Crop–Livestock Circular Optimization

MGSC 662 – Decision Analytics

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1 Introduction

Modern agriculture in Western Canada relies heavily on synthetic fertilizers to sustain high yields of cereal, oilseed, and pulse crops. While effective, these inputs are energy-intensive to manufacture and contribute to greenhouse gas emissions and nutrient losses. At the same time, Alberta hosts a large cattle population whose manure represents a major, but underutilized, nutrient source. Integrating grazing livestock into crop rotations can allow nutrients to circulate more efficiently between animals and fields.

This project studies how to design such integrated systems at the scale of Alberta's Census Consolidated Subdivisions (CCS). Our central decision problem is to plan multi-year crop rotations and pasture allocation so as to meet crop demand while minimizing synthetic fertilizer use. The optimization must incorporate agronomic rotation structure, field-size distributions, and cattle grazing capacity.

We developed three mathematical models that increase in realism to find the optimal crop rotation:

Model 1: **Base crop-rotation allocation model.** Aggregated land at the provincial level is allocated to crops across a 6-year horizon. Rotation effects appear through nutrient carryover. No CCS disaggregation, no field sizes, no cattle.

Model 2: **CCS-level model with fixed field identifiers.** Land is disaggregated to CCS units and each field has a unique identifier. Rotations operate at field-level. Computationally heavy and unrealistic because field-size distribution cannot be enforced.

Model 3: **CCS model with numbers of fields of fixed sizes.** Each CCS is endowed with fields in five size classes using proportions from Lesiv et al. Decision variables represent flows of fields between crops and years. This enforces realistic fragmentation and maintains empirical field-size distributions.

We integrated the cattle in our final model:

Final Model: **Integrated crop–livestock model (main focus).** Pasture is introduced as a grazing crop. Cattle numbers impose grazing capacity constraints. Manure nutrients offset synthetic fertilizer for crops following pasture. This model captures nutrient circularity and land–livestock interactions.

The remainder of the report focuses on the final integrated model. Section 2 presents the full mathematical formulation. Section 3 provides numerical results. Section 4 discusses extensions, and Section 5 concludes with recommendations.

2 Problem Description and Formulation

This section summarizes the real-world decision problem and presents the complete mathematical formulation of the final integrated crop–livestock model.

2.1 Sets and Indices

We model nutrients in fertilizer oxide form:

$$\mathcal{N} = \{N,\ P_2O_5,\ K_2O,\ S\}.$$

Other sets:

Symbol	Description
I	CCS units
J	Crops
J^{past}	Pasture crop
J^{grass}	Grass hay + pasture
S	Field-size classes
$K = \{0, \ldots, 5\}$	Years in rotation (cyclic)

2.2 Parameters

Parameter	Meaning
$A_{i,j}$	Baseline area (ha) of crop j in CCS i
β	Minimum required share of baseline area
$Up_{j,n}$	Uptake of nutrient n by crop j (kg/ha)
$Re_{j,n}$	Removal of nutrient n by crop j (kg/ha)
S_s	Field size (ha) in class s
p_s	CCS proportion of fields in size class s
c_i	Number of cattle in CCS i
ρ	Stocking rate (cows/ha of pasture)
α	Fraction of the year spent grazing
m_n	Manure nutrient excretion (kg per cow per year)

Baseline crop demand:

$$D_{i,j} = \beta A_{i,j}.$$

2.3 Decision Variables

We define the number of fields in CCS i, crop j, size s in year k, transitioning to crop l and size r in year k+1 as our decision variables (we don't index on i for simplicity:

$$x^k_{j_s,l_r} \geq 0$$

The derived cultivated area for one crop l in year k+1 is then equal to:

$$\sum_{s,r \in S^2} \sum_{j \in J} S_s\, x^k_{j_s,l_r}$$

2.4 Nutrient Balance and Objective

To induce a rotation and ensure that the solver takes into consideration its choice based on what remains in the soil, we introduce the quantity of nutrients to bring in the fields in kg with:

Non-pasture predecessor $j \notin J^{past}$:

$$q_{n,j_s,l_r} = S_s \left(Up_{l,n} - [Up_{j,n} - Re_{j,n}]\right)$$

Pasture predecessor $j \in J^{past}$ (with manure credit):

$$q_{n,j_s,l_r} = S_s \left(Up_{l,n} - [Up_{j,n} - Re_{j,n}] - \alpha \rho m_n\right)$$

2.5 Objective Function

Having $\Omega = I \times \mathcal{N} \times J^2 \times S^2 \times K$

$$\min_x \sum_{\Omega} x^k_{j_s,l_r} \times q_{n,j_s,l_r}$$

2.6 Constraints

(1) Crop demand: Since we don't have enough data to simulate weather constraints, the infrastructure and equipment available on Alberta's farms, or even the specific needs of the cattle (corn silage, etc.), we set the area allocated to each crop so that it reproduces the amounts already observed. This can be interpreted as an average representation of all the external constraints that lead to the current distribution of land across crops.

$$\sum_{r \in S} \sum_{j \in J, s \in S} x^k_{j_s,l_r} \times S_s \geq D_{i,j}, \quad j \notin J^{grass},$$

$$\sum_{l \in J^{grass}} \sum_{r \in S} \sum_{j \in J, s \in S} x^k_{j_s,l_r} \times S_s \geq \sum_{l \in J^{grass}} D_{i,l}.$$

(2) TMS conservation: The total cultivated area should remain constant across the rotation years.

$$\sum_{j \in J, s \in S} x^k_{j_s,l_r} \times S_s = \sum_{j \in J, s \in S} x^{k+1}_{l_r,j_s} \times S_r \text{ with } 5 + 1 = 0[5]$$

(3) Field-size distribution: To better reflect reality, we constrain field sizes to follow the distribution reported by Lesiv et al. (2018).

$$p_s \times \sum_{l \in J, r \in S} \sum_{j \in J, s \in S} x^k_{j_s,l_r} = \sum_{l \in J} \sum_{j \in J, s \in S} x^k_{j_s,l_r}$$

(4) Cattle grazing capacity: Whenever pasture is allocated, the calculated number of selected cows should not exceed the number of cows available.

$$\sum_{j \in J^{past}, s \in S, l \in J, r \in S} x^k_{j_s,l_r} \times S_s \times \rho \leq C_i$$

(5) Non-negativity:

$$x^k_{j_s,l_r} \geq 0, \qquad x^k_{j_s,l_r} \times q_{n,j_s,l_r} \geq 0.$$

3 Numerical Implementation and Results

The final integrated model was implemented in Python using `gurobipy`. Crop areas and cattle numbers at the CCS level are taken from the 2021 Agricultural Census [2], while agronomic nutrient coefficients (uptake and removal of N, P_2O_5, K_2O, and S) come from Alberta government publications [3]. Manure nutrient excretion values [5] and typical grazing durations [4] are taken from extension sources. Field-size distributions follow Lesiv et al. (2018) [1], ensuring that each CCS reproduces observed land fragmentation patterns.

We analyze two scenarios:

- **No Integration (crop-only):** All grass is treated as forage. No manure offsets. Cattle do not affect rotations.
- **With Integration (crop–livestock):** Pasture supports grazing. Manure nutrients reduce synthetic fertilizer requirements for the following crop, subject to grazing capacity constraints.

3.1 Objective Function Values

The optimization objective is the total amount of synthetic nutrients (N, P_2O_5, K_2O, S) required across all CCS units over the six-year rotation. Table 3.1 summarizes results.

Scenario	Synthetic Nutrients Required (Mt)
No Integration	**6.55**
With Integration	**6.35**

The integrated system reduces synthetic nutrient requirements by:

$$\frac{6.55 - 6.35}{6.55} \approx 3\%.$$

Though seemingly modest in percentage terms, this reduction corresponds to approximately CAD 200 million in direct fertilizer cost savings annually, along with roughly CAD 100 million in reduced environmental and social costs from lower greenhouse gas emissions.

3.2 Crop-Allocation Behavior

We now analyze how the structure of rotations changes between scenarios. The optimization produces a transition network in which each node represents a crop in a given year, and directed edges represent transitions between years, weighted by the number of fields undergoing that transition.

No Integration. In the crop-only scenario, the transition network is highly structured, symmetric, and visually "hexagonal" due to the cyclic six-year rotation. Key characteristics include:

- **Grass remains grass** in nearly all CCS units because there is no incentive to rotate grass into other crops.
- Low-density connectivity: only a few transitions dominate.

- Highly predictable crop sequences that are simple for farmers to execute.
- However, the system does **not exploit manure-based nutrient recycling**.

With Integration. Once cattle integration is activated, the rotation network becomes significantly more dynamic. Pasture provides manure nutrients to subsequent crops, which reshapes the optimal rotation structure:

- Pasture fields **rarely stay pasture**; they frequently transition into wheat or barley.
- Canola and rye **often shift into pasture**, enabling nutrient replenishment.
- The transition network becomes **much denser**: many more paths connect crop nodes across years.
- There is far more **reallocation of fields** across crop types, enabling nutrient savings.

The resulting network is more complex but also more agronomically efficient. Figure 2 shows an example for one representative CCS.

3.3 Visualization of Integrated Rotations

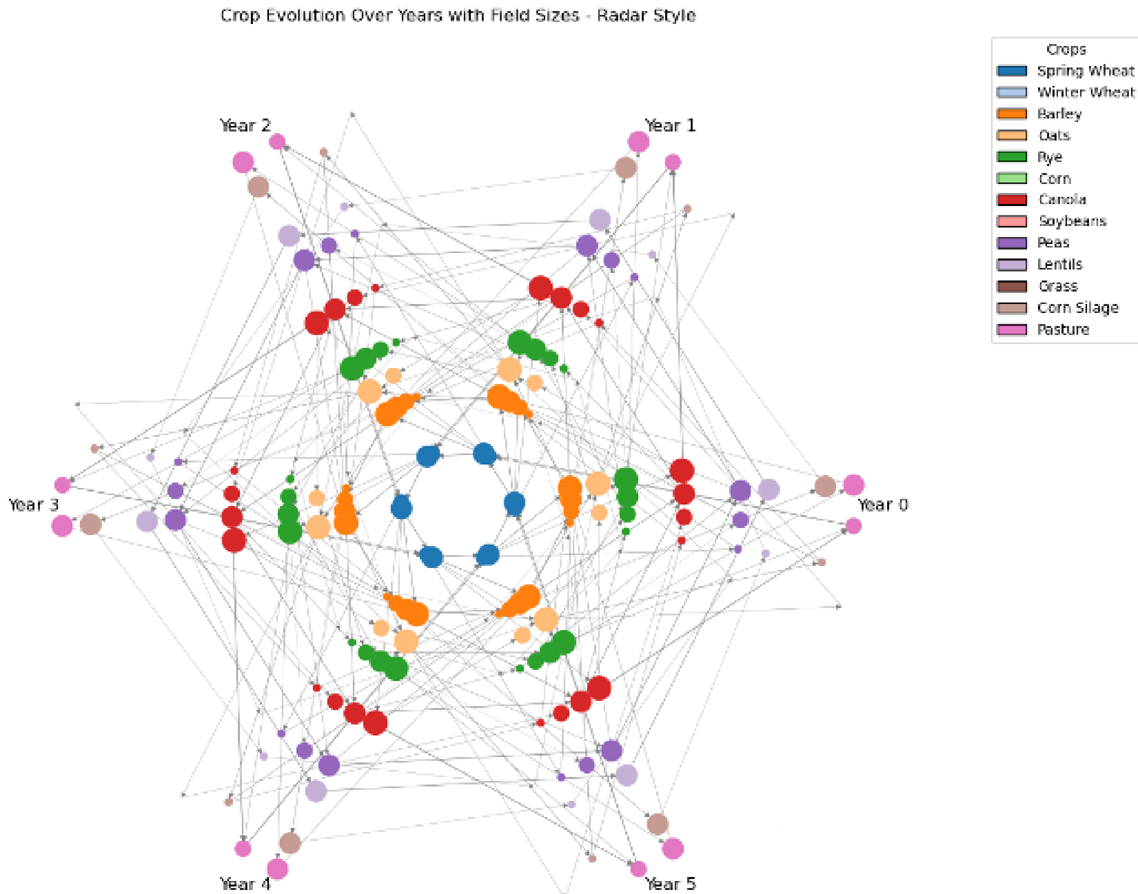

Figure 1: Rotation network for a representative CCS under the **integrated crop–livestock scenario**. Nodes represent crops in each year of the six-year cycle, color-coded by crop type and scaled by field size. Edges represent transitions between crops from one year to the next. Compared to the crop-only baseline, the integrated network is far denser: pasture regularly transitions into cereals, while cereals and oilseeds transition into pasture, reflecting the value of manure nutrients. The increased connectivity captures more dynamic and nutrient-efficient rotations.

3.4 Key Findings

The integrated model yields several notable insights:

- **Pasture plays a central role.** Pasture → wheat/barley transitions appear frequently, and canola/rye → pasture transitions provide manure nutrient replenishment.
- **Grass hay is not selected.** All grass fields in the solution are allocated as pasture rather than hay, because the manure benefit outweighs forage production.
- **Pulse crops (peas and lentils) show partial field stability.** These remain on specific suitable fields for several years but still rotate into cereals regularly.

- **Rotation diversity is consistently high.** Very few fields remain in the same non-pulse crop for consecutive years, even without cattle integration.
- **Grazing potential is limited geographically.** Alberta's pasture share is only about 13%, restricting the province-wide scale of manure recycling.

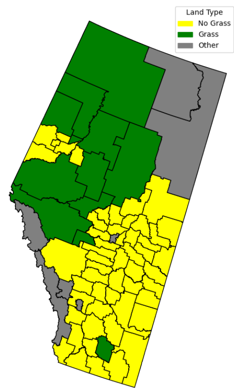

Figure 2: CCS where grass is still part of the rotation mix

Overall, the integrated model demonstrates that although cattle-grazing capacity constrains nutrient recycling, meaningful reductions in synthetic fertilizer usage are achievable with more dynamic, rotation-intensive land-use patterns.

4 Problem Extensions

The project developed a sequence of four increasingly realistic models, culminating in a fully integrated crop–livestock system with CCS-level resolution, field-size distributions, grazing capacity, and manure-based nutrient recycling. While the final model captures many essential agronomic and livestock interactions, several simplifying assumptions remain. This section outlines both the limitations of the current formulation and promising extensions for future work.

4.1 Model Limitations

Although the integrated model is considerably richer than the base crop-only formulation, several important agronomic and economic dimensions are not yet included:

- **Constant yields.** Crop yields are assumed to be fixed and do not depend on nutrient application, soil fertility, rotation history, or weather variability. This abstraction ignores the diminishing returns of fertilizer and the agronomic benefits of diversified rotations.
- **No explicit modeling of soil fertility decay.** Intensive cropping of cereals or oilseeds can reduce soil organic matter, nutrient reserves, and structure over time. The current model assumes that all land remains equally productive regardless of crop sequencing or nutrient removal patterns.
- **No fallow or soil-recovery periods.** The model does not allow for deliberate fallow years or green manure crops (e.g., clover, cover crops) that replenish soil nitrogen or organic matter. Some crop-intensive CCS units might require soil rest for sustainable long-term rotations.

- **No penalties or costs associated with pasture.** Choosing pasture is always either neutral or beneficial because of manure nutrients. In reality, pasture involves opportunity costs and may require fencing, water points, and grazing management.
- **Crop profitability is not considered.** The model minimizes synthetic nutrients but does not reflect crop prices, production costs, or profit margins. An economically optimal rotation may differ from a fertilizer-minimizing one.
- **Diseases and pest pressures are not modeled.** Continuous or frequent planting of the same crop increases risks of disease (e.g., blackleg in canola, aphanomyces in peas). These agronomic constraints are not included.
- **Livestock nutritional needs are simplified.** Cattle only influence the model through grazing capacity and manure nutrients; feed requirements are not modeled beyond baseline crop demand.

These limitations define important directions for refinement.

4.2 Proposed Model Extensions

Building on the limitations above, several promising extensions could significantly enhance the model's realism and usefulness for agricultural planning.

1. Soil Fertility Decay and Land Recovery (Professor's Recommendation)

One major extension concerns the **dynamic degradation and regeneration of soil quality**. Several mechanisms could be incorporated:

- **Soil nutrient depletion from repeated cropping.** Intensive cultivation of nutrient-demanding crops (e.g., canola, wheat) could impose a soil-fertility decay factor. Let's call γ_j the coefficient of soil depletion/ha for seeding a crop j year on year, we would have another objective:

$$\min \sum_{j,s,r \in J \times S^2, k \in K} \gamma_j \times x^k_{j_s,j_r} \times S_s$$

  As another minimisation problem, we would have just to add it to our current objective function and solve the problem with:

$$\min_x \sum_{\Omega} x^k_{j_s,l_r} \times q_{n,j_s,l_r} + \sum_{j,s,r \in J \times S^2, k \in K} \gamma_j \times x^k_{j_s,j_r} \times S_s$$

- **Mandatory fallow / soil-recovery periods.** Fields may be required to lie fallow after prolonged cereal/oilseed sequences. For example:

  If $j_{k-1}, j_{k-2}, j_{k-3}$ are cereals, then $x_{i,j,s,l,r,k}$ must include one fallow cycle.

  In our case, it would imply defining binary variables, which would substantially complexify the resolution complexity of our problem. We would add the following constraints:

$$\forall j, l \in (J \backslash J^{Grass})^2, x^k_{j_s,l_r} \leq M \times y^k_{j_s,l_r}, \text{with } y^k_{j_s,l_r} = \begin{cases} 1 & \text{if } x^k_{j_s,l_r} > 0 \\ 0 & \text{if } x^k_{j_s,l_r} = 0 \end{cases}$$

$$\forall j, l \in (J \backslash J^{Grass})^2, y^k_{j_s,l_r} + y^{k+1}_{j_s,l_r} + y^{k+2}_{j_s,l_r} + y^{k+3}_{j_s,l_r} \leq 3$$

- **Inclusion of cover crops and green manure.** These crops provide soil regeneration and could supply nitrogen biologically without synthetic inputs.
- **Soil-quality-based yield responses.** Introduce an endogenous yield function:

$$\text{Yield}_{i,j,k+1} = f(\sum_{s,rS^2} \gamma_j \times x^k_{j_s,j_r} \times S_s, Up_{j,N}, Up_{j,P_2O_5}, Up_{j,K_20}, Up_{j,S}).$$

  This links crop revenues, fertilizer use, and rotation decisions. The yield and uptake are most likely related by a non-linear expression, which would make our program even harder to solve.

Adding fertility decay and regeneration constraints would transform the model into a multi-period soil-health planning tool, aligning with the professor's suggestion that land must occasionally "rest" to avoid long-term degradation.

2. Economic Optimization

A natural next step is to extend the objective from fertilizer minimization to **profit maximization**, including:

- crop prices and revenue,
- fertilizer costs,
- cattle production costs and value of beef,
- penalties or incentives for stewardship practices.

This would allow the integrated model to recommend not just environmentally efficient rotations, but also economically viable ones.

3. Yield and Weather Variability

The model could incorporate:

- stochastic crop yields,
- drought probabilities,
- risk-averse objectives (CVaR),
- climate change scenarios.

This would provide more realistic recommendations for long-term crop diversification.

4. Detailed Livestock–Feed Interactions

Currently cattle contribute only manure. A next step would include:

- feed requirements of cattle,
- interactions between feed crops (e.g., silage, barley) and herd size,
- trade-offs between grazing vs. feedlot systems.

5. Infrastructure and Spatial Constraints

Future versions could incorporate:

- fencing and water access required for grazing,
- distance between pasture and feedlots,
- transportation costs for manure or forage.

6. Integration of Soil Condition and Environmental Metrics

Examples include:

- Soil organic carbon dynamics,
- Erosion risk and ground cover,
- Nutrient-leaching and water-quality impacts,
- Greenhouse-gas accounting (N_2O).

Each extension would further bridge agronomic science, livestock management, and quantitative optimization.

5 Recommendations and Conclusions

This project developed a sequence of four mathematical programming models culminating in an integrated crop–livestock system that incorporates CCS-level spatial structure, field-size distributions, agronomic rotations, grazing capacity, and nutrient recycling through manure. The final model demonstrates that strategically incorporating pasture into rotations and allowing cattle to graze can reduce synthetic fertilizer needs by approximately 3% at the provincial scale. Although the numerical magnitude is moderate, the economic and environmental impacts are substantial when applied across millions of hectares.

5.1 Recommendations

Based on our findings, we offer several recommendations for practitioners, policymakers, and analysts:

- **Promote targeted crop–livestock integration.** The fertilizer savings are largest in CCS units that have enough grassland to support grazing but also significant crop acreage. Efforts to encourage grazing in these areas could help close local nutrient loops.
- **Support transition toward managed pasture systems.** Many cattle in Alberta are currently raised in feedlots, which limits nutrient recycling. Investments in fencing, water access, and rotational grazing infrastructure would make integration more feasible.
- **Encourage diverse crop rotations.** The model consistently selects rotations that avoid repetitive monoculture. This supports agronomic best practices and could improve soil health, reduce disease pressure, and increase long-term resilience.

- **Use nutrient minimization as one dimension of planning.** While reducing synthetic fertilizer has clear environmental benefits, economic and logistical considerations must also be included. The model's output should be seen as a guide for sustainable land management rather than a strict prescription.

- **Perform CCS-specific planning.** Because grazing potential and crop composition vary widely across Alberta, integration strategies must be localized. Some CCS units have high manure-recycling potential; others remain crop-dominant.

5.2 Conclusions and Key Insights

The integrated crop–livestock model provides several important insights:

- **More dynamic land-use patterns emerge with integration.** Pasture fields frequently transition into cereals, and cereals often rotate into pasture. This creates a denser, more interconnected crop-transition network that enhances nutrient recovery.

- **Significant fertilizer savings are feasible.** Even a 3% reduction represents hundreds of millions of dollars in input cost savings and large reductions in emissions associated with fertilizer production and application.

- **Rotation diversity strengthens agronomic resilience.** In both scenarios, the optimization avoids repeated monoculture, demonstrating that rotational diversity is both agronomically and environmentally optimal.

- **Grazing potential is constrained by land availability.** Alberta's grass and pasture area (13%) limits the scale of manure recycling. Most cattle remain in feedlots, so the potential for circularity is geographically uneven.

5.3 What We Would Do Differently

If developing the project again, several improvements would be prioritized:

- **Model soil fertility decay and regeneration.** As suggested by the instructor, incorporating soil-quality dynamics, fallow periods, and cover crops would better capture long-term sustainability and prevent unrealistic continuous cropping.

- **Include economic optimization.** Integrating crop prices, production costs, and fertilizer costs would allow full profit-based decision making, rather than focusing purely on nutrient minimization.

- **Introduce stochastic crop yields.** Weather-driven yield variability could be included to study risk and resilience across CCS units.

- **Improve representation of grazing logistics.** Fencing, water infrastructure, and grazing accessibility are practical constraints that affect the feasibility of converting cropland to pasture.

- **Expand spatial coverage.** Incorporating multiple provinces (e.g., Saskatchewan, Manitoba) would allow comparison across regions with different livestock densities and cropping systems.

5.4 Final Remarks

The integrated crop–livestock optimization model demonstrates how mathematical programming can support more sustainable land-use decisions. By linking grazing, manure nutrients, crop rotations, and spatial field structures, the model highlights the potential to reduce synthetic fertilizer use and improve nutrient cycling across agricultural landscapes.

While further work is needed to incorporate soil dynamics, economics, and operational logistics, the modeling framework developed here provides a strong foundation for data-driven agricultural planning. It offers a clear quantitative perspective on how crop-livestock integration can contribute to both environmental sustainability and economic efficiency.

6 Appendices

6.1 Field size classes

Field Size (s)	**Surface (in ha)**	**Distribution (%) -** p_s
Very Small	0.32	4
Small	1.6	5
Medium	9.28	19
Large	58	61
Very Large	200	11

Table 1: Field size categories, surface, and distribution percentages.

6.2 Uptake and Removal table [3]

ID	**Crop**	**Category**	N **(Up)**	N **(Re)**	P_2O_5 **(Up)**	P_2O_5 **(Re)**
0	Spring Wheat	Cereals	85	60	32	23
1	Winter Wheat	Cereals	67	52	30	25
2	Barley	Cereals	111	77	45	34
3	Oats	Cereals	116	61	40	25
4	Rye	Cereals	92	58	45	24
5	Corn	Cereals	153	97	63	44
6	Canola	Oilseeds	112	68	52	37
7	Soybeans	Oilseeds	180	135	31	29
8	Peas	Oilseeds Pulse Crops	153	117	43	35
9	Lentils	Oilseeds Pulse Crops	92	62	25	18
10	Grass	Forages	102	102	30	30
11	Corn Silage	Forages	156	156	63	63

Table 2: Crop nitrogen and phosphorus requirements.

ID	**Crop**	**Category**	K_2O **(Up)**	K_2O **(Re)**	**S (Up)**	**S (Re)**
0	Spring Wheat	Cereals	73	18	9	5
1	Winter Wheat	Cereals	70	17	10	7
2	Barley	Cereals	106	26	13	7
3	Oats	Cereals	145	18	13	5
4	Rye	Cereals	130	20	15	5
5	Corn	Cereals	128	27	15	6
6	Canola	Oilseeds	81	18	19	11
7	Soybeans	Oilseeds	129	49	12	4
8	Peas	Oilseeds Pulse Crops	136	35	12	6
9	Lentils	Oilseeds Pulse Crops	76	33	9	5
10	Grass	Forages	130	130	13	13
11	Corn Silage	Forages	201	201	13	13

Table 3: Crop potassium and sulfur requirements.

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