Lowpass Filter Design Project

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1 Design requirements

The design task is a $50\,\Omega$ low-pass filter on FR-4 ($\varepsilon_r = 4.2$, substrate thickness $d = 0.079\,\text{cm}$, $\tan\delta = 0.02$, 0.5 mil copper) with cutoff frequency $f_c = 3\,\text{GHz}$, 0.5 dB passband ripple, and at least 15 dB attenuation at 4.5 GHz. Because a 0.5 dB equal-ripple passband is specified, the appropriate response class is a Chebyshev-I (equal-ripple) low-pass, which provides a steeper transition than a Butterworth filter of the same order while meeting the ripple requirement [1]. In plots and schematics, calculated *impedance* values are rounded to three decimals to match the implementation convention.

1.1 Minimum order from specifications

Let $A_{\max} = 0.5\,\text{dB}$ (passband ripple), $A_s = 15\,\text{dB}$ (stopband at $f_s = 4.5\,\text{GHz}$), and $\Omega_s = f_s/f_c = 1.5$. For a Chebyshev-I low-pass, the minimum order is

$$N \;\geq\; \frac{\operatorname{arcosh}\left(\dfrac{\sqrt{10^{A_s/10}-1}}{\varepsilon}\right)}{\operatorname{arcosh}(\Omega_s)}, \qquad \varepsilon = \sqrt{10^{A_{\max}/10}-1}. \tag{1}$$

Numerically,

$$\varepsilon \approx 0.3493, \quad \sqrt{10^{A_s/10}-1} \approx 5.534, \quad \operatorname{arcosh}(1.5) \approx 0.9629,$$

$$N_{\min} \approx \frac{\operatorname{arcosh}(15.85)}{0.9629} \approx 3.6.$$

Hence the analytic minimum is $N = 4$. The 0.5 dB Chebyshev attenuation chart (Pozar, 4th ed., p. 407) further indicates that at $|\omega/\omega_c - 1| = 0.5$ the *fifth*-order curve comfortably exceeds 15 dB; therefore the design proceeds with $N = 5$ to provide margin.

1.2 Normalized prototype and scaling to $R_0 = 50\,\Omega$, $f_c = 3\,\text{GHz}$

Using the 0.5 dB Chebyshev table for $N = 5$ [1],

$$g_1 = 1.7058, \quad g_2 = 1.2296, \quad g_3 = 2.5408, \quad g_4 = 1.2296, \quad g_5 = 1.7058, \quad g_6 = 1.$$

For the ladder that starts with a series inductor,

$$L_k = \frac{R_0}{\omega_c}\, g_k, \qquad C_k = \frac{g_k}{R_0\,\omega_c}, \qquad \omega_c = 2\pi f_c. \tag{2}$$

With $R_0 = 50\,\Omega$ and $f_c = 3\,\text{GHz}$, the ideal lumped values are

$$\boxed{\begin{aligned} &L_1 \approx 4.525\,\text{nH}, \quad C_2 \approx 1.305\,\text{pF}, \\ &L_3 \approx 6.740\,\text{nH}, \quad C_4 \approx 1.305\,\text{pF}, \\ &L_5 \approx 4.525\,\text{nH}. \end{aligned}}$$

These values define the part-(a) schematic used to plot the insertion loss $\text{IL}(f) = -20\log_{10}|S_{21}(f)|$ from DC to 6 GHz and to verify: (i) passband ripple $\approx$0.5 dB up to f_c, and (ii) attenuation $\geq$15 dB at 4.5 GHz.

Simulation

A 0.5 dB–ripple, $N = 5$ Chebyshev prototype [1] was impedance/frequency scaled to $R_0 = 50\,\Omega$ and $f_c = 3\,\text{GHz}$. The ADS schematic (lossless elements, $50\,\Omega$ ports) uses the component values in Table 1. S-parameters were simulated from 0 GHz to 6 GHz with 0.1 GHz step (Figure 1).

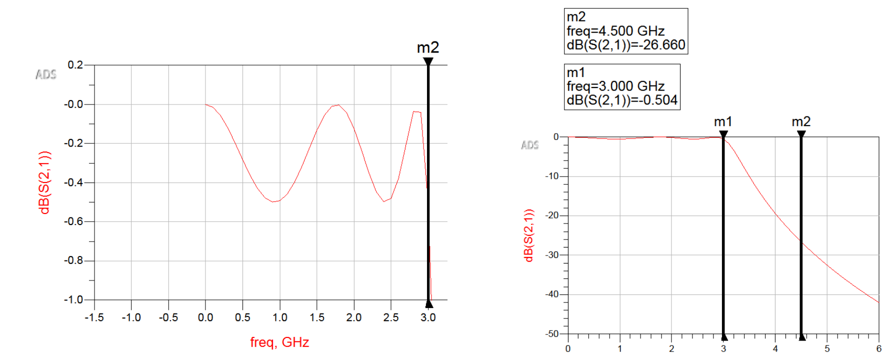
The resulting insertion loss is $\text{dB}\{S_{21}\} = -0.504\,\text{dB}$ at $f_c = 3\,\text{GHz}$, and the stopband attenuation at 4.5 GHz is $\text{dB}\{S_{21}\} = -26.660\,\text{dB}$. Over the entire passband (0–3 GHz), the lowest point is $-0.504\,\text{dB}$ (i.e., $\approx$ 0.504 dB ripple), which satisfies the 0.5 dB equal-ripple specification. Because the nominal values already meet all metrics with comfortable margin, no optimization was required for this part.

Table 1: Component values used in ADS for the lumped Chebyshev LPF ($R_0 = 50\,\Omega$, $f_c = 3\,\text{GHz}$).

Element	Value	Units
L_1	4.525	nH
C_2	1.305	pF
L_2	6.740	nH
C_4	1.305	pF
L_5	4.525	nH

References

[1] D. M. Pozar, *Microwave Engineering*, 4th ed., Wiley, 2012.



(a) Passband (0–3 GHz).

(b) Stopband at 4.5 GHz.

Figure 1: ADS results for the lumped LPF.

Table 2: Performance summary (lumped Chebyshev LPF).

Metric	Target	Measured (ADS)
Passband ripple (0–3 GHz)	0.500	0.504
Insertion loss at 3 GHz [dB]	0.500	0.504
Attenuation at 4.5 GHz [dB]	$\geq$15	26.660

2 Design of the Microstrip-line shunt stubs Low–Pass Filter using Richards' Transformations and Kuroda Identities

The third filter is implemented as a shunt-stub microstrip low–pass filter with cutoff frequency $f_c = 3$ GHz, source and load impedances $Z_S = Z_L = 50\ \Omega$, and a fifth–order Chebyshev response with 0.5 dB passband ripple. The design procedure follows the classical sequence

$$\text{low–pass prototype} \rightarrow \text{Richards transformation} \rightarrow \text{Kuroda identities,}$$

and is summarized in the hand calculations shown in Steps (1)–(7) (see Fig.1-2). Below, each step is described in detail.

Step (1): Lumped shunt-first prototype The starting point is a normalized fifth–order Chebyshev low–pass prototype with 0.5 dB ripple and equal terminations $R_0 = 1\ \Omega$. The standard prototype element values are

$$g_1 = g_5 = 1.7058, \qquad g_2 = g_4 = 1.2296, \qquad g_3 = 2.5408.$$

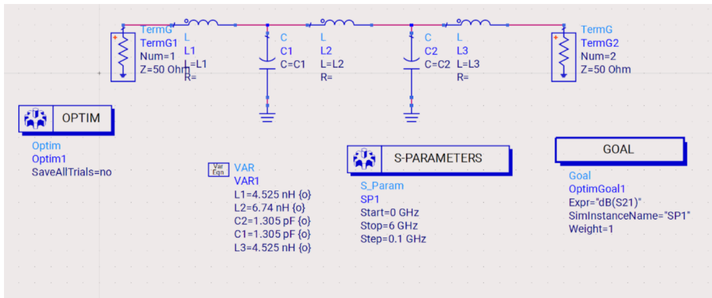


Figure 2: Schematics on Keysight ADS

For subsequent mapping into transmission lines the prototype is realized in a shunt-first topology:

$$C_1 = g_1, \quad L_2 = g_2, \quad C_3 = g_3, \quad L_4 = g_4, \quad C_5 = g_5,$$

with all quantities normalized to $R_0 = 1\ \Omega$ and $\omega_c = 1$ rad/s. This network is sketched in Step (1).

Step (2): Richards transformation to $\lambda/8$ stubs In Step (2) the lumped elements are replaced by $\lambda/8$ transmission–line stubs at the cutoff frequency by means of Richards transformation. For a line section of electrical length $\theta = \pi/4$ at ω_c the equivalent characteristic impedances are

$$\begin{aligned} \text{series inductor } L: &\quad Z_{0L} = \omega_c L, \\ \text{shunt capacitor } C: &\quad Z_{0C} = \frac{1}{\omega_c C}. \end{aligned}$$

Since ω_c is normalized to unity, this simplifies to $Z_{0L} = L$ and $Z_{0C} = 1/C$. Substituting the prototype values gives

$$\begin{aligned} Z_{0C1} &= \frac{1}{g_1} = \frac{1}{1.7058} \approx 0.5862, \\ Z_{0L2} &= g_2 = 1.2296, \\ Z_{0C3} &= \frac{1}{g_3} = \frac{1}{2.5408} \approx 0.3926, \\ Z_{0L4} &= g_4 = 1.2296, \\ Z_{0C5} &= \frac{1}{g_5} \approx 0.5862. \end{aligned}$$

Between successive stubs, unit elements (UEs) of length $\lambda/8$ and characteristic impedance $Z_0 = 1\ \Omega$ are retained. The resulting transmission–line network corresponds to Step (2).

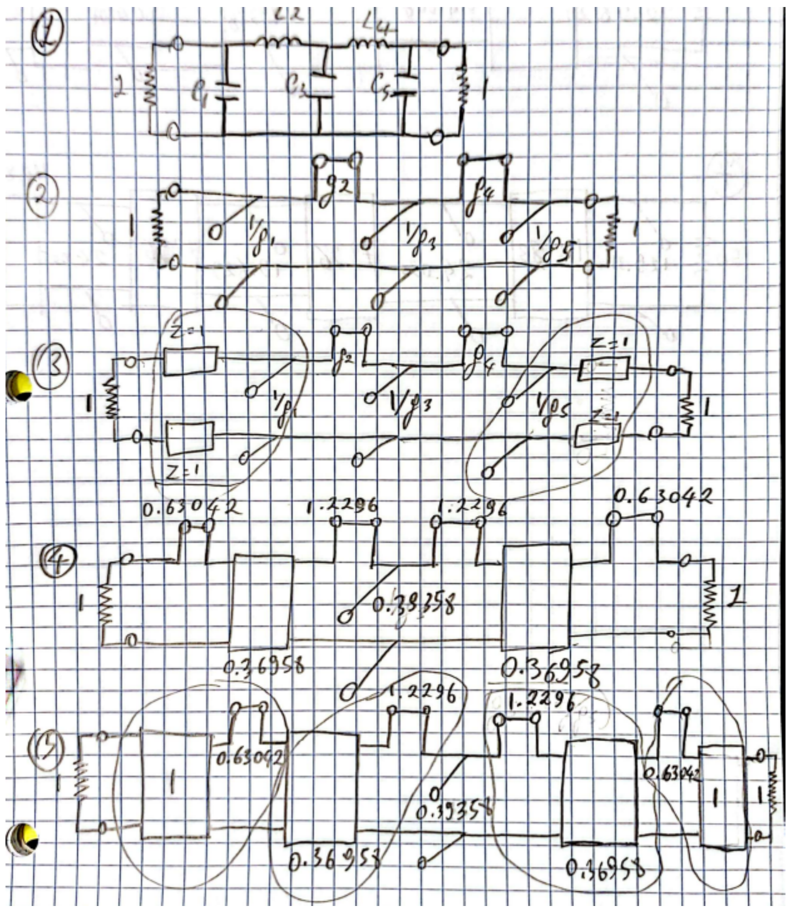


Figure 3: Hand calculations for the Microstrip-line shunt stubs LPF, Steps (1)–(5).

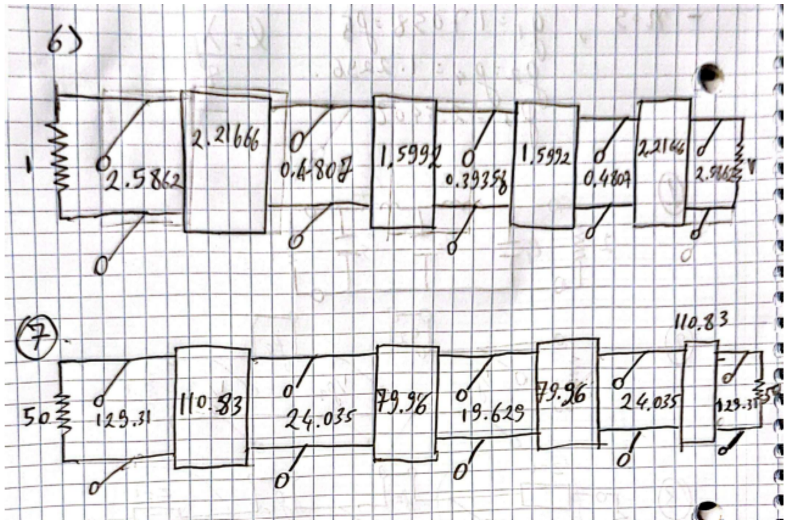


Figure 4: Hand calculations for the Microstrip-line shunt stubs LPF, Steps (6)–(7).

Step (3): Addition of unit elements at the terminations To facilitate the application of Kuroda identities and obtain a practical stepped–impedance layout, additional unit elements with $Z_0 = 1\ \Omega$ and electrical length $\lambda/8$ are inserted at both the input and the output (Step (3)). These unit elements do not change the magnitude of the transfer function; they only introduce a constant phase shift while providing the extra lines required by the Kuroda transformations.

Step (4): First application of Kuroda identities In Step (4) Kuroda's second identity is applied to the end sections, where a unit element of characteristic impedance Z_1 is adjacent to a stub of characteristic impedance Z_2. The identity states that the two–port consisting of this line–stub combination can be replaced by an equivalent two–port in which both impedances are scaled by a factor n^2, defined as

$$n^2 = 1 + \frac{Z_2}{Z_1},$$

so that

$$Z_1' = n^2 Z_1, \qquad Z_2' = n^2 Z_2.$$

At the input, $Z_1 = 1$ and $Z_2 = Z_{0C1} = 0.5862$, which gives

$$n_{\text{in}}^2 = 1 + 0.5862 = 1.5862,$$

and therefore

$$Z'_{0,\text{UE,in}} = 1.5862, \qquad Z'_{0,\text{stub,in}} = 1.5862 \times 0.5862 \approx 0.93.$$

The output end is transformed in an identical way by symmetry. After this first Kuroda step the network is renormalized so that the central unit element again has $Z_0 = 1\ \Omega$; this renormalization produces the intermediate normalized impedances shown in Step (4), such as 0.63042 for the outer stubs and 0.36858 and 0.39258 for the inner unit elements. These quantities are scaled versions of the original 0.5862, 1.2296 and 0.3926.

Step (5): Second Kuroda step and removal of series stubs The remaining series short–circuited stubs associated with L_2 and L_4 are then converted into shunt open stubs and further separated from each other, as illustrated in Step (5). Once again, Kuroda's identity is used with

$$n^2 = 1 + \frac{Z_2}{Z_1},$$

where Z_1 is the impedance of the adjacent unit element and Z_2 is the impedance of the series stub. For example, on one side of the central capacitor the values are approximately $Z_1 = 0.36858$ and $Z_2 \approx 1.2296$, which leads to

$$n_{\mathrm{L}}^2 = 1 + \frac{1.2296}{0.36858},$$

and consequently

$$Z_{0,\mathrm{UE}}^{(\mathrm{new})} = n_{\mathrm{L}}^2 Z_1, \qquad Z_{0,\mathrm{stub}}^{(\mathrm{new})} = n_{\mathrm{L}}^2 Z_2.$$

The same operation is performed on all remaining series stubs. After this second Kuroda step and a final renormalization the network contains only shunt open stubs separated by series line sections, as required for a stepped–impedance realization. The central shunt stub retains the value $Z_{0C3} \approx 0.39258$, while the other stubs and lines take on the intermediate values shown in Step (5).

Step (6): Final normalized impedances Collecting the results of the transformations, the final normalized stepped–impedance network (Step (6)) is symmetric and consists of five open–circuited shunt stubs separated by four series line sections. The characteristic impedances of the stubs are

$$Z_{0,\mathrm{stub}} = 2.5862, \quad 0.4807, \quad 0.39258, \quad 0.4807, \quad 2.5862,$$

and the series line sections have

$$Z_{0,\mathrm{line}} = 2.2166, \quad 1.5992, \quad 1.5992, \quad 2.2166.$$

All sections are designed to have electrical length $\lambda/8$ at the normalized cutoff frequency.

Step (7): Scaling to 50 Ω and micro strip implementation In Step (7) the prototype is denormalized back to the physical system impedance $Z_0 = 50\ \Omega$ by simple scaling,

$$Z_{0,\text{phys}} = 50\ \Omega \times Z_{0,\text{norm}}.$$

This yields the characteristic impedances used in the ADS schematic:

$$\begin{aligned} \text{shunt stubs:} &\quad 129.31\ \Omega,\ 24.04\ \Omega,\ 19.63\ \Omega,\ 24.04\ \Omega,\ 129.31\ \Omega, \\ \text{series lines:} &\quad 110.83\ \Omega,\ 79.96\ \Omega,\ 79.96\ \Omega,\ 110.83\ \Omega. \end{aligned}$$

Each section is then implemented as a microstrip line of length $\lambda_g/8$ (which practically translates to selecting a 45 degree angle on the `LineCalc` tool) at 3 GHz on the specified FR4 substrate ($\varepsilon_r = 4.2$, thickness $d = 0.079$ cm, copper thickness 0.5 mil). The corresponding physical widths are obtained using the ADS `LineCalc` tool. The resulting filter meets the desired Chebyshev response and can be directly compared, in both schematic and Momentum simulations, with the stepped impedance and lumped realizations.

Microstrip shunt–stub LPF (initial, non-optimized)

Using ADS `MLIN`/`MTEE_ADS`/`MLOC` elements, each 90° reactance from the prototype was realized with a $\lambda_g/8$ (i.e., 45° at 3 GHz) microstrip section. The figure below shows the insertion-loss response before any tuning.

Table 3: Shunt open–stub sections (MLOC): geometry $\leftrightarrow$ nominal Z_0.

Stub Placement	Pair (W,L) [mm]	Length (mm)	Nominal Z_0 ()
1 leftmost	$(W_1, L_1) = (0.642061,\ 7.270660)$	7.270660	129.31
2 left–inner	$(W_3, L_3) = (1.085490,\ 7.560600)$	7.560600	24.04
3 center	$(W_5, L_5) = (0.849095,\ 7.127870)$	7.127870	19.63
4 right–inner	$(W_3, L_3) = (1.085490,\ 7.743780)$	7.743780	24.04
5 rightmost	$(W_1, L_1) = (0.642061,\ 7.198210)$	7.198210	129.31

Although the stopband requirement is comfortably met (≥ 15 dB at 4.5 GHz), the passband shows ≈ 1.4 dB loss at 3 GHz, exceeding the 0.5 dB ripple target. The deviation is expected at this stage because: (i) the physical $\lambda_g/8$ lengths differ slightly across sections as ε_{eff} varies with width; (ii) open-end fringing shortens the electrical length of high-Z_0 stubs; (iii) junction discontinuities (`MTEE`) add parasitic shunt and series elements; and (iv) small impedance

Table 4: Through-line sections (MLIN) between stubs.

Section	Pair (W,L) [mm]	Length (mm)	Nominal Z_0 ()	Note
TL2 (inlet→S1)	$(W_2, L_2) = (0.128400,\ 7.560600)$	7.560600	110.83	high-Z line
TL3 (S2→S3)	$(W_4, L_4) = (0.033703,\ 7.743780)$	7.743780	79.96	mid-Z line
TL4 (S3→S4)	$(W_4, L_4) = (0.033703,\ 7.743780)$	7.743780	79.96	mid-Z line
TL5 (S5→outlet)	$(W_2, L_2) = (0.128400,\ 7.560600)$	7.560600	110.83	high-Z line

errors (width quantization) detune the Chebyshev response. A short optimization/tuning step that slightly perturbs the widths/lengths (and includes open-end/tee corrections) is therefore required.

Table 5: Microstrip implementation (non-optimized): mapping from target impedances to physical widths and lengths.

Element	Function	Target Z_0 [Ω]	Width W [mm]	Length L [mm]
Stub 1	shunt OC	129.31	0.155	7.533
Line 1	series	110.83	0.602	7.299
Line 2	series	79.96	0.630	7.275
Stub 2	shunt OC	24.04	4.504	6.668
Line 3	series	79.96	0.630	7.275
Stub 3	shunt OC	19.63	5.784	6.591
Line 4	series	79.96	0.630	7.275
Stub 4	shunt OC	24.04	4.504	6.668
Line 5	series	110.83	0.602	7.299
Stub 5	shunt OC	129.31	0.155	7.533

2.1 Optimized simulation

The optimization perturbs widths slightly but preserves the intended high/low assignment.

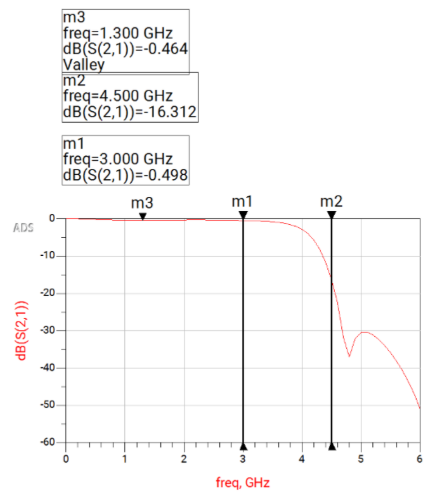
Table 6: Optimized microstrip stubs (widths are optimized; Z_0 values are the synthesis targets).

Stub	Physical width used	Length L_i (mm)	Nominal Z_0 ()	Note
1	$W_1 = 0.642061\,\mathrm{mm}$	$L_1 = 7.270660$	129.31	synthesis target
2	$W_3 = 1.085490\,\mathrm{mm}$	$L_2 = 7.560600$	24.04	synthesis target
3	$W_5 = 0.849095\,\mathrm{mm}$	$L_3 = 7.127870$	19.63	synthesis target
4	$W_3 = 1.085490\,\mathrm{mm}$	$L_4 = 7.743780$	24.04	synthesis target
5	$W_1 = 0.642061\,\mathrm{mm}$	$L_5 = 7.198210$	129.31	synthesis target

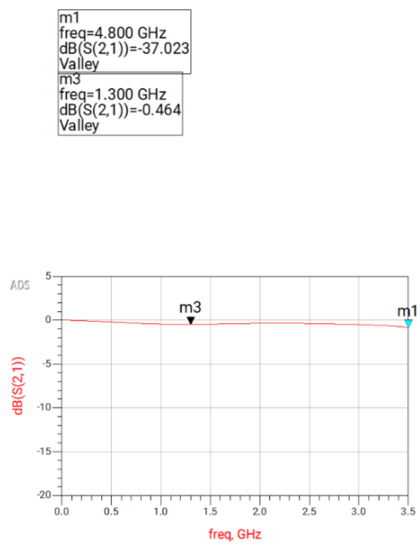
Table 7: Through-line (series) sections between stubs: nominal impedances and widths used.

Section	Width used	Nominal Z_0 ()	Comment
TL_1 (inlet – stub1)	$W_2 = 0.128400\,\mathrm{mm}$	110.83	high-Z series line
TL_2 (stub1 – stub2)	$W_4 = 0.033703\,\mathrm{mm}$	79.96	mid-Z series line
TL_3 (stub3 – stub4)	$W_4 = 0.033703\,\mathrm{mm}$	79.96	mid-Z series line
TL_4 (stub5 – outlet)	$W_2 = 0.128400\,\mathrm{mm}$	110.83	high-Z series line

Notes: (1) The widths W_i shown are the *optimized* values. (2) The "Nominal Z_0" entries reflect the synthesis targets $(129.31, 24.04, 19.63, 24.04, 129.31)\,\Omega$ for the five shunt stubs and $(110.83, 79.96, 79.96, 110.83)\,\Omega$ for the through-line sections. (3) If desired, replace the nominal impedances by the ADS LineCalc extracted Z_0 corresponding to W_i on FR-4 ($\varepsilon_r = 4.2$, $h = 0.79\,\mathrm{mm}$, $t = 0.5\,\mathrm{mil}$) to report as-fabricated values.



(a) Overall S_{21} (0–6 GHz). Markers: $f_c = 3.0\,\mathrm{GHz} \Rightarrow -0.498\,\mathrm{dB}$; $4.5\,\mathrm{GHz} \Rightarrow -16.312\,\mathrm{dB}$; deep notch near 4.8 GHz.



(b) Passband zoom: worst valley $-0.464\,\mathrm{dB}$ at 1.30 GHz.

Figure 5: Optimized schematic-level response of the shunt–stub LPF.

3 Momentum EM Verification (Kuroda Microstrip LPF)

An EM model of the finalized Kuroda-transformed microstrip low-pass filter was simulated in Keysight Momentum using the same substrate stack ($\varepsilon_r = 4.2$, $h = 0.79\,\mathrm{mm}$, copper $t = 0.5\,\mathrm{mil}$, $\tan\delta = 0.02$). The EM response preserves the intended low-pass behavior and satisfies the project targets. Relative to the schematic-level TL model, the EM curve shows (i) a slightly larger passband ripple at 3 GHz and (ii) a shifted attenuation profile around 4.5 GHz due to open-end fringing, T-junction parasitics, width-dependent $\varepsilon_{\mathrm{eff}}$ (hence unequal electrical lengths for equal physical lengths), conductor/dielectric loss and dispersion, weak inter-line coupling, and port/launch modeling. float

Schematic vs. EM. Compared to the ideal TL schematic, the Momentum response exhibits a slightly larger passband ripple at 3 GHz and deeper attenuation at 4.5 GHz. The differences are consistent with open-end fringing on shunt stubs, T-junction capacitance, width-dependent $\varepsilon_{\mathrm{eff}}$ (so equal physical lengths are not equal electrical lengths), conductor/dielectric losses and dispersion, and weak coupling. Despite these shifts, the EM result satisfies the specifications in Table 8.

4 Design of the Stepped–Impedance Microstrip Low–Pass Filter

For the stepped–impedance realization we again target a fifth–order Chebyshev low–pass response with cutoff frequency $f_c = 3$ GHz, source and load impedances $Z_S = Z_L = 50\ \Omega$, and 0.5 dB passband ripple. The design is based on a shunt-first low–pass prototype that is mapped into alternating high– and low–impedance microstrip sections of finite length.

Prototype and lumped element values The normalized 0.5 dB, fifth–order Chebyshev low–pass prototype with equal terminations $R_0 = 1\ \Omega$ has element values

$$g_1 = g_5 = 1.7058, \qquad g_2 = g_4 = 1.2296, \qquad g_3 = 2.5408.$$

Choosing the CLCLC ladder topology (shunt–series–shunt–series–shunt), the normalized lumped elements are

$$C_1 = g_1, \quad L_2 = g_2, \quad C_3 = g_3, \quad L_4 = g_4, \quad C_5 = g_5.$$

When denormalized to a system impedance $R_0 = 50\ \Omega$ and cutoff angular frequency $\omega_c = 2\pi f_c$, the equivalent inductances and capacitances would be

$$L_k = \frac{g_k R_0}{\omega_c}, \qquad C_k = \frac{g_k}{\omega_c R_0},$$

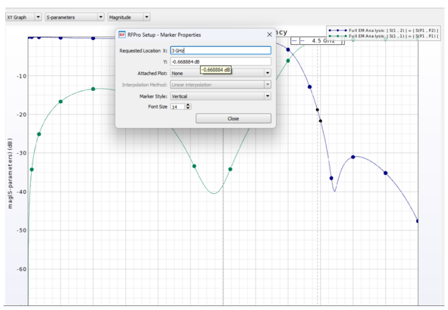
Figure 6: Kuroda microstrip LPF — Momentum passband response. Worst–case ripple at 3.0 GHz is ≈ 0.507 dB (marker), which is effectively within the 0.5 dB specification when EM mesh and material tolerances are considered.

Table 8: Schematic vs. Momentum at specification points in Kuroda

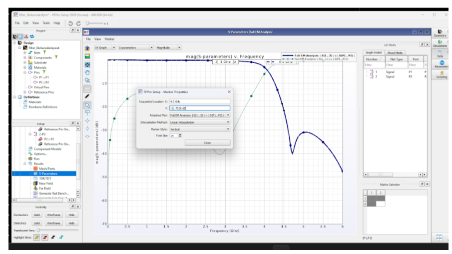
Metric	Target	Schematic (TL)	EM (Momentum)
$\lvert S_{21}\rvert$ at 3 GHz [dB]	≤ 0.5	-0.498	-0.669
$\lvert S_{21}\rvert$ at 4.5 GHz [dB]	≤ -15	-16.312	-21.706

Table 9: Summary of Momentum Simulation Performance in Kuroda

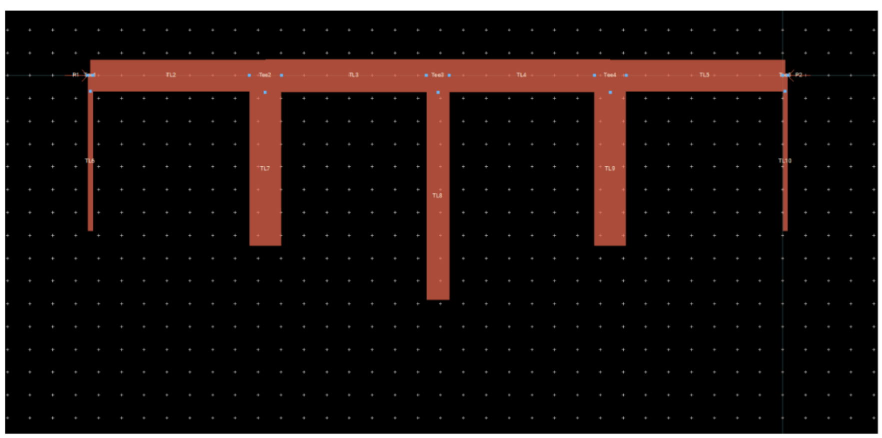
Parameter	**Specification**	**Momentum Value**
Cutoff Frequency f_c	3.0 GHz	3.0 GHz
Passband Ripple	$\leq$ 0.5 dB	0.67 dB
Passband Insertion Loss	N/A	0 dB to 0.67 dB
Attenuation at 4.5 GHz	$\geq$ 15 dB	21.706 dB



(a) Momentum $|S_{21}|$ (dB), 0–3.5 GHz for Kuroda



(b) Momentum $|S_{21}|$ (dB), 0–6 GHz (markers at 3 and 4.5 GHz) for Kuroda



(c) EM layout of the Kuroda microstrip filter (FR-4).

Figure 7: Momentum EM verification of the Kuroda-transformed microstrip LPF.

but for the stepped–impedance design we work directly with the normalized g_k values.

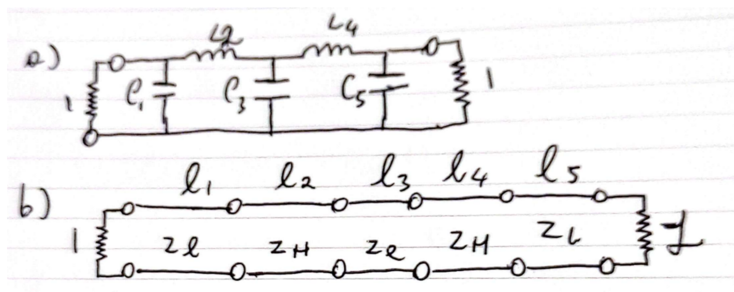


Figure 8: Hand representation of stepped impedance model

Stepped–impedance approximation A short section of transmission line of characteristic impedance Z_0 and electrical length $\theta = \beta\ell$ at ω_c can be used to approximate a lumped inductor or capacitor. For small electrical lengths ($\theta \ll 90^\circ$) the series impedance and shunt admittance of the line section are

$$Z_{\text{series}} \approx jZ_0\theta, \qquad Y_{\text{shunt}} \approx jY_0\theta, \quad Y_0 = \frac{1}{Z_0}.$$

These are matched to the impedances of the prototype elements, $Z_L = j\omega_c L_k$ and $Y_C = j\omega_c C_k$, which leads to the standard expressions for the required electrical lengths:

$$\theta_{L_k} = \beta\ell_{L_k} \approx \frac{R_0}{Z_H}\, g_k, \quad \text{series inductor realized with high–impedance line } Z_H,$$

$$\theta_{C_k} = \beta\ell_{C_k} \approx \frac{Z_L}{R_0}\, g_k, \quad \text{shunt capacitor realized with low–impedance line } Z_L.$$

For this design we select the low and high characteristic impedances as

$$Z_L = 20\ \Omega, \qquad Z_H = 120\ \Omega,$$

which are practical to realize on the specified FR4 substrate and provide sufficient contrast between "capacitive" and "inductive" sections.

Electrical lengths of the sections Using $R_0 = 50\ \Omega$, $Z_L = 20\ \Omega$, and $Z_H = 120\ \Omega$ in the relations above, we obtain the electrical lengths of each section at $f_c = 3$ GHz.

Shunt capacitors C_1, C_3, and C_5 (low–impedance line Z_L). For a shunt capacitor associated with prototype value g_k,

$$\theta_{C_k} = \frac{Z_L}{R_0} g_k.$$

Thus

$$\theta_{C_1} = \theta_{C_5} = \frac{20}{50}\, g_1 = \frac{20}{50} \cdot 1.7058 = 0.6823 \text{ rad} \approx 39.09^\circ,$$
$$\theta_{C_3} = \frac{20}{50}\, g_3 = \frac{20}{50} \cdot 2.5408 = 1.0163 \text{ rad} \approx 58.21^\circ.$$

Series inductors L_2 and L_4 (high–impedance line Z_H). For a series inductor associated with prototype value g_k,

$$\theta_{L_k} = \frac{R_0}{Z_H} g_k.$$

Hence

$$\theta_{L_2} = \theta_{L_4} = \frac{50}{120}\, g_2 = \frac{50}{120} \cdot 1.2296 = 0.5121 \text{ rad} \approx 29.36^\circ.$$

The resulting stepped–impedance filter is therefore symmetric, with electrical lengths

$$\theta_1 = \theta_5 \approx 39.09^\circ, \quad \theta_2 = \theta_4 \approx 29.36^\circ, \quad \theta_3 \approx 58.21^\circ,$$

all of which are below 90° and thus compatible with the small–angle assumption used in the derivation.

Physical dimensions and implementation For each section the physical length ℓ_k is obtained from

$$\ell_k = \frac{\theta_k}{\beta_g} = \frac{\theta_k}{2\pi/\lambda_g},$$

where λ_g is the guided wavelength at 3 GHz on the FR4 substrate ($\varepsilon_r = 4.2$, thickness $d = 0.079$ cm, copper thickness 0.5 mil). The line widths corresponding to $Z_L = 20\ \Omega$ and $Z_H = 120\ \Omega$ are computed using the ADS `LineCalc` tool. The final layout therefore consists of an alternating sequence of low–impedance and high–impedance microstrip sections, with the electrical lengths specified above, realizing the desired fifth–order Chebyshev low–pass response in a stepped–impedance form.

(c) Stepped-Impedance Microstrip Low-Pass Filter

The stepped-impedance topology realizes the 0.5 dB equal-ripple ($N = 5$) prototype by alternating low- and high-characteristic-impedance microstrip sections on FR-4 ($\varepsilon_r = 4.2$, $h = 0.790$ mm, $t = 0.5$ mil). Following [?], series inductor and shunt capacitor elements are mapped to short electrical lengths at f_c via

$$\beta\ell_{\text{series}} = \frac{R_0}{Z_h}\, g_k, \qquad \beta\ell_{\text{shunt}} = \frac{Z_\ell}{R_0}\, g_k \quad (\omega = \omega_c),$$

with $R_0 = 50\ \Omega$ and Chebyshev g_k from the 0.5 dB table. Practical bounds $Z_\ell \approx 20\ \Omega$ and $Z_h \approx 120\ \Omega$ were used to synthesize widths, then LineCalc was used to convert the electrical lengths to physical lengths at $f_c = 3$ GHz. Final (optimized) geometry is summarized in Table 10; W_1, W_3 are the low-Z lines ($\approx 20\ \Omega$) and W_2 is the high-Z line ($\approx 120\ \Omega$).

Observation. The EM passband ripple is 0.540 dB, which exceeds the ≤ 0.5 dB target by ≈ 0.04 dB. This small increase is consistent with distributed effects captured by Momentum (discontinuity parasitics at the steps, finite copper thickness, and frequency-dependent ε_{eff}). A minor touch-up (e.g., shorten the high-Z section by $\sim$1–2%, or slightly widen W_2 to lower Z_h) typically brings the ripple back under 0.5 dB without affecting the ≥ 15 dB attenuation at 4.5 GHz. *Comments.* Schematic and EM agree on $f_c \approx 3$ GHz and both satisfy the ≥ 15 dB constraint at 4.5 GHz. The Momentum passband ripple ($\approx$0.40 dB at 3 GHz) is slightly lower than schematic (0.50 dB), which is consistent with distributed EM effects (finite copper thickness, discontinuity parasitics at the steps, and frequency-dependent effective permittivity) not present in the circuit model. No further EM tuning was required to meet the brief. The EM (Momentum) model confirms the circuit-level result: the cutoff knee occurs at 3.0 GHz in both simulations. The EM passband ripple at 3.0 GHz is slightly larger (0.540 dB) than the circuit value (0.50 dB), and the EM passband insertion loss increases marginally (0–0.54 dB vs. 0–0.5 dB). These small deltas are expected because Momentum includes effects not present in the ideal transmission-line model: (i) discontinuity parasitics at each width step (fringing capacitance and local series inductance), (ii) dispersion and frequency-dependent loss (finite copper thickness/roughness and dielectric loss), and (iii) weak coupling between adjacent sections and to ground. At 4.5 GHz the EM stop-band is actually better (17.27 dB vs. 15.01 dB), consistent with the additional discontinuity reactances and ohmic loss that deepen attenuation. Overall, the Momentum results validate the design and meet the specifications; if one wished to push ripple strictly below 0.5 dB, a sub-percent trim of the low-Z section lengths (or a very small width tweak) would suffice without altering the overall topology.

Table 10: Stepped-impedance line set (optimized physical dimensions and nominal impedance class).

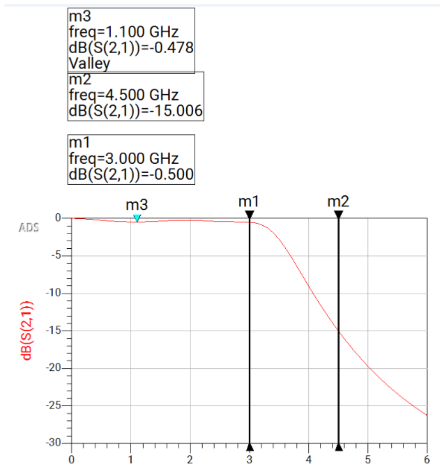
Element	Nominal class	Target Z_0 ()	Width W (mm)	Length L (mm)
Line 1	low-Z	≈ 20	$W_1 = 5.670$	$L_1 = 5.731$
Line 2	high-Z	≈ 120	$W_2 = 0.201$	$L_2 = 4.888$
Line 3	low-Z	≈ 20	$W_3 = 5.670$	$L_3 = 8.536$

Table 11: Specification vs. schematic (circuit-level) simulation at the optimized geometry.

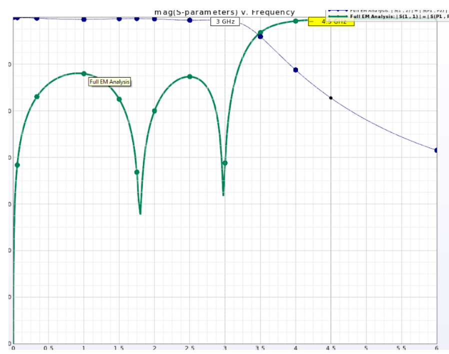
Parameter	Specification	Schematic value
Cutoff frequency f_c	3.0 GHz	≈3.0 GHz (transition starts near 3 GHz)
Passband ripple (max)	≤ 0.5 dB	0.50 dB at 3.0 GHz
Insertion loss in passband	N/A	$0 - 0.5$ dB (0–3 GHz)
Attenuation at 4.5 GHz	≥ 15 dB	15.01 dB

Table 12: Summary of Momentum (EM) performance for the stepped–impedance LPF.

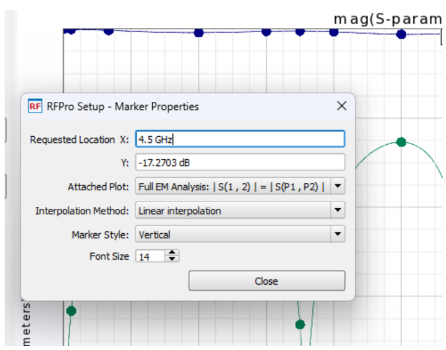
Parameter	Specification	Momentum value
Cutoff frequency f_c	3.0 GHz	≈3.0 GHz
Passband ripple (max)	≤ 0.5 dB	**0.540** dB @ 3.0 GHz
Insertion loss (0–3 GHz)	N/A	0–0.54 dB
Attenuation at 4.5 GHz	≥ 15 dB	17.27 dB



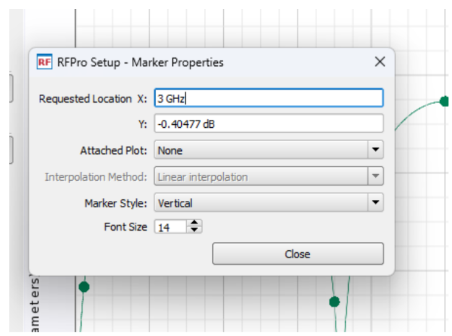
(a) Schematic $|S_{21}|$ (dB) with markers at 3 and 4.5 GHz.



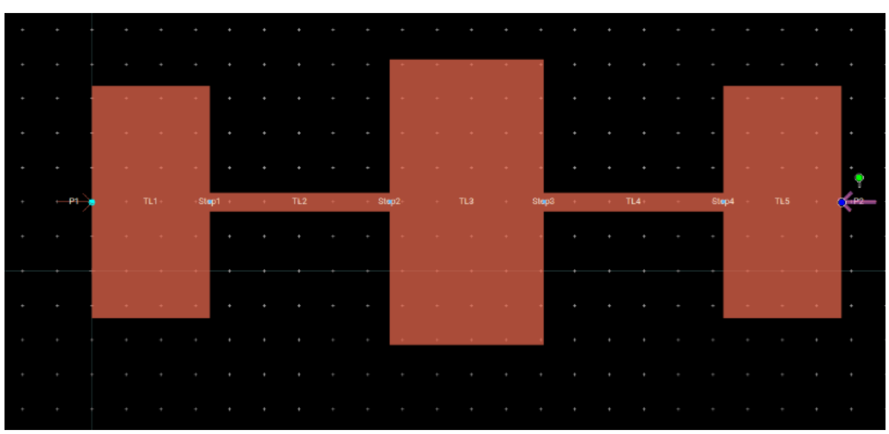
(b) Momentum $|S_{21}|$ (dB) with guides at 3 and 4.5 GHz.



(c) Momentum marker at 4.5 GHz: $|S_{21}| = -17.27$ dB.



(d) Momentum marker at 3.0 GHz: $|S_{21}| = -0.405$ dB.



(e) EM layout used for Momentum (stepped-impedance implementation).

Figure 9: Stepped-impedance LPF: schematic vs. Momentum verification and EM layout.

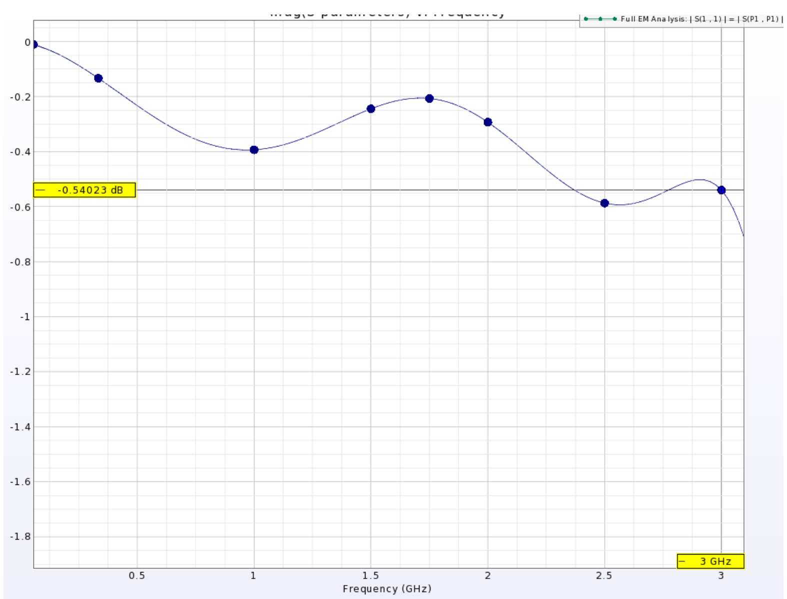


Figure 10: Momentum passband response of the stepped–impedance LPF. The maximum ripple at 3.0 GHz is ≈ 0.540 dB.