



UNIVERSIDADE ESTADUAL PAULISTA
“JÚLIO DE MESQUITA FILHO”

FCT - Faculdade de Ciências e Tecnologia
DMC - Departamento de Matemática e Computação
Pós-Graduação em Matemática Aplicada e Computacional

1ª Tarefa de Métodos Computacionais para Equações Diferenciais

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24 de março de 2025

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1 Descrição da Tarefa

São oferecidas quatro funções, as quais precisamos aproximar as derivadas em um determinado intervalo. Pede-se para realizar a aproximação de cada uma utilizando os métodos de diferenças finitas avançada, atrasada e centrada. Todas as funções apresentadas possuem solução analítica, de forma que é possível saber o erro de cada método. Com isso em mente, pede-se para avaliar o comportamento de cada método com diferentes passos (representado pela letra h).

Tendo em vista que os métodos não mudam de uma função para outra, e que todos os cálculos são feitos de forma explícita, a última etapa para automatizar esse processo é fazer a leitura da função cuja derivada será aproximada (esse processo é chamado de *parsing* na programação). A Linguagem de Programação utilizada (Julia) permite fazer a instanciação dessas funções a partir de uma entrada de texto. Portanto, para realizar a atividade, foi criado um arquivo JSON contendo cada uma das funções e intervalos pedidos.

Esse arquivo é lido por um *script*, que realiza a leitura, instanciação e aproximação de cada uma das funções no intervalo pedido, para os seguintes passos de malha:

1. $h = 0.1$ ou $h = 1e-1$;
2. $h = 0.001$ ou $h = 1e-3$;
3. $h = 0.00001$ ou $h = 1e-5$;

Esses valores foram escolhidos pois permitem visualizar de forma intuitiva o aumento da precisão dos métodos. Caso utilizássemos valores menores, não seria possível ver muita diferença. Os resultados para cada malha são salvos em arquivos tabulados (CSV), e as representações via gráficos são salvas em arquivos de imagem (PNG).

2 Resultados

Como as tabelas possuem números muito longos em precisão, as mesmas estão muito pequenas. No entanto, o *zoom* pode ser utilizado para permitir a leitura.

2.1 Item a

2.1.1 $h = 0.1$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.0	1.0806046117362795	0.9810949928088142	0.09950961892746535	0.0920869833856961	1.111916509439987	0.0313118977571914	0.02897627626020236	1.046505751514064	0.0340988605848731	0.03155535562746864
0.1	0.8362409206562051	0.5730287705875092	0.2632121500686959	0.31475636215237013	0.9810949928088142	0.14485407215260904	0.17322050209997	0.7770618816981617	0.05917903895804344	0.07076793002620006
0.2	0.23537881805947697	-0.28298971828060404	0.518368536340081	2.2022735121777033	0.5730287705875092	0.33764995252803226	1.4344959130634805	0.1450195261534526	0.09035929190602437	0.383887995571115
0.3	-0.9062676053964771	-1.753890988474264	0.847541483077787	-0.935200020425536	-0.28298971828060404	0.6232778871158731	-0.6877415494104517	-1.018399463377434	0.11213179798095685	-0.12372923550754199
0.4	-2.710516031683998	-3.842218996913317	1.137058680074237	-0.410883354936971	-1.753890988474264	0.95670694339644	-0.3529611823082844	-2.7890154940827983	0.0749946230889016	-0.12896107639233437
0.5	-4.95669946591047	-5.88358781019881	0.92688834228834	-0.1869970835760737	-3.842218996913317	1.132477566219138	-0.22847412355897578	-4.853904854945071	0.10279461096530855	-0.1020738519991450956
0.6	-6.534699419430502	-6.141350807630008	0.3933486118004034	-0.06019383395522325	-5.88358781019881	0.651111609231692	-0.09963910616847259	-6.01246308914408	0.5222301105160936	-0.07991647006184806
0.7	-4.954592833203829	-1.7947225417461221	3.1598702914577075	-0.6377658866903124	-6.141350807630008	1.186757974426179	-0.23952684193804394	-3.9680366746880646	0.9865561585157647	-0.19911952237613434
0.8	2.3608884506417414	7.397640572121265	5.036752121479523	2.1334138510908565	-1.7947225417461143	4.155610992387856	1.7601894707300845	2.8014590151875756	0.4405705645458342	0.1866121901803863
0.9	11.770844500682328	11.252757231552685	0.5180872691296425	0.04401445190271694	7.397640572121265	4.373203928561063	0.3715284768486712	9.325198901836975	2.4456455988453527	0.20777146437569408
1.0	6.62585846208666	-5.046535688809928	11.672394535018594	1.7616424988735715	11.252757231552685	4.6268983853440195	0.6983092294505403	3.103110771371379	3.522748074837287	0.5316666347115157

Tabela 1: Item a , com $h = 0.1$

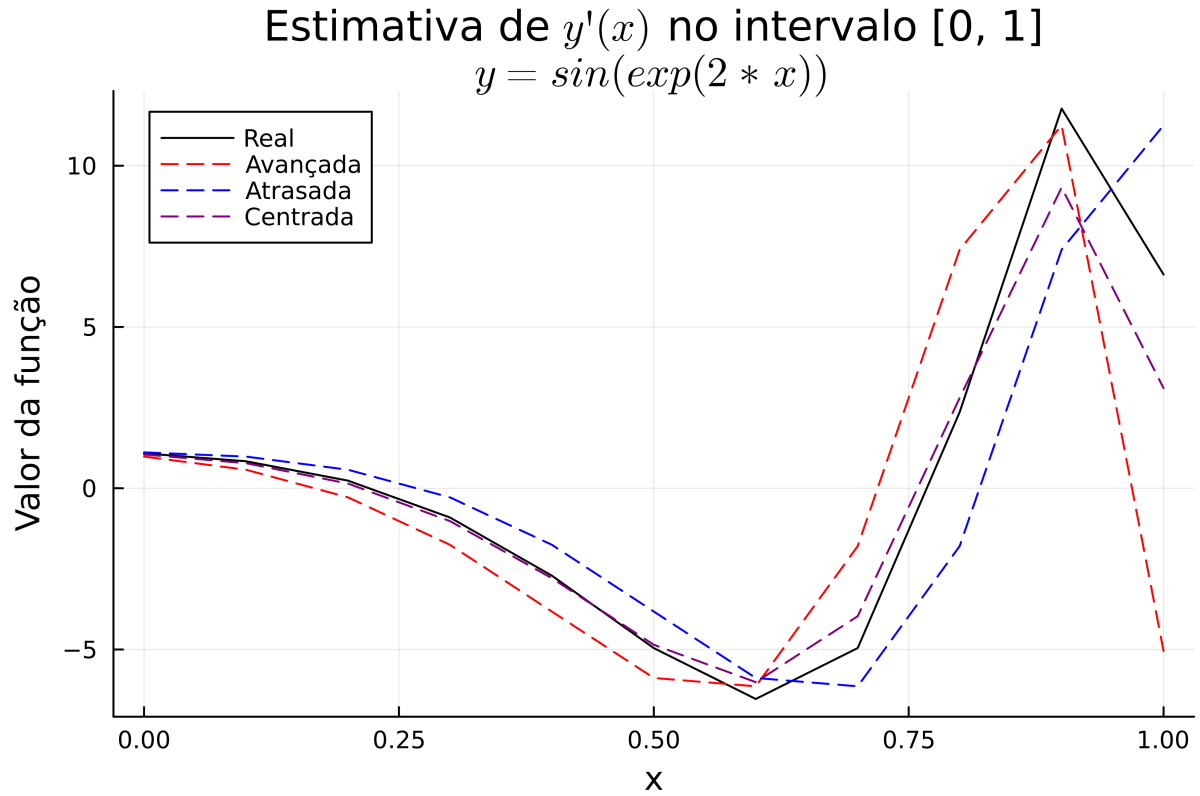


Figura 1: Gráfico plotado para o item a quando $h = 0.1$

2.1.2 $h = 0.01$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.0	1.0806046117362795	1.074239438408764	0.0063651733275156275	0.00589038142016466	1.0802905195665986	0.005691907830319032	0.005267336237972799	1.0802679789876812	0.00033663274859829784	0.00031152259109503056
0.1	0.8362409206562051	0.8159740464263598	0.0202668742298453	0.024235688220018838	0.8553315289701713	0.01900606831396614	0.02282907693513202	0.8356527876982656	0.00058813295793958	0.0007033056424434088
0.2	0.23537881805947697	0.19244583722723974	0.04293298083223723	0.1823995089540668	0.27649850766409134	0.04111968960461437	0.17469579439482102	0.23447217344566554	0.0009066456138114298	0.003851857279622897
0.3	-0.9062676053964771	-0.9807946218594157	0.0745270194629386	-0.0822351135792107	-0.834032999274313	0.0723520546904583	-0.07970626456010481	-0.9074135123934235	0.001145906969463886	-0.0012644243365529458
0.4	-2.710510031683998	-2.8170231804131	0.10631628636040211	-0.03929711979532717	-2.6057128460011182	0.1048034178378985	-0.0386654583604159	-2.71137240212714	0.0008564707888059075	-0.0003159807131905325
0.5	-4.95669946501047	-5.066001330127334	0.10930186421706445	-0.02205133980157245	-4.8456006866542385	0.11109877925623124	-0.022413862292904638	-4.955801008390886	0.0008984575195833955	-0.00018126124566699418
0.6	-6.534699419430502	-6.555610064042017	0.02091064461151504	-0.003199639778306957	-6.503488003454177	0.031211415976324375	-0.004776258856454709	-6.529540633748097	0.005150385682404668	-0.0007881595390738758
0.7	-4.954592833302829	-4.73336594165803	0.22122689154579955	-0.04465087222974603	-5.155209788143877	0.20061695494004805	-0.040491108289582994	-4.944287864900954	0.01030496830287575	-0.0020798819700815177
0.8	2.3668884506417414	2.8664278327579673	0.505539382116226	0.21413099038152744	1.866972550244017	0.49391590039772426	0.26920763971862671	2.366700191500092	0.005811740859250847	0.002461675331450365
0.9	11.770844500682328	12.032372652684879	0.2615281520025512	0.02221829979891341	11.460220063251358	0.31062443743097035	0.026389307701156377	11.74629635796812	0.0245481427142078	0.002085503951121332
1.0	6.625858846208666	5.673776242439332	0.952082603769334	0.1436919538837019	7.4917147253815	0.865855796294837	0.13067828635150125	6.582745484138741	0.04311336206992511	0.006506833766100328

Tabela 2: Item a , com $h = 0.01$

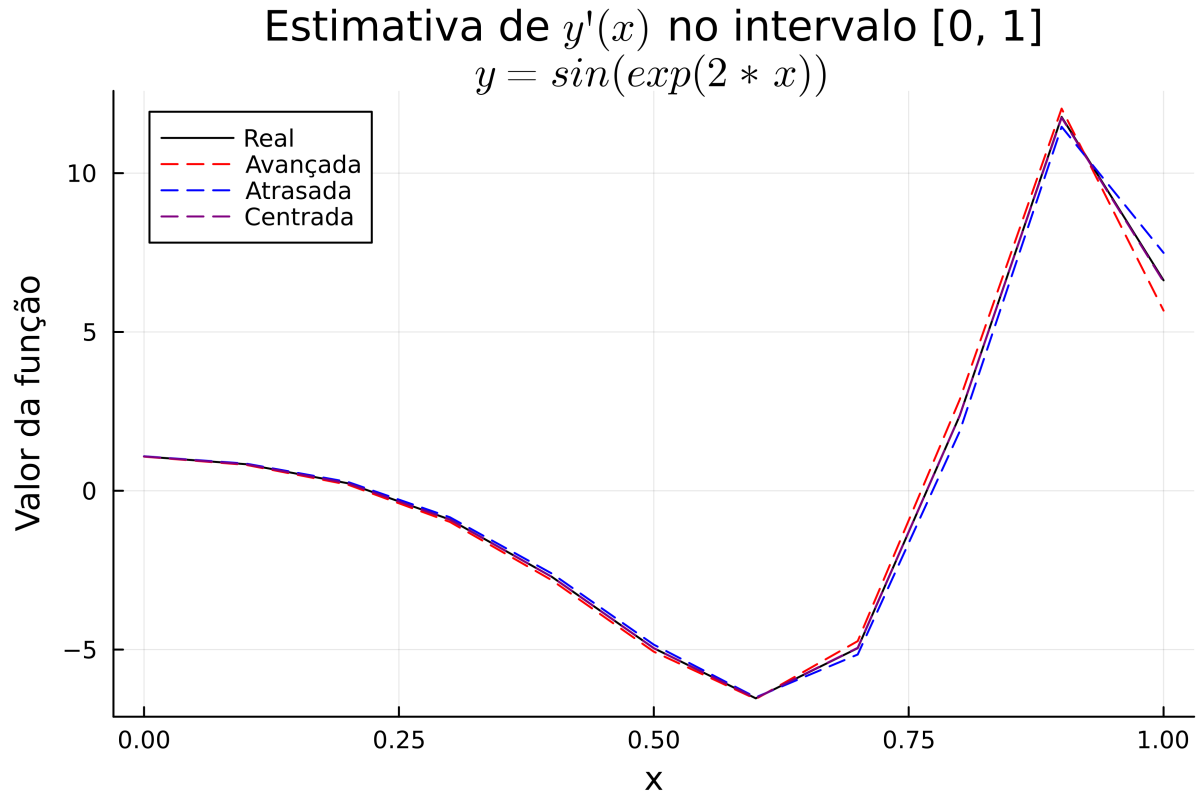


Figura 2: Gráfico plotado para o item a quando $h = 0.01$

2.1.3 $h = 0.0001$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.0	1.0806046117362795	1.080544344362428	6.02674000367287e-5	5.5771925625870736e-5	1.0806648118177886	6.020008150908751e-5	5.570962853134601e-5	1.0806045780770157	3.36592638205957e-8	3.114854726236372e-8
0.1	0.8362409206562051	0.83604411480581571	0.00019677259804806724	0.00023530610998282427	0.8364375756364506	0.0001966549802454587	0.00023516545936443998	0.8362408618473038	5.880890130427474e-8	7.032530919214873e-8
0.2	0.23537881805947697	0.23495854432176166	0.0004202737377153043	0.001785520639368267	0.23579891046177082	0.0004200924022938479	0.0017847602411525253	0.23537872739176624	9.066771072818725e-8	3.851991078707718e-7
0.3	-0.9062676539647771	-0.9070015994581052	0.00073394061628099	-0.000808094395716903	-0.905333035641397	0.000733674822374387	-0.0008089556501819866	-0.9062677200111224	1.1461464533013555e-7	-1.3646887591228204e-7
0.4	-2.710516031683908	-2.711572921493266	0.001056889893579193	-0.0002899219916073791	-2.7094593133114575	0.0010567183724505824	-0.00038985874279964913	-2.7105161174023618	8.571845396844855e-8	-3.162403864970304e-8
0.5	-4.95669946591047	-4.957802103273123	0.0011026373626537023	-0.0002224539474779645	-4.9555966491221515	0.0011028167883182905	-0.0002224901469476226	-4.956699376197637	8.971283271819175e-8	-1.809930848847881e-8
0.6	-6.534699419430502	-6.534960880683338	0.00026146125283599275	-4.001121337862225e-5	-6.534436928266884	0.00026249116361753977	-4.01688198292701e-5	-6.53469904475111	5.149553903294191e-7	-7.88032252596707e-8
0.7	-4.954592833203829	-4.95248337476939	0.0021094584344396594	-0.00042575818143982636	-4.956700229792332	0.0021073965885030077	-0.0004253420330284304	-4.954591802280861	1.0309229683258536e-6	-2.0807420569799262e-7
0.8	2.3608884506417414	2.365890245945579	0.005001795303837486	0.002118607214363681	2.355887820618685	0.005000630023056285	0.002118113637133037	2.360888903282132	5.826403900066306e-7	2.467886148717683e-7
0.9	11.770844506682328	11.77371304772079	0.002868547038461955	0.00024369934020415204	11.767971044262802	0.002873456419525411	0.0002441164199695989	11.770842045991797	2.454690530839798e-6	2.085398826479702e-7
1.0	6.625858846208666	6.616756547860314	0.00910228348361745	0.0013737537366284378	6.634952905214509	0.00093650005842802	0.0013724488539686255	6.6258545265537411	4.319671254471302e-6	6.51941339061742e-7

Tabela 3: Item a , com $h = 0.0001$

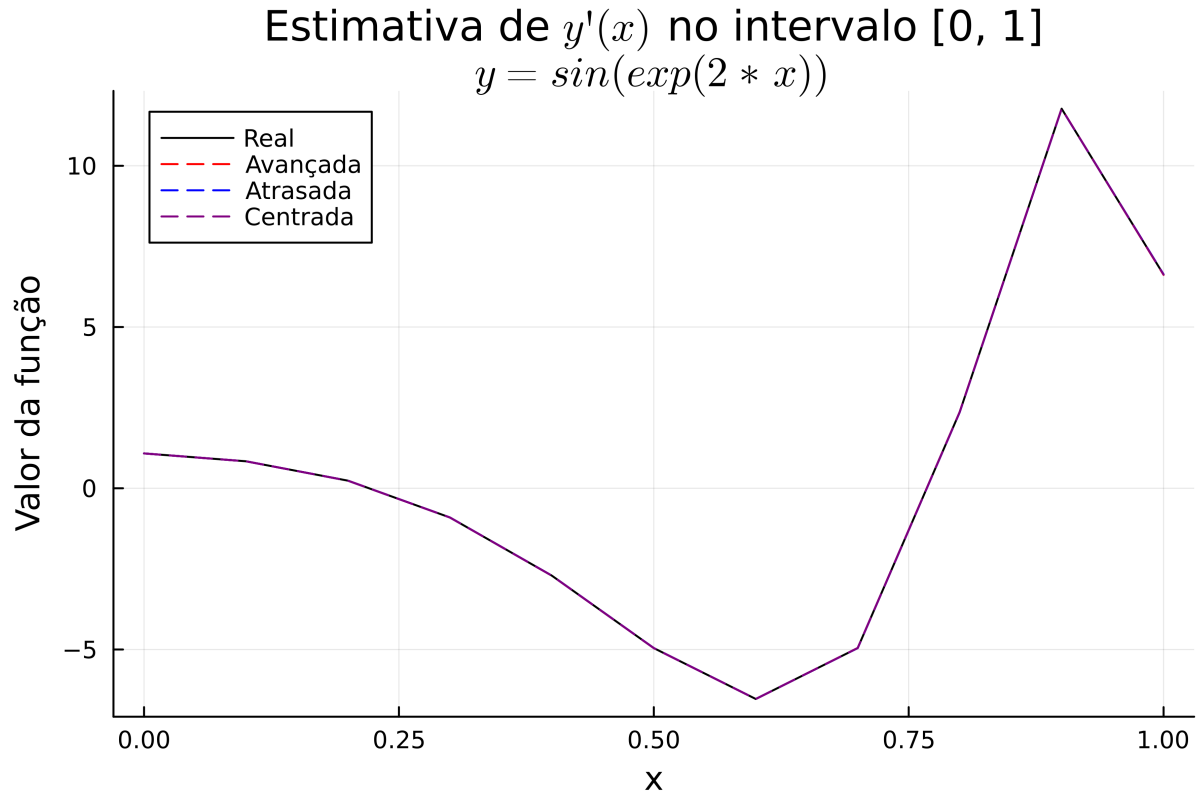


Figura 3: Gráfico plotado para o item a quando $h = 0.0001$

2.2 Item b

2.2.1 $h = 0.1$

x	Valor Real	Valor Avançada	Erro Avançada	Erro Avançada Relativa	Valor Atrasada	Erro Atrasada	Erro Atrasada Relativa	Valor Centrada	Erro Centrada	Erro Centrada Relativa
6.0	0.5503869781155519	0.5520725092326364	0.0016855311170844711	0.0030624473036326076	0.546956365295554	0.00343061282900118913	0.006233992272200607	0.5495144372640882	0.0008725408514637101	0.0015853224842839995
6.1	0.5528888960903625	0.5519782131523817	0.0009106837560807791	0.0016471369963684973	0.5520725092326364	0.0008163876767260936	0.0014765854067420848	0.5520253611925091	0.0008635357168533808	0.0015618612015551906
6.2	0.5502110642487835	0.5467494709467272	0.0034615933020563094	0.006291391662184232	0.5519782131523818	0.0017671489035983035	0.0032117654813267607	0.5493638420495546	0.0008472221992289475	0.0015398130904286348
6.3	0.542451329161323	0.5365056663243414	0.005945662836081613	0.01096073051599694	0.5467494709467272	0.004298141785404219	0.00792355286888046	0.5416275686355344	0.000823760525788586	0.0015185888235580353
6.4	0.5297504163460656	0.5214084119839407	0.008342004362124844	0.015747046353759415	0.5365056663243414	0.006755249978275812	0.01275175963969959	0.528957039154141	0.0007933771919245158	0.0014976433570299131
6.5	0.5122905670967536	0.5016599984387509	0.01063056925799677	0.02075105404908492	0.5214084119839407	0.009117844287187093	0.017798188883665588	0.511342052113488	0.0007563621854047823	0.001476432581621345
6.6	0.490293801180282	0.4775014899681406	0.012792332183511066	0.026091156287360586	0.5016599984387509	0.011366197259428631	0.023182120889405924	0.4895807337177835	0.000713067492512738	0.0014543676889777217
6.7	0.46401989520944636	0.4492103406247838	0.014809464584662535	0.031915587262440086	0.477501489968135	0.013481663787367140	0.0290540697530828	0.46335590481079864	0.000663900398647721	0.0014307587546787013
6.8	0.4337634898005883	0.4170979934941327	0.01666549630645564	0.03842060860263522	0.4492103406247838	0.015446850824195502	0.035611228670483074	0.43315416705945825	0.0006093227411300695	0.0014047349660760752
6.9	0.3998522187923162	0.3815067548920442	0.018345463900271974	0.045880610480745224	0.4170979934941327	0.017245747701816485	0.043130371400474744	0.39930237419308845	0.0005498445992277445	0.0013751195401352384
7.0	0.36264275017752284	0.3428067058441936	0.01983604433332925	0.054608582347555556	0.3815067548920442	0.018864004714521376	0.052018149281316026	0.3621567303681189	0.00048601980940393696	0.0013402165331197658

Tabela 4: Item b , com $h = 0.1$

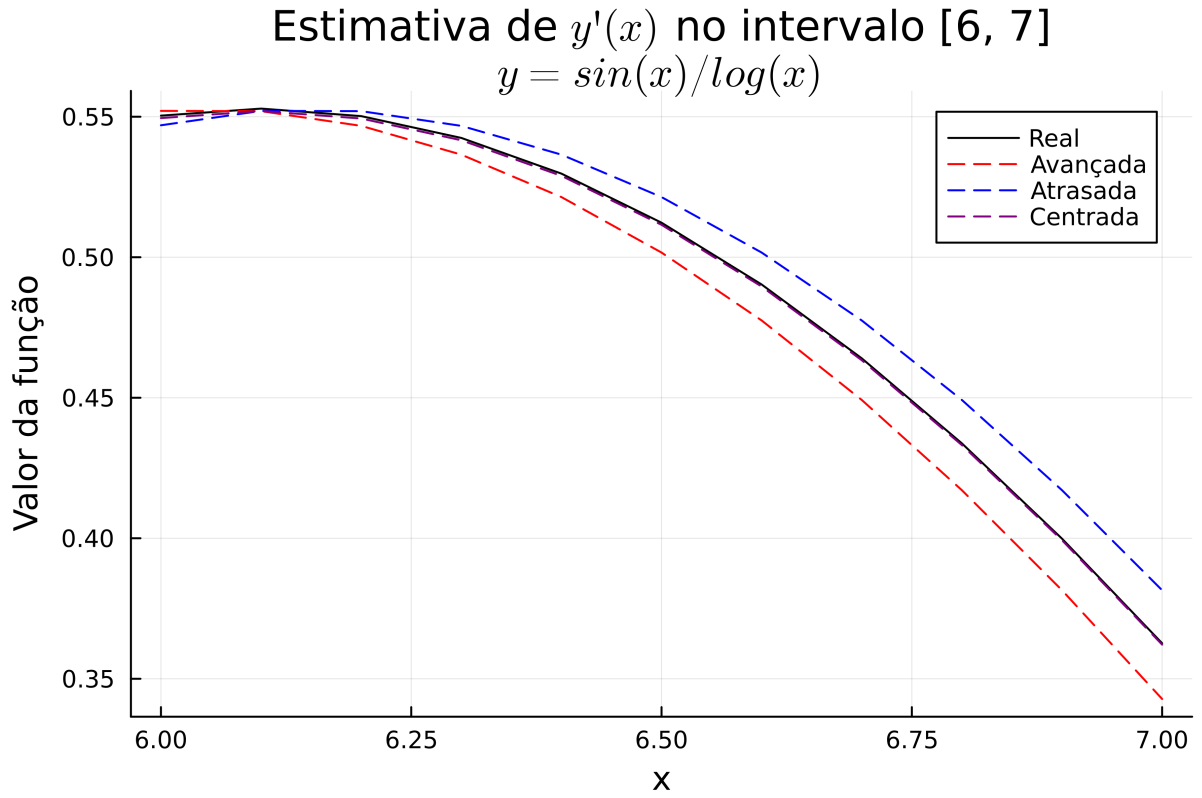


Figura 4: Gráfico plotado para o item b quando $h = 0.1$

2.2.2 $h = 0.01$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
6.0	0.5503869781155519	0.5506339246796915	0.00024694656413959404	0.00044867806463209676	0.5501225734003995	0.00026440471515243313	0.0004803978394578264	0.5503782490400455	8.720075506419548e-6	1.5850887412864822e-5
6.1	0.5528888960903625	0.5528752292356817	1.366767368082833e-5	2.4720470527135457e-5	0.5528852866259967	3.6102833658180344e-6	6.52985325984921e-6	0.5528802579308392	8.638978523323182e-6	1.5625161893492332e-5
6.2	0.5502110642487835	0.5499406582787525	0.00027040597003102906	0.0004914586194303832	0.5504645186099539	0.0002534544421703977	0.0004606494827879283	0.5502025884848532	8.475763930371194e-6	1.5404568321328397e-5
6.3	0.542451329161323	0.5418302303904059	0.000510987709171055	0.0009606369141407943	0.5429559438623471	0.0005046167010240854	0.0009302524925218024	0.5424430881263764	8.241034946565584e-6	1.519221080959824e-5
6.4	0.5297504163460656	0.528986781065073	0.000763632809925473	0.001441500103886738	0.530081775115548	0.000747611654892469	0.0014115348330387403	0.529742479288314	7.937057751947e-6	1.4983630174861873e-5
6.5	0.5122905670967536	0.511294584726725	0.0009959829700286749	0.0019441758892038259	0.5132714171933395	0.0009808494965858339	0.0019146350888241221	0.5122830009000322	7.566736721420497e-6	1.4770400234851811e-5
6.6	0.4902938011803282	0.4890775949321413	0.001212606248186913	0.0024805662344884447	0.49149574026016163	0.0012019390798334029	0.0024514670121055316	0.49028666759615147	7.133584176755026e-6	1.4549611191456447e-5
6.7	0.46401980520944636	0.4625973210604234	0.0014224841490229623	0.003065567747438906	0.4654290059887156	0.0014092007792692596	0.0030369410127265864	0.4640131635245695	6.64168487685135e-6	1.431336508115956e-5
6.8	0.4337634898005883	0.43215036293379194	0.0016131268667963772	0.0037189088171942987	0.43536442535632713	0.0016009355557388139	0.0036908029222902164	0.43375739414505954	6.095655528781663e-6	1.4052947452041168e-5
6.9	0.3998522187923162	0.39806562778818555	0.0017865910041306443	0.004468128273807584	0.4010278085971048	0.001775589804788591	0.004440615110631247	0.39984671819264517	5.5005996710266736e-6	1.3756581588168434e-5
7.0	0.36264275017752284	0.3607012567805379	0.0019414933969849613	0.005333735386977958	0.36457451945789976	0.001931769280376916	0.005326920997128071	0.3626378881192188	4.862658304022643e-6	1.3407294924943464e-5

Tabela 5: Item b , com $h = 0.01$

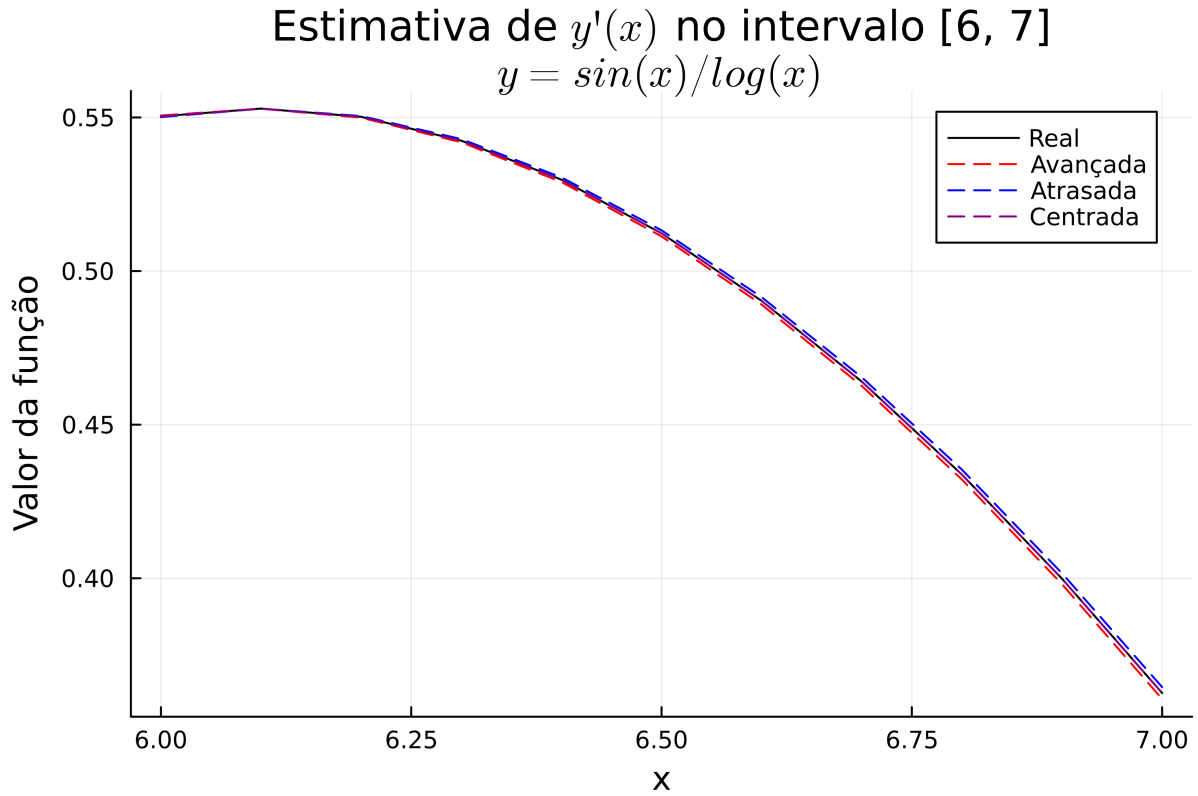


Figura 5: Gráfico plotado para o item b quando $h = 0.01$

2.2.3 $h = 0.0001$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
6.0	0.5503869781155519	0.5503895339845077	2.555868955811391e-6	4.643767126472231e-6	0.5503844204984842	2.557617067688866e-6	4.646943276975392e-6	0.550386977241496	8.740559387376834e-10	1.588075251580858e-9
6.1	0.5528888960903625	0.5528888457255465	5.118381596336263e-8	9.257522849433061e-8	0.5528889463629616	4.945359910468738e-8	8.944581701881152e-8	0.5528888960442541	8.651084293376243e-10	1.564705737759543e-9
6.2	0.5502110642487835	0.5502084440479893	2.620200794223315e-6	4.762173944649293e-6	0.5502136827519044	2.6185031208880716e-6	4.759088449926352e-6	0.5502110633999469	8.488366676218106e-10	1.5427473614707256e-9
6.3	0.542451329161323	0.5424461966911614	5.129470161624461e-6	9.456092898795306e-6	0.5424564569807451	5.127819422079405e-6	9.45304978790901e-6	0.5424513283339532	8.253697725280063e-10	1.5215554431474984e-9
6.4	0.5297504163406656	0.5297428584833332	7.557861532392351e-6	1.26683453034814e-5	0.5297579726178114	7.556271145867704e-6	1.4283833519918396e-5	0.5297504155511723	7.9489836222826e-10	1.5065032149011747e-9
6.5	0.5122905676967536	0.5122806826760029	9.885020750721729e-6	1.92957305366065e-5	0.512300451201847	9.883505093344347e-6	1.9292771947335187e-5	0.5122905669389249	7.578286886911201e-10	1.479294635656284e-9
6.6	0.4902938011803282	0.4902817066233414	1.2091556986837393e-5	2.4661859802690352e-5	0.4903058913986067	1.2090128278463563e-5	2.4658945818523328e-5	0.49029380046597404	7.143541869147896e-10	1.4569920835120915e-9
6.7	0.4640198052094636	0.4640056459900843	1.415921936204212e-5	3.051425651034662e-5	0.4640339630984647	1.4157889018318048e-5	3.0511389512625543e-5	0.46401980445442745	6.651718620354075e-10	1.433498860539296e-9
6.8	0.4337634898005883	0.43374741873547507	1.6071065113254512e-5	3.70502946678218e-5	0.4337795596442984	1.606984371005371e-5	3.7047478840234824e-5	0.4337634891898867	6.107016004008869e-10	1.407913793486034e-9
6.9	0.3998522187923162	0.39983440718260876	1.78116097074299e-5	4.454548173129251e-5	0.39987002930064186	1.781050832566633e-5	4.454272725923557e-5	0.3998522182416253	5.506908817842771e-10	1.3772360284695748e-9
7.0	0.36264275017752284	0.36262338321269016	1.9366964832678857e-5	5.340507930517909e-5	0.3626621161684218	1.93659908989674e-5	5.340239364908648e-5	0.362642749690556	4.86968557283546e-10	1.3428280463072046e-9

Tabela 6: Item b , com $h = 0.0001$

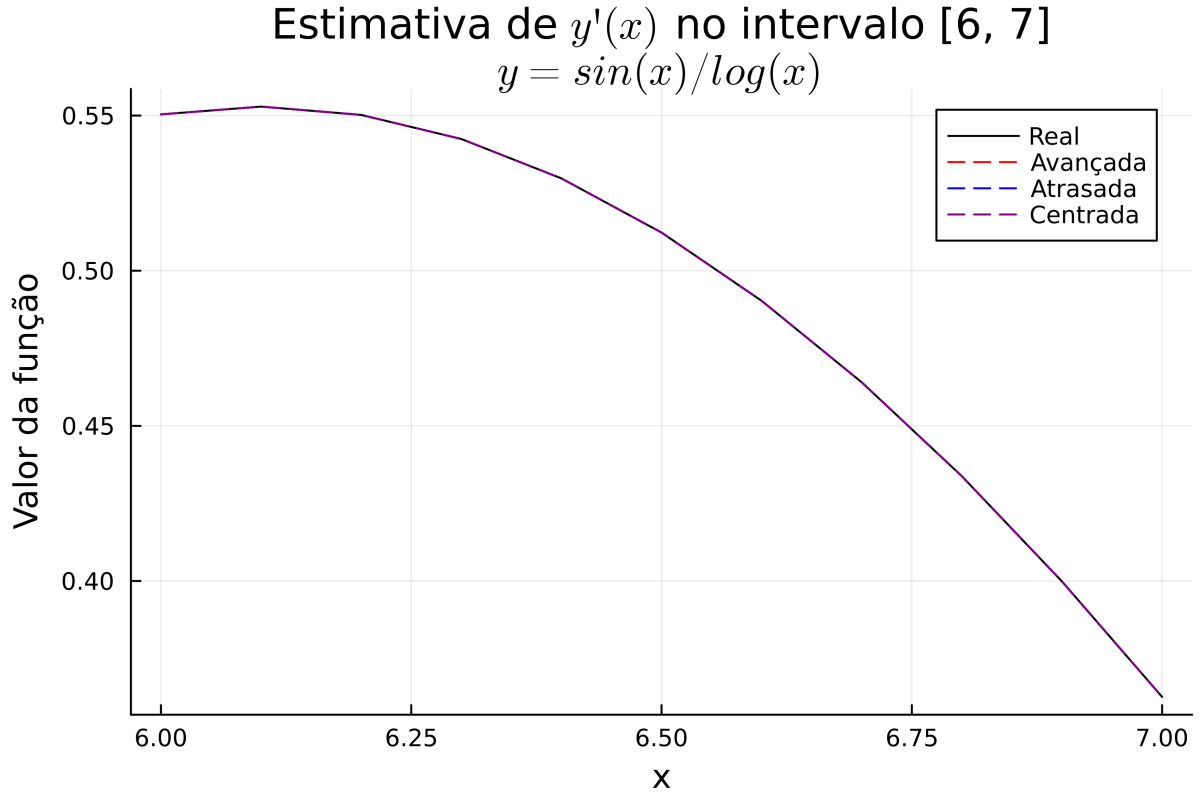


Figura 6: Gráfico plotado para o item b quando $h = 0.0001$

2.3 Item c

2.3.1 $h = 0.1$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	0.3505571987255205	0.43878616695145756	0.08822896822593707	0.25168208938997894	0.24507057507277078	0.10547762365274971	0.3008856301802453	0.34193287101211417	0.008624327713406321	0.024601770395133157
0.6	0.519468208692017	0.5865753619694744	0.06710715328027261	0.12918433151011727	0.43878616695145756	0.08068204173774418	0.1553166110806529	0.512680764460466	0.006787444228735784	0.013066139785267817
0.7	0.6475109374964576	0.697029217996468	0.04951828049968919	0.07647481707636807	0.5865753619694744	0.060935575526983254	0.09410740730123439	0.6418022899829606	0.005708647513496978	0.008816295112433077
0.8	0.74122950427624	0.775416585531954	0.034187081255714014	0.04612212689657485	0.6970292179964471	0.04420028627979289	0.059631040082452534	0.7362229017642005	0.0050066025120395485	0.00675445659293899
0.9	0.8048700861243837	0.8253172712700314	0.020447185145647695	0.025404329839247883	0.775416585531954	0.029453500592429682	0.03659410518566343	0.8003669284009927	0.004503157723391049	0.0055948876732078425
1.0	0.8414709848078965	0.8494113373039316	0.007940352496035086	0.009436276044441184	0.8253172712700314	0.0161537135378651	0.019196994108540665	0.8373643042869816	0.004106680520914896	0.004880359032049608
1.1	0.8531208370883756	0.8498968341596621	0.0035210028787134614	0.004129267444351137	0.8494113373039316	0.0040094997344439776	0.004698150736930839	0.8496540857317969	0.003766751306578664	0.004413799090640923
1.2	0.842764862185707	0.8287241741225301	0.01404009379604056	0.016660274206398352	0.849896834159662	0.007131965941091334	0.008462589976975598	0.8393105429109061	0.003454363927745574	0.004098846615161312
1.3	0.8113807375850768	0.7877324013837544	0.02364833620132234	0.02914579445367066	0.8287241741225301	0.01734343683745332	0.021375213921238527	0.8082282879031423	0.0031524496819345105	0.0038852902662160653
1.4	0.761081889001233	0.7287293785377741	0.03235251046345888	0.04250858012915674	0.787732401383755	0.02665051238252203	0.03501661617187537	0.7582308899607645	0.002850990404684826	0.0037459819786407555
1.5	0.6936781248575005	0.6535380854697875	0.040140039387712934	0.05786551132192433	0.7287293785377741	0.03505125368027362	0.05052956468459209	0.6911337320037808	0.002544392853719657	0.0036679733186661197

Tabela 7: Item c , com $h = 0.1$

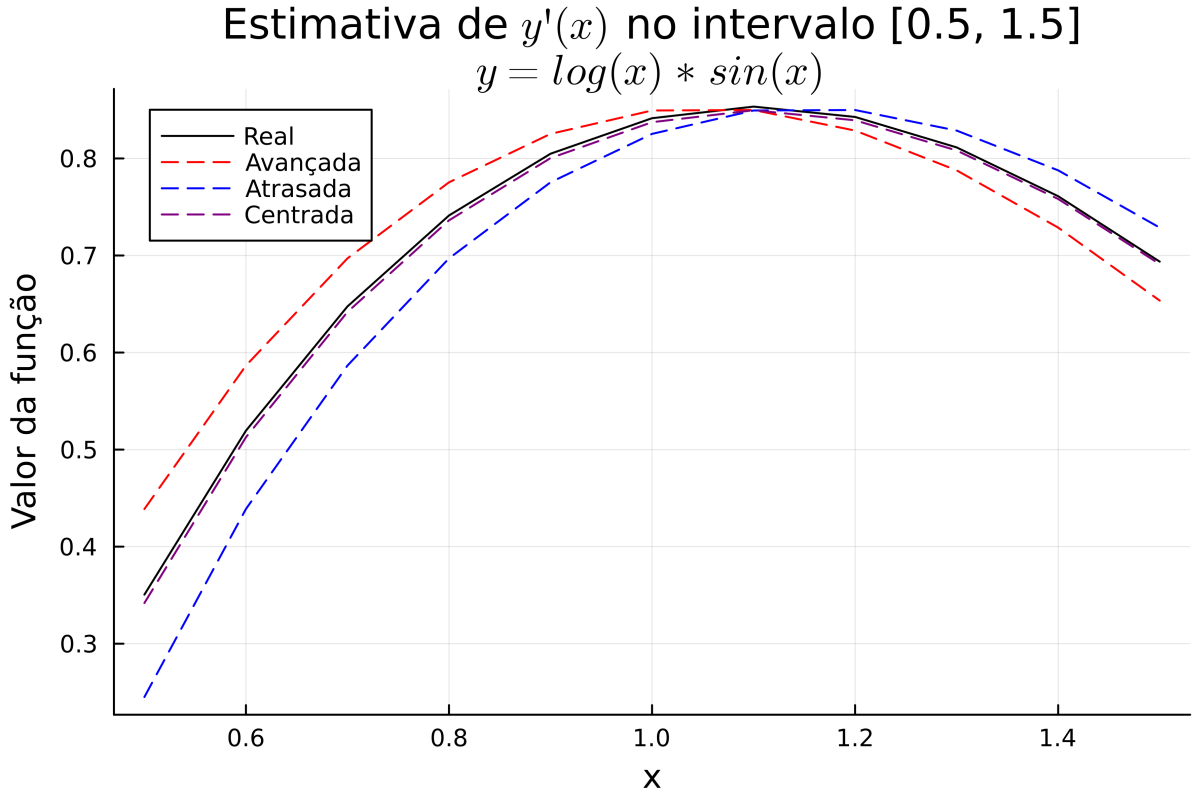


Figura 7: Gráfico plotado para o item c quando $h = 0.1$

2.3.2 $h = 0.01$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	0.3505571987255205	0.36009701560301144	0.009539816877490948	0.027213296181546793	0.3408464187191318	0.009710780006388697	0.02770095863913324	0.3504717171610716	8.548156444887489e-5	0.0002438448418326522
0.6	0.5194682086892017	0.5267565261747542	0.007288317485552476	0.014030343654606748	0.5120448430434743	0.007423365645727431	0.01429031752387534	0.5194006846091143	6.752408008747768e-5	0.00012998693463429518
0.7	0.6475109374964576	0.6529557860735197	0.00544848577062067	0.008408890509423795	0.6419522697586971	0.005558667737760481	0.008584670027741255	0.6474540279161084	5.690958034920701e-5	8.788975915872981e-5
0.8	0.7412250427624	0.7450845328222239	0.003855028545983852	0.005200856851681888	0.7372745318408175	0.003954972435422532	0.005335692133655587	0.7411795323315207	4.99719447193403e-5	6.741764115684936e-5
0.9	0.804870061213837	0.8073092991757902	0.002439213051406496	0.003030567408892453	0.8023409116978893	0.002520174429494927	0.0031423387079433673	0.8048251051368398	4.498068754388296e-5	5.5885610521992105e-5
1.0	0.8414709848078965	0.8426257030947885	0.0011547182868919537	0.0013722615606948942	0.8402341867187447	0.0012367980891517805	0.0014698047959837083	0.8414299449067665	4.103990113002425e-5	4.8771617644530014e-5
1.1	0.8534208370383756	0.8533994830808788	2.1353730296769236e-5	2.5021336918457532e-5	0.8533668817823705	5.3955256005044916e-5	6.322233259769672e-5	0.8533831825452247	3.7654403150851565e-5	4.412183475801208e-5
1.2	0.8427648682185707	0.8416641525354207	0.0011007156831499554	0.001306076848548088	0.8437965062595976	0.0010316380410269055	0.0012241113505449906	0.8427303293975691	3.453882106152495e-5	4.09827490015487e-5
1.3	0.8113807375850768	0.8092921913713469	0.0020885462137298916	0.002574064328845246	0.813406234805597	0.002025497220520256	0.0024963585240497205	0.811349213088472	3.152449660481782e-5	3.885290239776285e-5
1.4	0.761081889001233	0.7580957183776327	0.0029861706236002927	0.003923586498056131	0.7640110342405027	0.0029291452392696815	0.003848659758693767	0.7610533763090677	2.851269216530561e-5	3.746336968118212e-5
1.5	0.6936781248575005	0.6898854332139603	0.0037926916435401248	0.005467509364403328	0.697419920597131	0.0037417957396305157	0.005394138297786424	0.6936526769055457	2.5447951954804537e-5	3.6685533308452255e-5

Tabela 8: Item c , com $h = 0.01$

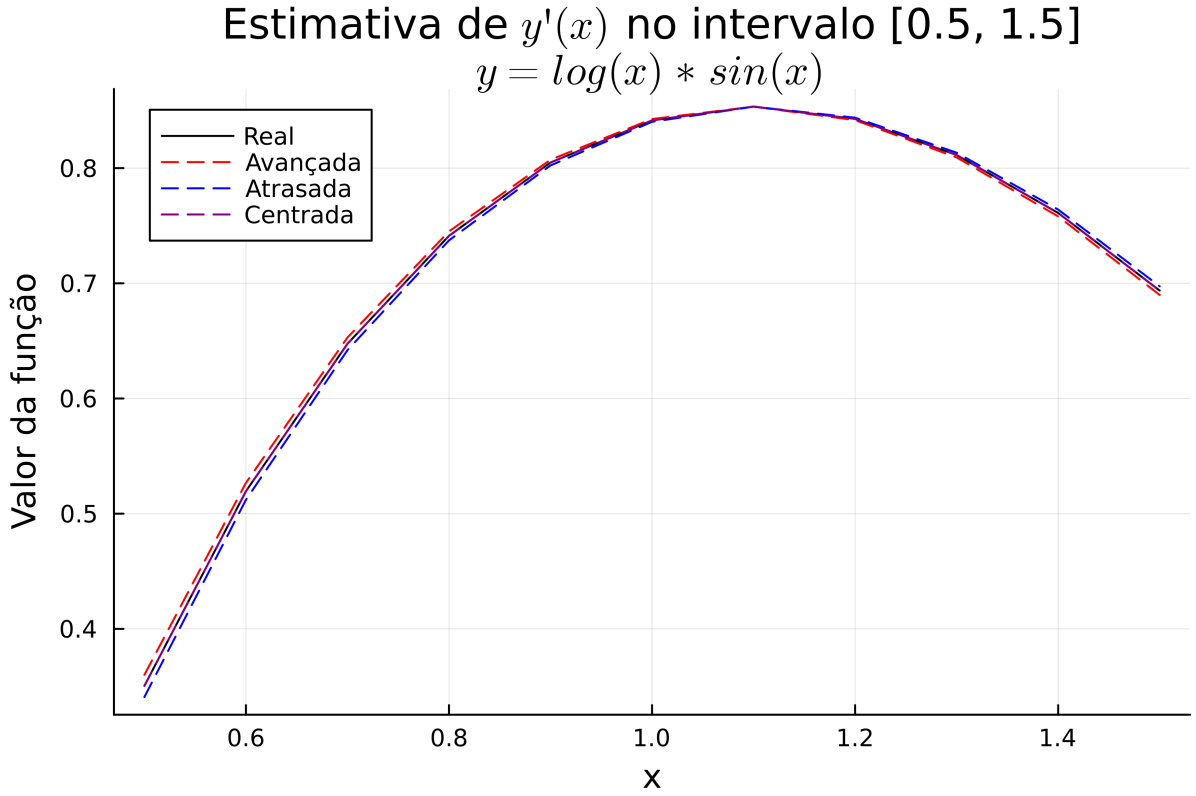


Figura 8: Gráfico plotado para o item c quando $h = 0.01$

2.3.3 $h = 0.0001$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	0.3505571987255205	0.3506534372060832	9.623848056272699e-5	0.0002745300365036288	0.3504609431498329	9.625557568759824e-5	0.000274578802082922	0.35055719017795806	8.54756243562340e-9	2.438278964659364e-8
0.6	0.519468208692017	0.5195417569997529	7.354831055117472e-5	0.00014158385310385517	0.5193946468740052	7.356181519657667e-5	0.00014160985016233932	0.519468201936879	6.75232270097581e-9	1.2998529242076739e-8
0.7	0.6475109374964576	0.6475659472918172	5.500979535955075e-5	8.49557784715289e-5	0.6474559163194282	5.5021177029379587e-5	8.497335541869606e-5	0.6475109318056227	5.690834914118019e-9	8.78878571590387e-9
0.8	0.74122950427624	0.7412685478425107	3.904356627071337e-5	5.2674058500729466e-5	0.7411904507154121	3.905366082789524e-5	5.268754225502838e-5	0.7412294992789614	4.997278590934684e-9	6.741875994544164e-9
0.9	0.804870061233837	0.8048940224711253	2.483634974159707e-5	3.085758828631496e-5	0.8048452107784235	2.484531590226405e-5	3.086876531704324e-5	0.8048700816202744	4.081093291538255e-9	5.588613364142994e-9
1.0	0.8414709848078965	0.8414820373852771	1.1952377380625051e-5	1.4204384460569087e-5	0.8414590240224041	1.1960785402370896e-5	1.4214138940393528e-5	0.8414709807038406	4.10405872922174e-9	4.877239913220667e-9
1.1	0.8534208370383756	0.8534209954769167	1.5843854117569833e-7	1.8565112814157112e-7	0.8534206710687775	1.6596950806491896e-7	1.9447568053398236e-7	0.8534208332728471	3.765528444610311e-9	4.412276196205633e-9
1.2	0.8427648682185707	0.8427542022351742	1.066598339649616e-5	1.2655942124215293e-5	0.8427755272941151	1.065907554442802e-5	1.2647745470167835e-5	0.8427648647646446	3.4539260340693545e-9	4.098327023728735e-9
1.3	0.8113807375890768	0.8113601634657241	2.0574119352634312e-5	2.5356923574337413e-5	0.8114013053994729	2.0567814396077466e-5	2.5349152923316524e-5	0.8113807344325985	3.152478278423132e-9	3.8853255104451885e-9
1.4	0.761081889001233	0.7610523088125154	2.958018871757062e-5	3.8865973747435604e-5	0.7611114634870075	2.9574485774563897e-5	3.885848054192232e-5	0.7610818861497615	2.8514715033622906e-9	3.746602756641961e-9
1.5	0.6936781248575005	0.6936404490998083	3.767575769220155e-5	5.4313025511567244e-5	0.6937157955255735	3.767066807303099e-5	5.4305688363417146e-5	0.6936781223126909	2.544809585280916e-9	3.668574075049125e-9

Tabela 9: Item c , com $h = 0.0001$

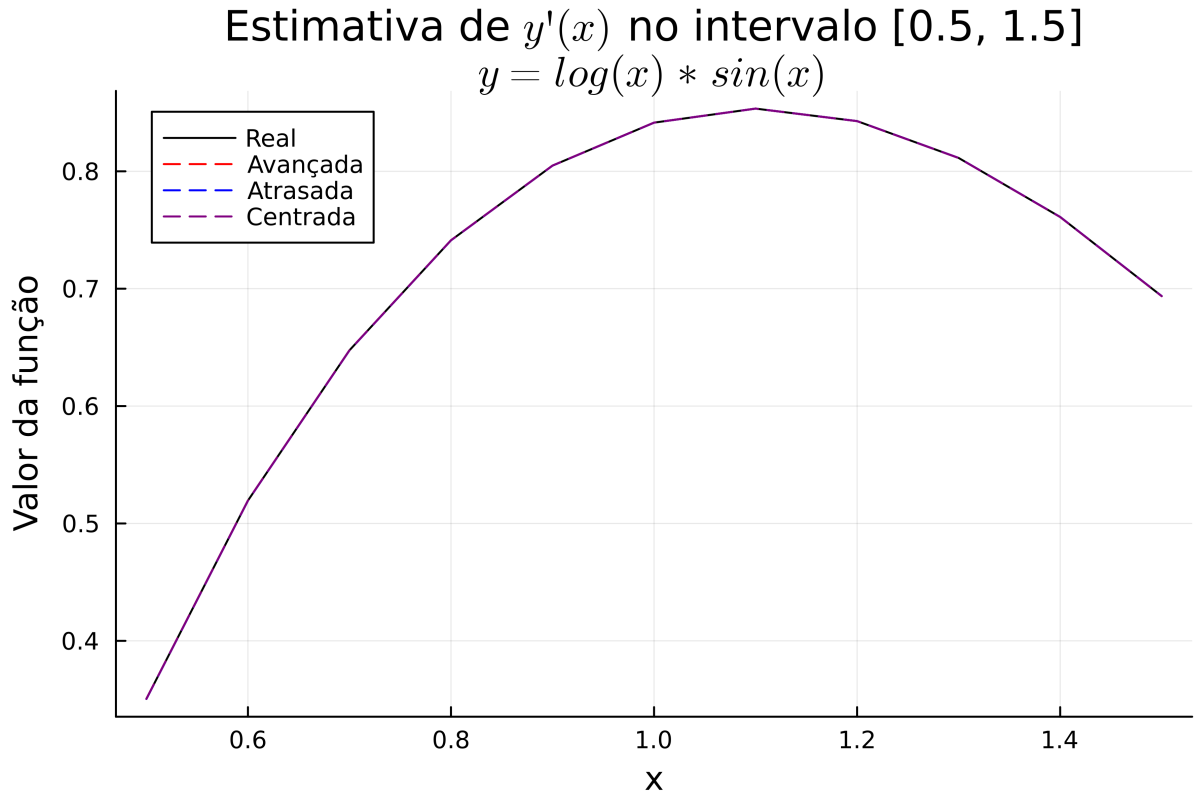


Figura 9: Gráfico plotado para o item c quando $h = 0.0001$

2.4 Item d

2.4.1 $h = 0.1$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	1.4174242246503913	1.4367254032496892	0.01930126858969783	0.013617143162863572	1.390243499963555	0.027180724695836433	0.01917613952334418	1.413484496606322	0.003039728053069302	0.002779498180240304
0.6	1.4516158335858316	1.456776886197375	0.005161052611543537	0.003555384621835191	1.4367254032496892	0.014890340336742414	0.01025776930247432	1.4467511897232321	0.004864643862599438	0.0033511923403195644
0.7	1.4566392950360747	1.4451211577574397	0.011518137260634997	0.007907336634324247	1.456776886197375	0.00013759116130040994	9.445794972667025e-5	1.4509490219864074	0.005690273049667294	0.003906439342298788
0.8	1.427558074079829	1.397332624403389	0.03022544967644003	0.02117283368378737	1.445121157757442	0.01756308369561288	0.012302885616008048	1.4212268910894155	0.006331182990413575	0.004434974033889661
0.9	1.360543790846827	1.3103491211123052	0.05019466973443176	0.03689309382918847	1.397332624403389	0.03678883355656204	0.027039801147204535	1.353840872757892	0.006702918088934862	0.004926646340991968
1.0	1.253380767493447	1.1829468044047253	0.07043396308872163	0.05610518418038073	1.3103491211123052	0.05609833361894825	0.04545175344670039	1.2466479627585603	0.006732804734886688	0.0053717133713401605
1.1	1.1058997784971287	1.0161102722445436	0.08978950025258508	0.08119135928809502	1.1829468044047253	0.0770470259075966	0.06966098521320844	1.099528538246345	0.06371240172494241	0.005761137036443292
1.2	0.9202736505171314	0.8132339390501614	0.10703972045796994	0.11631292426008782	1.0161102722445438	0.09583661272741673	0.1041392543797289	0.9146721056518547	0.0056015538652766095	0.006086834940180458
1.3	0.7011160150997927	0.5801052128601647	0.12101080223962801	0.17259740133364954	0.8132339390501614	0.1121179239593687	0.1509135115226408	0.6966695759596631	0.004464439140129654	0.006341944905504369
1.4	0.45534477137668733	0.32464570109446367	0.13069907028222366	0.28703320757822404	0.5801052128601647	0.12476044148347737	0.2739911586253142	0.4523754569773142	0.0029693143993731463	0.006521024476454917
1.5	0.1918025795659312	0.05641990749124748	0.1353826720746837	0.7058438545564325	0.32464570109446367	0.13284312152853248	0.6926034145587093	0.19053280429285557	0.0012697752730756184	0.006620219998861587

Tabela 10: Item d , com $h = 0.1$

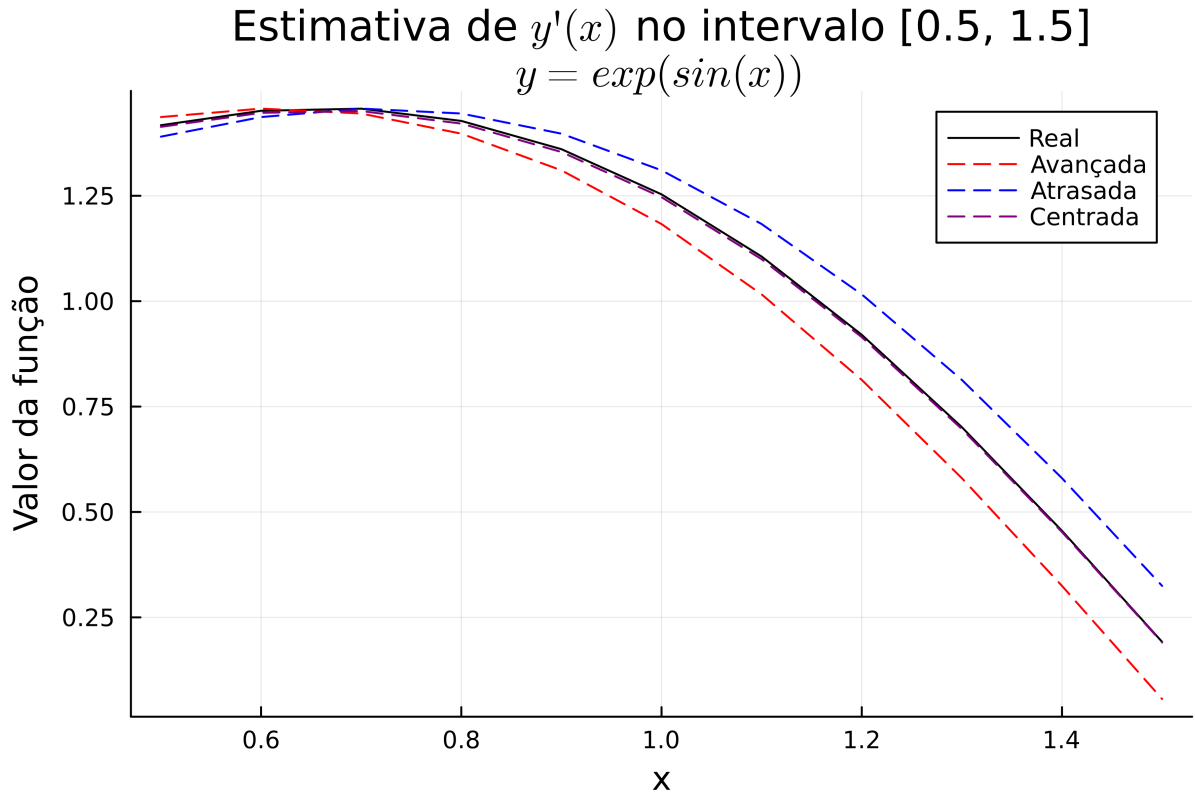


Figura 10: Gráfico plotado para o item d quando $h = 0.1$

2.4.2 $h = 0.01$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	1.417424246593913	1.4197324015811752	0.0023081769217838666	0.001628430558493188	1.4150372332210281	0.002386991438363184	0.0016840346008173952	1.4173848174011017	3.940725828965874e-5	2.780202116210364e-5
0.6	1.45161583355858316	1.4525917472509864	0.000975913665154815	0.0006722947232836937	1.4505425296473362	0.0010733039384953624	0.0007303856650378702	1.4515671384491613	4.86951366702737e-5	3.354547087708825e-5
0.7	1.456639295060747	1.45601805669616055	0.0006212380744692325	0.0004264872412726221	1.4571465447042353	0.0005072496681606253	0.0003482328603170512	1.4565823008329204	5.699420315430359e-5	3.9127190477785434e-5
0.8	1.427558074079829	1.42511809674716	0.002439974405113121	0.001709194497529429	1.4298711570762013	0.002313082966372229	0.0016203074595498952	1.4274946283754586	6.344570437044617e-5	4.444351898702991e-5
0.9	1.360543790846827	1.3561326732712351	0.00441117572591827	0.0032421724330139114	1.3648205091094656	0.004276718362638646	0.00314338890207207	1.3604765911918504	6.719965497659075e-5	4.939176190332246e-5
1.0	1.253380767493447	1.2469301815834285	0.006441585910018421	0.005139368719451888	1.2596873061541558	0.006306538660708849	0.00503162232290561	1.2533132438687922	6.752362465478612e-5	5.387319353066358e-5
1.1	1.1058997784971287	1.0974800263260469	0.008419752170181871	0.007613485718953629	1.1141916974825694	0.008291918085440683	0.007497893703088585	1.1058358619047581	6.391659237059422e-5	5.7796007932521854e-5
1.2	0.9202736595171314	0.9100496184402118	0.010224041076919543	0.0111097834553736	0.9303852834405824	0.010111623923450996	0.010987627233356409	0.9202174509403971	5.6208576734273485e-5	6.107811100859519e-5
1.3	0.7011160150997927	0.6893820028297526	0.011734012270040117	0.0167361920385886	0.7127607757910326	0.011644760691239875	0.016608892737363056	0.7010713893103926	4.4625789400121185e-5	6.364965061277257e-5
1.4	0.45534477137668733	0.44250215809413795	0.012842613282549376	0.028204152303585427	0.468127775300875	0.012783003924187686	0.028073241920700025	0.4553149666975065	2.980467918084484e-5	6.545519143820078e-5
1.5	0.1918025795659312	0.178346718013945	0.013467907764536685	0.0702175528348781	0.2052449946752688	0.013442415109337608	0.07008464192587589	0.19178983323833165	1.2746327599638754e-5	6.645545450110731e-5

Tabela 11: Item d , com $h = 0.01$

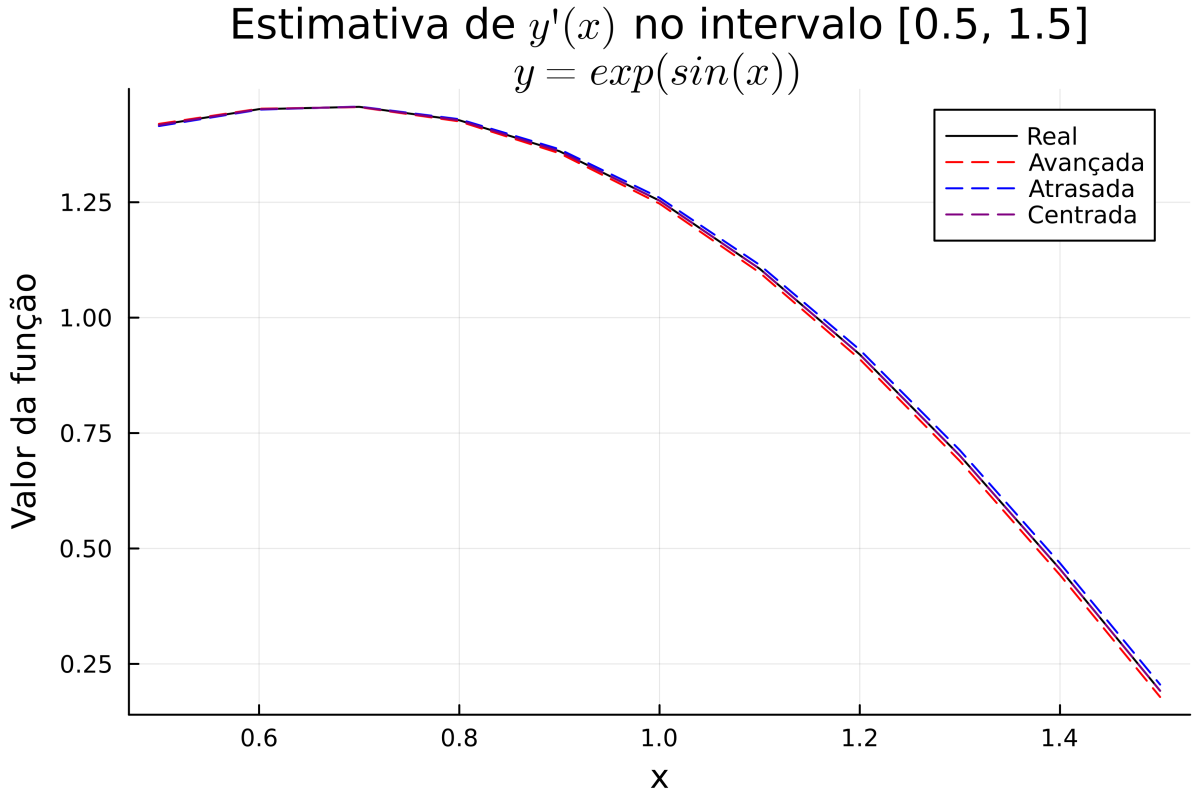


Figura 11: Gráfico plotado para o item d quando $h = 0.01$

2.4.3 $h = 0.0001$

x	ValorReal	ValorAvançada	ErroAvançada	ErroAvançadaRelativa	ValorAtrasada	ErroAtrasada	ErroAtrasadaRelativa	ValorCentrada	ErroCentrada	ErroCentradaRelativa
0.5	1.417424224659313	1.417447698039256	2.3474279864599268e-5	1.656122384266444e-5	1.4174007424982982	2.3482161093157927e-5	1.6566784089499187e-5	1.417424220718777	3.9406142793296794e-9	2.780123417374649e-9
0.6	1.4516158335585816	1.4516260770358613	1.0243450029756573e-5	7.056584664313594e-6	1.4516055803959205	1.0253189911102467e-5	7.063294346807091e-6	1.451615828715891	4.869940672946882e-9	3.35484124674845e-9
0.7	1.4566392950360747	1.4566336487775722	5.646258502522983e-6	3.87622283825808e-6	1.4566440298936135	5.634857538794336e-6	3.8683959426309345e-6	1.4566392893355928	5.700481864323592e-9	3.913447813573103e-9
0.8	1.427558974079829	1.4275343037573052	2.37032252387501e-5	1.6651037149011827e-5	1.427581831716651	2.3757636822052675e-5	1.66421908542595e-5	1.4275589677360781	6.342850911167375e-9	4.433147376162493e-9
0.9	1.360543790846827	1.3605603454841835	4.34453626444613e-5	3.192251560992346e-5	1.3605872227717697	4.343192402776194e-5	3.192247484785725e-5	1.3605437841279766	6.7188503649617815e-9	4.93835656745575e-9
1.0	1.253380767493447	1.253317019722644	6.374777080297088e-5	5.086065819444862e-5	1.2533445017564622	6.373426301520209e-5	5.084988112074674e-5	1.253380760739553	6.7538938860026e-9	5.388541186974588e-9
1.1	1.1058997784971287	1.1058162123323	8.356616482863011e-5	7.556395837441346e-5	1.1059833318771695	8.355338004073509e-5	7.555239784411624e-5	1.1058997721047348	6.392393947507458e-9	5.7802651486145085e-9
1.2	0.9202736595171314	0.9201719731422031	0.00010168637492824484	0.00011049580076169929	0.9203753346520571	0.00010167513492576052	0.00011048358700075105	0.9202736538971301	5.620001242157002e-9	6.106880474125298e-9
1.3	0.7011160150997927	0.7009991134365734	0.00011690166321931539	0.00016673654673638613	0.7012329078381896	0.00011689273839687875	0.00016672381728470562	0.7011160106373815	4.462411218320028e-9	6.364725840251809e-9
1.4	0.45534477137668733	0.45521663627479114	0.00012813510189618738	0.0002814023789244011	0.45547290051572986	0.0001281291390425321	0.00028138928367431787	0.4553447683952605	2.9814268276417977e-9	6.547625041631126e-9
1.5	0.191802579659312	0.19166802223580248	0.00013455733012870662	0.0007015407740251798	0.1919371343506171	0.00013455478468590854	0.0007015275028647673	0.1918025782932698	1.2727214004915055e-9	6.63538026334054e-9

Tabela 12: Item d , com $h = 0.0001$

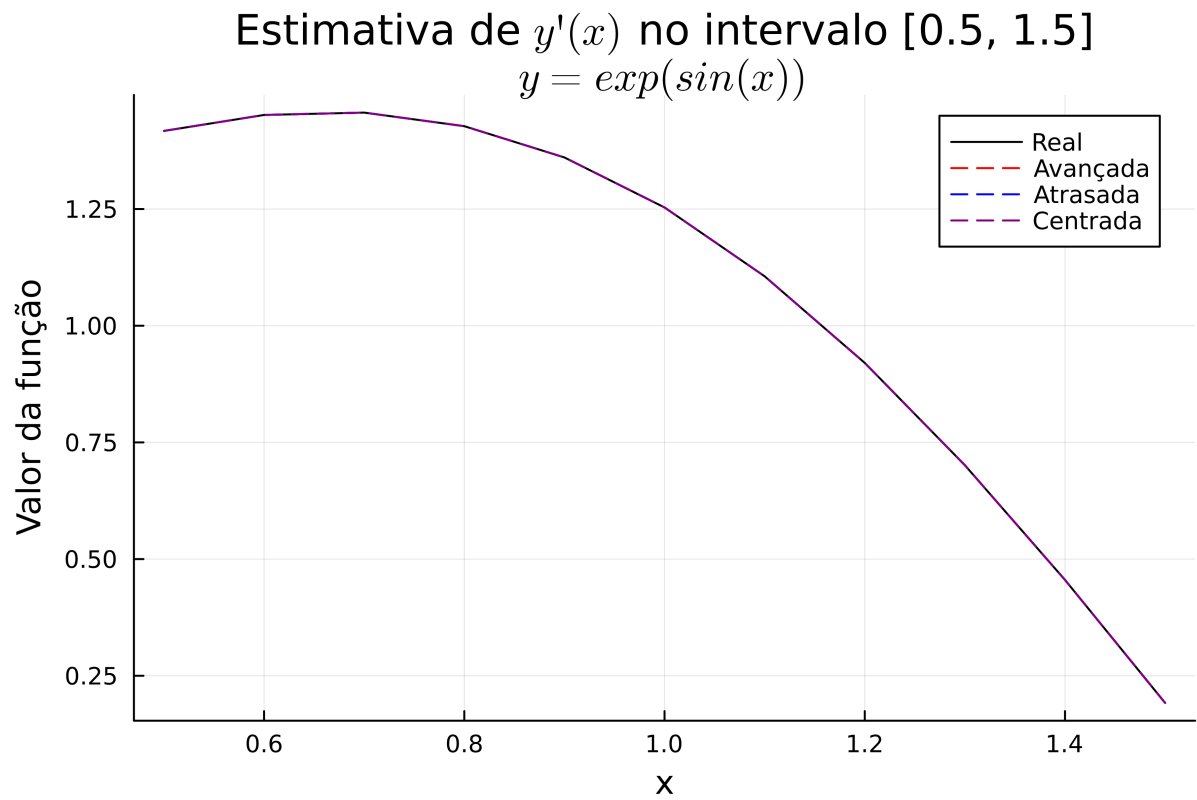


Figura 12: Gráfico plotado para o item d quando $h = 0.0001$