$$I = \int_a^b f(x)dx$$

$$f(x) \approx P_2(x) = \left(\frac{x-b}{a-b}\right)\left(\frac{x-x_m}{a-x_m}\right)f(a) + \left(\frac{x-a}{b-a}\right)\left(\frac{x-x_m}{b-x_m}\right)f(b) + \left(\frac{x-a}{x_m-a}\right)\left(\frac{x-b}{x_m-b}\right)f(x_m)$$

Donde $\forall x \ \in [a,b]$

Entonces

$$\int_a^b f(x)dx \approx \int_a^b \left(\frac{x-b}{a-b}\right)\left(\frac{x-x_m}{a-x_m}\right)f(a) + \left(\frac{x-a}{b-a}\right)\left(\frac{x-x_m}{b-x_m}\right)f(b) + \left(\frac{x-a}{x_m-a}\right)\left(\frac{x-b}{x_m-b}\right)f(x_m)$$

$$\int_a^b f(x)dx \approx \int_a^b \left(\frac{x-b}{a-b}\right)\left(\frac{x-x_m}{a-x_m}\right)f(a) + \int_a^b \left(\frac{x-a}{b-a}\right)\left(\frac{x-x_m}{b-x_m}\right)f(b) + \int_a^b \left(\frac{x-a}{x_m-a}\right)\left(\frac{x-b}{x_m-b}\right)f(x_m)$$

$$I \approx \frac{f(a)}{(a-b)(a-x_n)}\int_a^b x^2 - bx - xx_m + bx_m dx + \frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b x^2 - bx - ax + abdx + \frac{f(b)}{(b-a)(b-x_m)}\int_a^b x^2 - ax - xx_m + ax_m dx$$

$$I \approx \frac{f(a)}{2}\int_a^b x^2 - bx - xx_m + bx_m dx + \frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b x^2 - bx - ax + abdx + \frac{f(b)}{(b-a)(b-x_m)}\int_a^b x^2 - ax - xx_m + ax_m dx$$

$$I \approx \frac{f(a)}{2h^2}\left(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{x^2x_m}{2} + bx_m x\right)_a^b + \frac{f(x_m)}{-h^2}\left(\frac{x^3}{3} - \frac{ax^2}{2} - \frac{bx^2}{2} + abx\right)_a^b + \frac{f(b)}{2h^2}\left(\frac{x^3}{3} - \frac{ax^2}{2} - \frac{x_m x^2}{2} + ax_m x\right)_a^b$$

$$I \approx \frac{f(a)}{2h^2}\left(\left(\frac{b^3}{3} - \frac{b^3}{2} - \frac{x_m b^2}{2} + b^2 x_m\right) - \left(\frac{a^3}{3} - \frac{ba^2}{2} - \frac{x_m a^2}{2} + abx_m\right)\right) + \frac{f(x_m)}{-h^2}\left(\left(\frac{b^3}{3} - \frac{ab^2}{2} - \frac{b^3}{2} + ab^2\right) - \left(\frac{a^3}{3} - \frac{a^3}{2} - \frac{ba^2}{2} + ba^2\right)\right) +$$

$$a = -h$$

$$x_m = 0$$

$$b = h$$

$$\frac{f(b)}{2h^2}\left(\left(\frac{b^3}{3}-\frac{ab^2}{2}-\frac{x_m b^2}{2}+ax_m b\right)-\left(\frac{a^3}{3}-\frac{a^3}{2}-\frac{x_m a^2}{2}+a^2 x_m\right)\right)$$

$$I \approx \left[\left(\frac{h^3}{3}-\frac{h^3}{2}\right)-\left(-\frac{h^3}{3}-\frac{h^3}{2}\right)\right]\frac{f(a)}{2h^2}+\left[\left(\frac{h^3}{3}+\frac{h^3}{2}-\frac{h^3}{2}-h^3\right)+\left(\frac{h^3}{3}+\frac{h^3}{2}+\frac{h^3}{2}+h^3\right)\right]\frac{f(x_m)}{-h^2}$$
$$+\left[\left(\frac{h^3}{3}+\frac{h^3}{2}\right)-\left(-\frac{h^3}{2}+\frac{h^3}{2}\right)\right]\frac{f(b)}{2h^2}$$

$$I \approx \frac{1}{3}h(f(a)+4f(x_m)+f(b)$$