a) Calcular analítico del polinomio que interpola el conjunto soporte.

$$\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$$

$$P_2(x) = \sum_{i=0}^{2} \mathrm{L_i}(x) f(x_i),$$

$$L_i(x) = \prod_{j=0, j\neq i}^{2} \frac{x - x_j}{x_i - x_j}$$

$$P_2(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) f(x_1) + \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) f(x_2)$$

$$P_2(x) = \left(\frac{x - x_1}{-h}\right)\left(\frac{x - x_2}{-h - h}\right) f(x_0) + \left(\frac{x - x_0}{-h}\right)\left(\frac{x - x_2}{h}\right) f(x_1) + \left(\frac{x - x_0}{h + h}\right)\left(\frac{x - x_1}{h}\right) f(x_2)$$

$$P_2(x) = \left(\frac{(x - x_1)(x - x_2)}{2h^2}\right) f(x_0) + \left(\frac{(x - x_0)(x - x_2)}{-h^2}\right) f(x_1) + \left(\frac{(x - x_0)(x - x_1)}{2h^2}\right) f(x_2)$$

b) Derivar el polinomio interpolador para encontrar la derivada en el punto x_0:

$$f'(x_0) \approx p'(x_0) = \frac{1}{2h}(-3f(x_0) + 4f(x_1) - f(x_2))$$

$$P'_2(x) = \left(\frac{(x - x_2) + (x - x_1)}{2h^2}\right) f(x_0) + \left(\frac{(x - x_2) + (x - x_0)}{-h^2}\right) f(x_1) + \left(\frac{(x - x_1) + (x - x_0)}{2h^2}\right) f(x_2)$$

$$P'_2(x_0) = \left(\frac{(x_0 - x_2) + (x_0 - x_1)}{2h^2}\right) f(x_0) + \left(\frac{(x_0 - x_2) + (x_0 - x_0)}{-h^2}\right) f(x_1) + \left(\frac{(x_0 - x_1) + (x_0 - x_0)}{2h^2}\right) f(x_2)$$

$$P'_2(x_0) = \left(\frac{-h - h - h}{2h^2}\right) f(x_0) + \left(\frac{-h - h}{-h^2}\right) f(x_1) + \left(\frac{-h}{2h^2}\right) f(x_2)$$

$$P'_2(x_0) = \left(\frac{-3h}{2h^2}\right) f(x_0) + \left(\frac{-2h}{-h^2}\right) f(x_1) + \left(\frac{-h}{2h^2}\right) f(x_2)$$

$$P'_2(x_0) = \left(\frac{-3}{2h}\right) f(x_0) + \left(\frac{2}{h}\right) f(x_1) + \left(\frac{-1}{2h}\right) f(x_2)$$

$$P'_2(x_0) = \frac{1}{2h}\left((-3) f(x_0) + (4) f(x_1) + (-1) f(x_2)\right)$$

$$P'_2(x_0) = \frac{1}{2h}\left((-3) f(x_0) + (4) f(x_0 + h) + (-1) f(x_0 + 2h)\right)$$