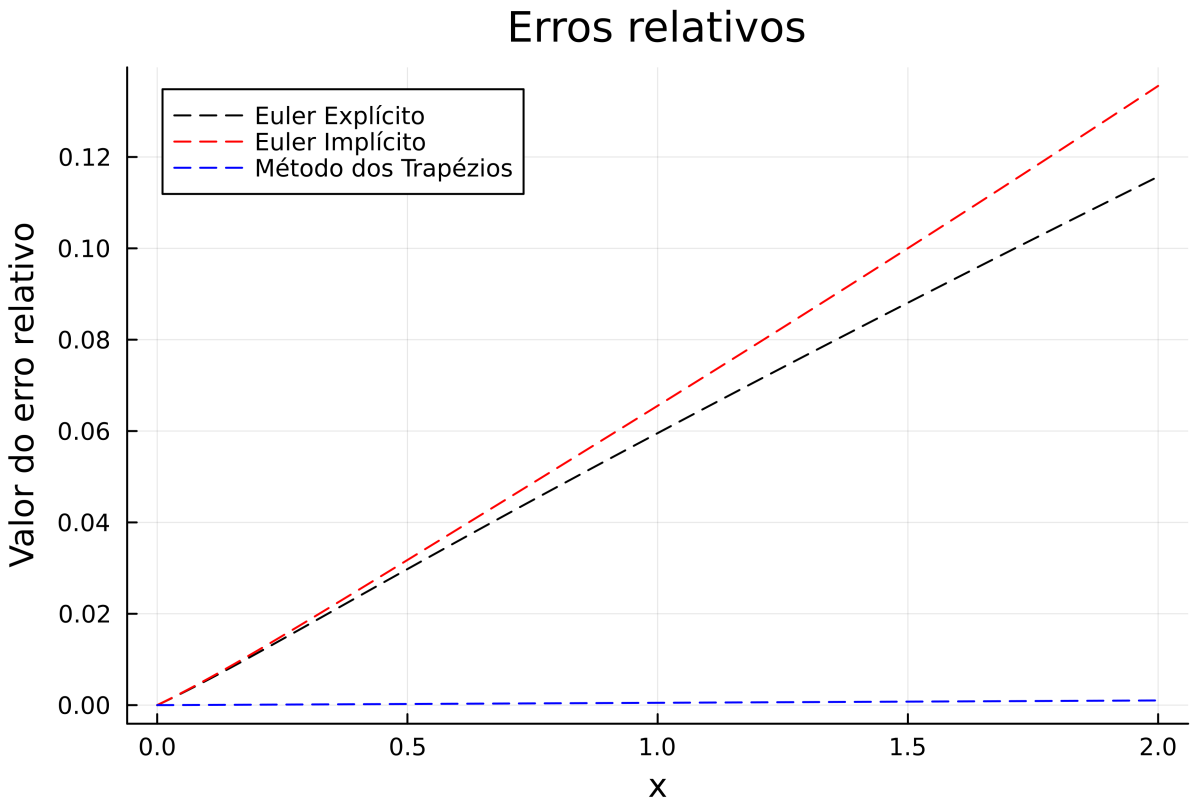

Figura 33: Gráfico plotado para o erro relativo do item c quando $h = 0.005$

2.3.4 $h = 0.001$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	1.2000	1.2000	1.2000	1.2000
0.2000	2.9183	2.9115	2.9251	2.9183
0.4000	7.5891	7.5523	7.6262	7.5890
0.6000	20.2855	20.1360	20.4373	20.2853
0.8000	54.7982	54.2567	55.3487	54.7972
1.0000	148.6132	146.7756	150.4863	148.6101
1.2000	403.6288	397.6423	409.7464	403.6187
1.4000	1096.8332	1077.8717	1116.2585	1096.8013
1.6000	2981.1580	2922.3251	3041.5808	2981.0590
1.8000	8103.2839	7923.5923	8288.2938	8102.9812
2.0000	22026.6658	21484.6140	22586.1576	22025.7515

Tabela 23: Valores obtidos para o item c com $h = 0.001$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	6.76e-03	2.32e-03	6.83e-03	2.34e-03	1.13e-05	3.87e-06
4.00e-01	3.67e-02	4.84e-03	3.72e-02	4.90e-03	6.13e-05	8.08e-06
6.00e-01	1.50e-01	7.37e-03	1.52e-01	7.48e-03	2.50e-04	1.23e-05
8.00e-01	5.41e-01	9.88e-03	5.51e-01	1.00e-02	9.07e-04	1.65e-05
1.00e+00	1.84e+00	1.24e-02	1.87e+00	1.26e-02	3.08e-03	2.07e-05
1.20e+00	5.99e+00	1.48e-02	6.12e+00	1.52e-02	1.00e-02	2.49e-05
1.40e+00	1.90e+01	1.73e-02	1.94e+01	1.77e-02	3.19e-02	2.91e-05
1.60e+00	5.88e+01	1.97e-02	6.04e+01	2.03e-02	9.90e-02	3.32e-05
1.80e+00	1.80e+02	2.22e-02	1.85e+02	2.28e-02	3.03e-01	3.74e-05
2.00e+00	5.42e+02	2.46e-02	5.59e+02	2.54e-02	9.14e-01	4.15e-05

Tabela 24: Erros obtidos para o item c com $h = 0.001$

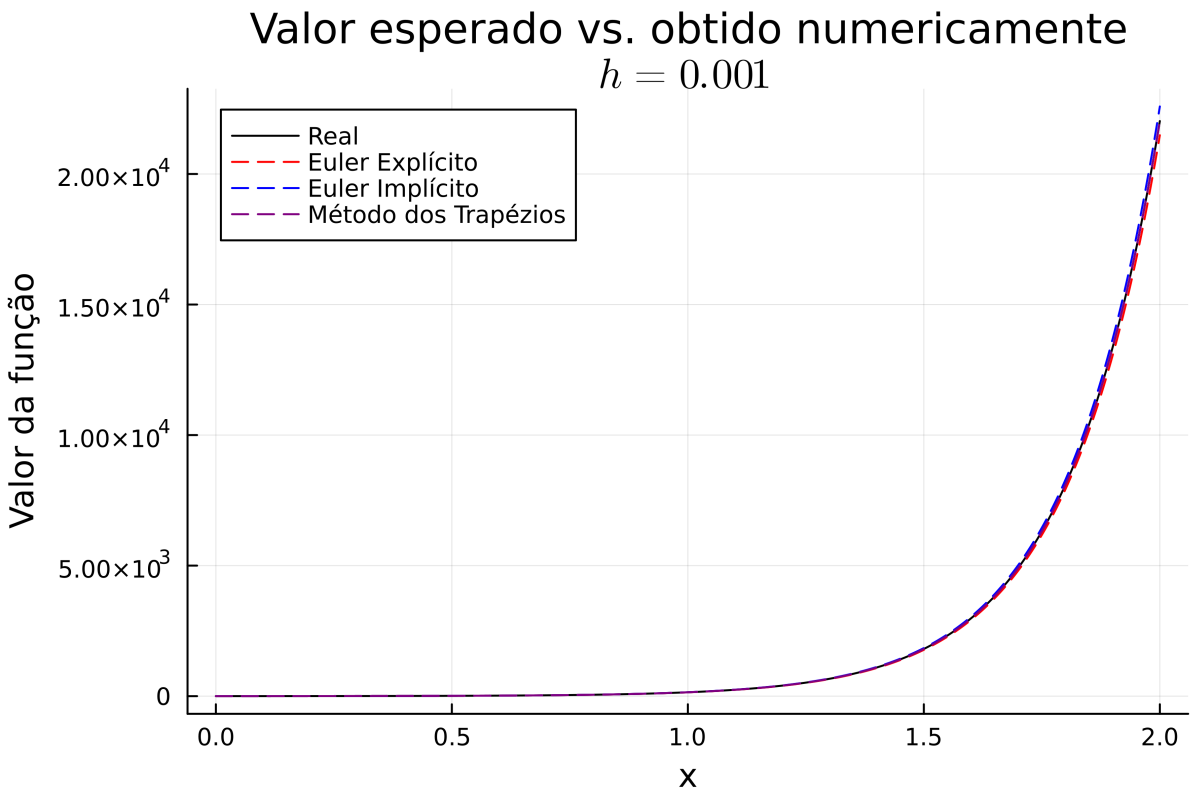

Figura 34: Gráfico plotado para o item c quando $h = 0.001$

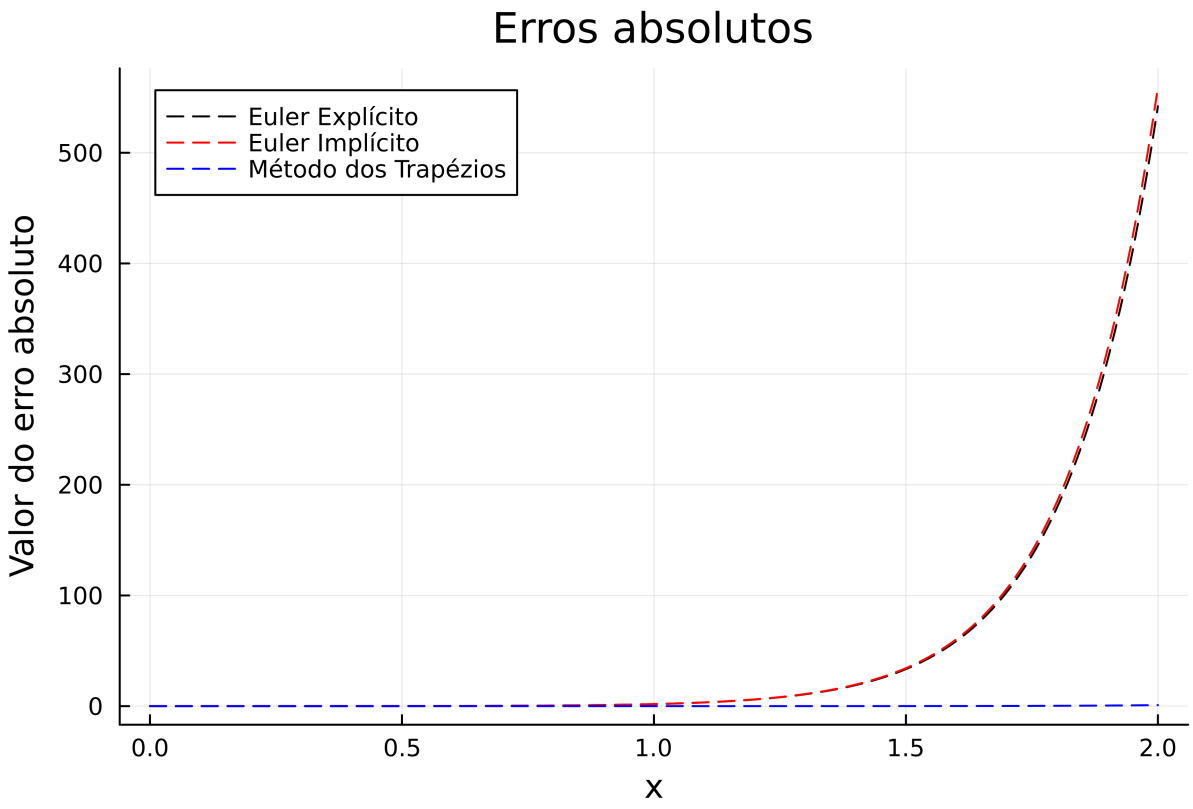

Figura 35: Gráfico plotado para o erro absoluto do item c quando $h = 0.001$

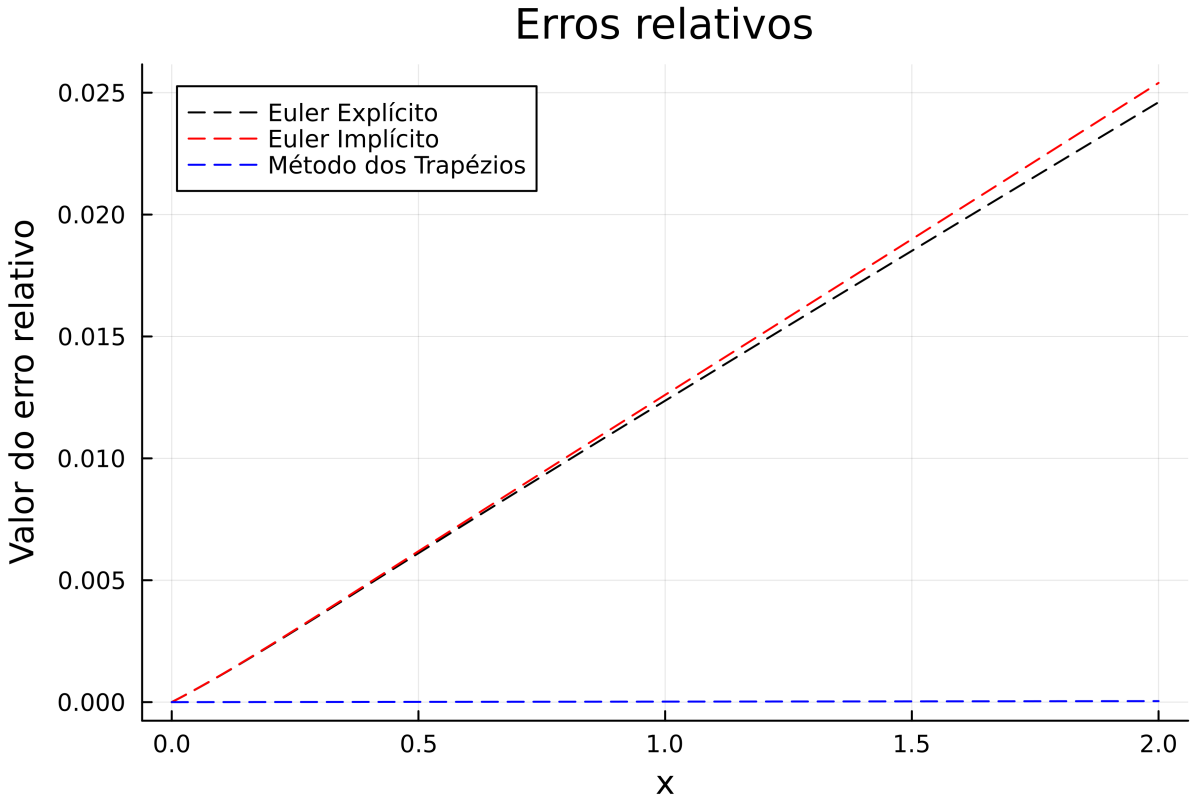

Figura 36: Gráfico plotado para o erro relativo do item c quando $h = 0.001$

2.4 Exercício 1, item d

2.4.1 $h = 0.1$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	0.5000	0.5000	0.5000	0.5000
0.2000	0.8293	0.8140	0.8462	0.8284
0.4000	1.2141	1.1815	1.2503	1.2122
0.6000	1.6489	1.5971	1.7073	1.6459
0.8000	2.1272	2.0538	2.2108	2.1228
1.0000	2.6409	2.5438	2.7528	2.6348
1.2000	3.1799	3.0569	3.3237	3.1720
1.4000	3.7324	3.5815	3.9115	3.7223
1.6000	4.2835	4.1030	4.5014	4.2708
1.8000	4.8152	4.6040	5.0750	4.7996
2.0000	5.3055	5.0635	5.6099	5.2866

Tabela 25: Valores obtidos para o item d com $h = 0.1$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	1.53e-02	1.84e-02	1.69e-02	2.03e-02	8.64e-04	1.04e-03
4.00e-01	3.25e-02	2.68e-02	3.62e-02	2.99e-02	1.88e-03	1.55e-03
6.00e-01	5.19e-02	3.15e-02	5.84e-02	3.54e-02	3.06e-03	1.86e-03
8.00e-01	7.34e-02	3.45e-02	8.35e-02	3.93e-02	4.45e-03	2.09e-03
1.00e+00	9.71e-02	3.68e-02	1.12e-01	4.24e-02	6.06e-03	2.30e-03
1.20e+00	1.23e-01	3.87e-02	1.44e-01	4.52e-02	7.94e-03	2.50e-03
1.40e+00	1.51e-01	4.04e-02	1.79e-01	4.80e-02	1.01e-02	2.71e-03
1.60e+00	1.80e-01	4.21e-02	2.18e-01	5.09e-02	1.26e-02	2.95e-03
1.80e+00	2.11e-01	4.38e-02	2.60e-01	5.40e-02	1.56e-02	3.23e-03
2.00e+00	2.42e-01	4.56e-02	3.04e-01	5.74e-02	1.89e-02	3.56e-03

Tabela 26: Erros obtidos para o item d com $h = 0.1$

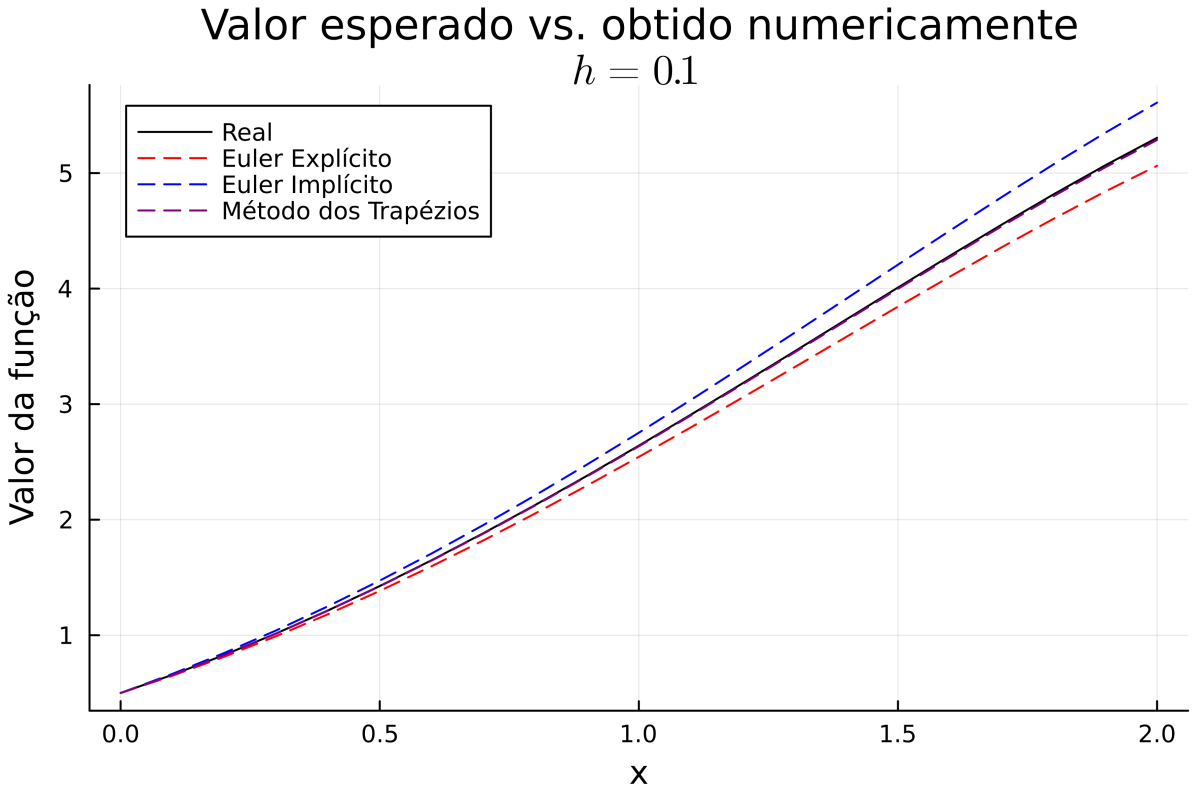

Figura 37: Gráfico plotado para o item d quando $h = 0.1$

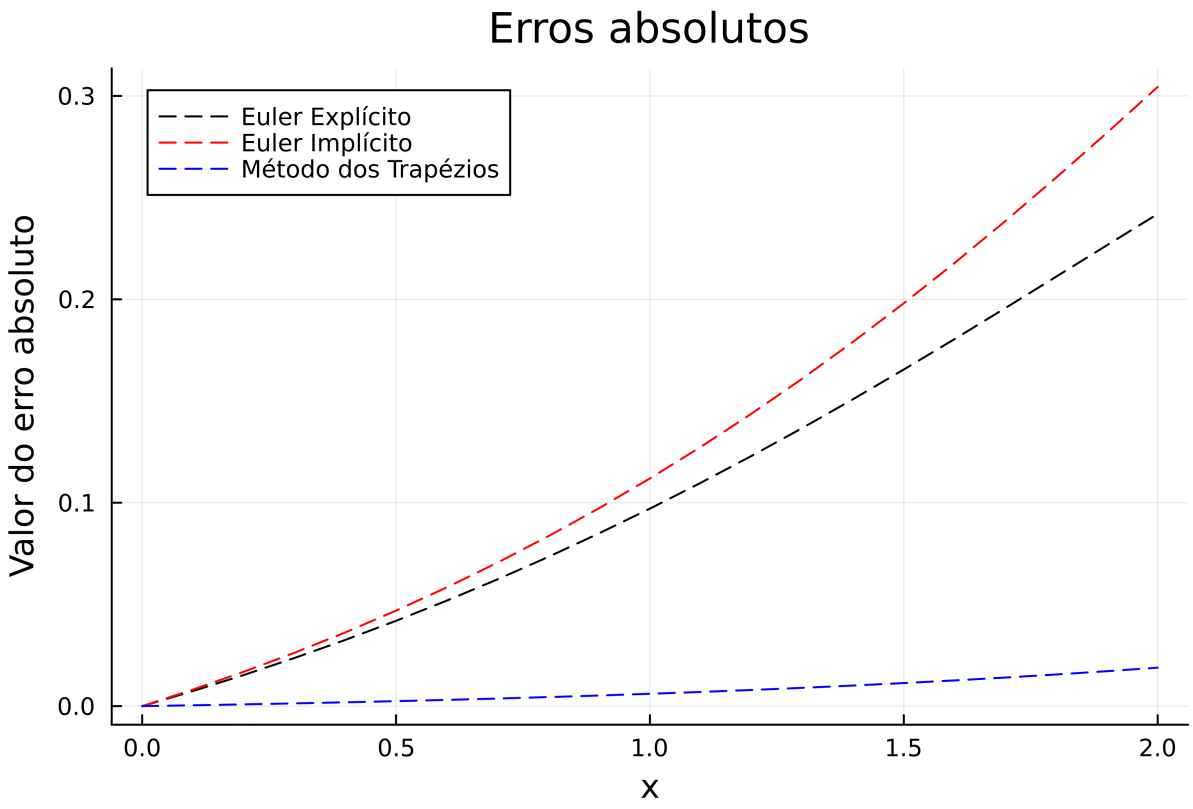

Figura 38: Gráfico plotado para o erro absoluto do item d quando $h = 0.1$

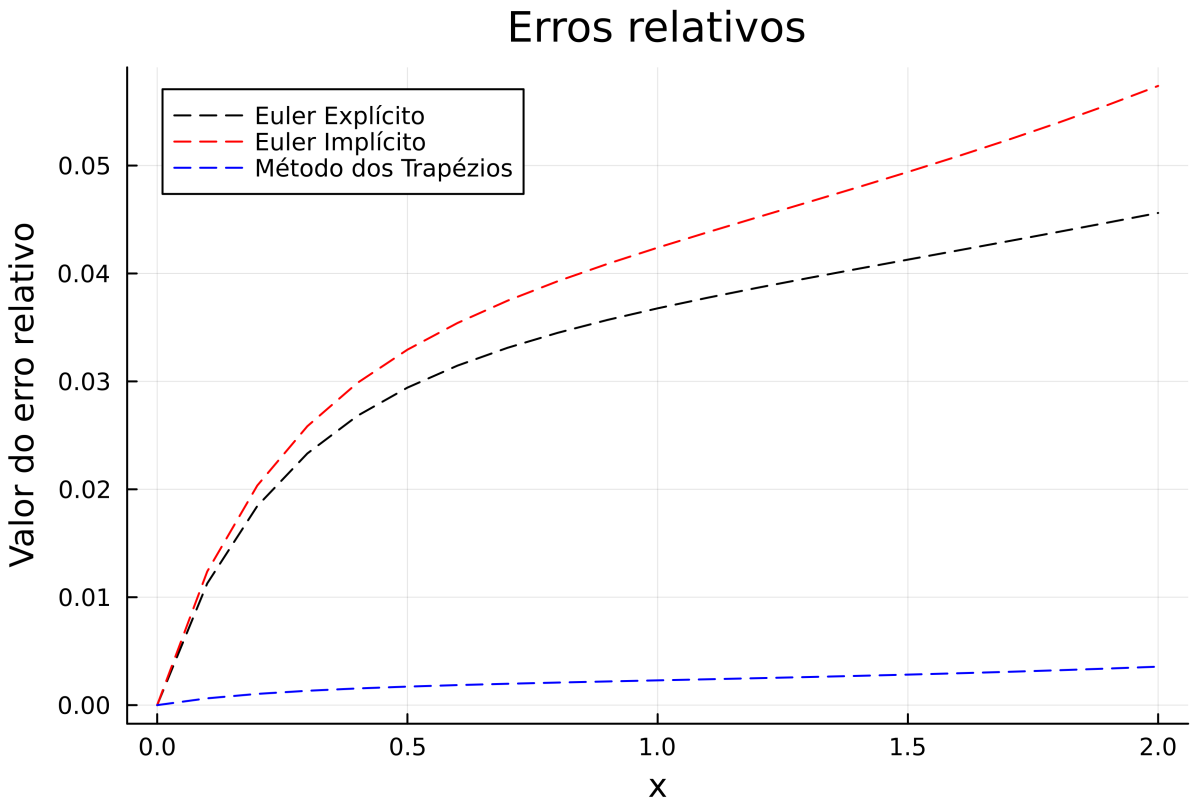

Figura 39: Gráfico plotado para o erro relativo do item d quando $h = 0.1$

2.4.2 $h = 0.01$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	0.5000	0.5000	0.5000	0.5000
0.2000	0.8293	0.8277	0.8309	0.8293
0.4000	1.2141	1.2107	1.2175	1.2141
0.6000	1.6489	1.6435	1.6545	1.6489
0.8000	2.1272	2.1195	2.1351	2.1272
1.0000	2.6409	2.6305	2.6513	2.6408
1.2000	3.1799	3.1668	3.1933	3.1799
1.4000	3.7324	3.7162	3.7489	3.7323
1.6000	4.2835	4.2639	4.3034	4.2834
1.8000	4.8152	4.7921	4.8387	4.8150
2.0000	5.3055	5.2788	5.3327	5.3053

Tabela 27: Valores obtidos para o item d com $h = 0.01$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	1.60e-03	1.92e-03	1.61e-03	1.94e-03	8.99e-06	1.08e-05
4.00e-01	3.41e-03	2.81e-03	3.44e-03	2.84e-03	1.95e-05	1.61e-05
6.00e-01	5.46e-03	3.31e-03	5.52e-03	3.35e-03	3.19e-05	1.93e-05
8.00e-01	7.75e-03	3.65e-03	7.86e-03	3.69e-03	4.62e-05	2.17e-05
1.00e+00	1.03e-02	3.91e-03	1.05e-02	3.96e-03	6.30e-05	2.39e-05
1.20e+00	1.31e-02	4.13e-03	1.33e-02	4.20e-03	8.25e-05	2.59e-05
1.40e+00	1.62e-02	4.35e-03	1.65e-02	4.42e-03	1.05e-04	2.81e-05
1.60e+00	1.95e-02	4.56e-03	1.99e-02	4.65e-03	1.31e-04	3.06e-05
1.80e+00	2.30e-02	4.78e-03	2.35e-02	4.88e-03	1.61e-04	3.35e-05
2.00e+00	2.66e-02	5.02e-03	2.73e-02	5.14e-03	1.96e-04	3.69e-05

Tabela 28: Erros obtidos para o item d com $h = 0.01$

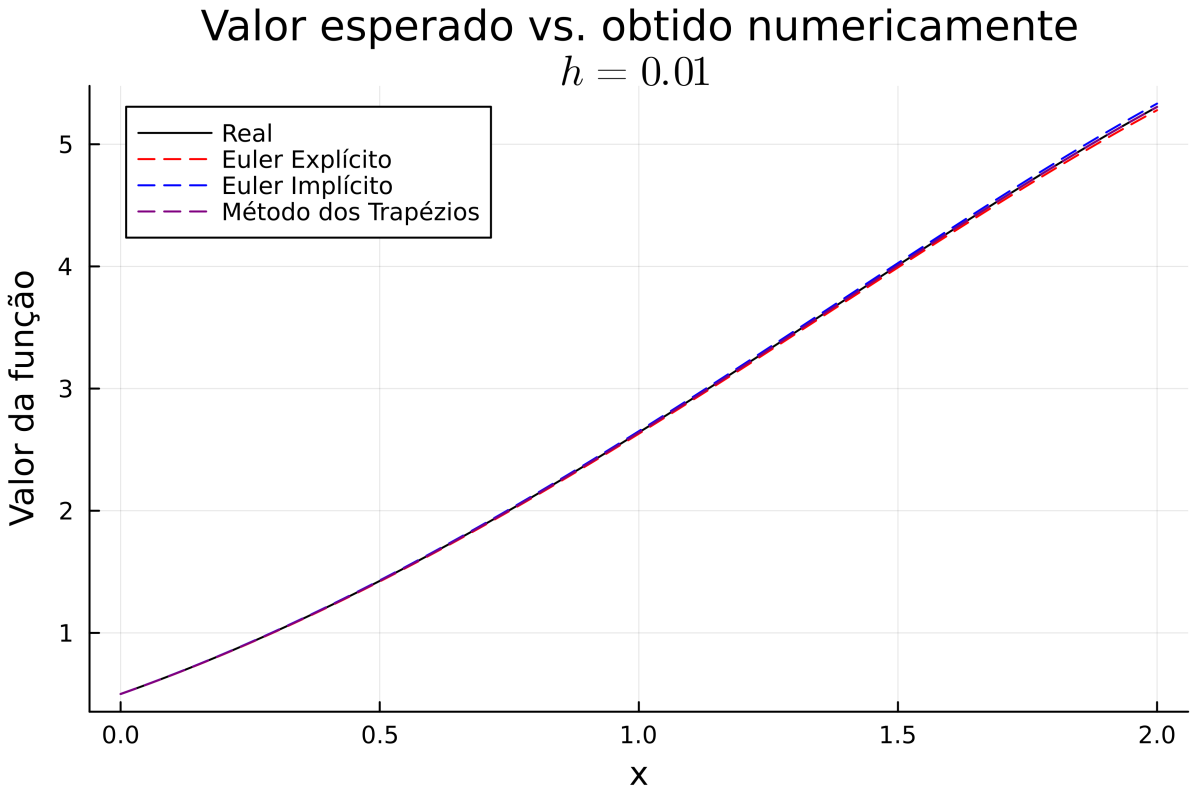

Figura 40: Gráfico plotado para o item d quando $h = 0.01$

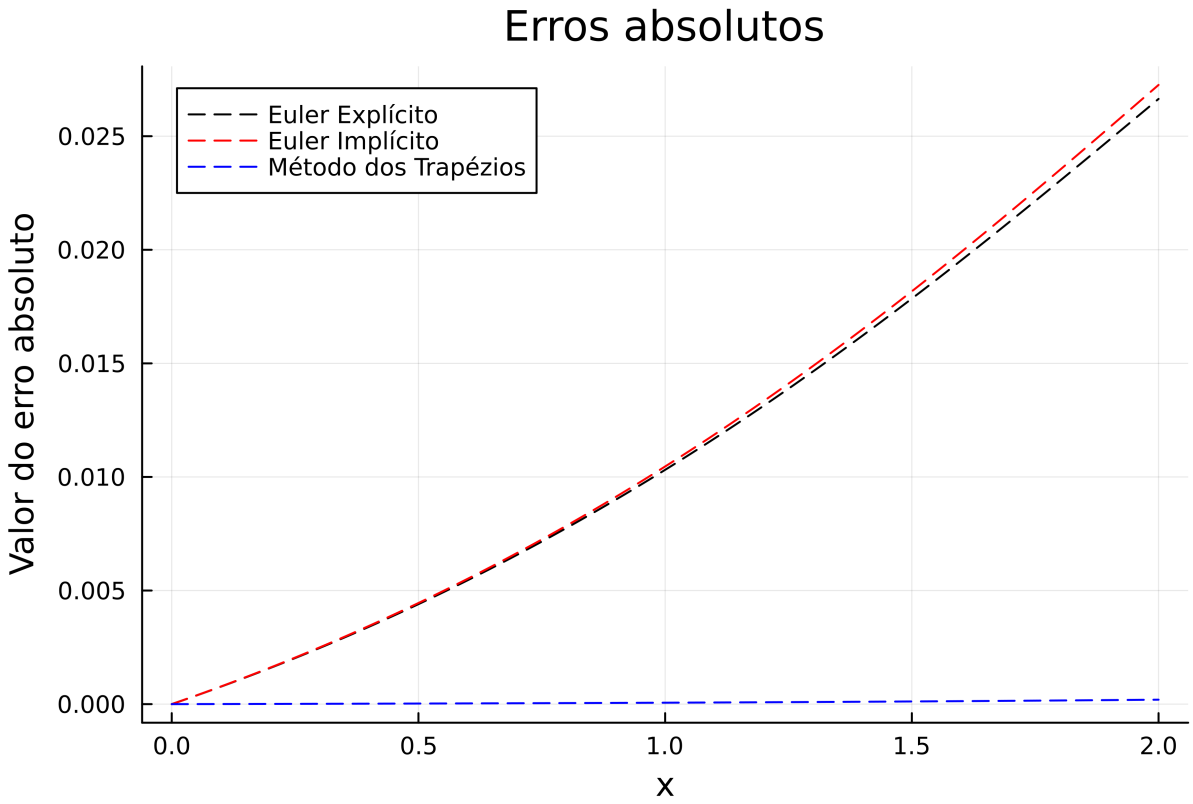

Figura 41: Gráfico plotado para o erro absoluto do item d quando $h = 0.01$

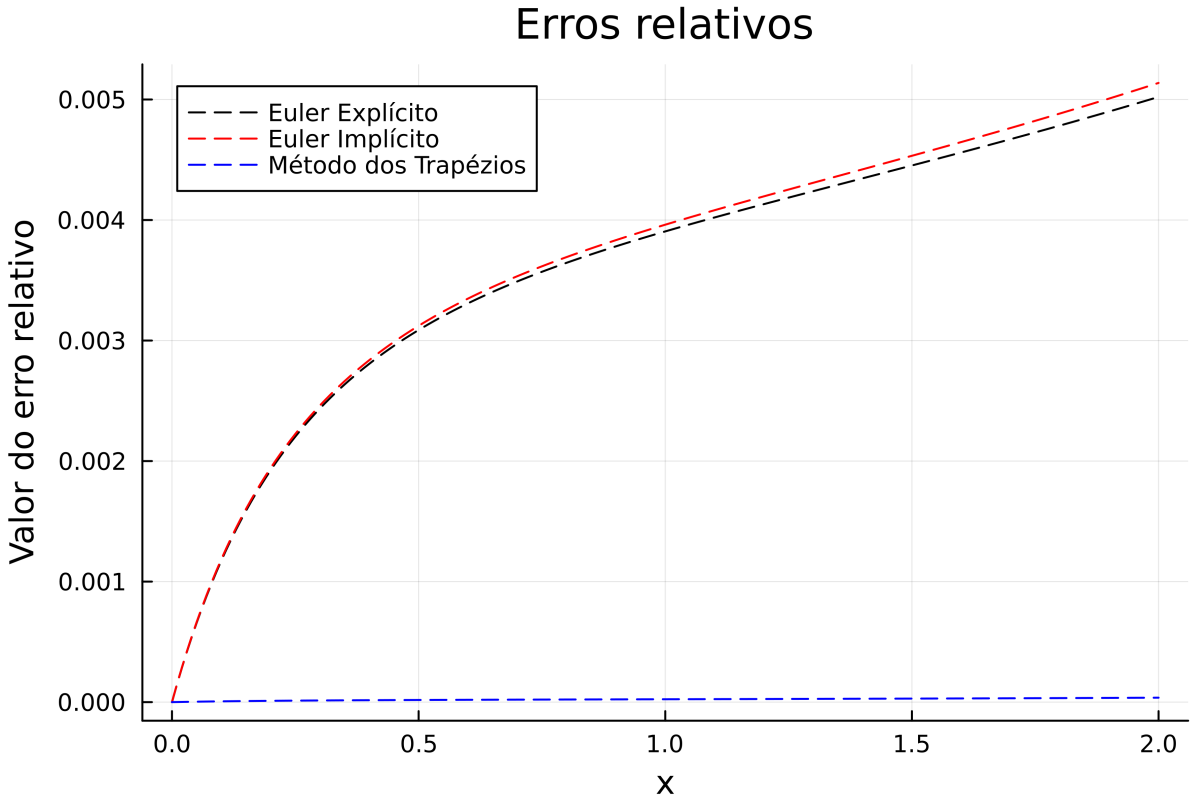

Figura 42: Gráfico plotado para o erro relativo do item d quando $h = 0.01$

2.4.3 $h = 0.005$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	0.5000	0.5000	0.5000	0.5000
0.2000	0.8293	0.8285	0.8301	0.8293
0.4000	1.2141	1.2124	1.2158	1.2141
0.6000	1.6489	1.6462	1.6517	1.6489
0.8000	2.1272	2.1233	2.1311	2.1272
1.0000	2.6409	2.6357	2.6461	2.6408
1.2000	3.1799	3.1733	3.1866	3.1799
1.4000	3.7324	3.7243	3.7406	3.7324
1.6000	4.2835	4.2737	4.2934	4.2835
1.8000	4.8152	4.8036	4.8269	4.8151
2.0000	5.3055	5.2921	5.3190	5.3054

Tabela 29: Valores obtidos para o item d com $h = 0.005$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	8.00e-04	9.64e-04	8.04e-04	9.69e-04	2.25e-06	2.72e-06
4.00e-01	1.71e-03	1.41e-03	1.72e-03	1.41e-03	4.89e-06	4.03e-06
6.00e-01	2.74e-03	1.66e-03	2.75e-03	1.67e-03	7.98e-06	4.84e-06
8.00e-01	3.89e-03	1.83e-03	3.91e-03	1.84e-03	1.16e-05	5.45e-06
1.00e+00	5.18e-03	1.96e-03	5.21e-03	1.97e-03	1.58e-05	5.98e-06
1.20e+00	6.59e-03	2.07e-03	6.65e-03	2.09e-03	2.07e-05	6.50e-06
1.40e+00	8.14e-03	2.18e-03	8.21e-03	2.20e-03	2.63e-05	7.05e-06
1.60e+00	9.81e-03	2.29e-03	9.91e-03	2.31e-03	3.28e-05	7.67e-06
1.80e+00	1.16e-02	2.40e-03	1.17e-02	2.43e-03	4.04e-05	8.38e-06
2.00e+00	1.34e-02	2.52e-03	1.36e-02	2.55e-03	4.90e-05	9.23e-06

Tabela 30: Erros obtidos para o item d com $h = 0.005$

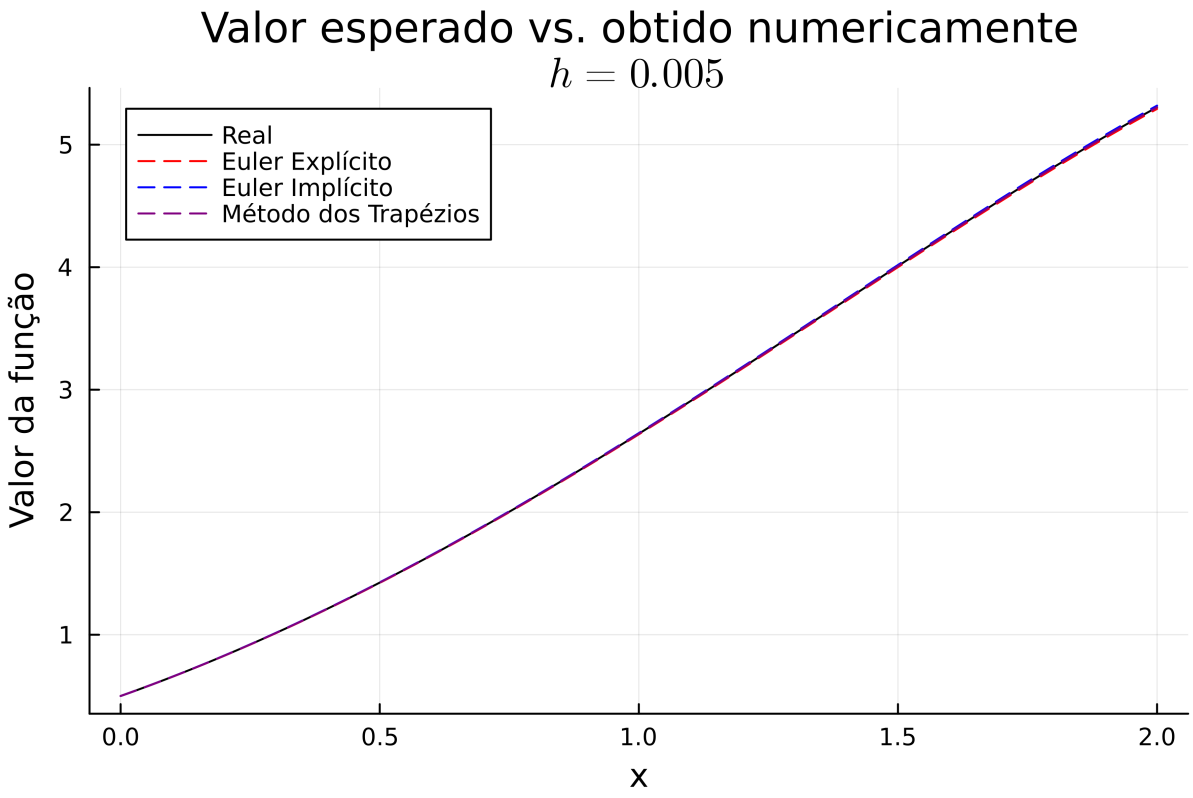

Figura 43: Gráfico plotado para o item d quando $h = 0.005$

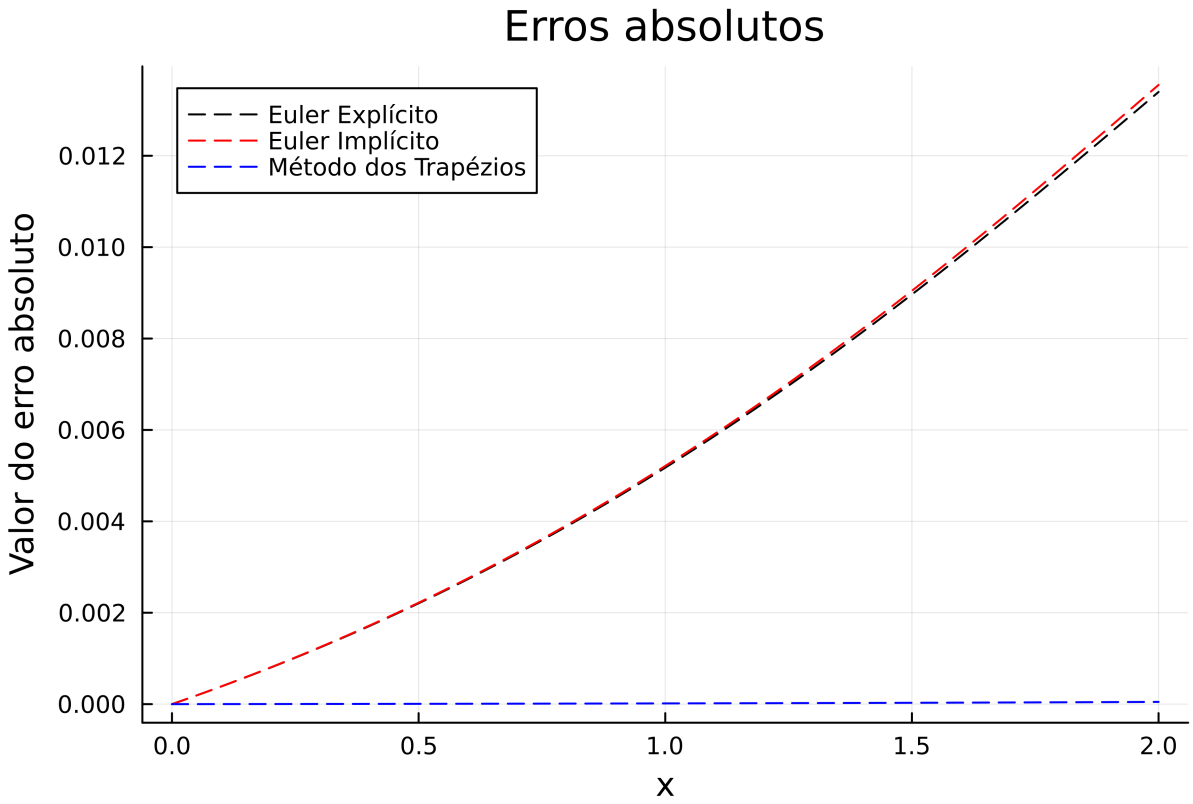

Figura 44: Gráfico plotado para o erro absoluto do item d quando $h = 0.005$

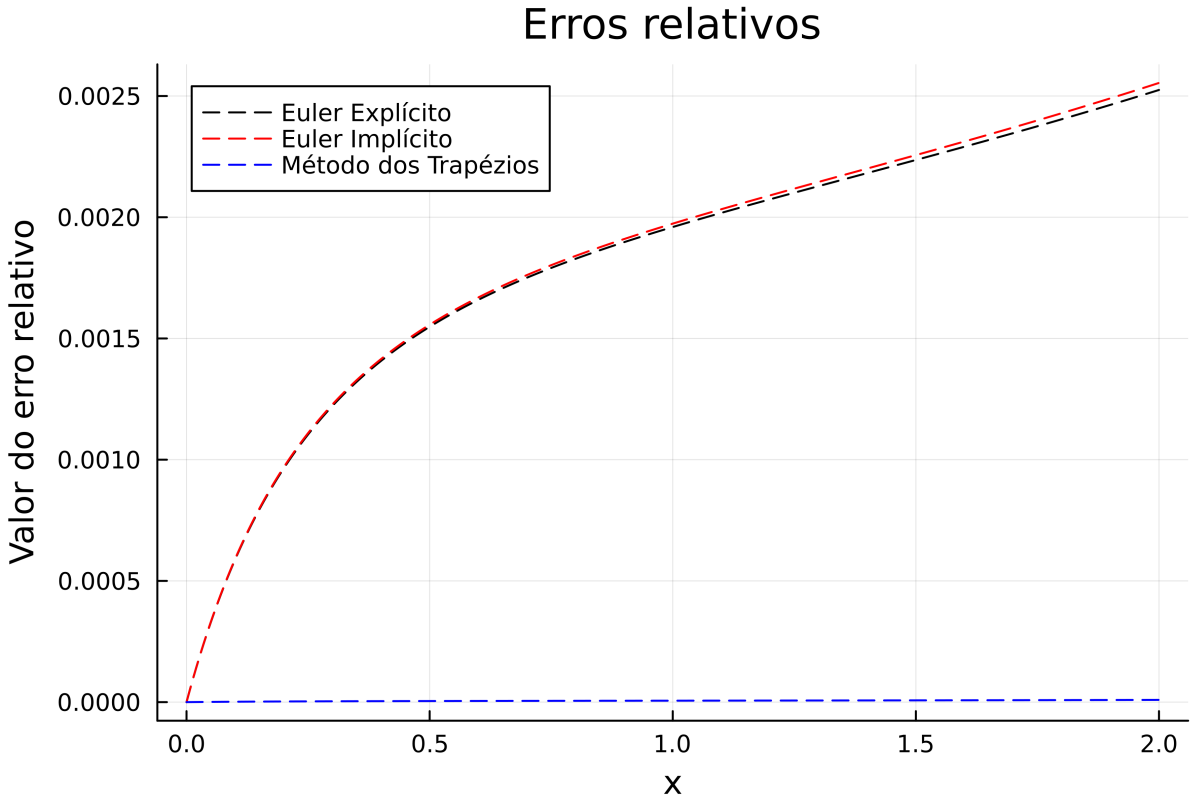

Figura 45: Gráfico plotado para o erro relativo do item d quando $h = 0.005$

2.4.4 $h = 0.001$

x	Valor real	Valor (Euler Explícito)	Valor (Euler Implícito)	Valor (Método dos Trapézios)
0.0000	0.5000	0.5000	0.5000	0.5000
0.2000	0.8293	0.8291	0.8295	0.8293
0.4000	1.2141	1.2137	1.2144	1.2141
0.6000	1.6489	1.6484	1.6495	1.6489
0.8000	2.1272	2.1264	2.1280	2.1272
1.0000	2.6409	2.6398	2.6419	2.6409
1.2000	3.1799	3.1786	3.1813	3.1799
1.4000	3.7324	3.7308	3.7340	3.7324
1.6000	4.2835	4.2815	4.2855	4.2835
1.8000	4.8152	4.8129	4.8175	4.8152
2.0000	5.3055	5.3028	5.3082	5.3055

Tabela 31: Valores obtidos para o item d com $h = 0.001$

x	Erro absoluto (Euler Explícito)	Erro relativo (Euler Explícito)	Erro absoluto (Euler Implícito)	Erro relativo (Euler Implícito)	Erro absoluto (Método dos Trapézios)	Erro relativo (Método dos Trapézios)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
2.00e-01	1.60e-04	1.93e-04	1.60e-04	1.93e-04	9.03e-08	1.09e-07
4.00e-01	3.42e-04	2.82e-04	3.43e-04	2.82e-04	1.96e-07	1.62e-07
6.00e-01	5.48e-04	3.33e-04	5.49e-04	3.33e-04	3.20e-07	1.94e-07
8.00e-01	7.80e-04	3.67e-04	7.81e-04	3.67e-04	4.64e-07	2.18e-07
1.00e+00	1.04e-03	3.93e-04	1.04e-03	3.94e-04	6.32e-07	2.39e-07
1.20e+00	1.32e-03	4.16e-04	1.33e-03	4.17e-04	8.28e-07	2.60e-07
1.40e+00	1.63e-03	4.38e-04	1.64e-03	4.39e-04	1.05e-06	2.82e-07
1.60e+00	1.97e-03	4.60e-04	1.97e-03	4.61e-04	1.32e-06	3.07e-07
1.80e+00	2.32e-03	4.83e-04	2.33e-03	4.84e-04	1.62e-06	3.36e-07
2.00e+00	2.69e-03	5.07e-04	2.70e-03	5.08e-04	1.96e-06	3.70e-07

Tabela 32: Erros obtidos para o item d com $h = 0.001$

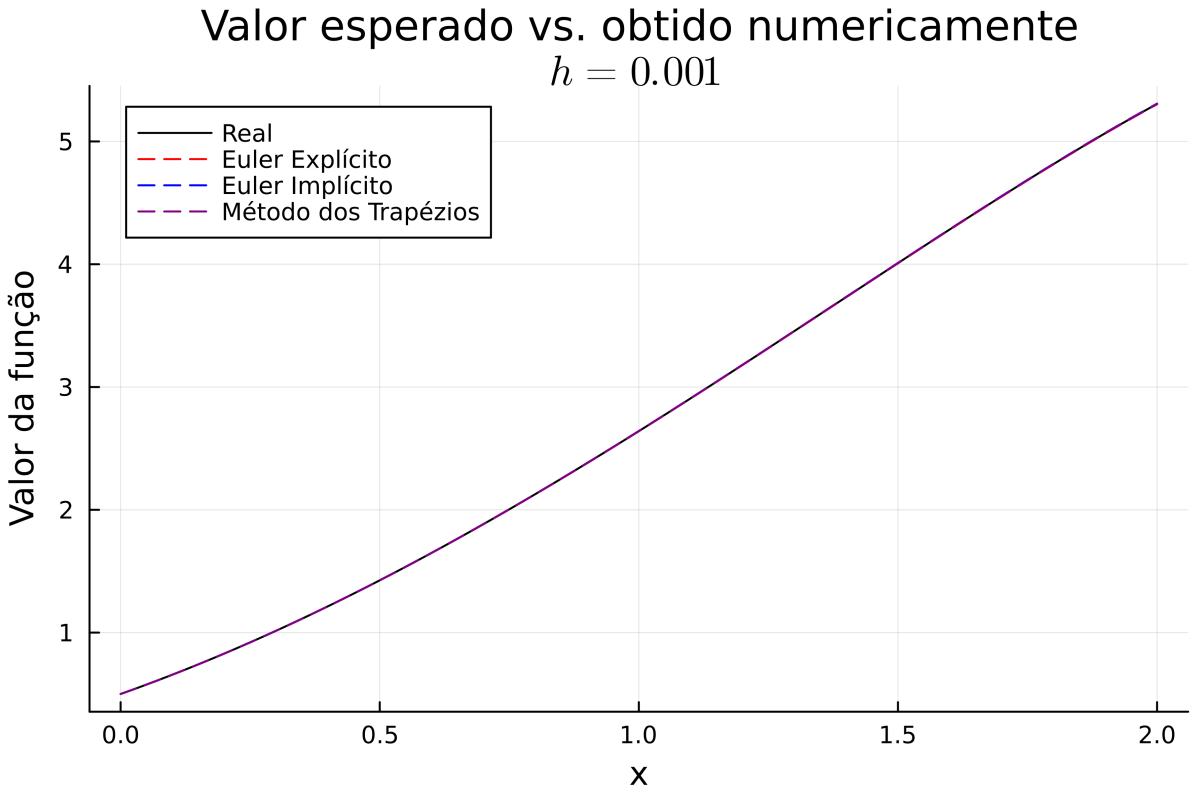

Figura 46: Gráfico plotado para o item d quando $h = 0.001$

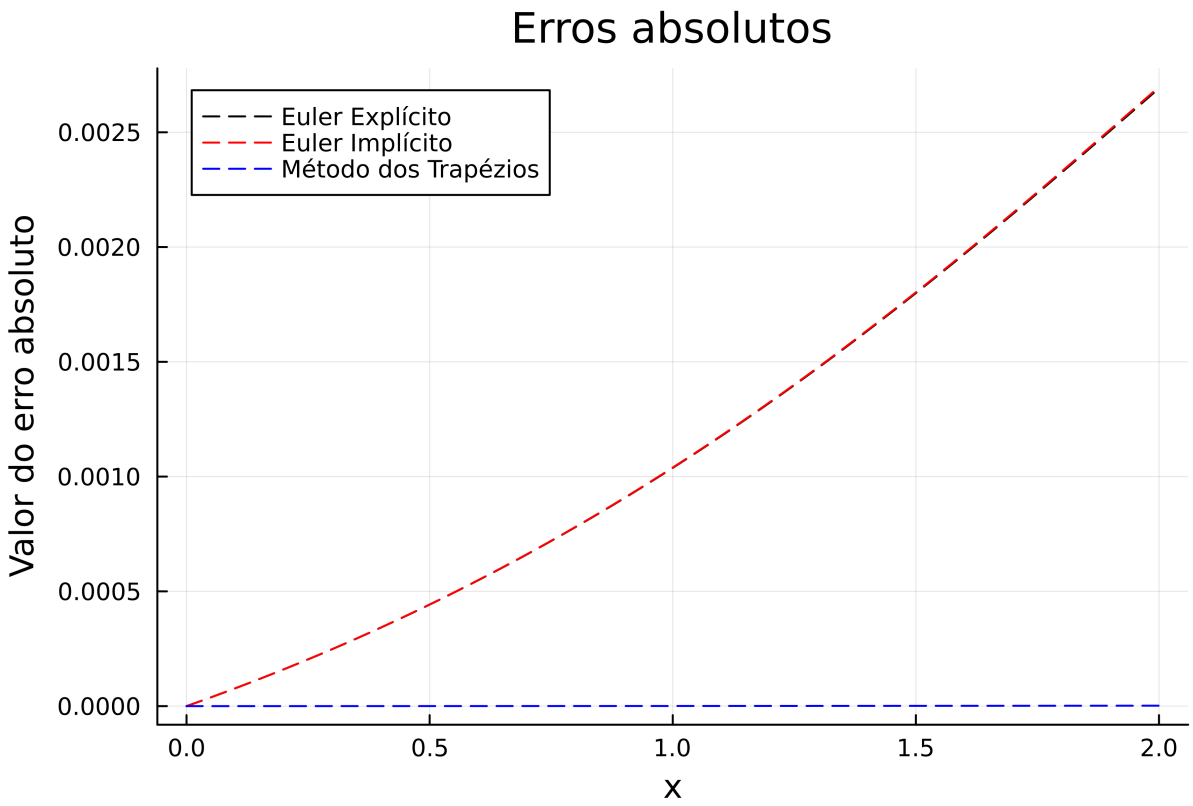

Figura 47: Gráfico plotado para o erro absoluto do item d quando $h = 0.001$

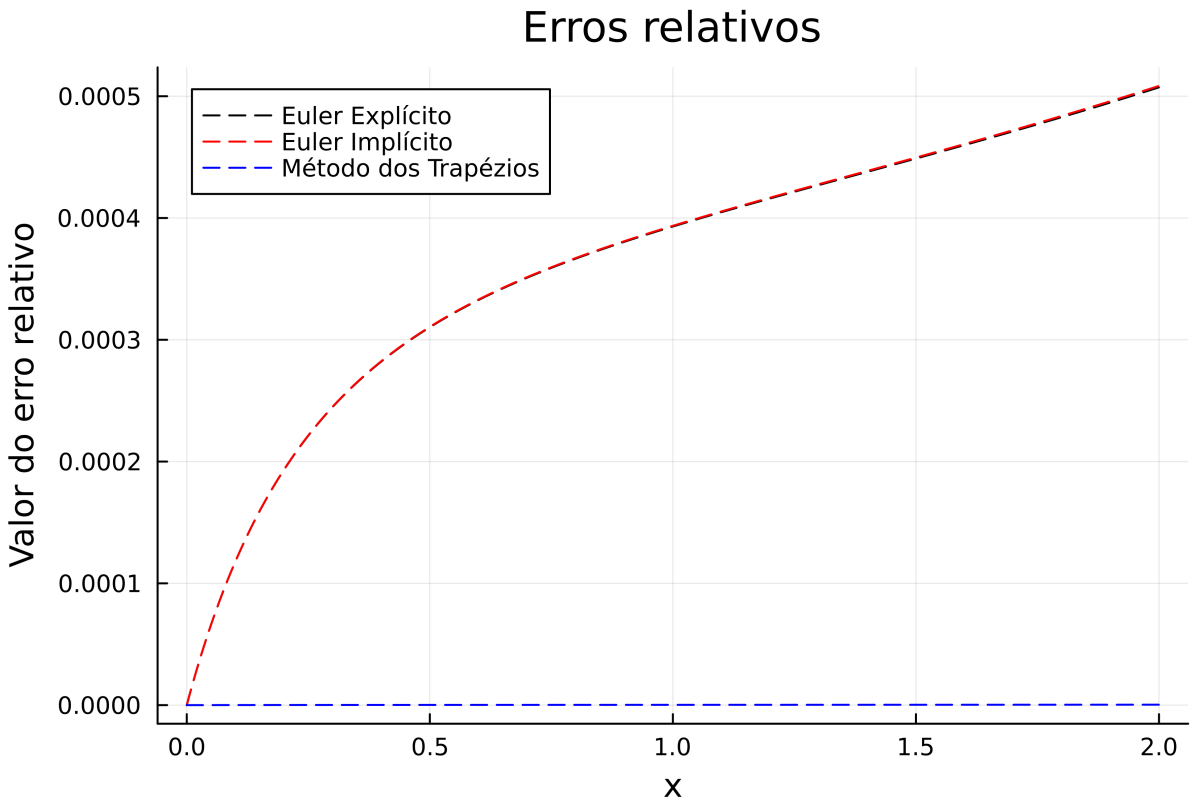

Figura 48: Gráfico plotado para o erro relativo do item d quando $h = 0.001$

2.5 Exercício 2, item a

2.5.1 $h = 0.1$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 33: Valores obtidos para o item a com $h = 0.1$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	4.08e-06	2.04e-06	8.20e-08	4.09e-08
2.00e-01	7.39e-06	3.66e-06	1.48e-07	7.35e-08
3.00e-01	1.00e-05	4.92e-06	2.01e-07	9.86e-08
4.00e-01	1.21e-05	5.85e-06	2.43e-07	1.17e-07
5.00e-01	1.37e-05	6.50e-06	2.75e-07	1.30e-07
6.00e-01	1.49e-05	6.92e-06	2.98e-07	1.39e-07
7.00e-01	1.57e-05	7.14e-06	3.15e-07	1.43e-07
8.00e-01	1.62e-05	7.21e-06	3.26e-07	1.45e-07
9.00e-01	1.65e-05	7.16e-06	3.31e-07	1.44e-07
1.00e+00	1.66e-05	7.01e-06	3.33e-07	1.41e-07

Tabela 34: Erros obtidos para o item a com $h = 0.1$

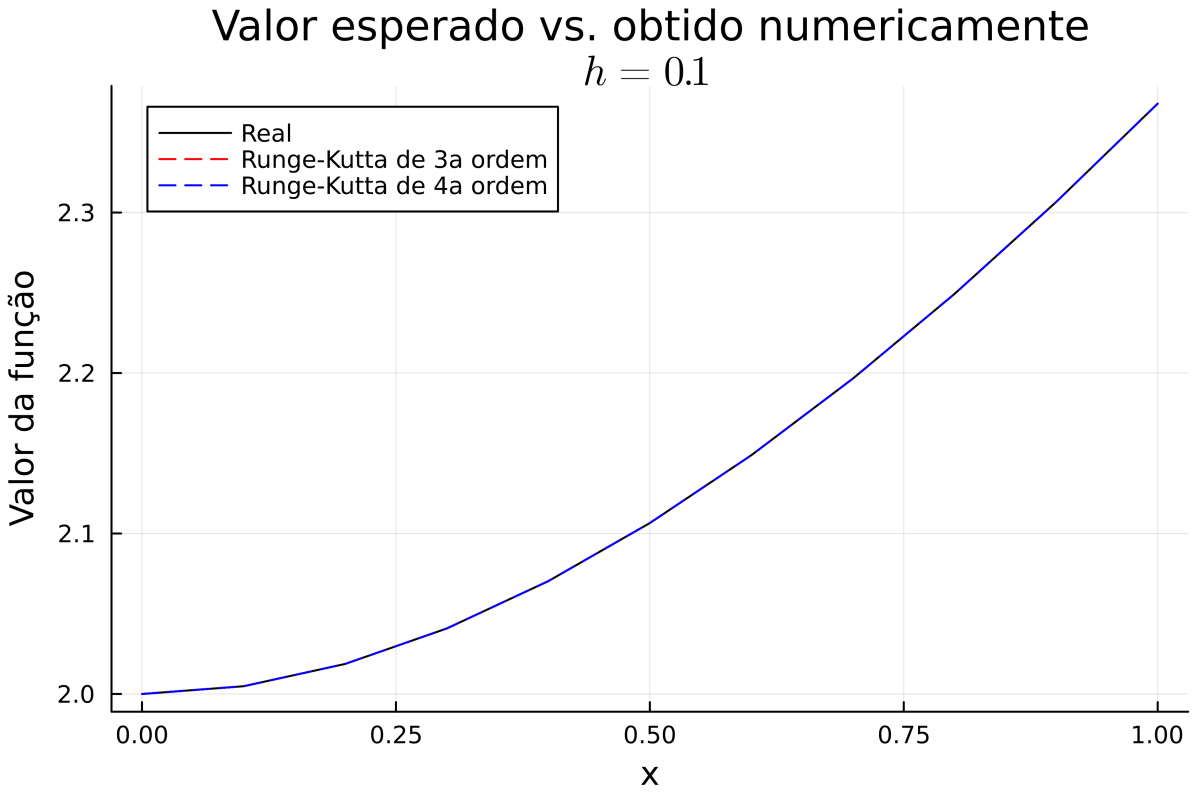

Figura 49: Gráfico plotado para o item a quando $h = 0.1$

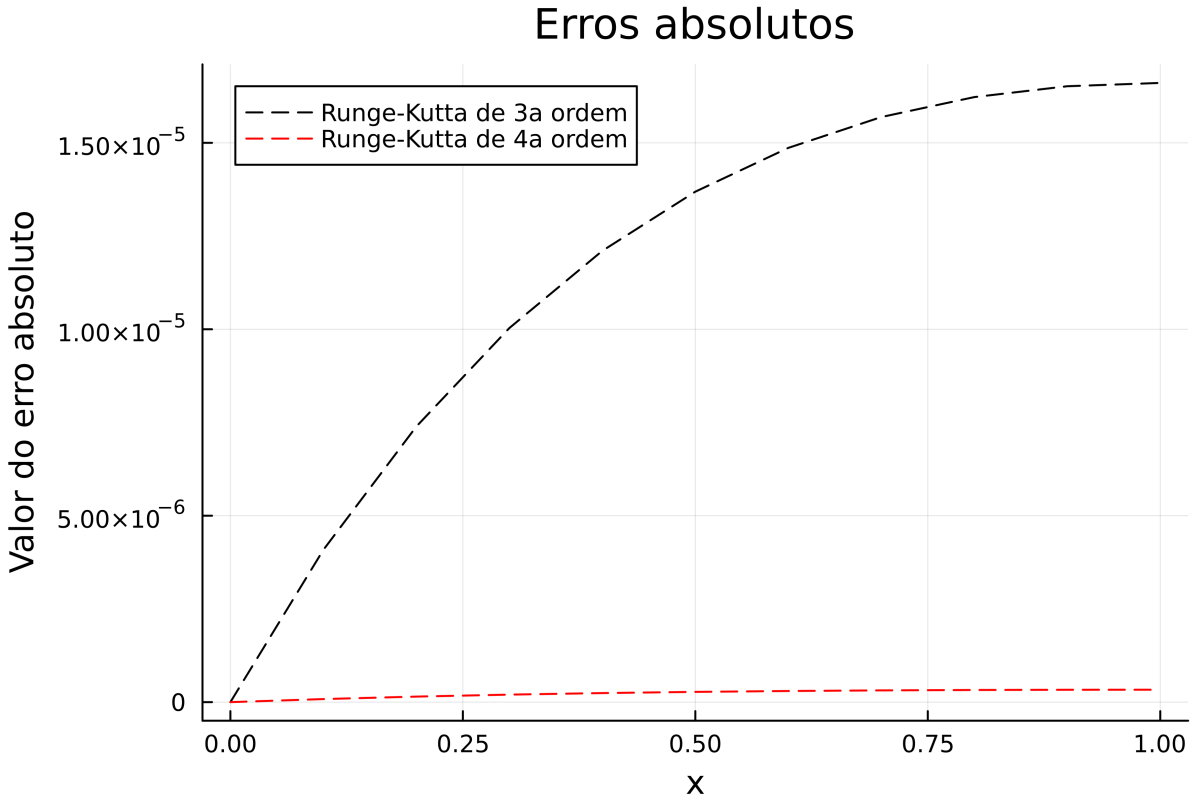

Figura 50: Gráfico plotado para o erro absoluto do item a quando $h = 0.1$

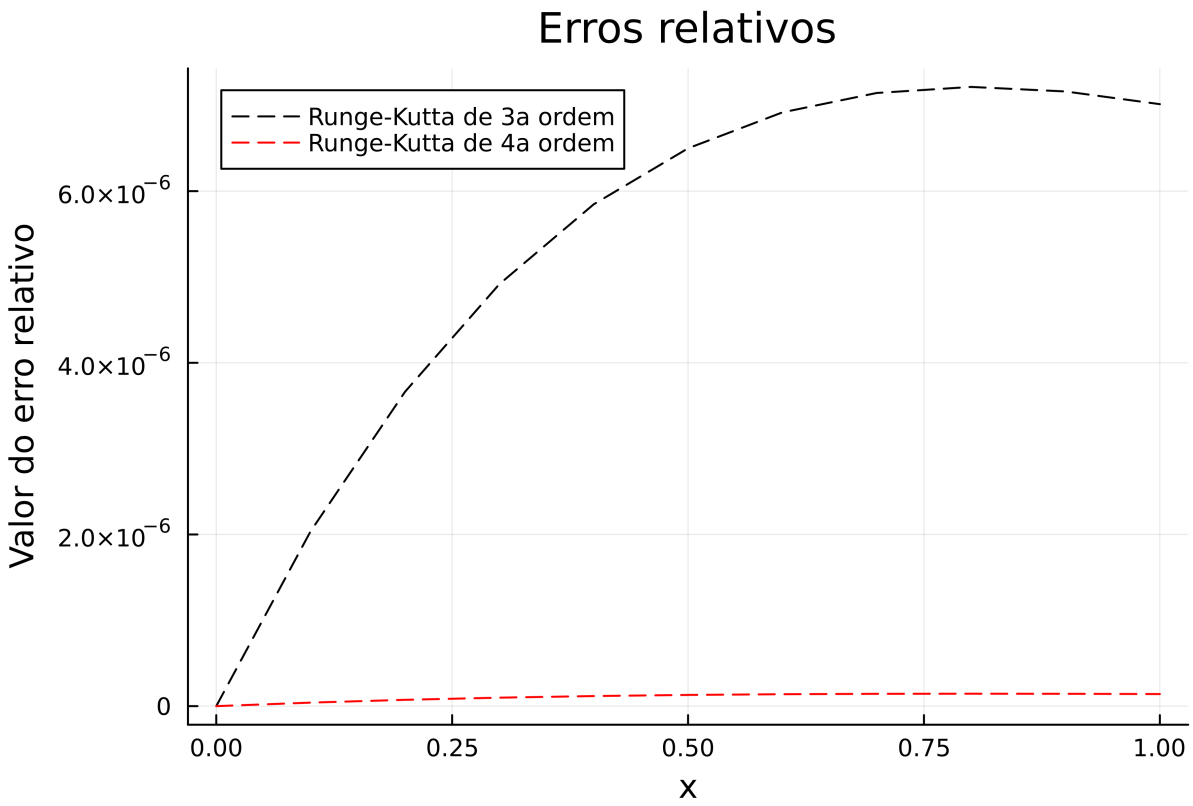

Figura 51: Gráfico plotado para o erro relativo do item a quando $h = 0.1$

2.5.2 $h = 0.01$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 35: Valores obtidos para o item a com $h = 0.01$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.80e-09	1.90e-09	7.60e-12	3.79e-12
2.00e-01	6.88e-09	3.41e-09	1.38e-11	6.82e-12
3.00e-01	9.33e-09	4.57e-09	1.87e-11	9.15e-12
4.00e-01	1.13e-08	5.44e-09	2.25e-11	1.09e-11
5.00e-01	1.27e-08	6.05e-09	2.55e-11	1.21e-11
6.00e-01	1.38e-08	6.44e-09	2.77e-11	1.29e-11
7.00e-01	1.46e-08	6.65e-09	2.92e-11	1.33e-11
8.00e-01	1.51e-08	6.71e-09	3.02e-11	1.34e-11
9.00e-01	1.54e-08	6.66e-09	3.07e-11	1.33e-11
1.00e+00	1.55e-08	6.53e-09	3.09e-11	1.31e-11

Tabela 36: Erros obtidos para o item a com $h = 0.01$

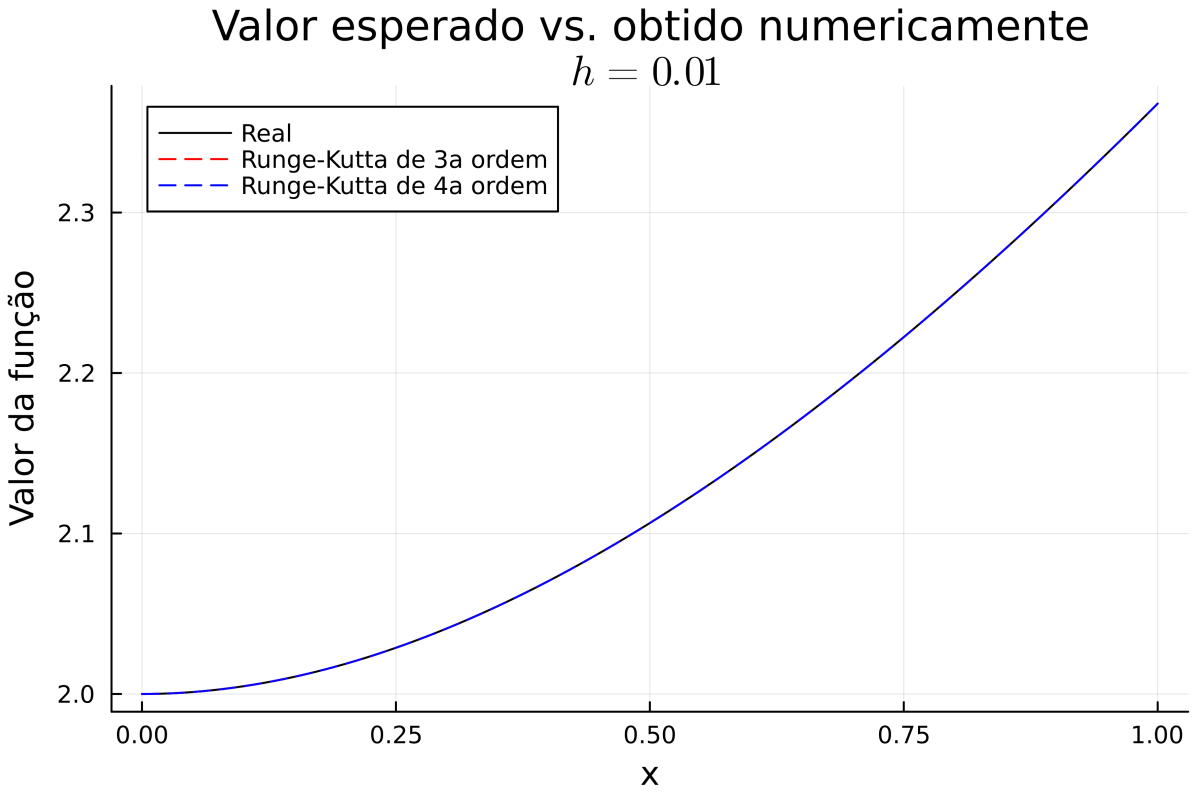

Figura 52: Gráfico plotado para o item a quando $h = 0.01$

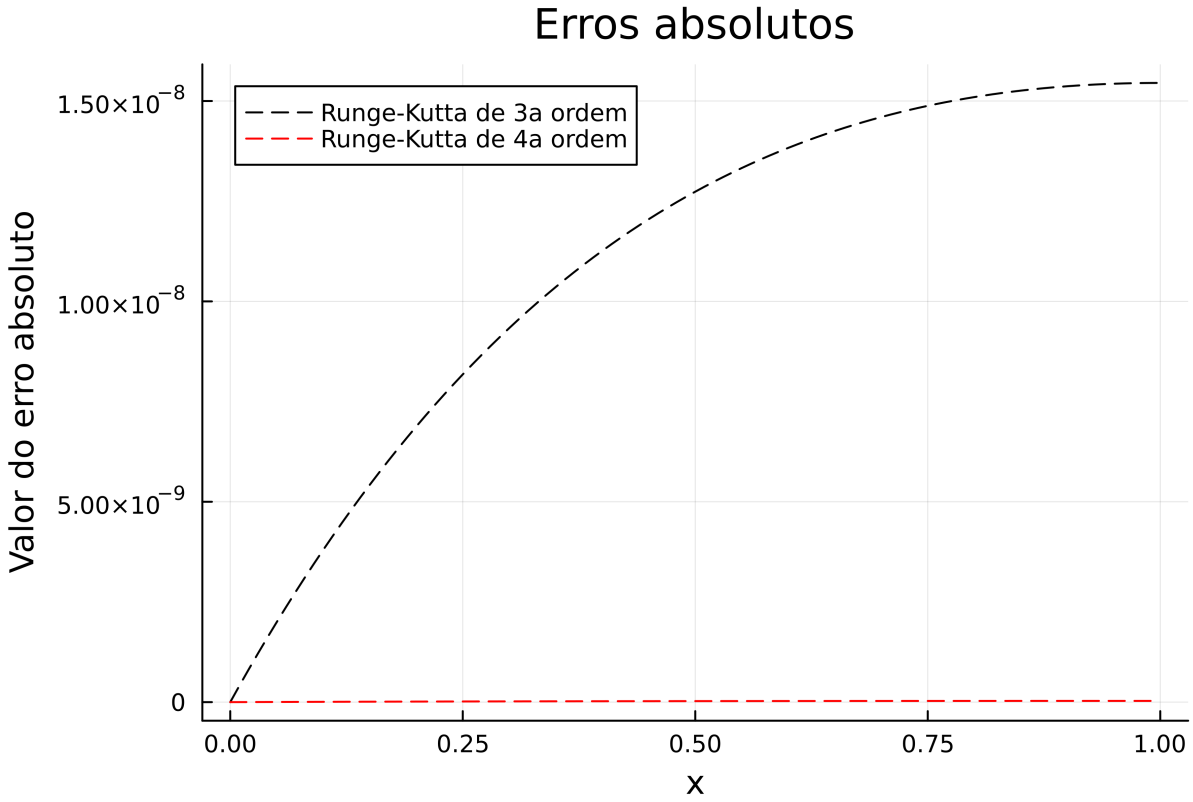

Figura 53: Gráfico plotado para o erro absoluto do item a quando $h = 0.01$

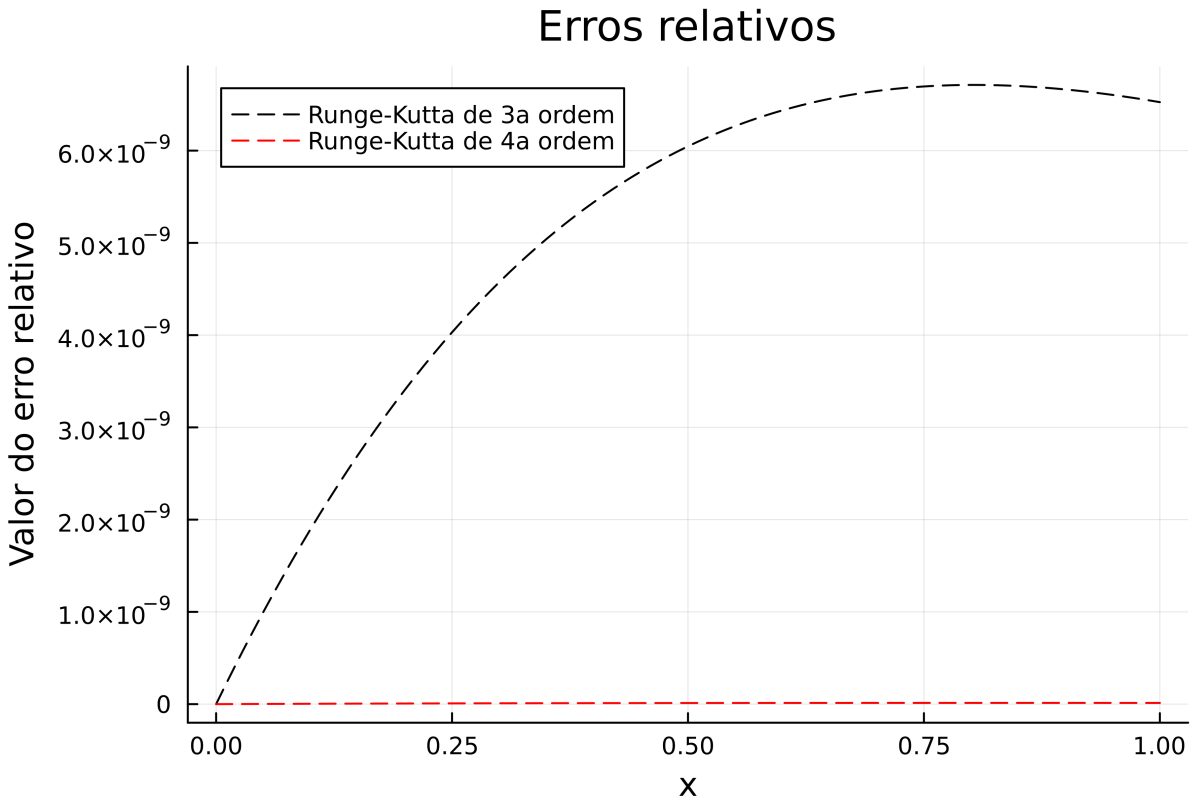

Figura 54: Gráfico plotado para o erro relativo do item a quando $h = 0.01$

2.5.3 $h = 0.005$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 37: Valores obtidos para o item a com $h = 0.005$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	4.73e-10	2.36e-10	4.73e-13	2.36e-13
2.00e-01	8.56e-10	4.24e-10	8.56e-13	4.24e-13
3.00e-01	1.16e-09	5.69e-10	1.16e-12	5.70e-13
4.00e-01	1.40e-09	6.77e-10	1.40e-12	6.77e-13
5.00e-01	1.59e-09	7.53e-10	1.59e-12	7.53e-13
6.00e-01	1.72e-09	8.01e-10	1.72e-12	8.02e-13
7.00e-01	1.82e-09	8.28e-10	1.82e-12	8.28e-13
8.00e-01	1.88e-09	8.36e-10	1.88e-12	8.36e-13
9.00e-01	1.91e-09	8.30e-10	1.91e-12	8.29e-13
1.00e+00	1.92e-09	8.12e-10	1.92e-12	8.13e-13

Tabela 38: Erros obtidos para o item a com $h = 0.005$

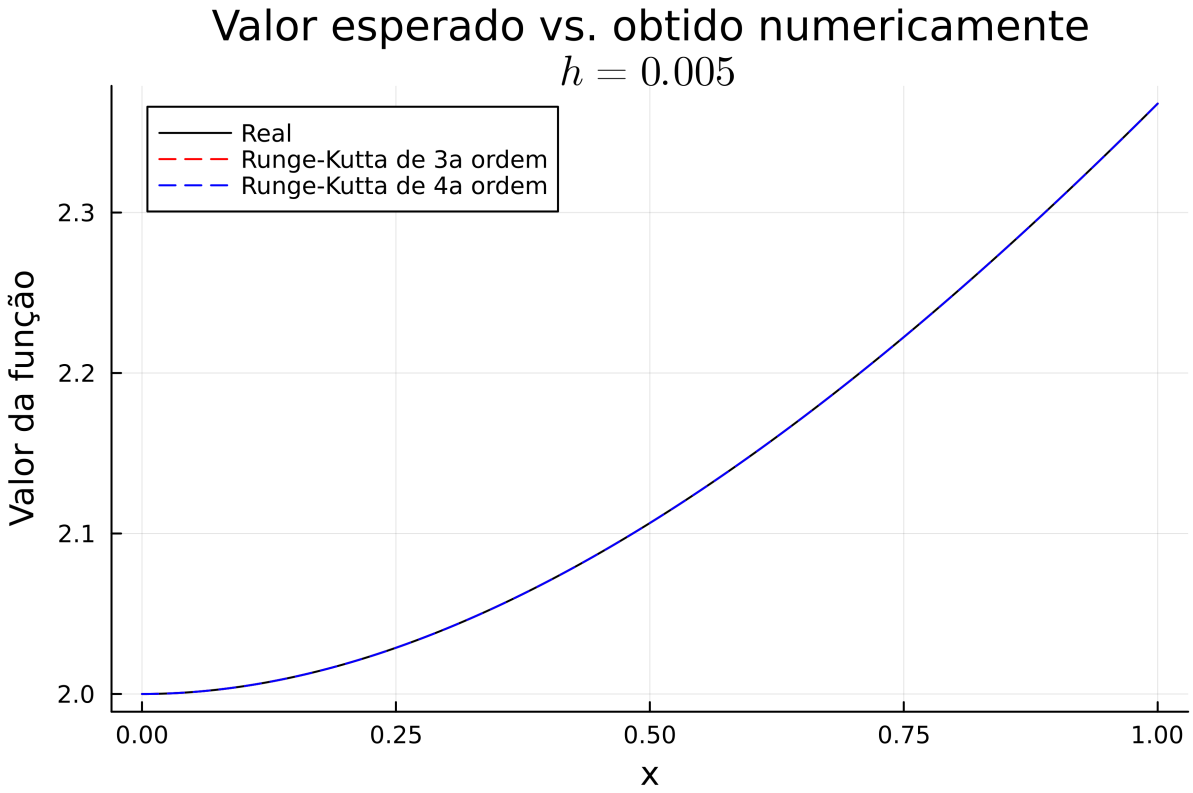

Figura 55: Gráfico plotado para o item a quando $h = 0.005$

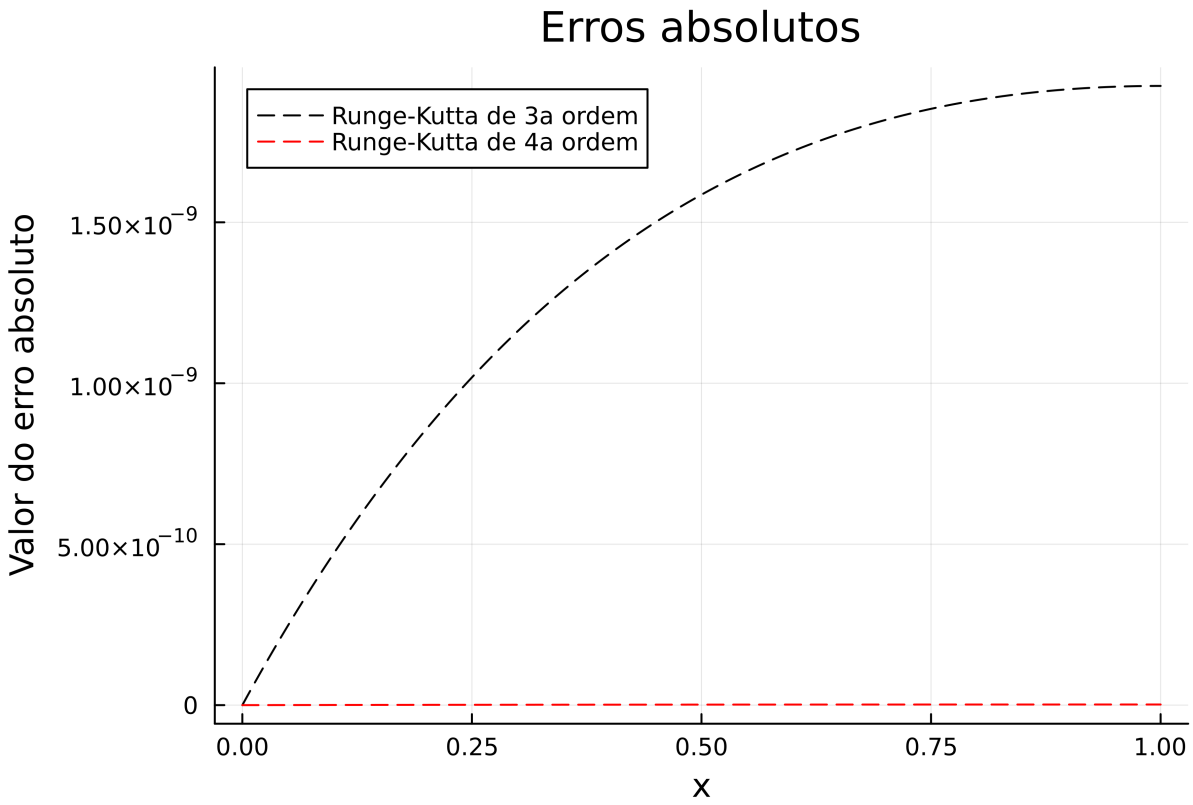

Figura 56: Gráfico plotado para o erro absoluto do item a quando $h = 0.005$

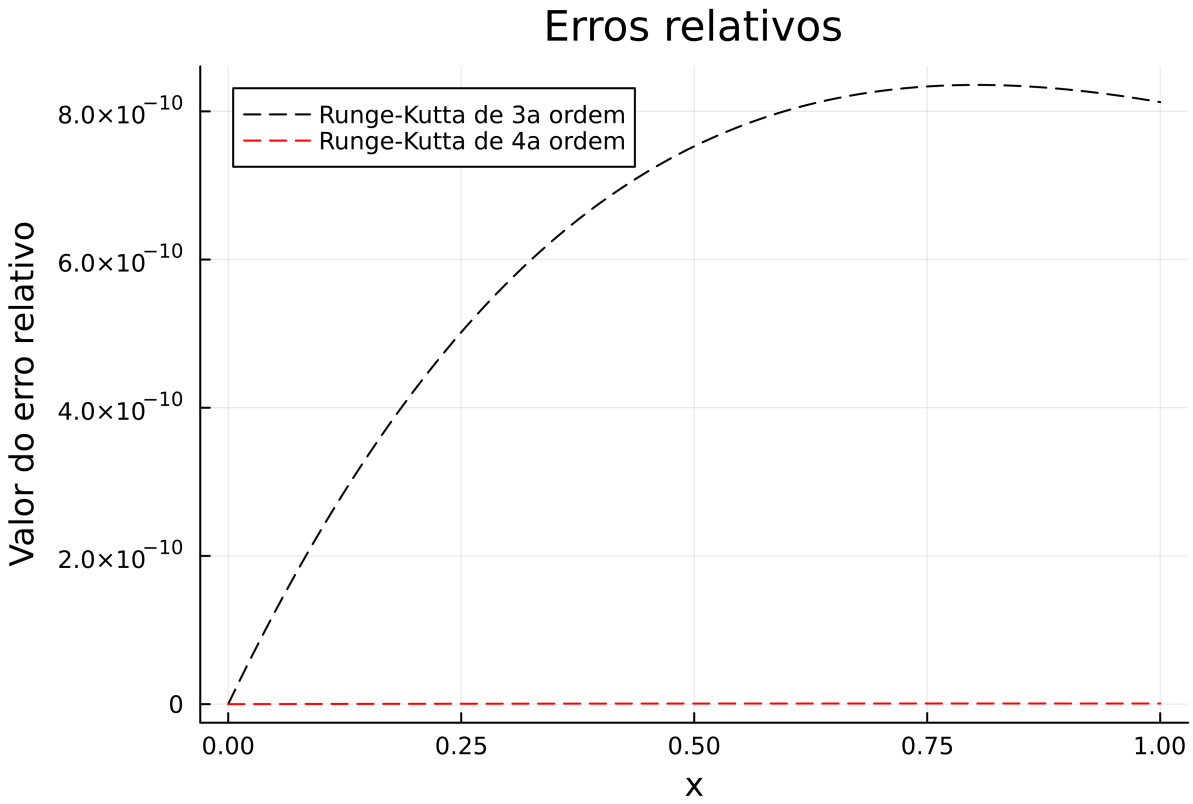

Figura 57: Gráfico plotado para o erro relativo do item a quando $h = 0.005$

2.5.4 $h = 0.001$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 39: Valores obtidos para o item a com $h = 0.001$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.77e-12	1.88e-12	4.44e-16	2.22e-16
2.00e-01	6.83e-12	3.38e-12	8.88e-16	4.40e-16
3.00e-01	9.27e-12	4.54e-12	1.33e-15	6.53e-16
4.00e-01	1.12e-11	5.40e-12	1.33e-15	6.44e-16
5.00e-01	1.26e-11	6.00e-12	1.33e-15	6.32e-16
6.00e-01	1.37e-11	6.39e-12	2.22e-15	1.03e-15
7.00e-01	1.45e-11	6.60e-12	4.44e-16	2.02e-16
8.00e-01	1.50e-11	6.67e-12	4.44e-16	1.97e-16
9.00e-01	1.53e-11	6.62e-12	4.44e-16	1.93e-16
1.00e+00	1.53e-11	6.48e-12	4.44e-16	1.88e-16

Tabela 40: Erros obtidos para o item a com $h = 0.001$

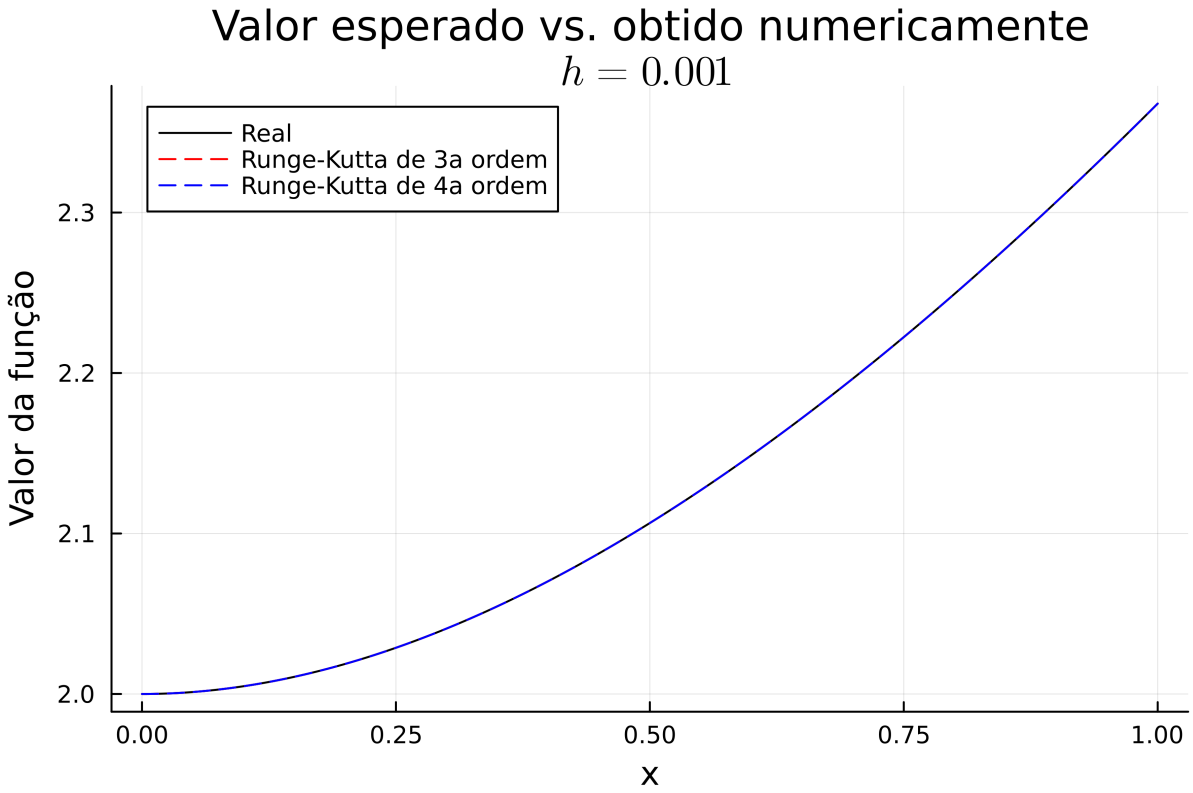

Figura 58: Gráfico plotado para o item a quando $h = 0.001$

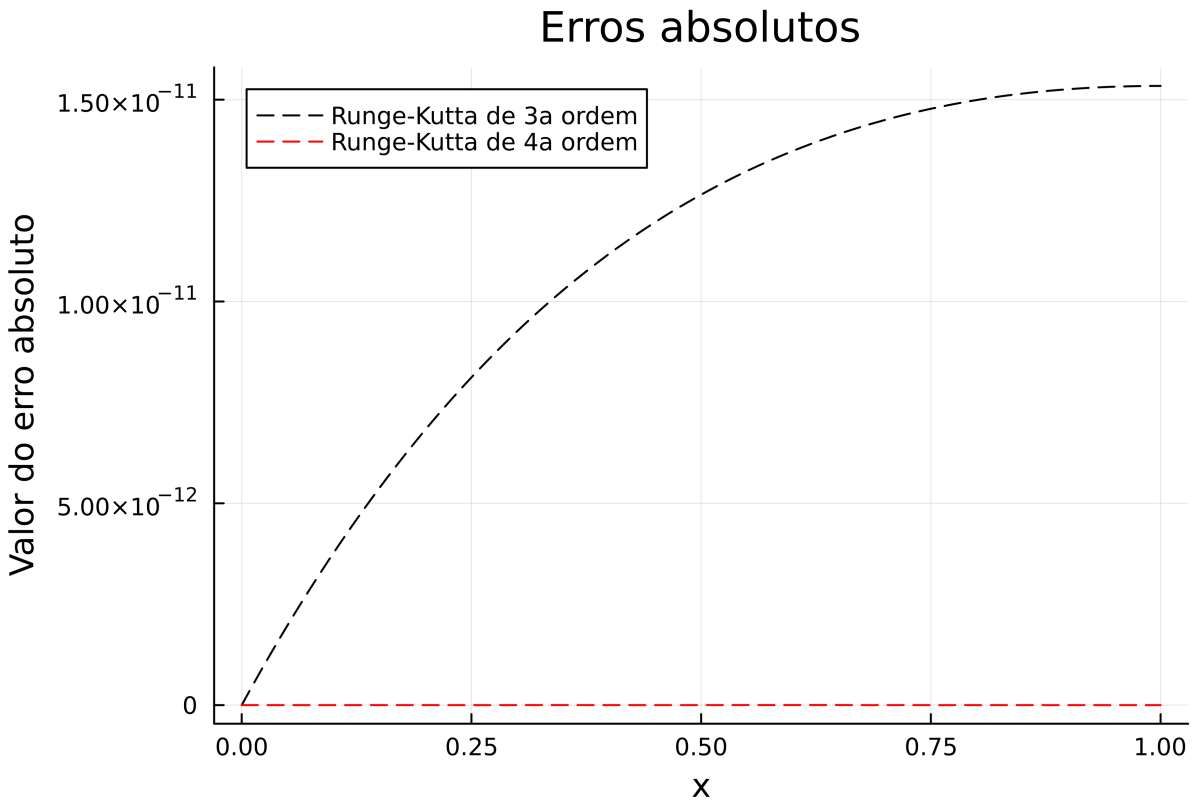

Figura 59: Gráfico plotado para o erro absoluto do item a quando $h = 0.001$

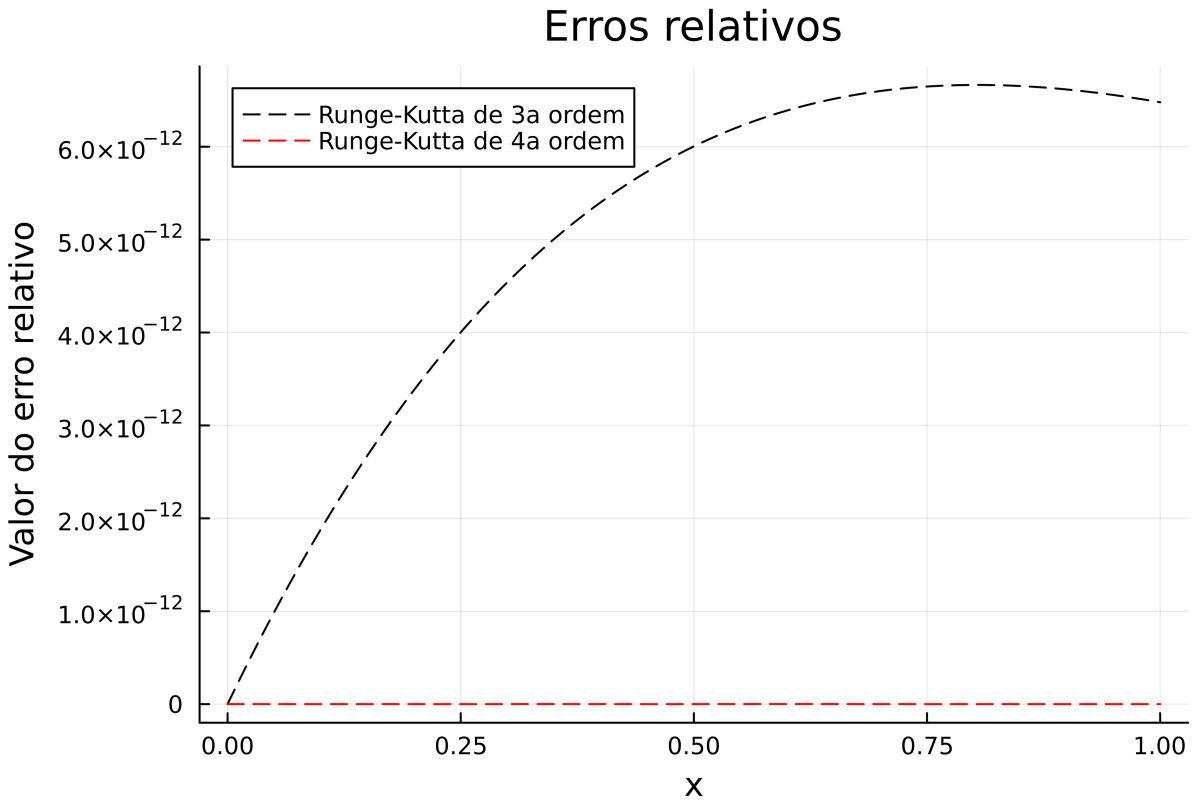

Figura 60: Gráfico plotado para o erro relativo do item a quando $h = 0.001$

2.6 Exercício 2, item b

2.6.1 $h = 0.1$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 41: Valores obtidos para o item b com $h = 0.1$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.88e-08	7.79e-08	5.26e-09	1.06e-08
2.00e-01	2.78e-07	5.66e-07	2.04e-08	4.16e-08
3.00e-01	8.03e-07	1.68e-06	4.20e-08	8.78e-08
4.00e-01	1.57e-06	3.40e-06	6.47e-08	1.40e-07
5.00e-01	2.44e-06	5.49e-06	8.26e-08	1.86e-07
6.00e-01	3.21e-06	7.57e-06	9.14e-08	2.16e-07
7.00e-01	3.71e-06	9.23e-06	8.89e-08	2.21e-07
8.00e-01	3.84e-06	1.01e-05	7.56e-08	2.00e-07
9.00e-01	3.59e-06	1.01e-05	5.38e-08	1.51e-07
1.00e+00	3.02e-06	9.06e-06	2.68e-08	8.04e-08

Tabela 42: Erros obtidos para o item b com $h = 0.1$

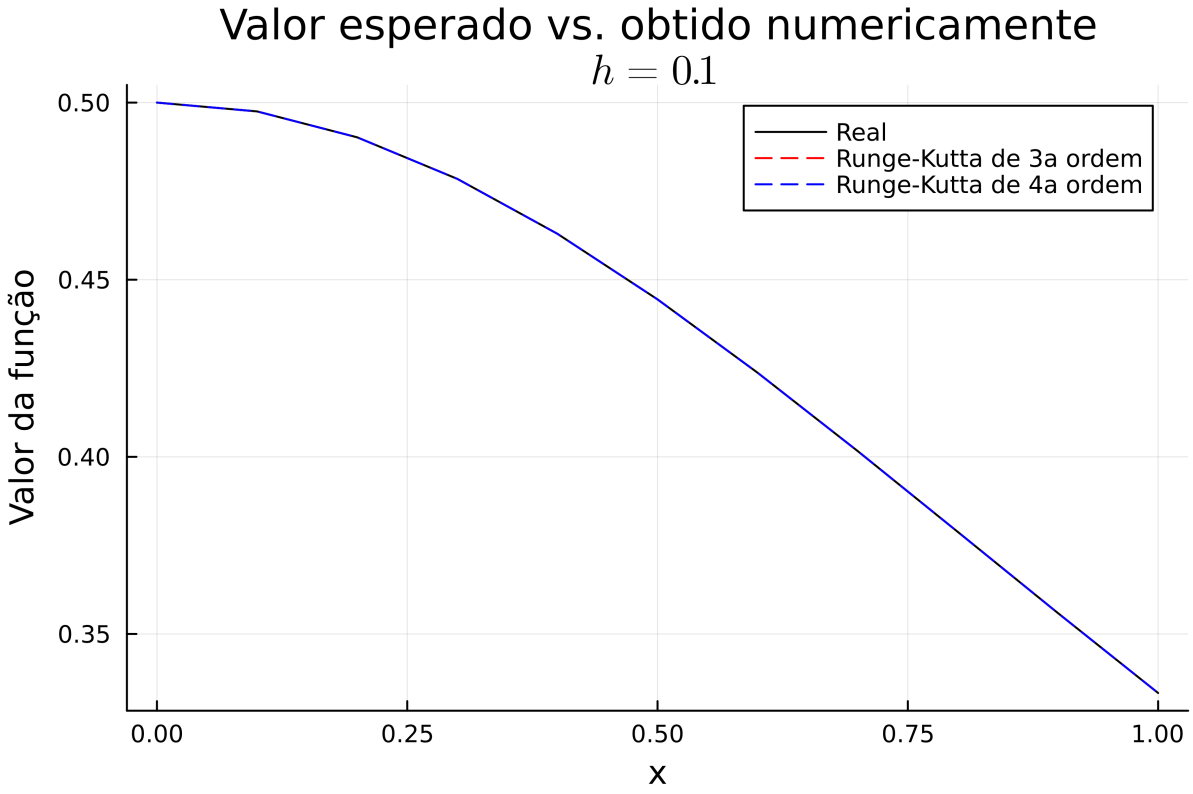

Figura 61: Gráfico plotado para o item b quando $h = 0.1$

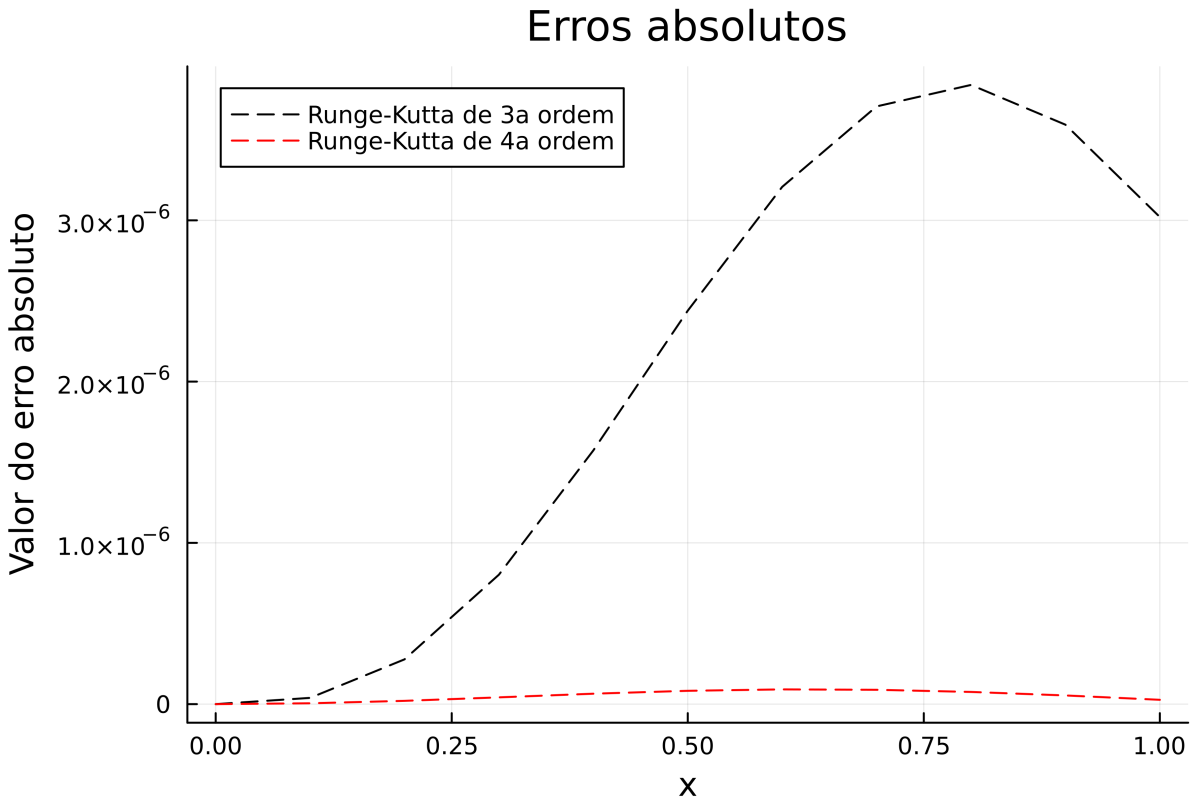

Figura 62: Gráfico plotado para o erro absoluto do item b quando $h = 0.1$

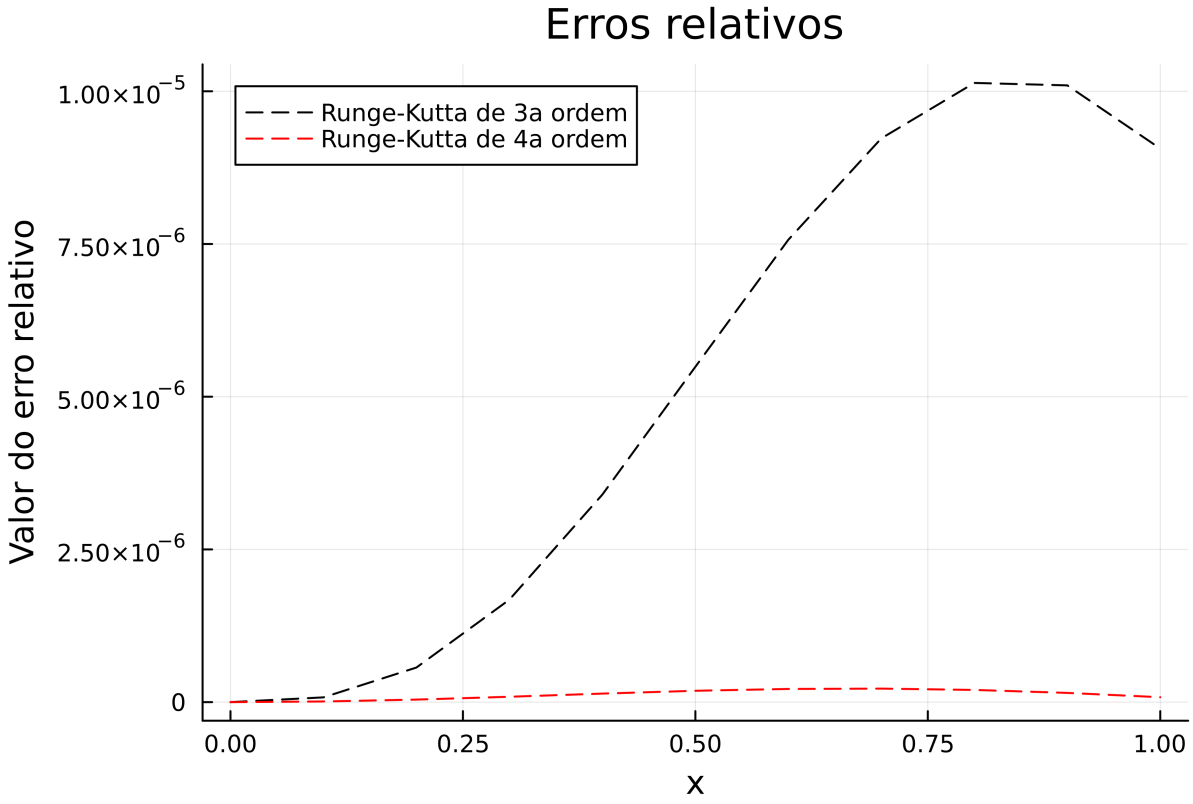

Figura 63: Gráfico plotado para o erro relativo do item b quando $h = 0.1$

2.6.2 $h = 0.01$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 43: Valores obtidos para o item b com $h = 0.01$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.19e-11	6.42e-11	5.10e-13	1.03e-12
2.00e-01	2.31e-10	4.72e-10	1.90e-12	3.87e-12
3.00e-01	6.80e-10	1.42e-09	3.78e-12	7.90e-12
4.00e-01	1.34e-09	2.90e-09	5.63e-12	1.22e-11
5.00e-01	2.08e-09	4.68e-09	6.95e-12	1.56e-11
6.00e-01	2.72e-09	6.41e-09	7.40e-12	1.75e-11
7.00e-01	3.11e-09	7.75e-09	6.86e-12	1.71e-11
8.00e-01	3.18e-09	8.40e-09	5.41e-12	1.43e-11
9.00e-01	2.92e-09	8.20e-09	3.28e-12	9.22e-12
1.00e+00	2.38e-09	7.13e-09	7.61e-13	2.28e-12

Tabela 44: Erros obtidos para o item b com $h = 0.01$

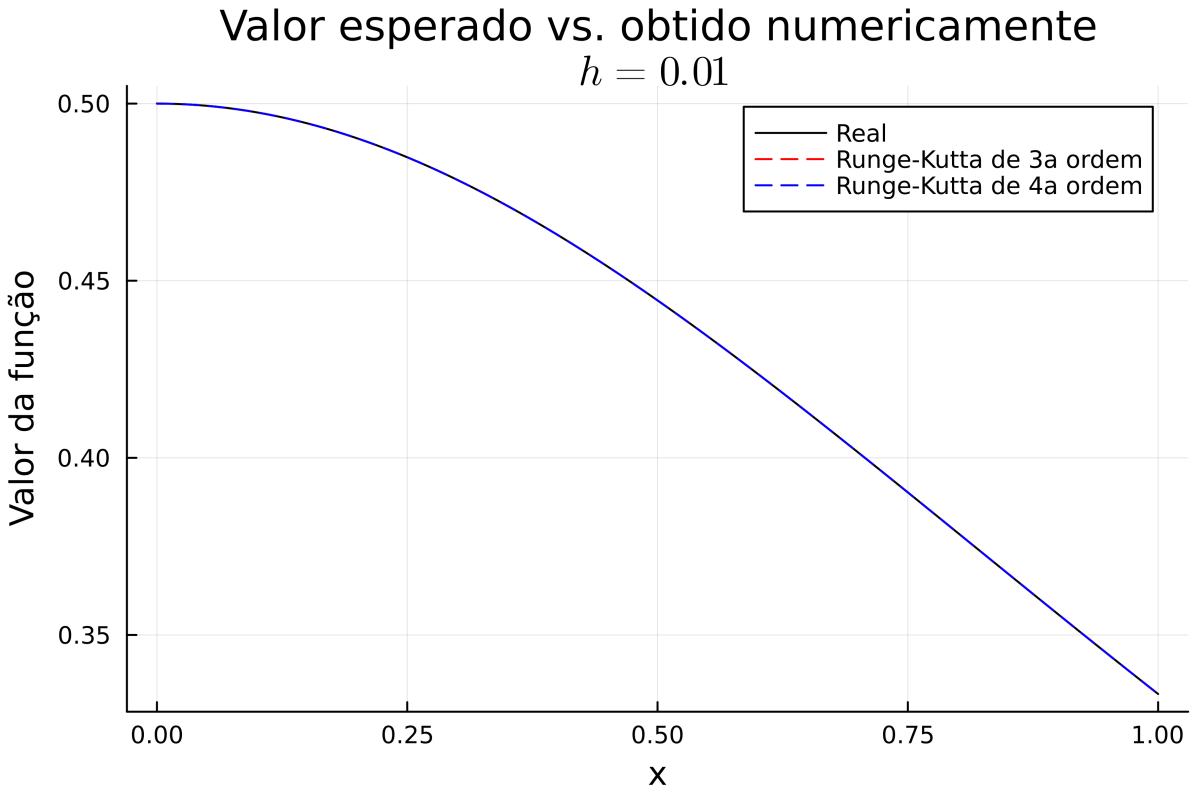

Figura 64: Gráfico plotado para o item b quando $h = 0.01$

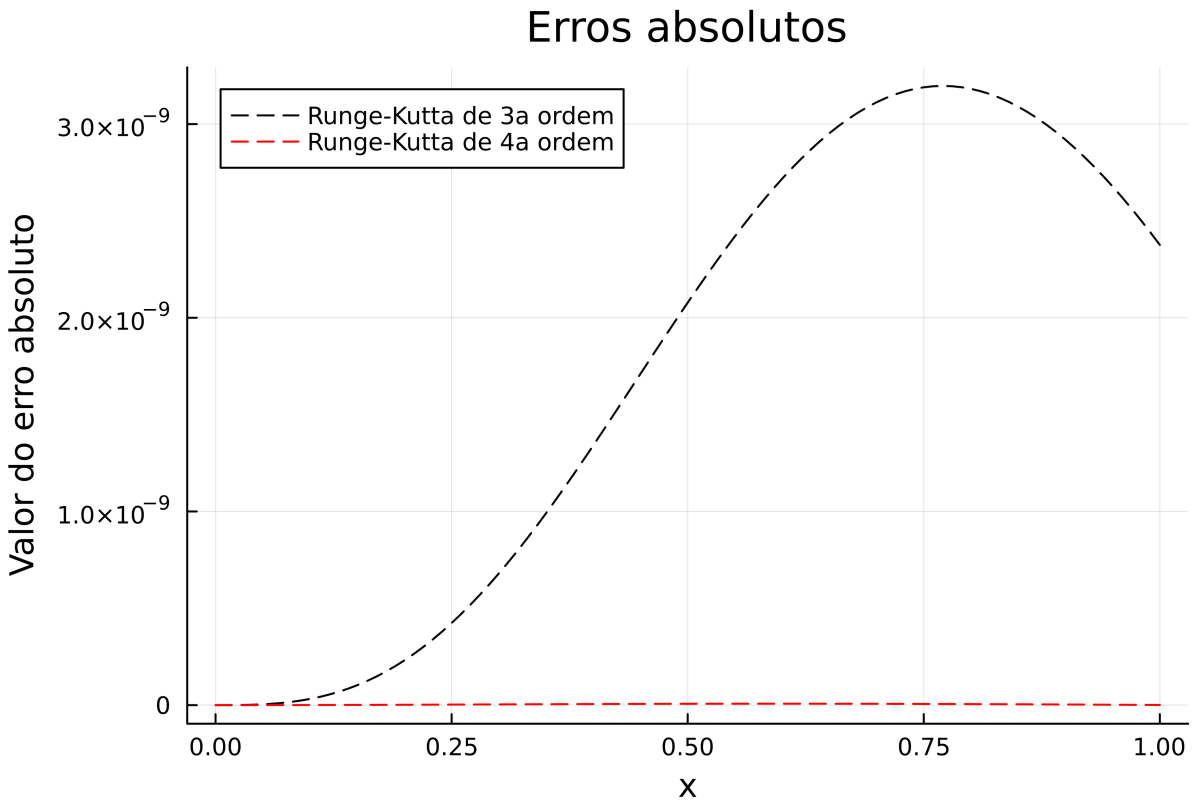

Figura 65: Gráfico plotado para o erro absoluto do item b quando $h = 0.01$

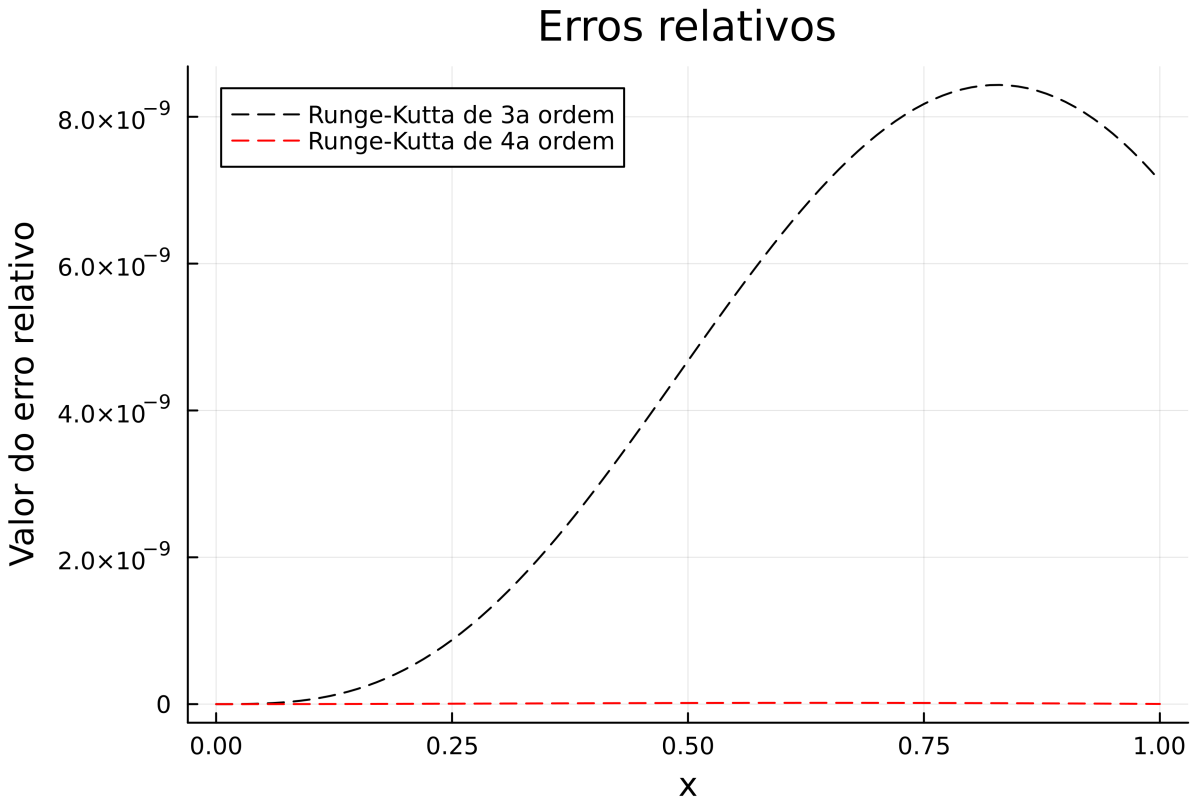

Figura 66: Gráfico plotado para o erro relativo do item b quando $h = 0.01$

2.6.3 $h = 0.005$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 45: Valores obtidos para o item b com $h = 0.005$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.90e-12	7.84e-12	3.18e-14	6.39e-14
2.00e-01	2.85e-11	5.81e-11	1.18e-13	2.41e-13
3.00e-01	8.41e-11	1.76e-10	2.34e-13	4.90e-13
4.00e-01	1.66e-10	3.59e-10	3.48e-13	7.53e-13
5.00e-01	2.57e-10	5.79e-10	4.30e-13	9.67e-13
6.00e-01	3.36e-10	7.94e-10	4.57e-13	1.08e-12
7.00e-01	3.85e-10	9.59e-10	4.22e-13	1.05e-12
8.00e-01	3.94e-10	1.04e-09	3.31e-13	8.74e-13
9.00e-01	3.61e-10	1.01e-09	1.98e-13	5.57e-13
1.00e+00	2.93e-10	8.79e-10	4.15e-14	1.24e-13

Tabela 46: Erros obtidos para o item b com $h = 0.005$

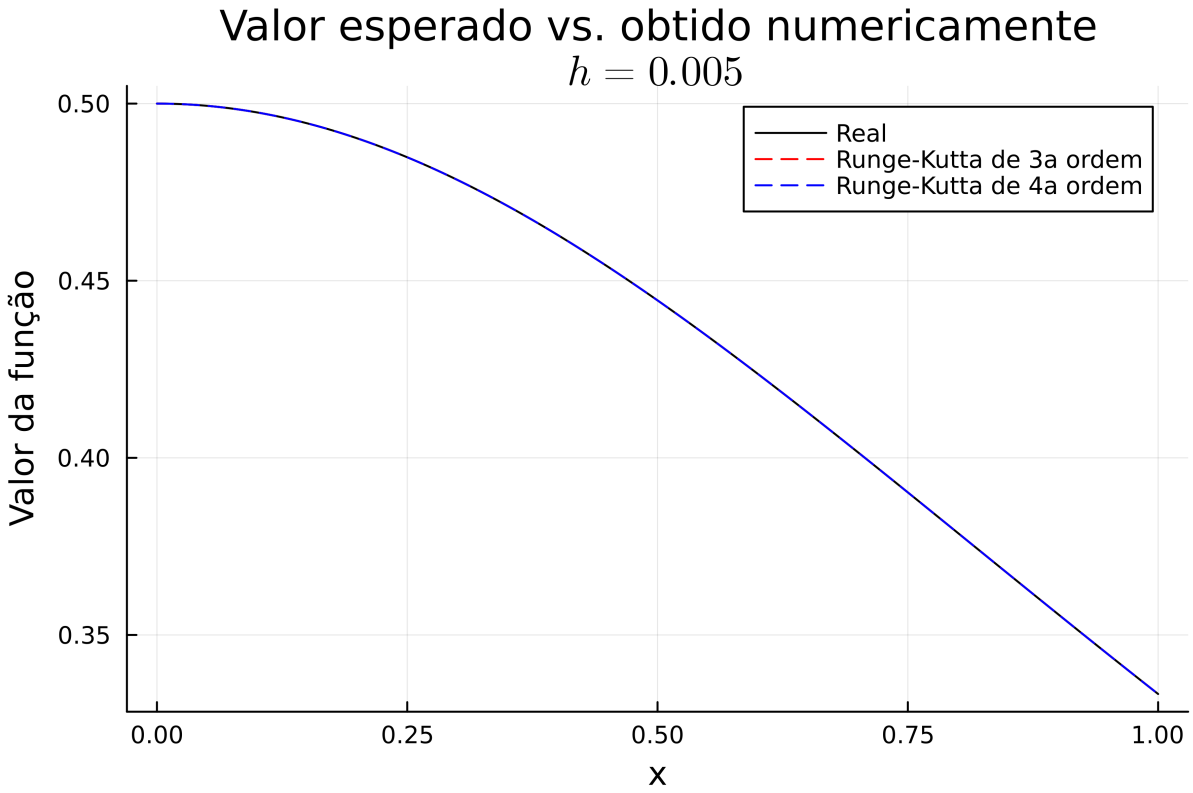

Figura 67: Gráfico plotado para o item b quando $h = 0.005$

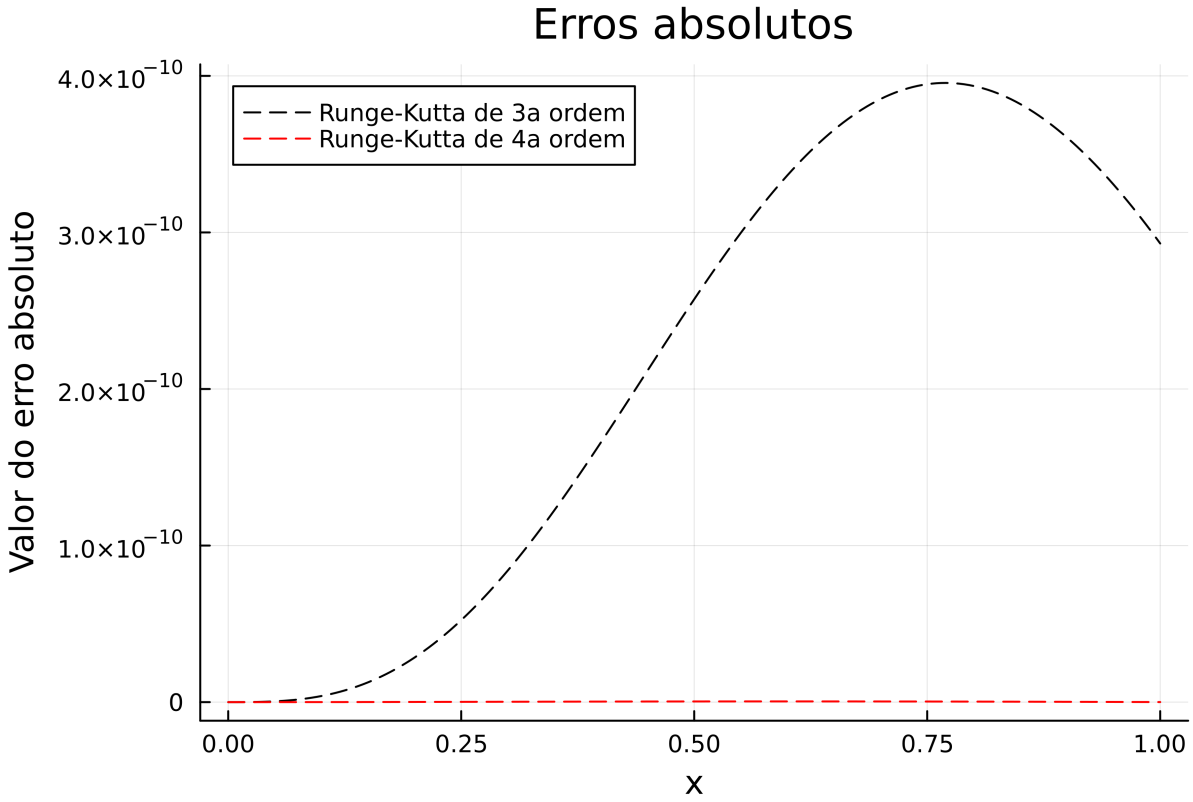

Figura 68: Gráfico plotado para o erro absoluto do item b quando $h = 0.005$

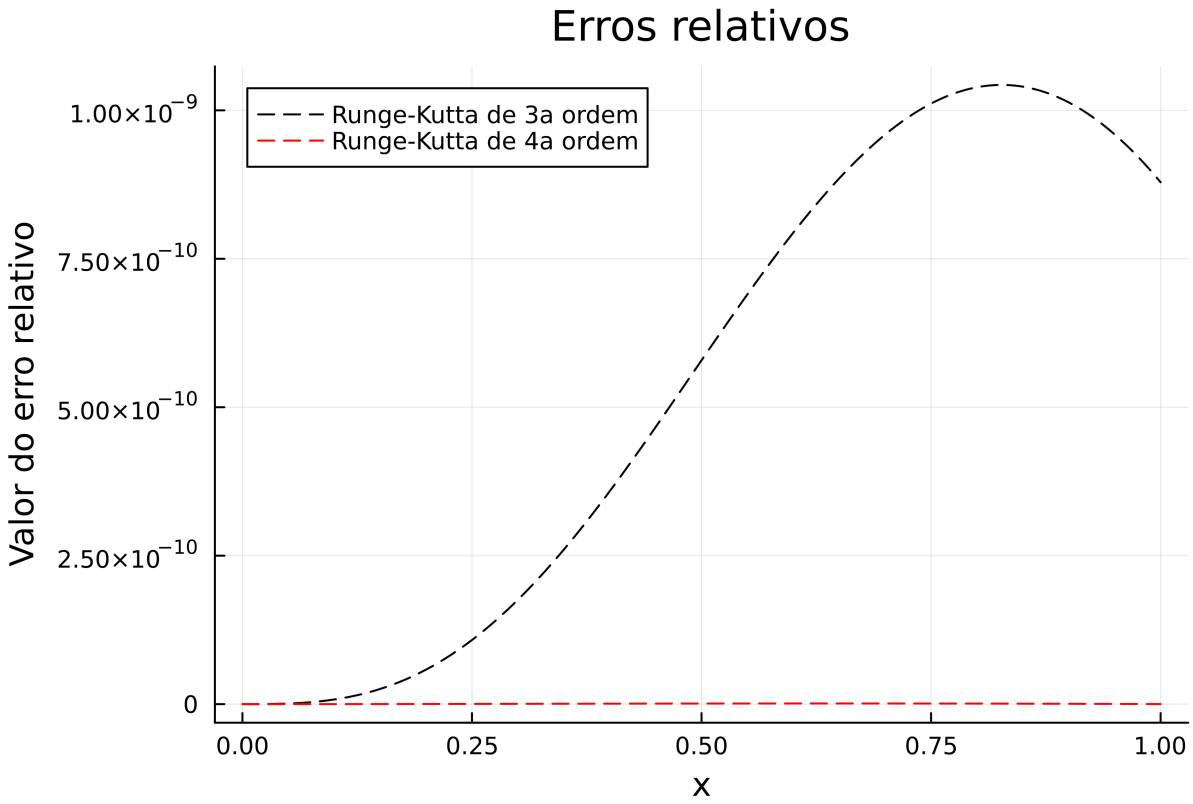

Figura 69: Gráfico plotado para o erro relativo do item b quando $h = 0.005$

2.6.4 $h = 0.001$

x	Valor real	Valor (Runge-Kutta de 3a ordem)	Valor (Runge-Kutta de 4a ordem)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 47: Valores obtidos para o item b com $h = 0.001$

x	Erro absoluto (Runge-Kutta de 3a ordem)	Erro relativo (Runge-Kutta de 3a ordem)	Erro absoluto (Runge-Kutta de 4a ordem)	Erro relativo (Runge-Kutta de 4a ordem)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.08e-14	6.19e-14	5.55e-16	1.12e-15
2.00e-01	2.26e-13	4.61e-13	5.55e-16	1.13e-15
3.00e-01	6.67e-13	1.40e-12	5.55e-16	1.16e-15
4.00e-01	1.32e-12	2.85e-12	5.55e-16	1.20e-15
5.00e-01	2.04e-12	4.60e-12	7.22e-16	1.62e-15
6.00e-01	2.67e-12	6.31e-12	9.44e-16	2.23e-15
7.00e-01	3.06e-12	7.61e-12	6.11e-16	1.52e-15
8.00e-01	3.12e-12	8.24e-12	5.00e-16	1.32e-15
9.00e-01	2.86e-12	8.03e-12	1.11e-16	3.12e-16
1.00e+00	2.32e-12	6.95e-12	0.00e+00	0.00e+00

Tabela 48: Erros obtidos para o item b com $h = 0.001$

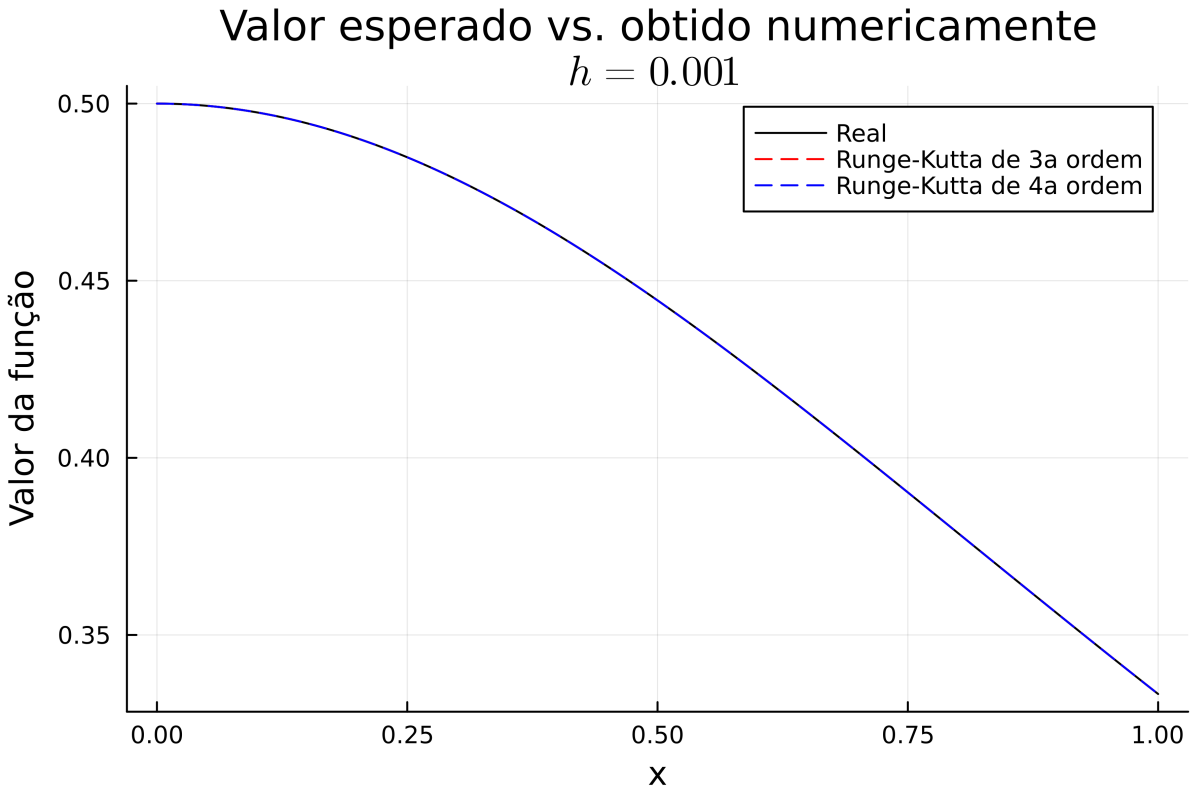

Figura 70: Gráfico plotado para o item b quando $h = 0.001$

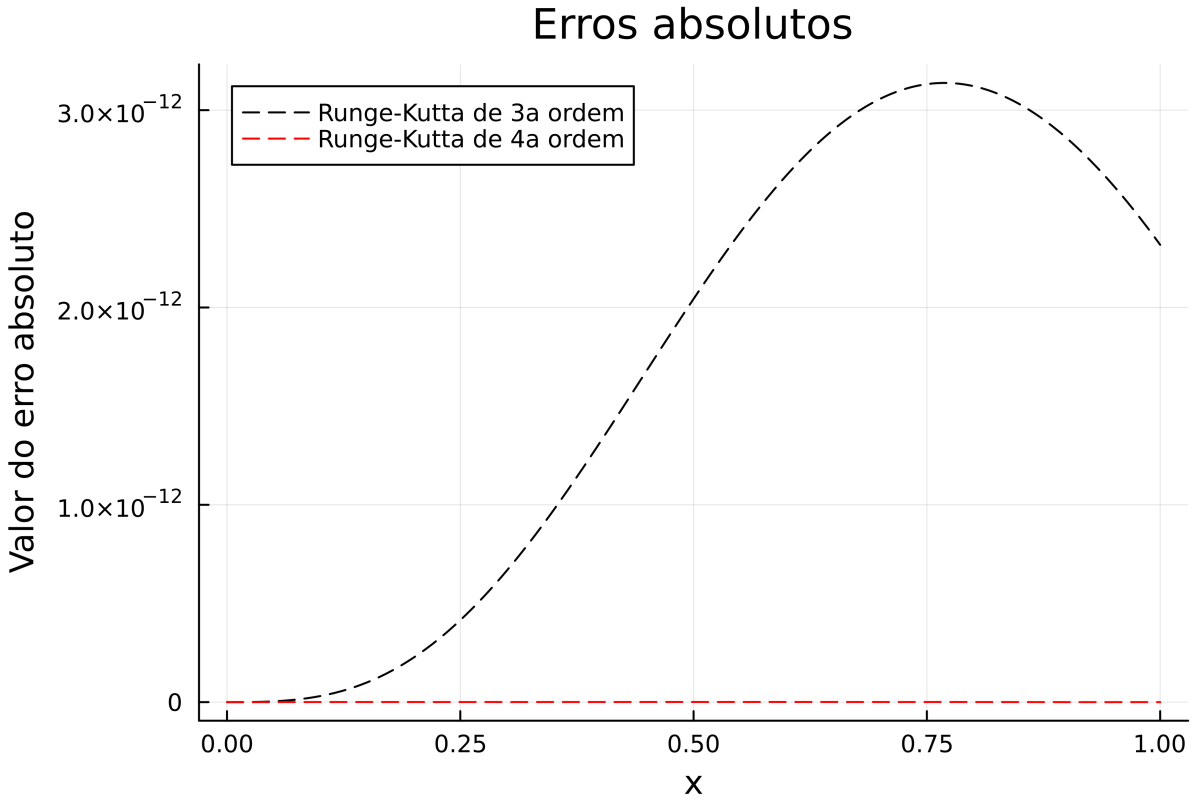

Figura 71: Gráfico plotado para o erro absoluto do item b quando $h = 0.001$

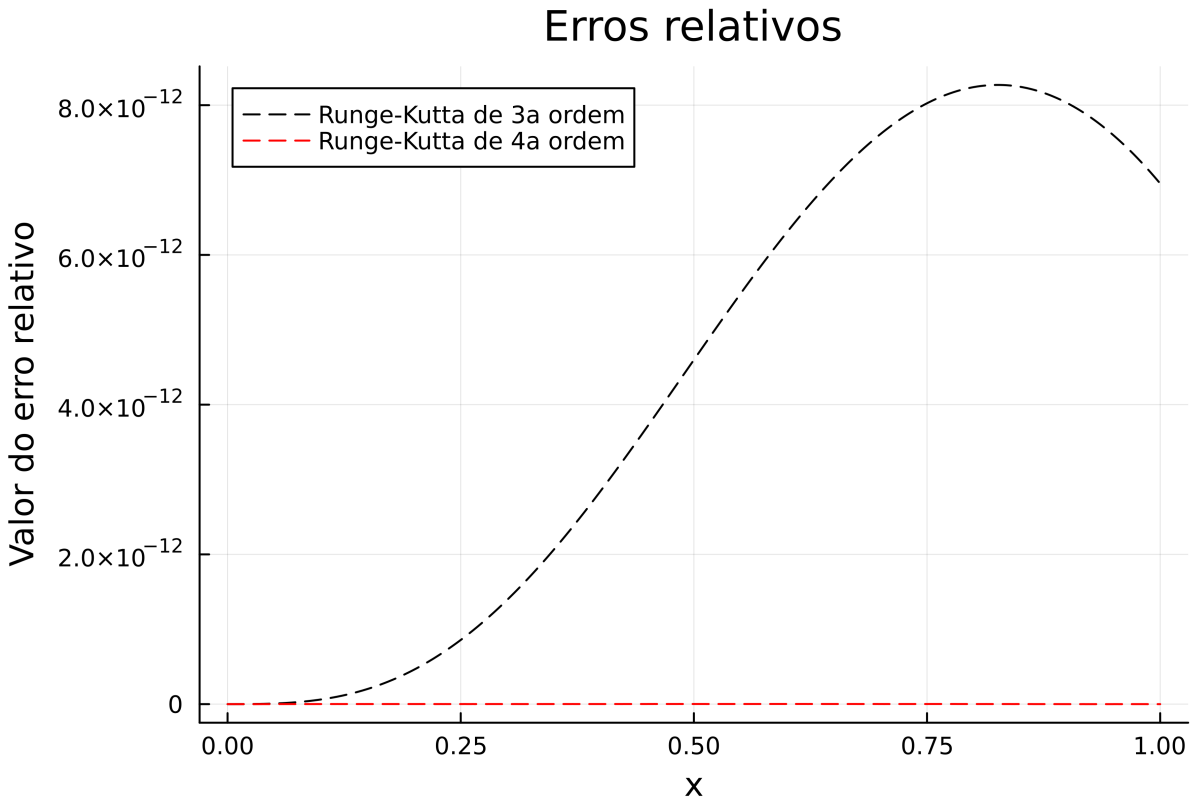

Figura 72: Gráfico plotado para o erro relativo do item b quando $h = 0.001$

2.7 Exercício 3, item a

2.7.1 $h = 0.1$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0050	2.0050
0.2000	2.0187	2.0190	2.0190
0.3000	2.0408	2.0412	2.0412
0.4000	2.0703	2.0708	2.0708
0.5000	2.1065	2.1071	2.1071
0.6000	2.1488	2.1494	2.1494
0.7000	2.1966	2.1972	2.1972
0.8000	2.2493	2.2500	2.2500
0.9000	2.3066	2.3072	2.3072
1.0000	2.3679	2.3685	2.3685

Tabela 49: Valores obtidos para o item a com $h = 0.1$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.63e-04	8.11e-05	1.63e-04	8.11e-05
2.00e-01	2.94e-04	1.46e-04	2.94e-04	1.46e-04
3.00e-01	3.99e-04	1.96e-04	3.99e-04	1.96e-04
4.00e-01	4.82e-04	2.33e-04	4.82e-04	2.33e-04
5.00e-01	5.45e-04	2.59e-04	5.45e-04	2.59e-04
6.00e-01	5.92e-04	2.75e-04	5.92e-04	2.75e-04
7.00e-01	6.25e-04	2.84e-04	6.25e-04	2.84e-04
8.00e-01	6.46e-04	2.87e-04	6.46e-04	2.87e-04
9.00e-01	6.58e-04	2.85e-04	6.58e-04	2.85e-04
1.00e+00	6.62e-04	2.79e-04	6.62e-04	2.79e-04

Tabela 50: Erros obtidos para o item a com $h = 0.1$

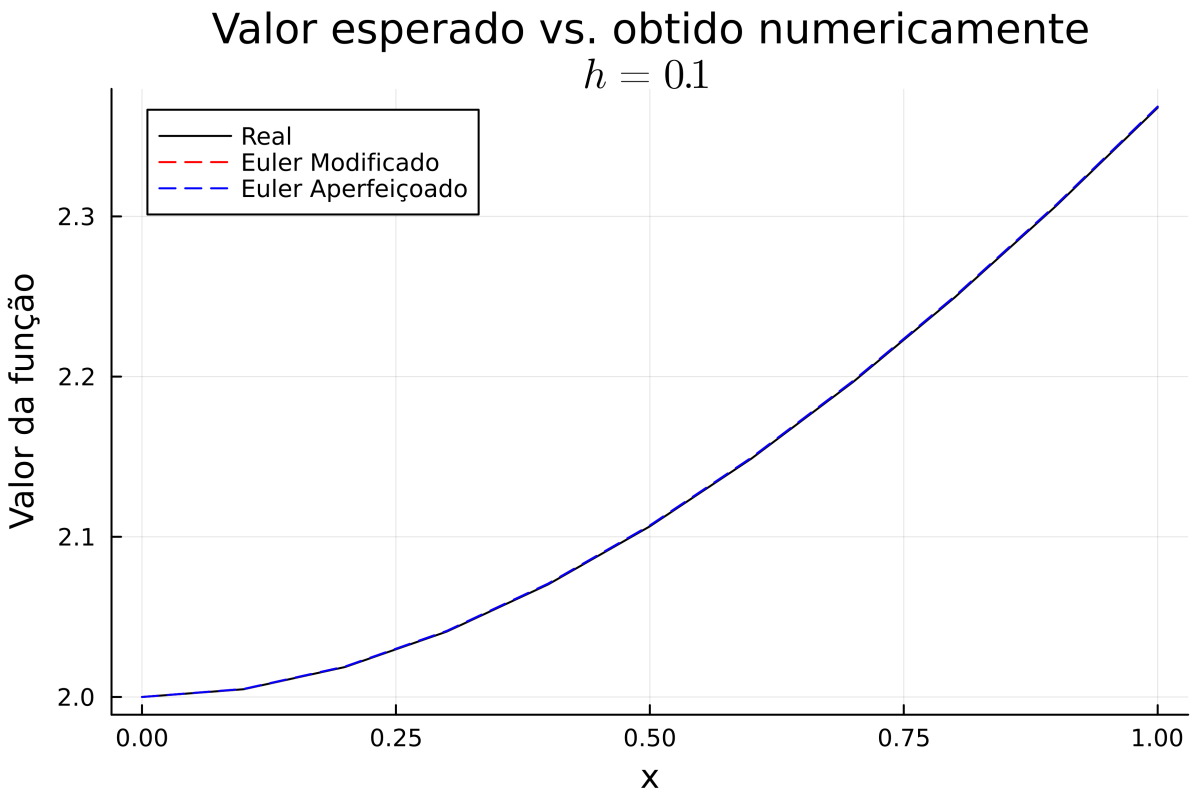

Figura 73: Gráfico plotado para o item a quando $h = 0.1$

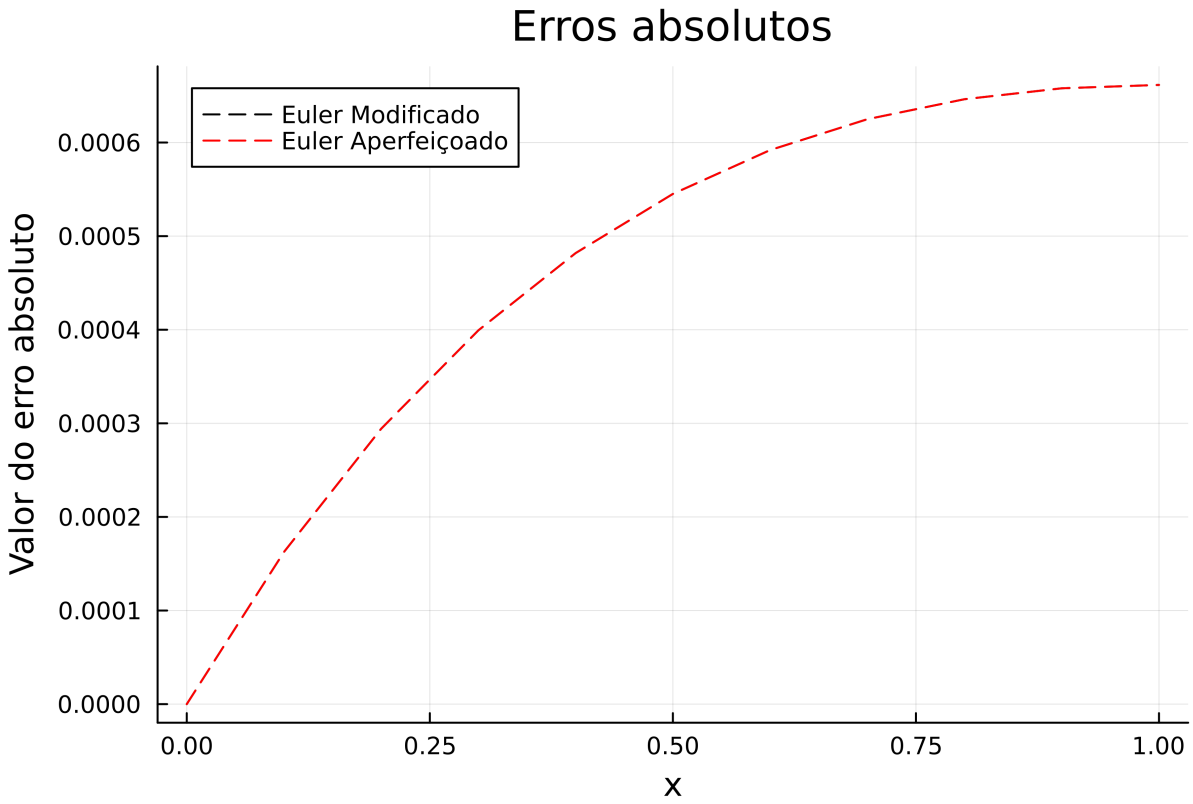

Figura 74: Gráfico plotado para o erro absoluto do item a quando $h = 0.1$

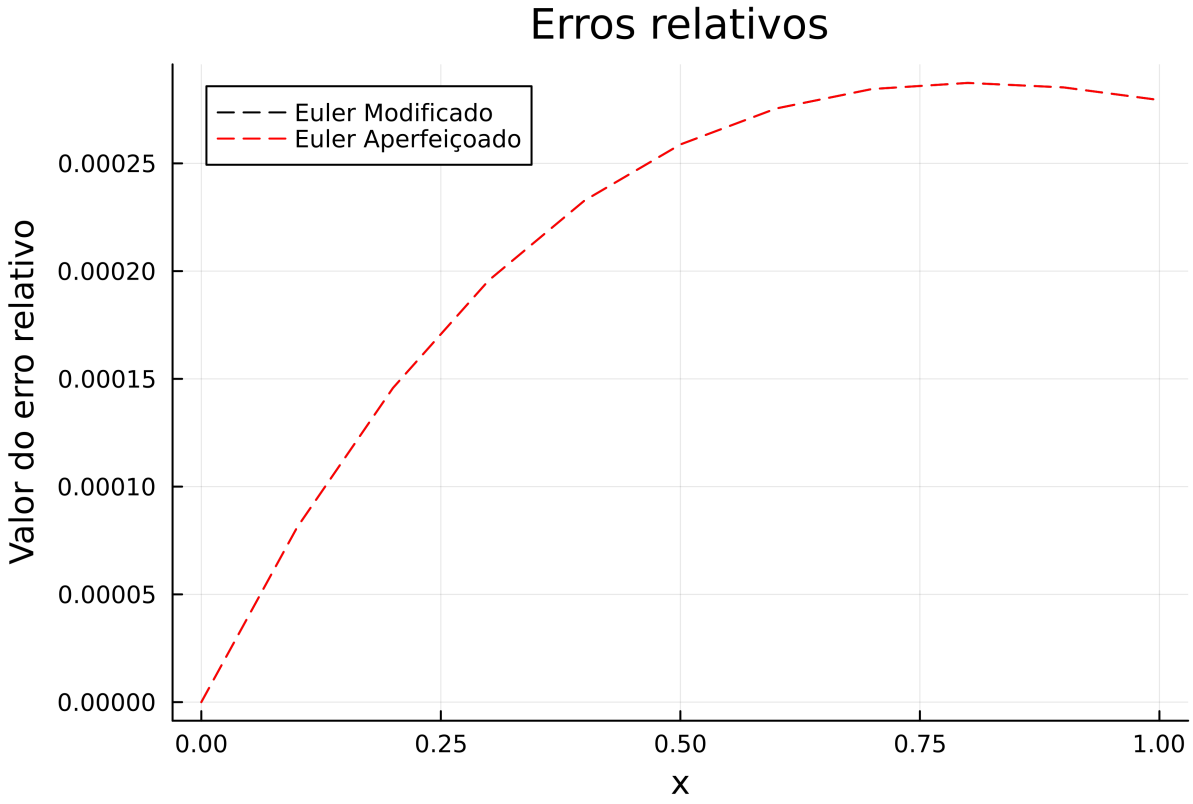

Figura 75: Gráfico plotado para o erro relativo do item a quando $h = 0.1$

2.7.2 $h = 0.01$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 51: Valores obtidos para o item a com $h = 0.01$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.52e-06	7.58e-07	1.52e-06	7.58e-07
2.00e-01	2.75e-06	1.36e-06	2.75e-06	1.36e-06
3.00e-01	3.73e-06	1.83e-06	3.73e-06	1.83e-06
4.00e-01	4.50e-06	2.17e-06	4.50e-06	2.17e-06
5.00e-01	5.09e-06	2.42e-06	5.09e-06	2.42e-06
6.00e-01	5.53e-06	2.57e-06	5.53e-06	2.57e-06
7.00e-01	5.84e-06	2.66e-06	5.84e-06	2.66e-06
8.00e-01	6.04e-06	2.68e-06	6.04e-06	2.68e-06
9.00e-01	6.14e-06	2.66e-06	6.14e-06	2.66e-06
1.00e+00	6.18e-06	2.61e-06	6.18e-06	2.61e-06

Tabela 52: Erros obtidos para o item a com $h = 0.01$

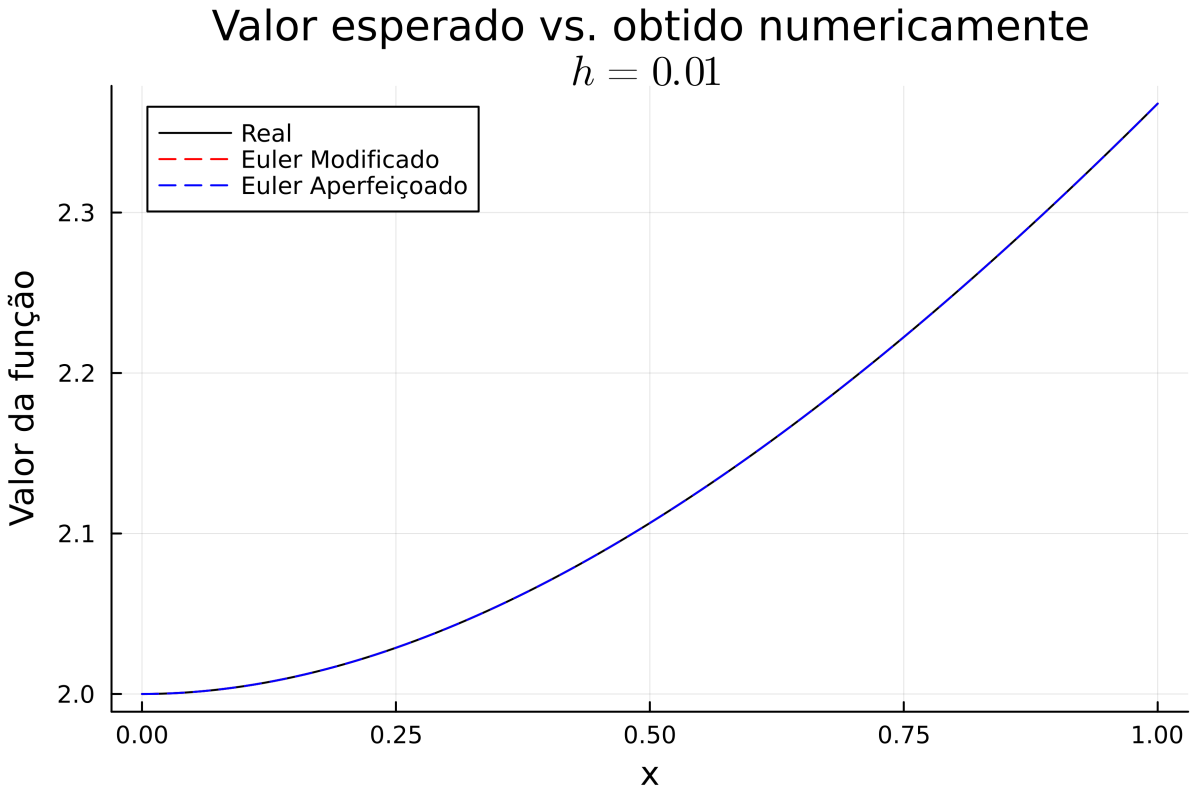

Figura 76: Gráfico plotado para o item a quando $h = 0.01$

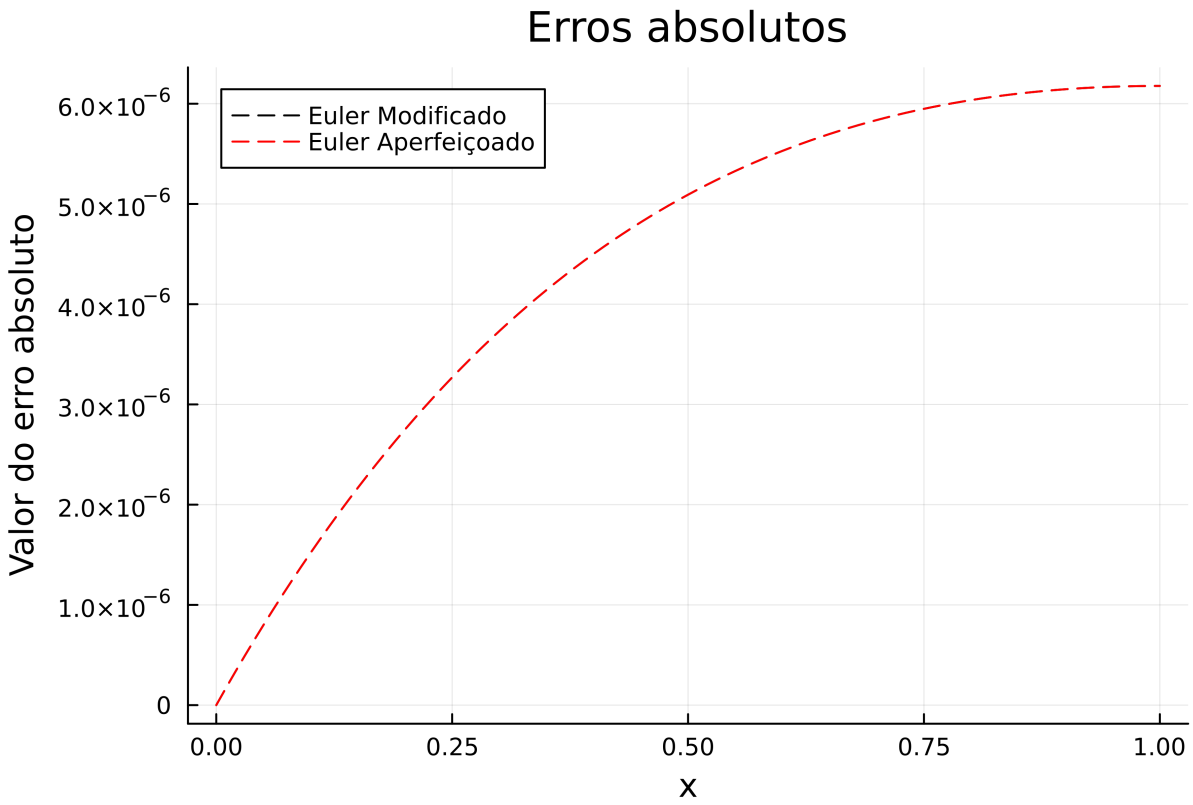

Figura 77: Gráfico plotado para o erro absoluto do item a quando $h = 0.01$

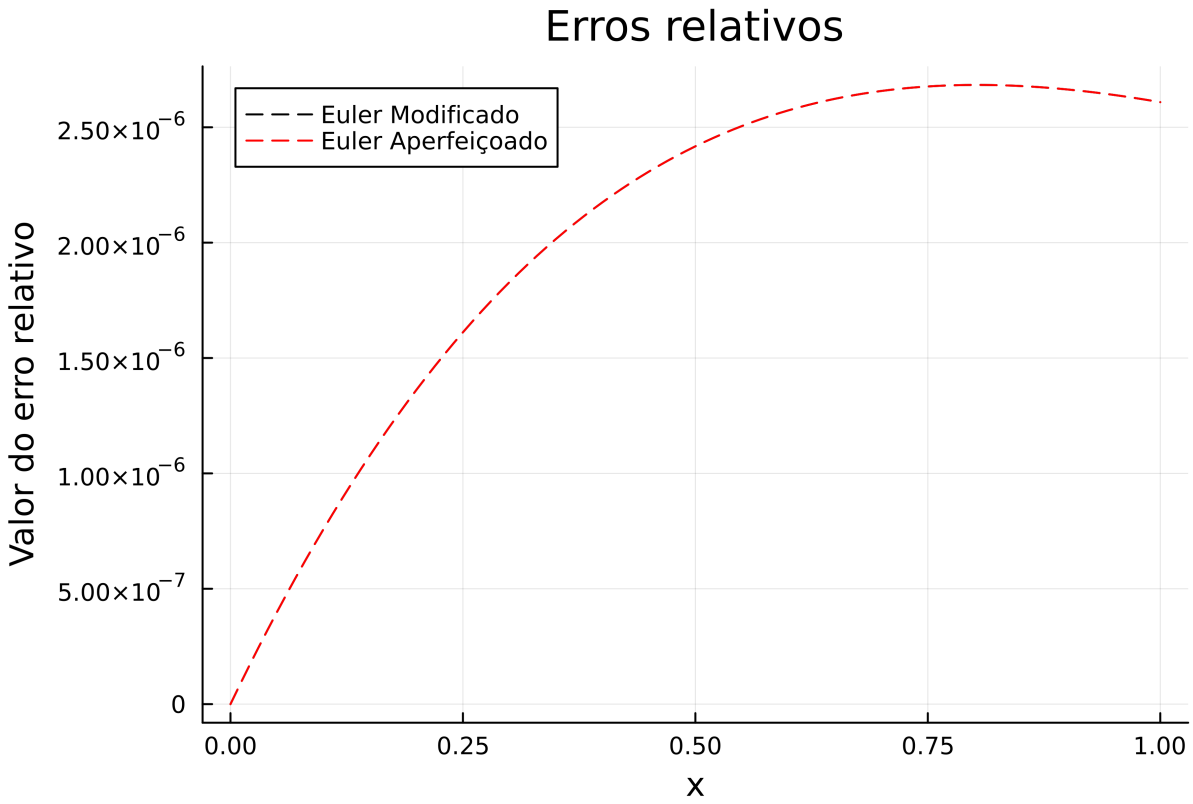

Figura 78: Gráfico plotado para o erro relativo do item a quando $h = 0.01$

2.7.3 $h = 0.005$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 53: Valores obtidos para o item a com $h = 0.005$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.78e-07	1.89e-07	3.78e-07	1.89e-07
2.00e-01	6.85e-07	3.39e-07	6.85e-07	3.39e-07
3.00e-01	9.30e-07	4.55e-07	9.30e-07	4.55e-07
4.00e-01	1.12e-06	5.42e-07	1.12e-06	5.42e-07
5.00e-01	1.27e-06	6.02e-07	1.27e-06	6.02e-07
6.00e-01	1.38e-06	6.41e-07	1.38e-06	6.41e-07
7.00e-01	1.45e-06	6.62e-07	1.45e-06	6.62e-07
8.00e-01	1.50e-06	6.68e-07	1.50e-06	6.68e-07
9.00e-01	1.53e-06	6.63e-07	1.53e-06	6.63e-07
1.00e+00	1.54e-06	6.50e-07	1.54e-06	6.50e-07

Tabela 54: Erros obtidos para o item a com $h = 0.005$

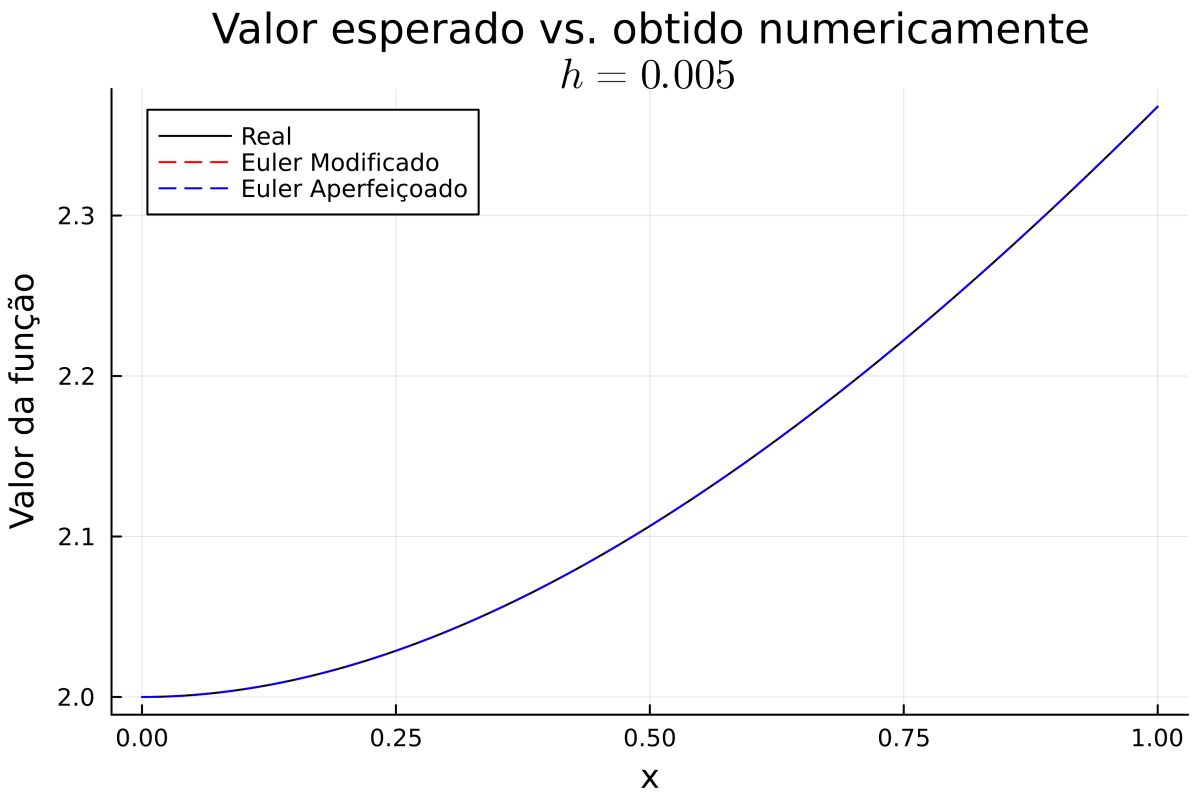

Figura 79: Gráfico plotado para o item a quando $h = 0.005$

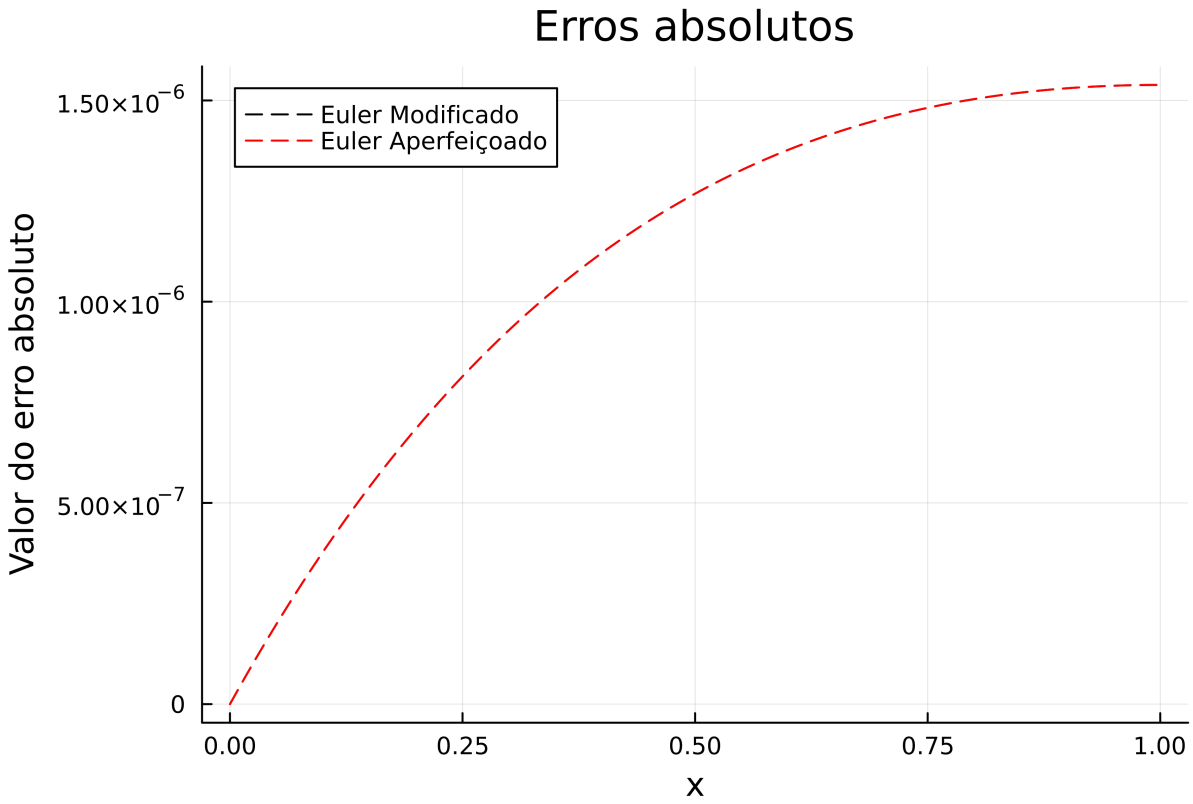

Figura 80: Gráfico plotado para o erro absoluto do item a quando $h = 0.005$

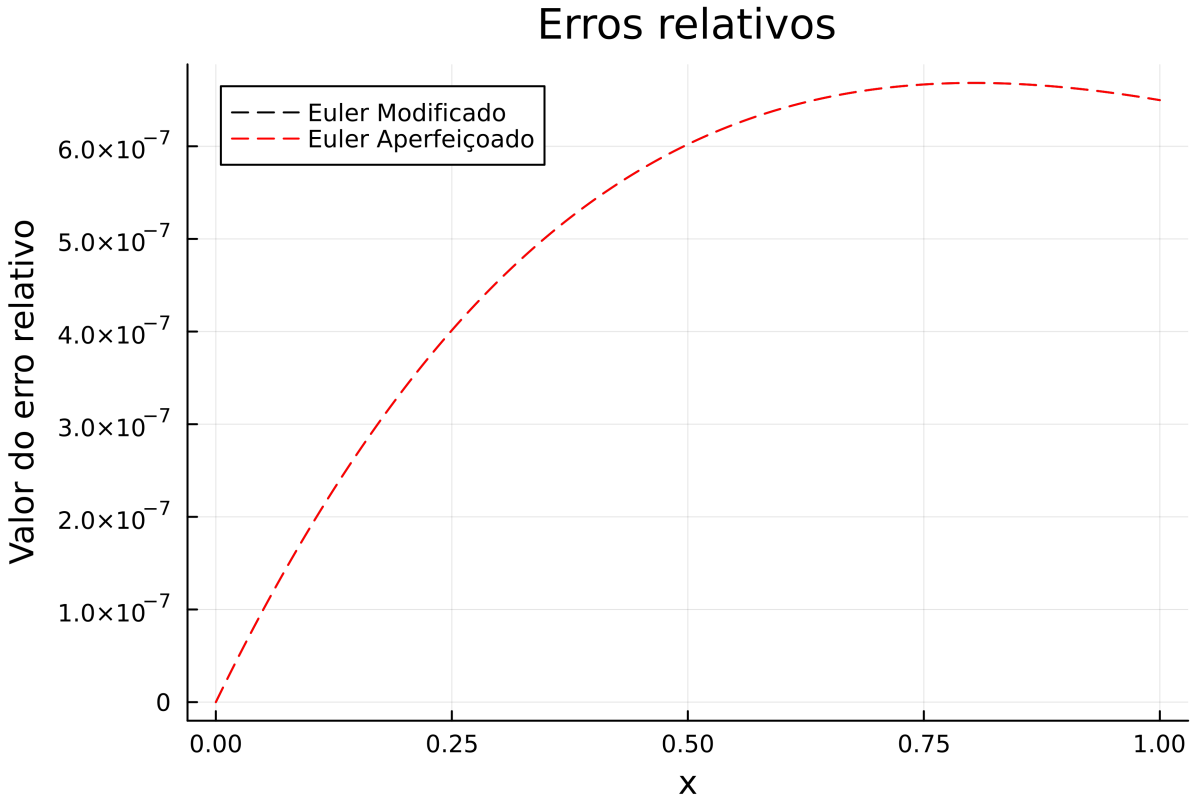

Figura 81: Gráfico plotado para o erro relativo do item a quando $h = 0.005$

2.7.4 $h = 0.001$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	2.0000	2.0000	2.0000
0.1000	2.0048	2.0048	2.0048
0.2000	2.0187	2.0187	2.0187
0.3000	2.0408	2.0408	2.0408
0.4000	2.0703	2.0703	2.0703
0.5000	2.1065	2.1065	2.1065
0.6000	2.1488	2.1488	2.1488
0.7000	2.1966	2.1966	2.1966
0.8000	2.2493	2.2493	2.2493
0.9000	2.3066	2.3066	2.3066
1.0000	2.3679	2.3679	2.3679

Tabela 55: Valores obtidos para o item a com $h = 0.001$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.51e-08	7.53e-09	1.51e-08	7.53e-09
2.00e-01	2.73e-08	1.35e-08	2.73e-08	1.35e-08
3.00e-01	3.71e-08	1.82e-08	3.71e-08	1.82e-08
4.00e-01	4.47e-08	2.16e-08	4.47e-08	2.16e-08
5.00e-01	5.06e-08	2.40e-08	5.06e-08	2.40e-08
6.00e-01	5.49e-08	2.56e-08	5.49e-08	2.56e-08
7.00e-01	5.80e-08	2.64e-08	5.80e-08	2.64e-08
8.00e-01	6.00e-08	2.67e-08	6.00e-08	2.67e-08
9.00e-01	6.10e-08	2.65e-08	6.10e-08	2.65e-08
1.00e+00	6.14e-08	2.59e-08	6.14e-08	2.59e-08

Tabela 56: Erros obtidos para o item a com $h = 0.001$

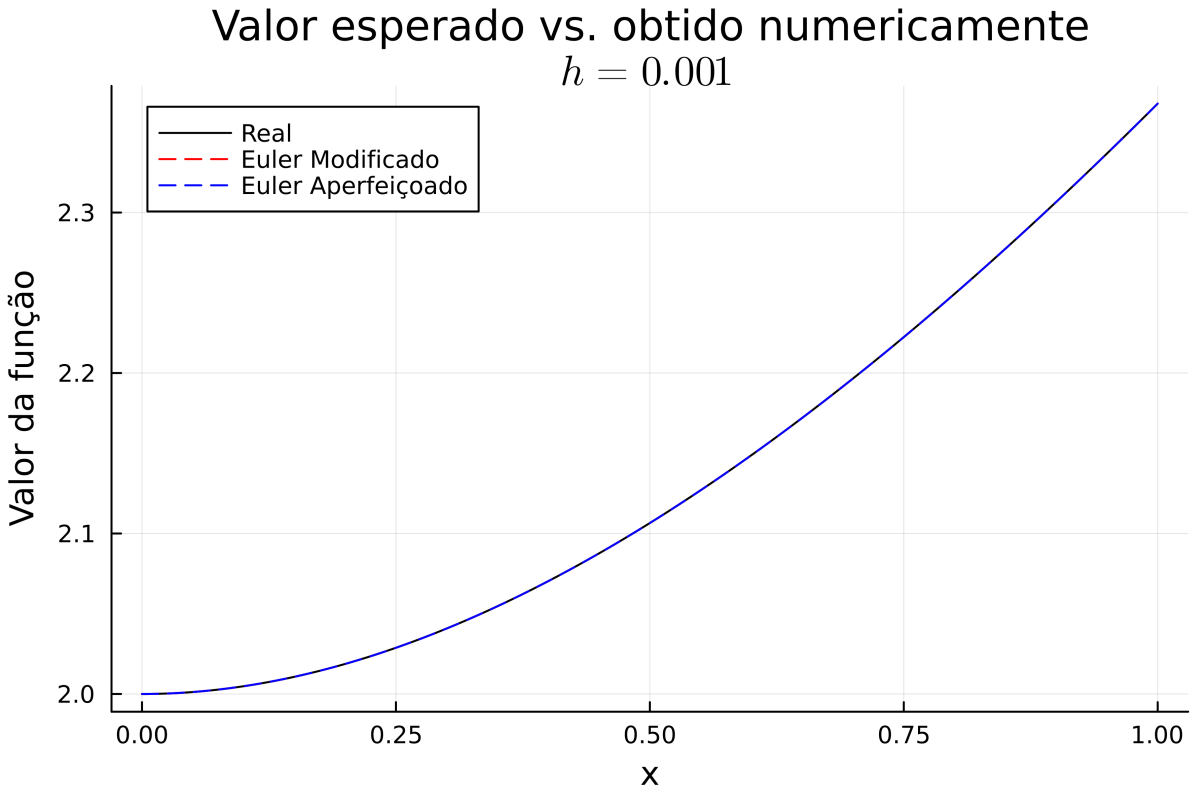

Figura 82: Gráfico plotado para o item a quando $h = 0.001$

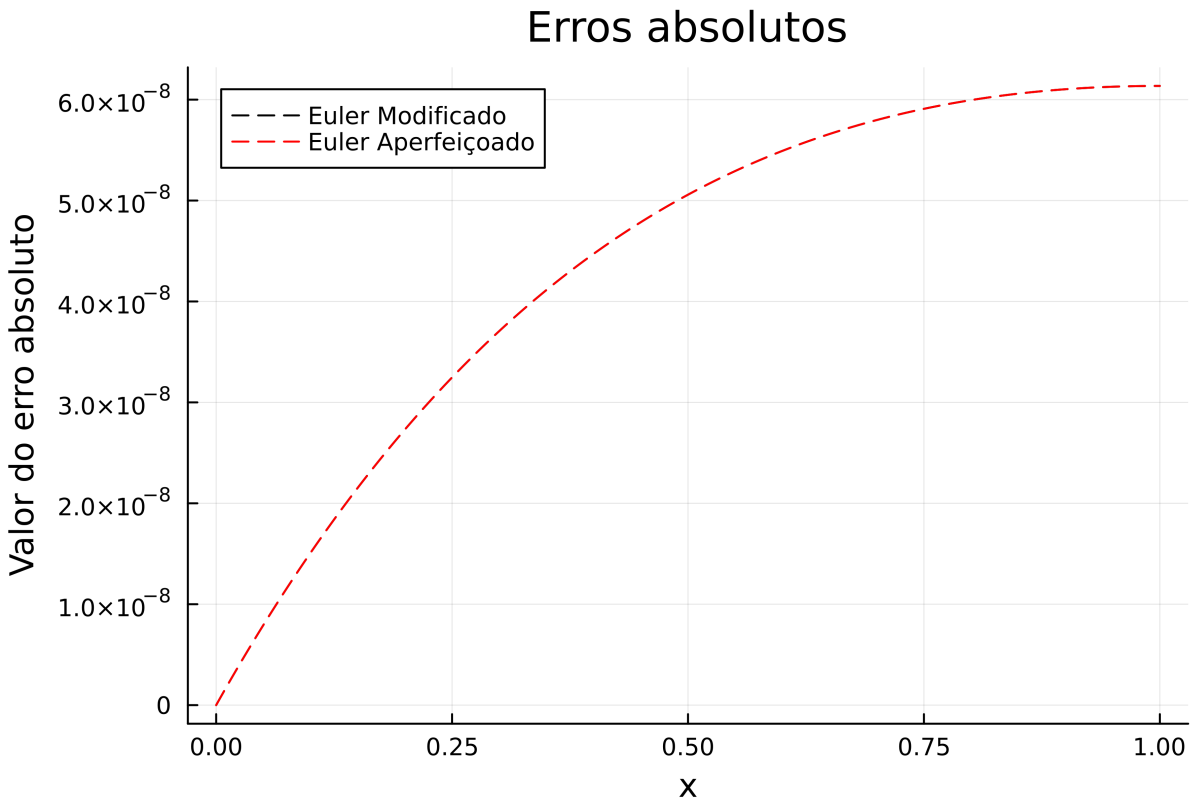

Figura 83: Gráfico plotado para o erro absoluto do item a quando $h = 0.001$

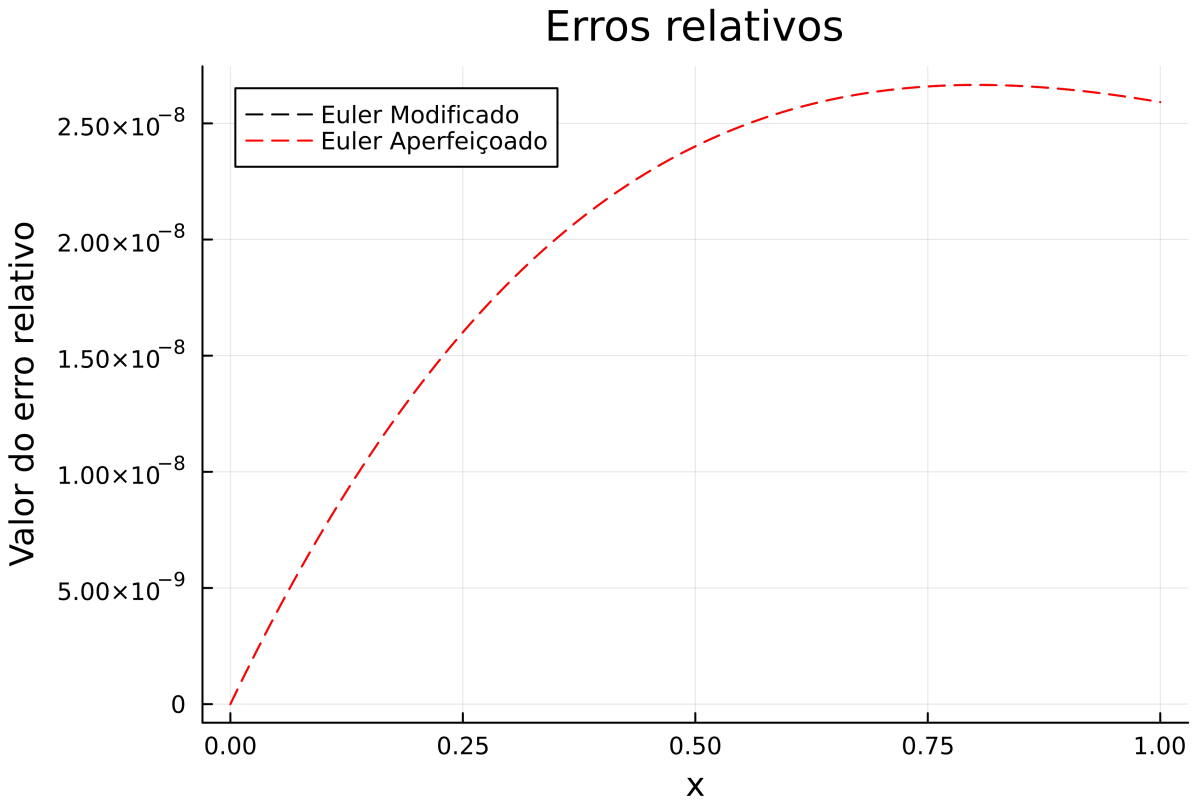

Figura 84: Gráfico plotado para o erro relativo do item a quando $h = 0.001$

2.8 Exercício 3, item b

2.8.1 $h = 0.1$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4901	0.4902
0.3000	0.4785	0.4784	0.4784
0.4000	0.4630	0.4628	0.4629
0.5000	0.4444	0.4442	0.4444
0.6000	0.4237	0.4235	0.4237
0.7000	0.4016	0.4013	0.4016
0.8000	0.3788	0.3785	0.3788
0.9000	0.3559	0.3556	0.3560
1.0000	0.3333	0.3331	0.3335

Tabela 57: Valores obtidos para o item b com $h = 0.1$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.24e-05	2.50e-05	1.24e-05	2.50e-05
2.00e-01	4.76e-05	9.70e-05	2.32e-05	4.74e-05
3.00e-01	9.83e-05	2.05e-04	2.97e-05	6.20e-05
4.00e-01	1.55e-04	3.34e-04	2.89e-05	6.24e-05
5.00e-01	2.06e-04	4.64e-04	1.91e-05	4.29e-05
6.00e-01	2.45e-04	5.79e-04	2.60e-08	6.13e-08
7.00e-01	2.67e-04	6.65e-04	2.70e-05	6.72e-05
8.00e-01	2.71e-04	7.15e-04	5.96e-05	1.57e-04
9.00e-01	2.59e-04	7.28e-04	9.48e-05	2.66e-04
1.00e+00	2.35e-04	7.04e-04	1.30e-04	3.89e-04

Tabela 58: Erros obtidos para o item b com $h = 0.1$

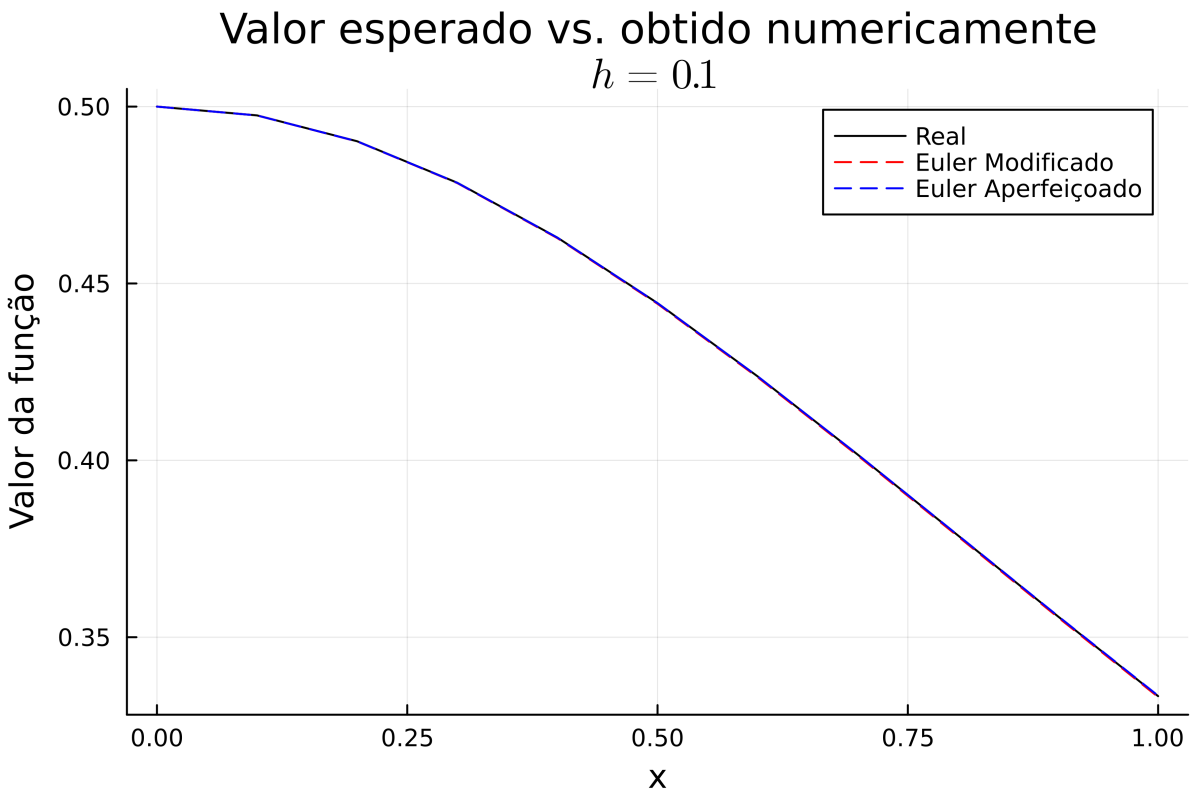

Figura 85: Gráfico plotado para o item b quando $h = 0.1$

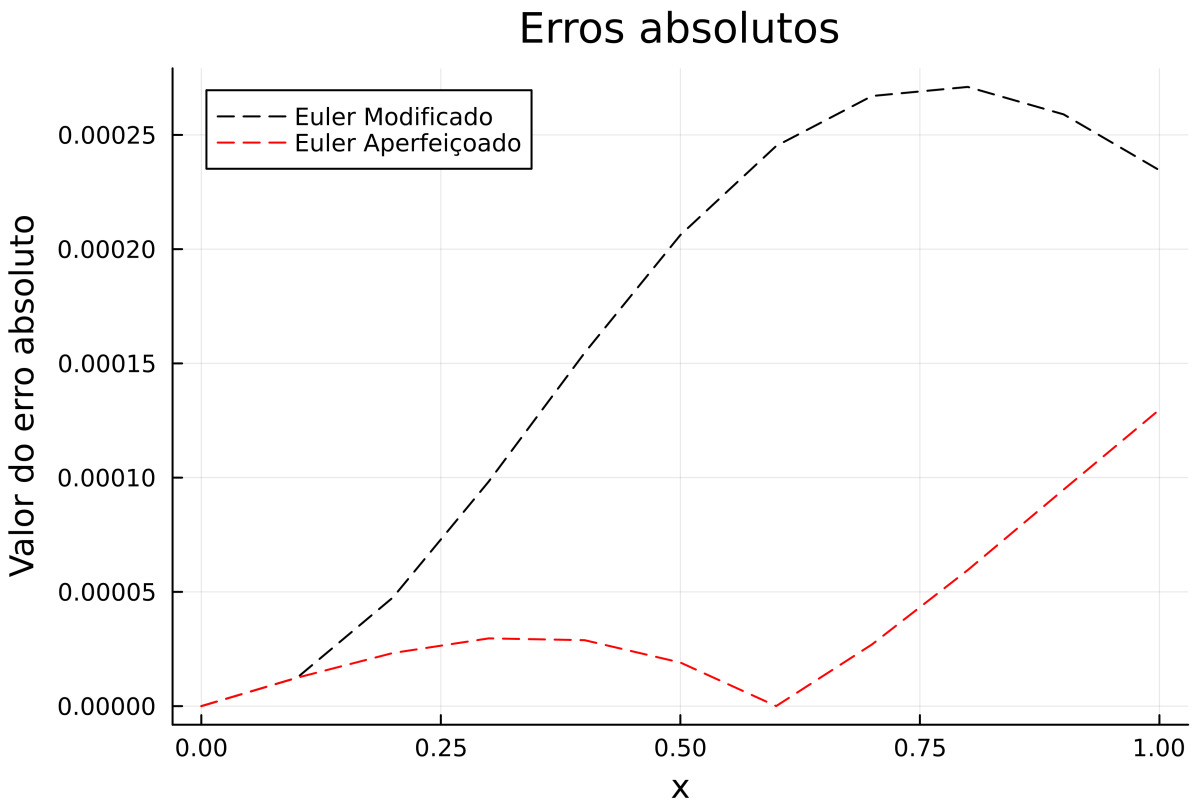

Figura 86: Gráfico plotado para o erro absoluto do item b quando $h = 0.1$

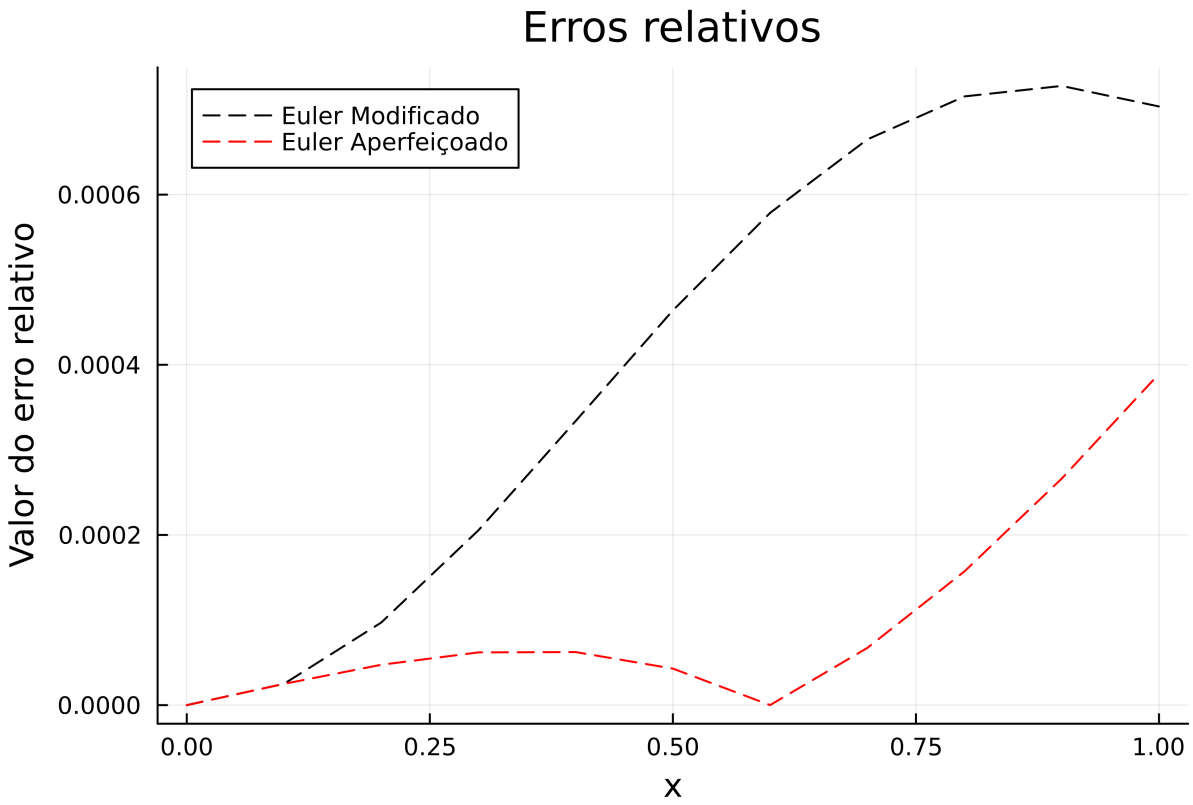

Figura 87: Gráfico plotado para o erro relativo do item b quando $h = 0.1$

2.8.2 $h = 0.01$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 59: Valores obtidos para o item b com $h = 0.01$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.23e-07	2.47e-07	1.17e-08	2.35e-08
2.00e-01	4.63e-07	9.45e-07	1.42e-08	2.90e-08
3.00e-01	9.46e-07	1.98e-06	1.08e-08	2.26e-08
4.00e-01	1.47e-06	3.18e-06	8.43e-08	1.82e-07
5.00e-01	1.94e-06	4.37e-06	2.20e-07	4.96e-07
6.00e-01	2.29e-06	5.40e-06	4.22e-07	9.96e-07
7.00e-01	2.47e-06	6.16e-06	6.79e-07	1.69e-06
8.00e-01	2.49e-06	6.57e-06	9.72e-07	2.57e-06
9.00e-01	2.36e-06	6.64e-06	1.28e-06	3.59e-06
1.00e+00	2.12e-06	6.37e-06	1.57e-06	4.72e-06

Tabela 60: Erros obtidos para o item b com $h = 0.01$

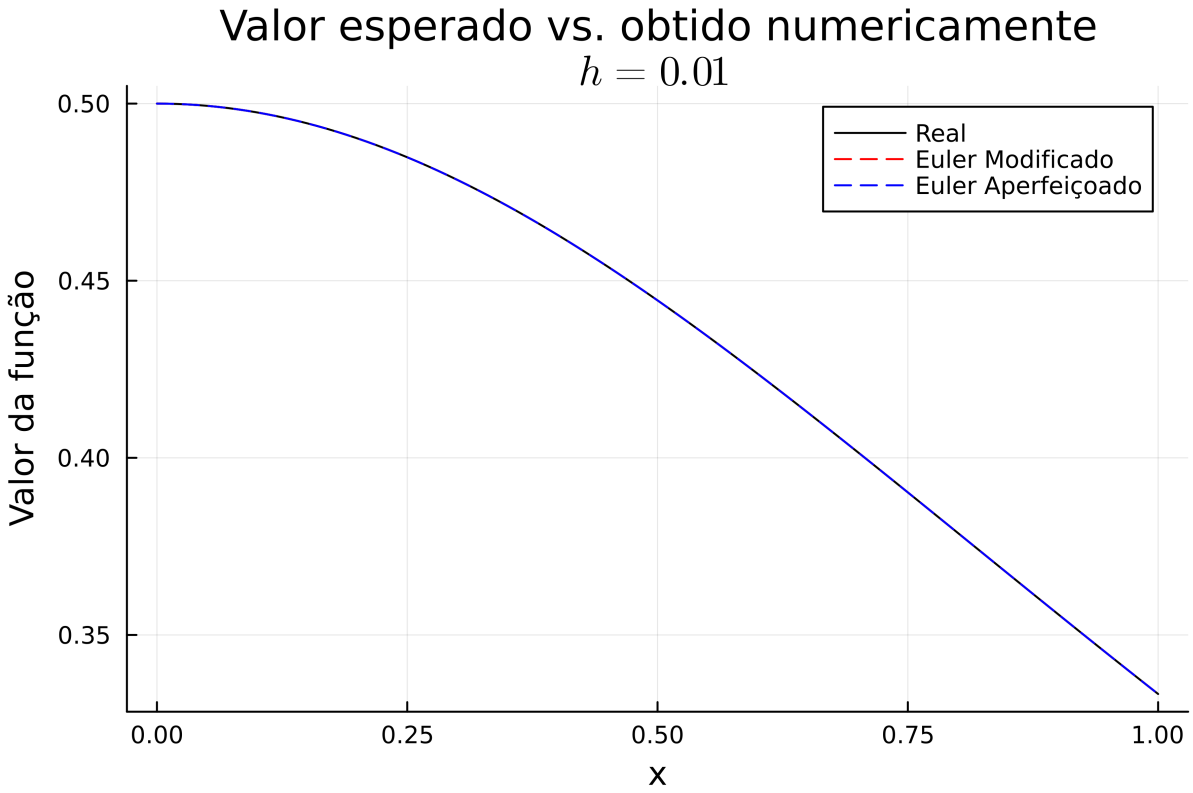

Figura 88: Gráfico plotado para o item b quando $h = 0.01$

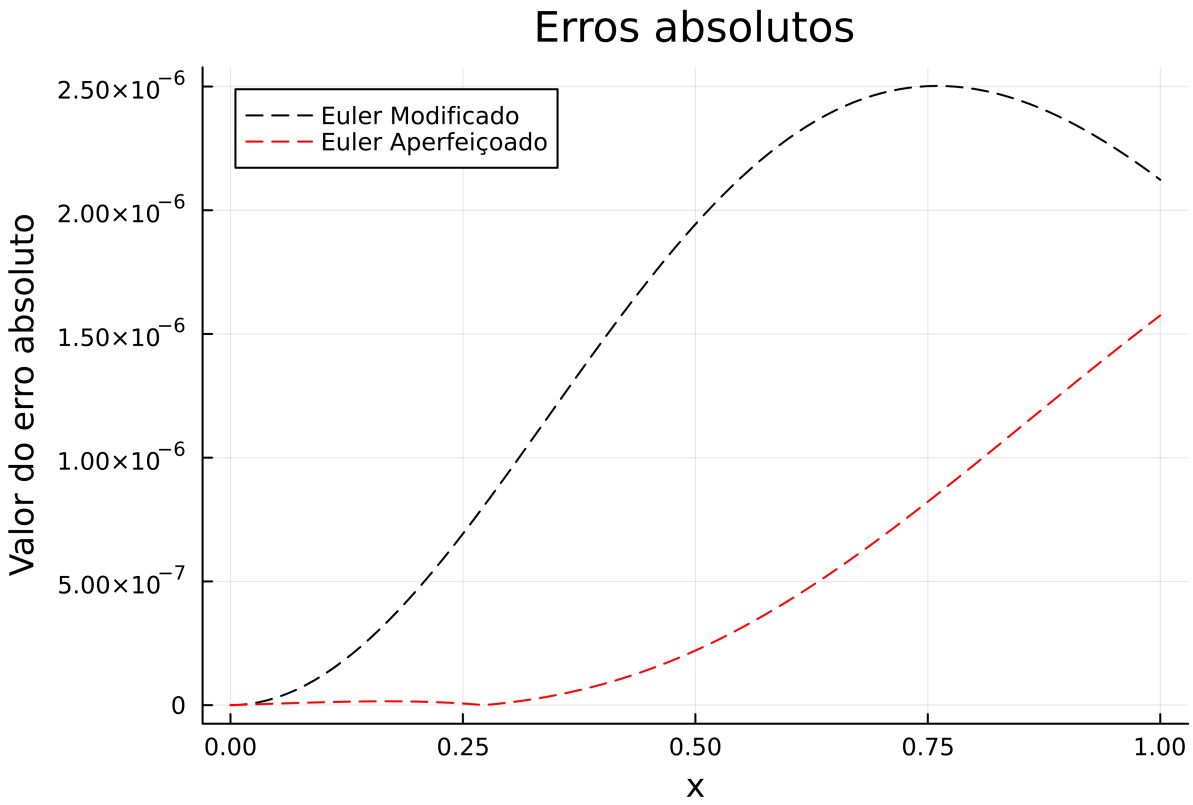

Figura 89: Gráfico plotado para o erro absoluto do item b quando $h = 0.01$

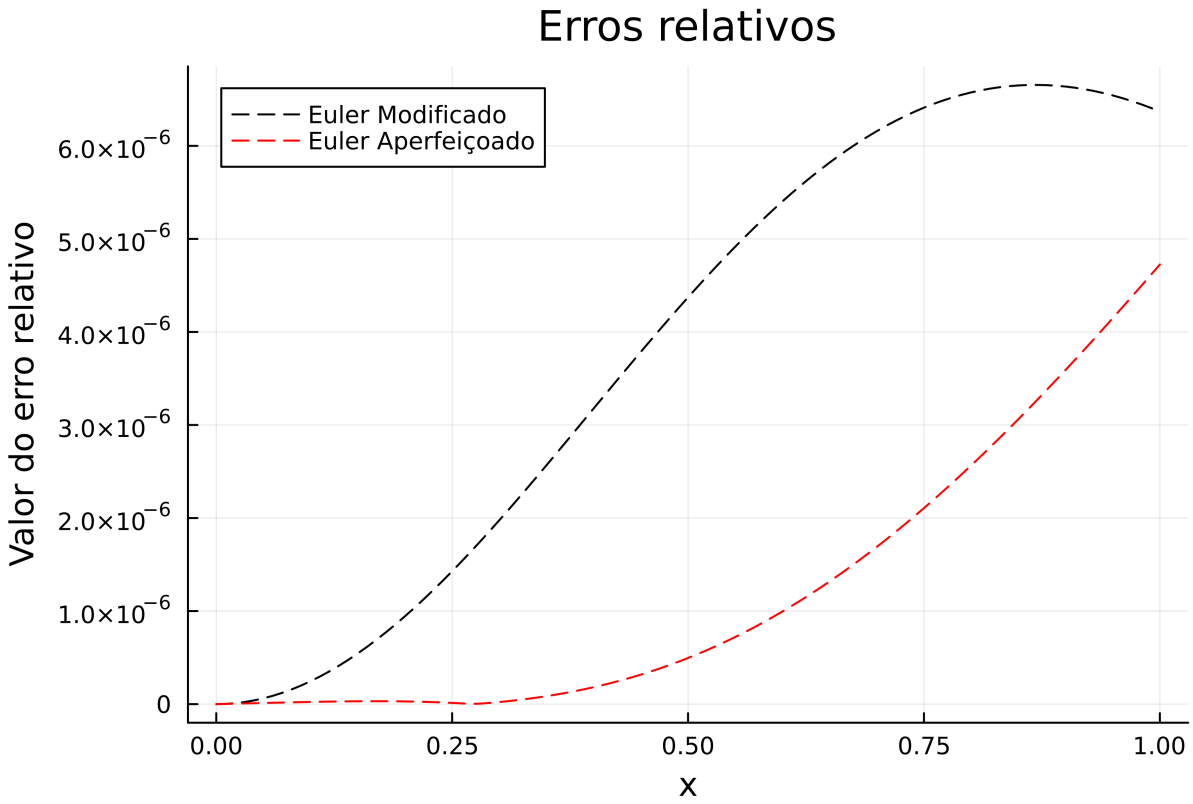

Figura 90: Gráfico plotado para o erro relativo do item b quando $h = 0.01$

2.8.3 $h = 0.005$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 61: Valores obtidos para o item b com $h = 0.005$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	3.07e-08	6.16e-08	1.39e-09	2.79e-09
2.00e-01	1.16e-07	2.36e-07	6.06e-10	1.24e-09
3.00e-01	2.36e-07	4.93e-07	6.83e-09	1.43e-08
4.00e-01	3.67e-07	7.93e-07	2.60e-08	5.62e-08
5.00e-01	4.84e-07	1.09e-06	6.05e-08	1.36e-07
6.00e-01	5.70e-07	1.35e-06	1.11e-07	2.62e-07
7.00e-01	6.16e-07	1.53e-06	1.75e-07	4.36e-07
8.00e-01	6.20e-07	1.64e-06	2.48e-07	6.54e-07
9.00e-01	5.88e-07	1.65e-06	3.24e-07	9.10e-07
1.00e+00	5.28e-07	1.58e-06	3.97e-07	1.19e-06

Tabela 62: Erros obtidos para o item b com $h = 0.005$

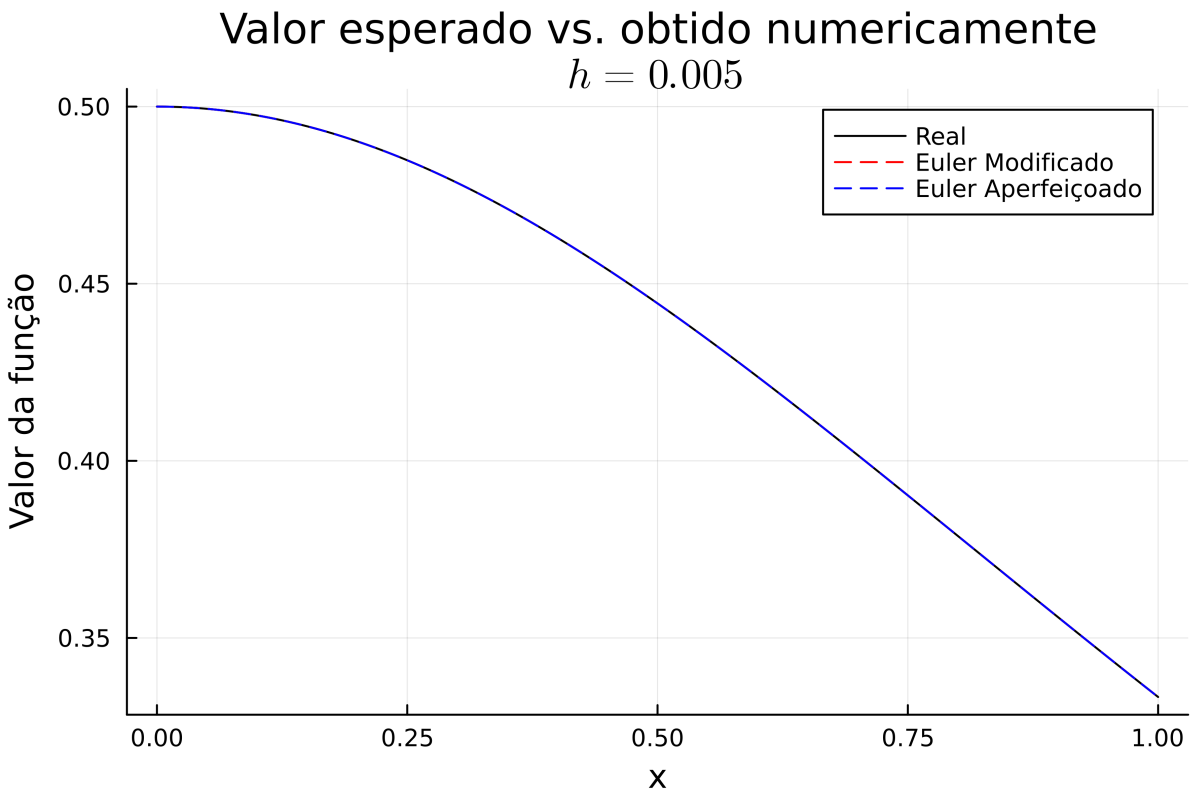

Figura 91: Gráfico plotado para o item b quando $h = 0.005$

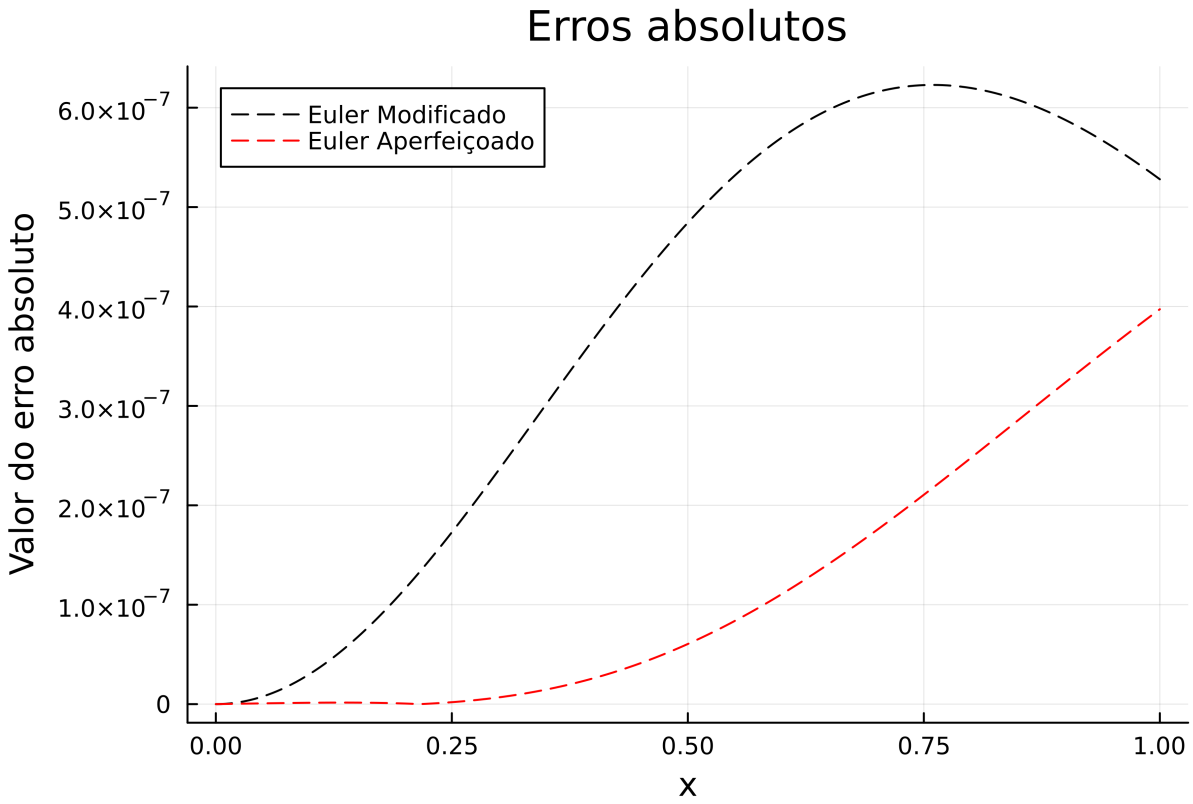

Figura 92: Gráfico plotado para o erro absoluto do item b quando $h = 0.005$

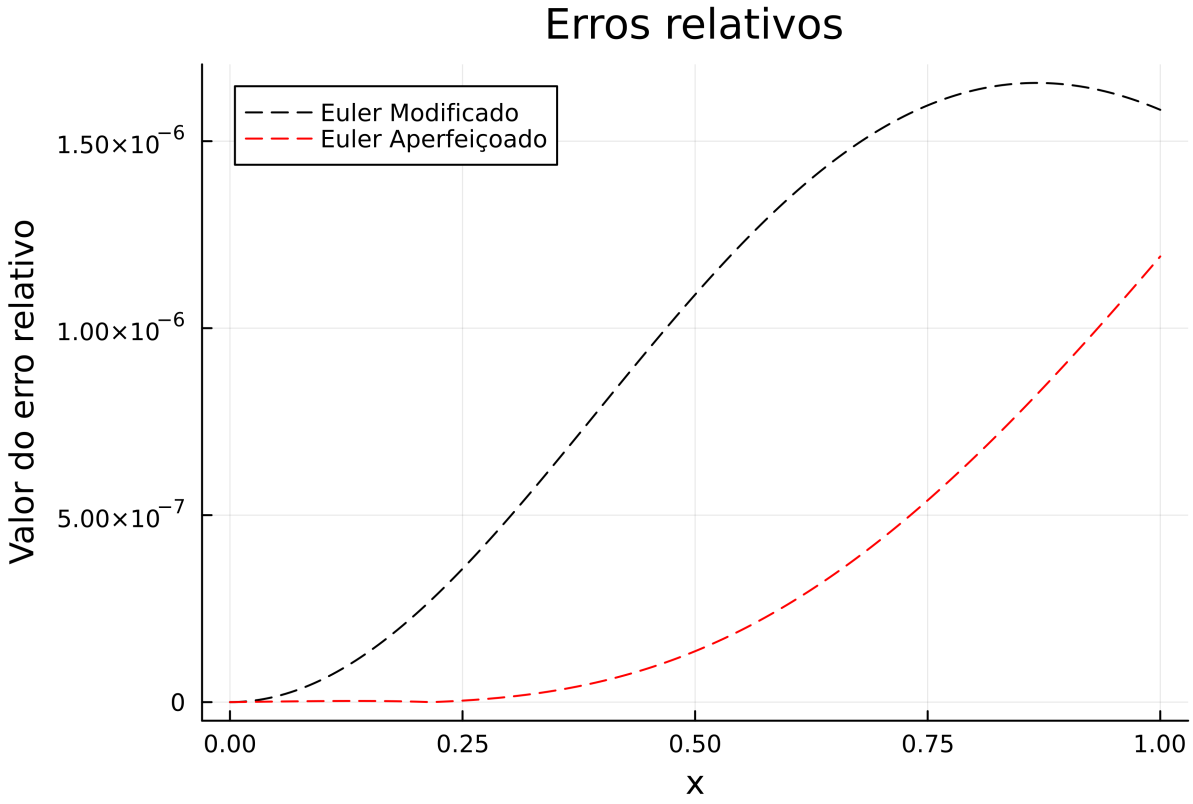

Figura 93: Gráfico plotado para o erro relativo do item b quando $h = 0.005$

2.8.4 $h = 0.001$

x	Valor real	Valor (Euler Modificado)	Valor (Euler Aperfeiçoado)
0.0000	0.5000	0.5000	0.5000
0.1000	0.4975	0.4975	0.4975
0.2000	0.4902	0.4902	0.4902
0.3000	0.4785	0.4785	0.4785
0.4000	0.4630	0.4630	0.4630
0.5000	0.4444	0.4444	0.4444
0.6000	0.4237	0.4237	0.4237
0.7000	0.4016	0.4016	0.4016
0.8000	0.3788	0.3788	0.3788
0.9000	0.3559	0.3559	0.3559
1.0000	0.3333	0.3333	0.3333

Tabela 63: Valores obtidos para o item b com $h = 0.001$

x	Erro absoluto (Euler Modificado)	Erro relativo (Euler Modificado)	Erro absoluto (Euler Aperfeiçoado)	Erro relativo (Euler Aperfeiçoado)
0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
1.00e-01	1.23e-09	2.46e-09	6.17e-12	1.24e-11
2.00e-01	4.62e-09	9.42e-09	7.01e-11	1.43e-10
3.00e-01	9.42e-09	1.97e-08	4.04e-10	8.45e-10
4.00e-01	1.46e-08	3.16e-08	1.20e-09	2.59e-09
5.00e-01	1.93e-08	4.35e-08	2.59e-09	5.83e-09
6.00e-01	2.27e-08	5.37e-08	4.61e-09	1.09e-08
7.00e-01	2.46e-08	6.11e-08	7.17e-09	1.78e-08
8.00e-01	2.47e-08	6.52e-08	1.01e-08	2.66e-08
9.00e-01	2.34e-08	6.58e-08	1.31e-08	3.68e-08
1.00e+00	2.10e-08	6.31e-08	1.60e-08	4.80e-08

Tabela 64: Erros obtidos para o item b com $h = 0.001$

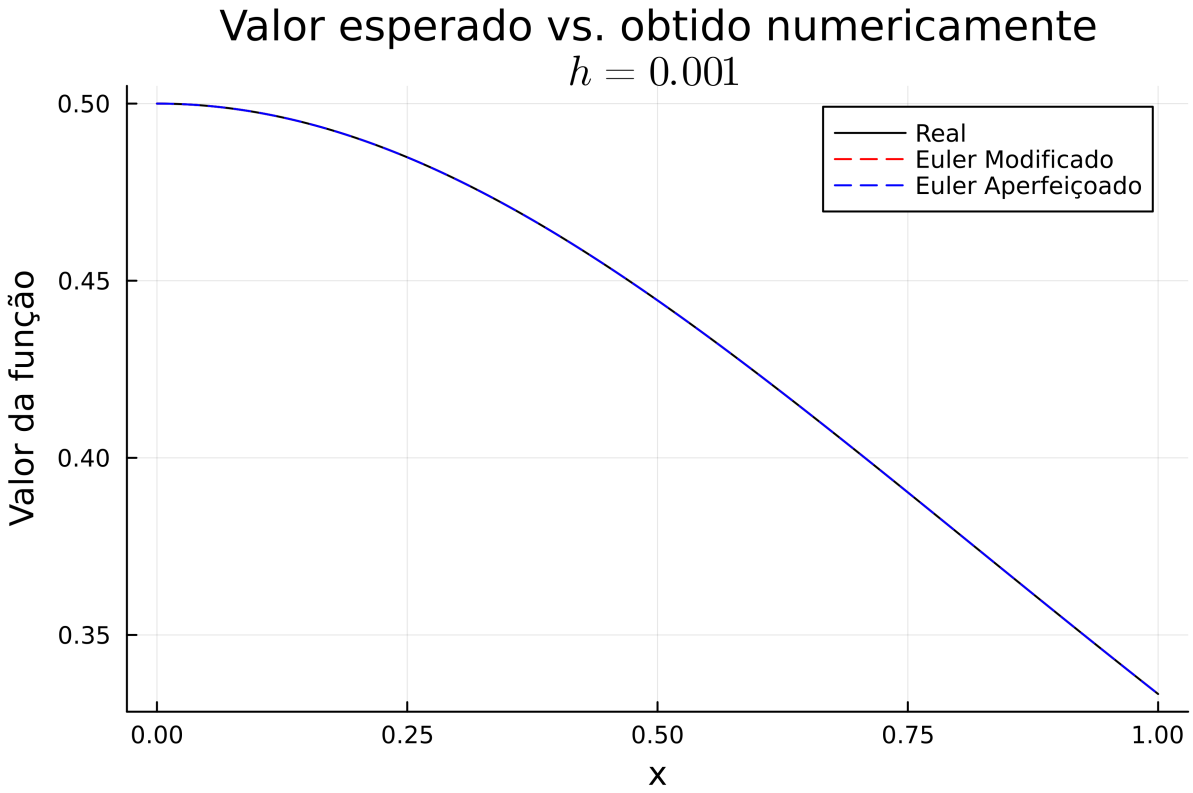

Figura 94: Gráfico plotado para o item b quando $h = 0.001$

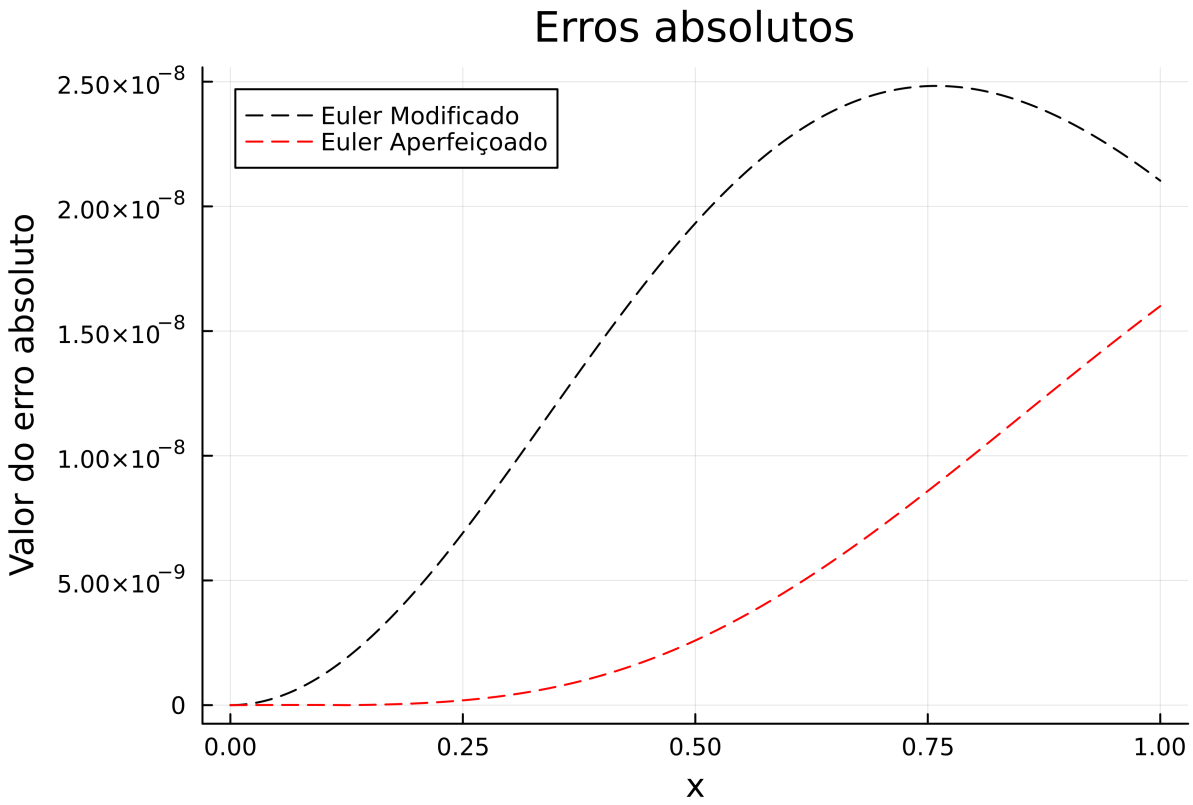

Figura 95: Gráfico plotado para o erro absoluto do item b quando $h = 0.001$

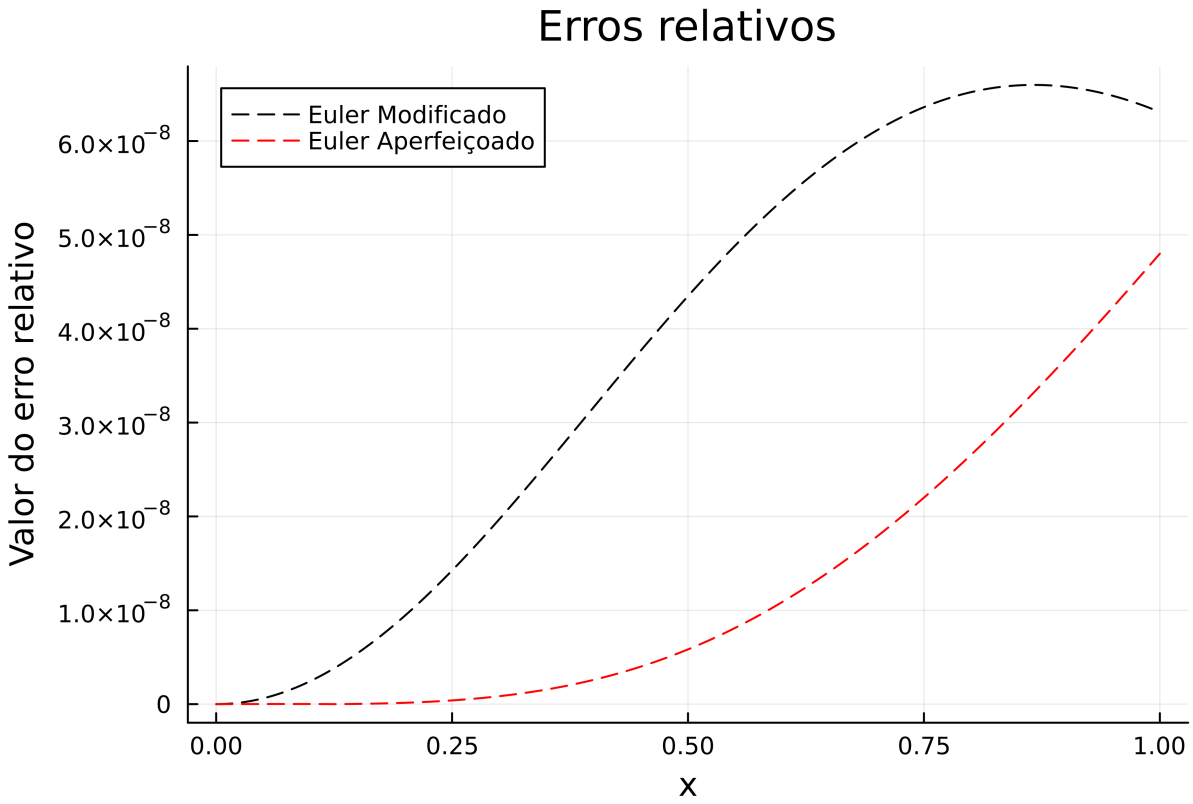

Figura 96: Gráfico plotado para o erro relativo do item b quando $h = 0.001$

Nota-se que apesar de obter erros parecidos, os mesmos são quase sempre pelo menos uma ordem de grandeza superiores aos erros obtidos pelos métodos Runge-Kutta de Terceira e Quarta ordem. Isso é esperado devido a maior ordem dos métodos, pois os métodos de Euler Modificado e Euler Aperfeiçoado são de segunda ordem.