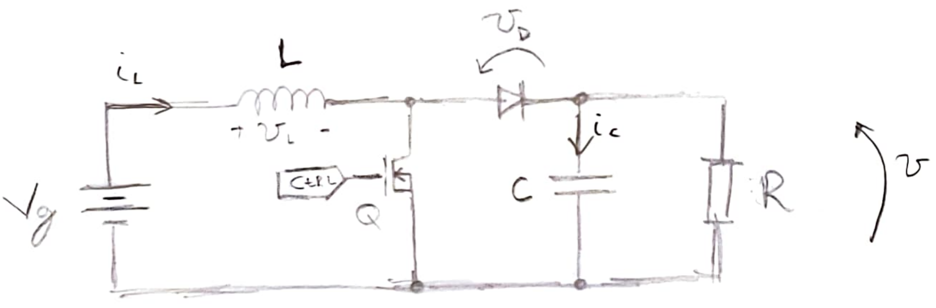
DC BOOST CONVERTER ANALYSIS



1.) Q ON:

- $v_L = V_g$
- $i_c = -\dfrac{v}{R}$

2.) Q OFF:

- $V_g - v_L - v_D - v = 0$
  $\rightarrow v_L = V_g - (v + v_D)$
- $i_c = i_L - \dfrac{v}{R}$

ASSUMING WE DESIRE SMALL RIPPLE:

$v \cong V = cte$; $v_D \cong V_D = cte$

$i_L \cong I = cte$

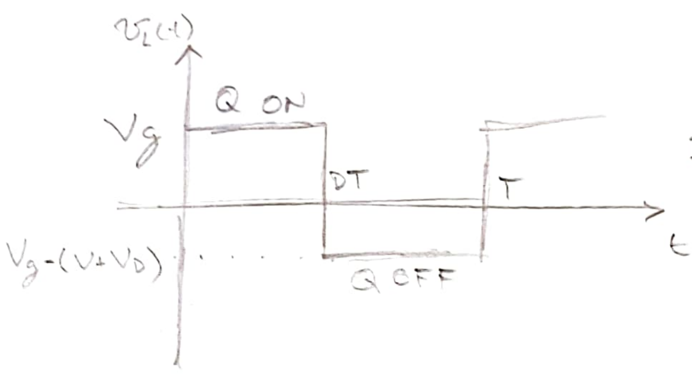
$\rightarrow$ 1.) Q ON:

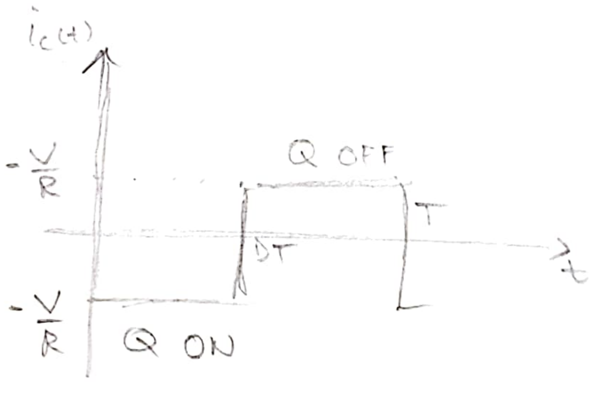
- $v_L = V_g$
- $i_c \cong -\dfrac{V}{R}$

2.) Q OFF:

- $v_L \cong V_g - (V + V_D)$
- $i_c \cong I - \dfrac{V}{R}$

THEN:





$0 < D < 1$: DUTY CYCLE

CONVERSION RATIO: M(D)

$$M(D) = \frac{V}{V_g} \quad (1)$$

ON STEADY-STATE CONDITION (INDUCTOR VOLT-SECOND BALANCE), GIVES US:

$$i_L(T) - i_L(0) = 0 \Rightarrow \frac{1}{L}\int_0^T v_L(t)\,dt = 0$$

$$\Rightarrow V_g \cdot DT + (V_g - V - V_D)\cdot(1-D)\cdot T = 0$$

$$\Rightarrow V_g \cdot T - V(1-D)\cdot T - V_D(1-D)T = 0$$

$$\Rightarrow \frac{V_g}{(1-D)} = V + V_D \Rightarrow \frac{V}{V_g} = \frac{1}{1-D} - \frac{V_D}{V_g}$$

$$\Rightarrow \boxed{M(D) = -\frac{V_D}{V_g} + \frac{1}{1-D}} \quad (2)$$

WE ALSO KNOW THAT, ON STEADY-STATE:

$$\int_0^T i_C(t)\,dt = 0 \Rightarrow -\frac{V}{R}\cdot DT + \left(I - \frac{V}{R}\right)(1-D)T = 0$$

$$\Rightarrow -\frac{V}{R}\cdot T + I(1-D)T = 0 \Rightarrow I = \frac{V}{R}\cdot\frac{1}{1-D}$$

$$\overset{(1)}{\Longrightarrow} \boxed{I = \frac{V_g}{R}\cdot\frac{1}{(1-D)^2} - \frac{V_D}{R}\cdot\frac{1}{(1-D)}} \quad (3)$$

AC ANALYSIS:

1.) Q ON:

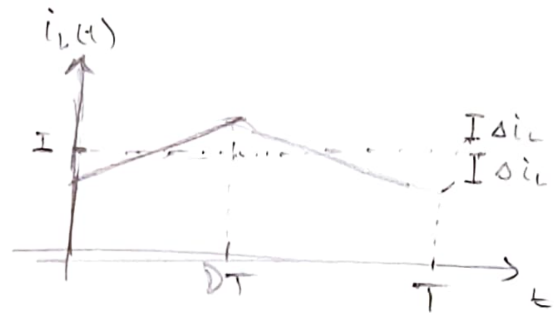
$$v_L(t) = V_g \Rightarrow L\frac{di_L}{dt} = V_g \Rightarrow \frac{di}{dt} = \frac{V_g}{L}$$

2.) Q OFF:

$$v_L(t) \cong V_g - (V+V_D) \Rightarrow \frac{di}{dt} = \frac{V_g - (V+V_D)}{L}$$

ASSUMING

$$i_L \in [I - \Delta i_L, I + \Delta i_L]$$



$$\rightarrow 2\Delta i_L = \frac{V_g}{L} \cdot DT \Rightarrow \boxed{\Delta i_L = \frac{V_g}{2L} \cdot DT} \quad (4)$$

OR

$$-2\Delta i_L = \frac{V_g - (V+V_D)}{L}(1-D) \cdot T$$

LIKEWISE, FOR THE CAPACITOR WE HAVE:

1.) Q ON:

$$i_C(t) = C\frac{dv}{dt} \Rightarrow \frac{dv}{dt} \cong -\frac{V}{RC}$$

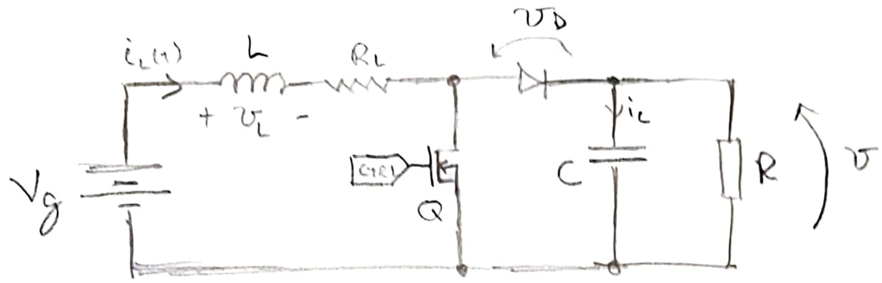
2.) Q OFF:

$$i_C(t) = C\frac{dv}{dt} \Rightarrow \frac{dv}{dt} \cong \frac{I}{C} - \frac{V}{RC}$$

SIMILAR TO THE PREVIOUS ANALYSIS:

$$2\Delta v = -\frac{V}{RC} DT \Rightarrow \boxed{\Delta v \cong \frac{V}{2RC} \cdot DT} \quad (5)$$

INCLUDING INDUCTOR LOSS:



1.) Q ON:

- $v_L = V_g - i_L \cdot R_L$
- $i_C = -\dfrac{v}{R}$

2.) Q OFF:

- $v_L = V_g - (v + v_D) - i_L \cdot R_L$
- $i_C = i_L - \dfrac{v}{R}$

ASSUMING SMALL RIPPLE:

1.) Q ON:

- $v_L = V_g - I \cdot R_L$
- $i_C = -\dfrac{V}{R}$

2.) Q OFF:

- $v_L = V_g - (V + V_D) - I R_L$
- $i_C = I - \dfrac{V}{R}$

ON STEADY-STATE:

- $(V_g - I \cdot R_L) \cdot DT + [V_g - (V + V_D) - I R_L](1-D)T = 0$

$$\Rightarrow \boxed{0 = V_g - I R_L - (1-D)(V + V_D)} \quad (6)$$

- $-\dfrac{V}{R} \cdot DT + \left(I - \dfrac{V}{R}\right)(1-D)T = 0 \Rightarrow \boxed{I = \dfrac{V}{R} \cdot \dfrac{1}{1-D}} \quad (7)$

FROM (7) IN (6):

$$V_g = V \cdot \frac{R_L}{R} \cdot \frac{1}{1-D} + (1-D)V + (1-D)V_D$$

$$V_g - (1-D)V_D = V\left[\frac{R_L}{R}\cdot\frac{1}{1-D} + (1-D)\right]$$

$$\Rightarrow \boxed{\frac{V}{V_g} = -\frac{V_D}{V_g}\cdot\frac{1}{1+\frac{R_L}{R}\cdot\frac{1}{(1-D)^2}} + \frac{1}{(1-D)}\cdot\frac{1}{1+\frac{R_L}{R}\frac{1}{(1-D)^2}}} \quad (8)$$

EFFICIENCY:

$$\eta = \frac{P_{OUT}}{P_{IN}};$$

$$\left.\begin{array}{l} \cdot\ P_{IN} = V_g \cdot I \\ \cdot\ P_{OUT} = V\cdot\frac{V}{R} \end{array}\right\} \Rightarrow \eta = \frac{V^2}{V_g}\cdot\frac{1}{RI} \overset{(4)}{=} \frac{V}{V_g}\cdot(1-D)$$

$$\overset{(5)}{\Rightarrow} \boxed{\eta = -\frac{V_D}{V_g}\cdot\frac{(1-D)}{1+\frac{R_L}{R}\cdot\frac{1}{(1-D)^2}} + \frac{1}{1+\frac{R_L}{R}\cdot\frac{1}{(1-D)^2}}} \quad (9)$$

INCLUDING SEMICONDUCTORS LOSSES:

$$\boxed{\frac{V}{V_g} = -\frac{V_D}{V_g}\cdot\frac{1}{1+\frac{R_L+D\cdot R_{ON}+(1-D)R_D}{R(1-D)^2}} + \frac{1}{(1-D)}\cdot\frac{1}{1+\frac{R_L+DR_{ON}+(1-D)R_D}{R(1-D)^2}}} \quad (10)$$

LET $\boxed{\gamma(D) = \frac{1}{1+\frac{R_L+DR_{ON}+(1-D)R_D}{R(1-D)^2}}}$ THE LOSS FACTOR (11)

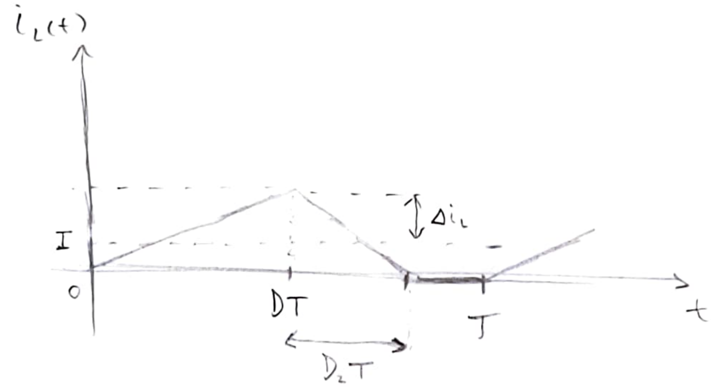
THEN:

$$\boxed{M(D) = \gamma(D) \cdot \left[-\frac{V_D}{V_g} + \frac{1}{1-D} \right]} \quad (12)$$

$$\boxed{\eta = \gamma(D) \left[1 - (1-D) \cdot \frac{V_D}{V_g} \right]} \quad (13)$$

DISCONTINUOUS MODE ANALYSIS:

DISCONTINUOUS CONDUCTION MODE (DCM) HAPPENS WHEN $i_L(t)$ GOES TO ZERO, THEN NO CURRENT FLOWS THROUGH THE DIODE



THEN, THE CONDITION TO STAY IN CONTINUOUS CONDUCTION MODE (CCM) IS:

$$I > \Delta i_L \quad (14)$$

ASSUMING ALL LOSSES:

<u>Q ON:</u> $v_L(t) \cong V_g - R_L \cdot I = L \cdot \frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} \cong \frac{(V_g - R_L \cdot I)}{L}$

THEN:

$$\boxed{\Delta i_L = \frac{V_g}{2L} \cdot DT - \frac{R_L I}{2L} \cdot DT} \quad (15)$$

FROM (15) IN (14):

$$I > \frac{V_g}{2L} DT - \frac{R_L I}{2L} \cdot DT$$

$$\Rightarrow I \left[1 + \frac{R_L DT}{2L} \right] > \frac{V_g}{2L} \cdot DT$$

$$\xrightarrow{(1),(7)} \frac{V_g}{R} \cdot \frac{M(D)}{1-D} \left[1 + \frac{R_L DT}{2L} \right] > \frac{V_g}{2L} DT$$

$$\Longrightarrow \boxed{\frac{2L}{RT} > \frac{D(1-D)}{M(D)}\left[1 - \frac{R_L}{R}\cdot\frac{M(D)}{1-D}\right]} \quad (16)$$

LET

$$\begin{cases} K = \dfrac{2L}{RT} & (17) \\ K_{CRIT}(D) = \dfrac{D(1-D)}{M(D)}\left[1 - \dfrac{R_L}{R}\cdot\dfrac{M(D)}{1-D}\right] & (18) \end{cases}$$

ASSUMING NO LOSSES:

$$K_{CRIT}(D) = \frac{D(1-D)}{M(D)}$$

ASSUMING IDEAL DIODE, FROM (2):

$$M(D) = \frac{1}{1-D} \quad \therefore \quad K_{CRIT}(D) = D(1-D)^2$$

$$\Longrightarrow \max_{0<D<1}\left\{K_{CRIT}(D)\right\} = \frac{4}{27}$$

THEN $\boxed{\dfrac{2L}{RT} > \dfrac{4}{27}}$ (19) GUARANTEE CCM WHEN NO LOSSES ARE CONSIDERED AND ASSUMING IDEAL DIODE.