Wireless Power Transfer via SCMR for Embedded Systems

Robert St. Denis
University of California, Davis
One Shields Ave
Davis, CA 95616
+1-(732)-503-7007
rstdenis@ucdavis.edu

Waqas Haque
University of California, Davis
One Shields Ave
Davis, CA 95616
+1-(650)-798-4568
whaque@ucdavis.edu

ABSTRACT
In this paper, we briefly review the methods behind mid-range wireless power transfer using Strongly Coupled Magnetic Resonance (SCMR) to power embedded systems applications. From these reviewed methods, we then develop a mathematical model for SCMR to calculate resonant frequencies and other electrical characteristics for a given coil in MATLAB. The results from the model are verified experimentally through the implementation of several design iterations. The MATLAB model is accurate to within 10% of the experimental values. We also discuss the optimal design for maximum wireless power transfer and the constraints associated. Previous work shows wireless power transfer over 2 meters at about 40% efficiency with 60cm diameter resonating coils. We present a significantly scaled down design (Design III) with wireless power transfer via SCMR over 10cm using 8.25cm diameter coils, roughly 7.5 times smaller. The smaller design makes it an excellent choice for powering small scale embedded systems applications.

Categories and Subject Descriptors
K.4 [**Computers and Society**]:– Use/Abuse of power

General Terms
Algorithms, Measurement, Design, Experimentation.

Keywords
SCMR, WPT, Strongly Coupled Magnetic Resonance, Wireless Power Transfer, power, mid-range.

1. INTRODUCTION
The transmission of power wirelessly has long been dream of pioneers in the field that would eventually become Electrical Engineering, namely Nikola Tesla. While Tesla was indeed able to demonstrate mid-to-long-range wireless power transfer over a century ago, safe and practical methods for achieving similar results have only been discovered recently in 2007. This method, which exploits electromagnetic resonance, or rather, an object's tendency to oscillate at large amplitudes at specific frequencies, was deemed Strongly Coupled Magnetic Resonance (SCMR) by its discoverers [1]. Although there are many electromagnetic phenomena capable of wireless energy transfer from one point to another (for example, transformers, which demonstrate a high efficiency at very close distances), SCMR is the only method capable of efficiently transferring energy in the mid-range (that does not require an uninterrupted line-of-sight) where the transfer distance is several times greater than the radius of the transmitting device.

Previous work done and a basic understanding of SCMR systems is presented in section 2. Mathematical modelling and MATLAB implementation of resonating coils and their associated geometric and electrical characteristics are discussed in section 3. We also analyze the effects of varying geometric parameters of a coil on electrical characteristics. Experiments using multiple coil designs and respective results are presented in section 4. In section 5, we discuss possible improvements and future work.

2. BACKGROUND
Over the past 20 years there has been an inordinate increase in the amount of mobile electronics (smart phones, tablets, laptops) being manufactured and purchased across the globe. While these devices have produced endless conveniences for the end-user, they still present the inconvenience of statically tethering the user to an outlet when the battery runs low. Thus, a recent interest in midrange wireless power transfer has led to the discovery and development of SCMR systems.

The idea behind wireless power transfer via SCMR is quite simple from an intuitive standpoint - objects that share the same resonant frequency will efficiently exchange energy, and lose only a relatively small amount of that energy to any off-resonant objects encountered by the resonating field. Thus, SCMR system consists of a 2 or more identically tuned (usually identically sized) resonating coils – a transmitter and a receiver. In order to excite the resonant frequency in the transmitting resonator however, a single turn loop, which is connected to a source outputting power at the resonant frequency of the dual-coil system, is inductively coupled to the transmitting coil. In order to use the power obtained at the receiving end resonator, one or several loops will be inductively coupled to the receiver. This is so that the receiving resonator has no direct contact with the electrical load, as it would ultimately alter the resonant frequency otherwise, preventing the coupled magnetic resonance from occurring. A simple illustration of an SCMR system is shown below in Figure B-1 [3].

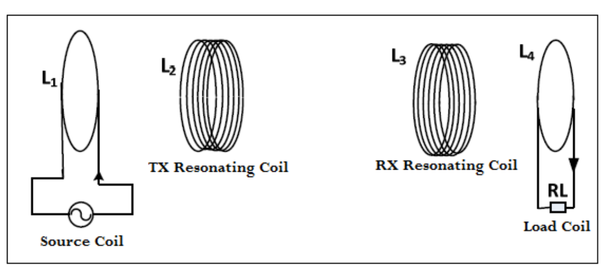

Figure B-1. Basis for an SCMR system

In the earliest SCMR experiments performed at MIT [1], large 5-turn, resonating coils with a radius of 30 cm (2 feet in diameter) were used to transfer 60W over 2 meters at 40% efficiency (60W received / 150W transmitted). As revolutionary as this first SCMR system was, its large size makes it impractical for use in everyday low-power, mobile electronics. The most recent work in SCMR was performed in 2014 [3] with the goal of creating SCMR systems to charge batteries in medical devices implanted under the skin. This however, is still only intended for use in a controlled environment, with a stationary patient, positioned directly in front of the resonating source. As wonderful as the application of preventing invasive surgeries is, SCMR in this application context defeats the original motivation of allowing an end user to use their mobile device without the requirement of fixing themselves to a power source. Our goal is to design and build a scaled-down version of the original SCMR system, with the intended application of mobile, embedded-systems.

3. SCMR MODELING

3.1 Mathematical Model

Highly resonant systems that operate in a strongly coupled regime are able to transfer energy efficiently versus non-resonant objects that only interact weakly. Calculating the resonant frequency of loops and helices require careful consideration of their geometry. In this section, we analytically derive the equations used to calculate the electrical characteristics and eventually the resonant frequency and quality factor, of the coils. We will also see that in order to obtain the maximum power transfer efficiency, the quality factor, Q needs to be maximized. The equation for maximum Q is later presented and used to work our way back to the optimal geometry required for maximum power transfer. The following characteristics of a coil/loop are considered:

i. Capacitance, C
ii. Inductance, L
iii. Resonant Frequency, f_o
iv. Loss (Impedance), Γ
v. Quality factor, Q

These key characteristics are directly related to the geometric parameters of the helix under consideration, as well as the physical properties of the helix material (in our case, copper).

i. Cross sectional radius of wire, r_c
ii. Radius of coil, r
iii. Number of turns, N
iv. Pitch, s

In their resonant state, the transmitting (TX) and receiving (RX) helices are effectively represented by a pair of equivalent RLC circuits. Since helical coils exhibit both distributed capacitance and inductance, they can be designed to self-tune to a desired resonant frequency, f_o without the need for external passive elements.

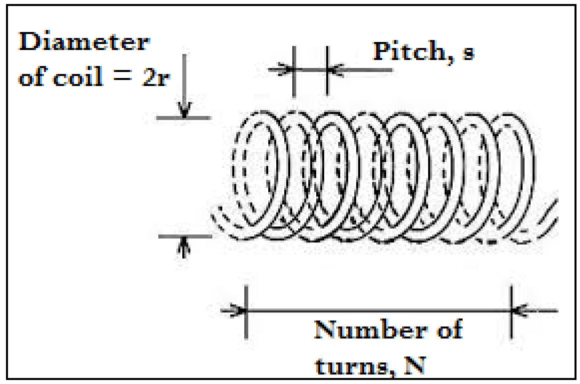

Figure 1. Coil geometry

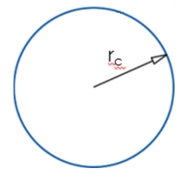

Figure 2. Wire Cross-section

3.1.1 Inductance and Capacitance

The inductance and capacitance of the helical coil depend largely on the underlying geometry. They are provided by equations derived in [3][4]:

$$L = \mu_o r N^2 \left[\ln\left(\frac{8r}{r_c}\right) - 2 \right] \tag{1}$$

$$C = \frac{2\pi^2 r \varepsilon_o}{\ln\left[\frac{s}{2r_c} + \sqrt{\left(\frac{s}{2r_c}\right)^2 - 1}\right]} \tag{2}$$

where μ_o is permeability of free space and ε_o is permittivity of free space. It is important to note that (1) and (2) can be used to describe a loop (N=1), which will have negligible self-capacitance. Therefore, in order to realize a resonating loop element, it must be connected in parallel to an external capacitor for the reduction of a resonant RLC circuit to remain true. In this paper, we will be focusing on helical coils with N>1, since this increases both the inductance and consequent coupling factor, as well as induces a capacitance (between successive rungs of the helix) that avoids the need to apply an external capacitor.

Through our experimentation, we found that it is better to avoid adding an external capacitance if possible, since the auxiliary capacitance adds to the intrinsic loss of the system and lowers the frequency, ultimately lowering the Q. However, external capacitors are sometimes necessary to correct for mismatch errors in the resonating coils - if Δf_o between the coils is too large, the maximum possible efficiency will be nearly zero.

3.1.2 Resonant Frequency

Setting capacitance $1/\omega_o C$ equal to the inductance $\omega_o L$ and solving for f_o, the resonant frequency of the oscillating circuit is obtained as:

$$f_o = \frac{1}{2\pi\sqrt{LC}} \tag{3}$$

Substituting (1) and (2) into (3) yields an equation for the resonant frequency dependent solely on the geometry and physical parameters of the coil. This resonant frequency is the frequency at which the source coil needs to be operated. The source coil and the TX resonant coil are inductively coupled.

We have calculated the resonant frequency at which power will be transferred between the TX and RX resonating coil but we have not considered efficiency. To maximize the efficiency, we need to consider the quality factor, Q of the circuit, in our case, the coil.

3.1.3 *Quality Factor and Efficiency*

SCMR requires that both RX and TX resonant coils exhibit maximum Q-factor at their resonant frequency f_o, in order to achieve maximum wireless power efficiency. This can also be seen from the following equation from [3] that describes the efficiency of an SCMR system at its operational frequency, f_o, as follows:

$$\eta(f_o) = \frac{k_{TX\_RX}^2(f_o)\, Q_{TX}(f_o)\, Q_{RX}(f_o)}{1 + k_{TX\_RX}^2(f_o)\, Q_{TX}(f_o)\, Q_{RX}(f_o)} \tag{4}$$

where k_{TX_RX} is the mutual coupling between the RX and TX resonant coils. Q_{RX} and Q_{TX} are the Q factors of the RX and TX resonant coils, respectively. Since we want TX and RX coils to resonate at the same resonant frequency, their geometry will be similar and they will exhibit the same Q factor. Therefore, we define, $Q_{RX}=Q_{TX}=Q$ and equation (4) can be rewritten as:

$$\eta(f_o) = \frac{k_{TX\_RX}^2(f_o)\, Q^2(f_o)}{1 + k_{TX\_RX}^2(f_o)\, Q^2(f_o)} \tag{5}$$

Equation (5) shows that in order to maximize efficiency, the Q factor needs to be maximized. Since the Q factor is a function of f_o, we need to calculate f_{max}, the resonant frequency at which the Q factor maximizes. The Q factor is given by:

$$Q = \frac{2\pi f_o}{2\Gamma} \tag{6}$$

where Γ is the loss (impedance) of the coil. It is a function of R_z, ohmic resistance and R_r, radiation resistance. They are given by the following set of equations:

$$\Gamma = \frac{R_z + R_r}{2L} \tag{7a}$$

$$R_r = \left(\frac{\pi}{6}\right)\eta_o N^2 \left(\frac{2\pi f_o r}{c}\right)^4 \tag{7b}$$

$$R_z = \frac{\left(\sqrt{\mu_o \rho \pi f_o}\right) N r}{r_c} \tag{7c}$$

where η_o is characteristic impedance of free space, ρ is coil material resistivity and c is the speed of light

Substitution of equations (1)-(7) in (6) gives the following expression for Q:

$$Q(f_o, r, r_c, N) = \frac{2\pi f_o \mu_o r N^2 \left[\ln\left(\frac{8r}{r_c}\right) - 2\right]}{\left(\frac{\mu_o \rho \pi r^2 f_o N}{r_c^2}\right)^{1/2} + 20\pi^2 N^2 \left(\frac{2\pi f_o r}{c}\right)^4} \tag{8}$$

3.1.4 *Maximizing Q and Efficiency*

Using calculus, the maxima from equation (8) can be calculated to get f_{max}. For maximum efficiency, $f_o = f_{max}$. Working our way back to the geometry of the coil, it can be seen that for maximum efficiency, the following ratio between r and r_c must be implemented:

$$r = \frac{e^{13/3}}{8} r_c \approx 9.52 r_c \tag{9}$$

Maximum wireless power efficiency can be achieved in SCMR systems if the radius of the coil is approximately 9.52 times the radius of the wire cross section. We call this, the optimal geometry. Figure 3 from [3] shows the relationship between efficiency and r/r_c ratio.

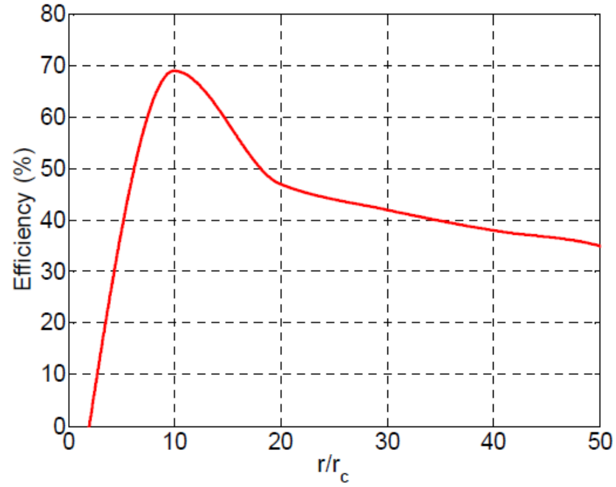

Figure 3. Efficiency peaks for $r/r_c \simeq 9.52$

These parameters can be set for any wire but as we will see later on, physically realizing it for our experiments proved very difficult.

3.2 MATLAB Implementation, Analysis and Results

The mathematical equations discussed are implemented in MATLAB. The function requires geometric parameters of the coil and provides the inductance, capacitance, resonant frequency, Q factor and other characteristics of the coil. In this section, we will go through a system level implementation block diagram of the MATLAB model followed by some results.

The block diagram in Figure 4 shows the overall implementation of the mathematical model and the dependencies between characteristics. The model uses the coil geometric parameters as inputs, discussed earlier, and calculates the inductance and capacitance. This allows calculation of the resonant frequency. Equations (6) and (7) require inductance and resonant frequency values and therefore are calculated next. All these values are then printed out in the command window for review and analysis.

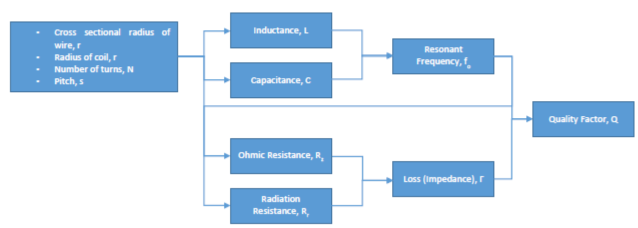

Figure 4. MATLAB System level block diagram

3.2.1 *Analysis and Results*

Equation (3) shows that any increase in L or C decreases the resonant frequency. Since L and C are functions of geometric parameters we consider the overall effect of varying each of them on the resonant frequency. The default values are given in the table.

Table 1. Default Values for Geometry

Parameter	Default Value
Number of Turns, N	10
Pitch, s	$2.007r_c$ mm
Radius of Loop, r	4.12 cm
Cross Sectional radius of wire, r_c	0.574 mm (17 AWG)

3.2.1.1 Varying Number of turns, N

Equations (1), (2) and (3) show that varying the number of turns in the coil increases the inductance and decreases the resonant frequency. The capacitance however remains unchanged. Figure 5 show the change in L and f_o with change in number of turns, N.

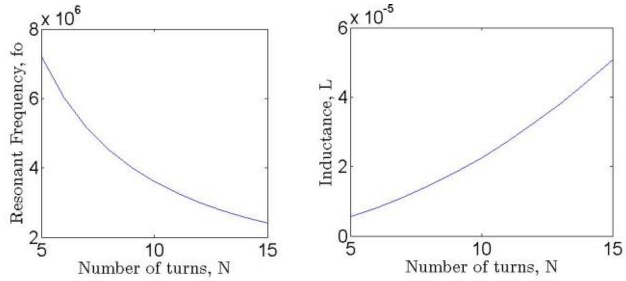

Figure 5. Varying Number of turns, N

3.2.1.2 Varying Pitch, s

Varying the pitch as a huge effect on the capacitance of the coil. As the pitch increases, the capacitance between turns decreases and increases the resonant frequency. A 1/1000th change in pitch has a large effect on C and f_o. Figure 6 shows this phenomenon. Pitch is varied from $2.001r_c$ to $2.009r_c$ Since inductance, L is not dependent on pitch, it remains unchanged.

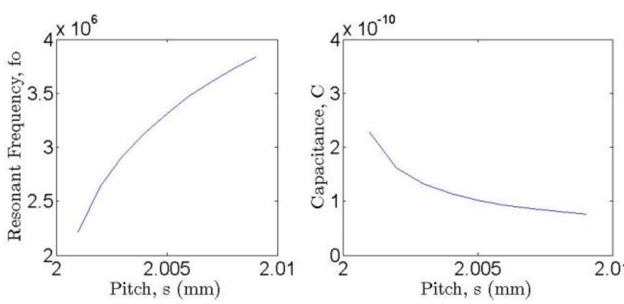

Figure 6. Varying pitch, s

3.2.1.3 Varying Loop Radius, r

Capacitance and inductance are directly proportional to the loop radius therefore we will see an increase as loop radius is increased. Resonant frequency $\propto 1/LC$ and will decrease. We will vary the loop radius, r from 3.0cm to 5.0cm.

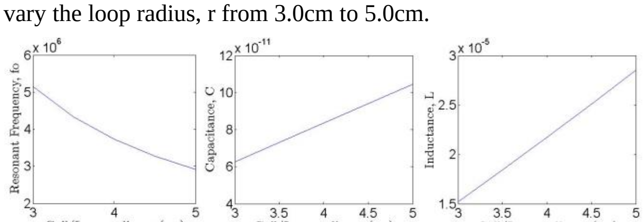

Figure 7. Varying Loop Radius, r

3.2.1.4 Varying Cross Sectional Radius of wire, r_c

Varying the cross sectional radius of the wire affects the inductance and therefore the resonant frequency. The effect on capacitance is negligible. We will vary the cross sectional radius from 15 AWG to 20 AWG.

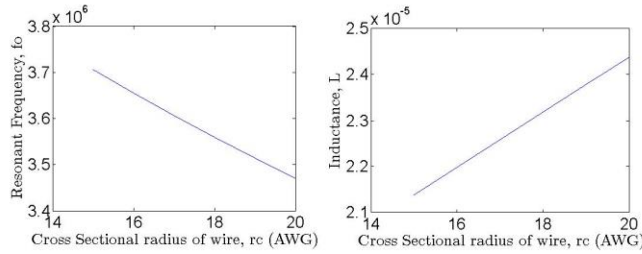

Figure 8. Varying Cross Sectional Radius, r_c

4. EXPERIMENTATION via RAPID PROTOTYPING AND CHALLENGES

4.1 Designs & Comparison of Results

4.1.1 Design I – Copper tube

Cross-sectional radius	= 2.59470 mm (4 AWG)
Loop radius	= 1.90500 cm (0.750000 in)
Loop diameter	= 3.81000 cm (1.500001 in)
Pitch	= 0.80000 cm (0.314961 in)
N	= 3.25 turns
Length of wire/tube	= 62.840 cm (2.061670 ft)
Estimated L	= 1.367826 uH
Estimated C	= 3.333478 pF
Estimated Q	= 8025.117251

Calculated resonant frequency: **74.5 MHz**
True resonant frequency: **16.1 MHz**

Based on the results of design 1, there is a clear mismatch between the model and the true resonant frequency coil helix, as even offsetting the pitch and capacitance parameters still left us results far from "the ballpark" After careful consideration of what may have been causing the mismatch, we returned to Eq (1): The derivation of the formula in [4] assumes solid copper, hence our next design is a thick solid copper wire.

Figure 9. Design I – 3.25 turns of ¼" OD copper tube

4.1.2 Design II – Solid Copper Wire (Thick)

Cross-sectional radius = 2.59470 mm (4 AWG)
Loop radius = 1.90500 cm (0.750000 in)
Loop diameter = 3.81000 cm (1.500001 in)
Pitch = 0.80000 cm (0.314961 in)
N = 5.25 turns
Length of wire/tube = 62.840 cm (2.061670 ft)
Estimated L = 1.367826 uH
Estimated C = 3.333478 pF
Estimated Q = 8025.117251
Calculated resonant frequency: **74.534254 MHz**
True resonant frequency: **14.1 MHz**

Figure 10. Design II using 4 AWG solid copper

Based on the results comparison of Design II, it is again obvious an incongruity is present. After exhaustively searching our MATLAB model for possible discrepancies, we eventually came to the realization which is shown graphically in Figure 6 – for a seemingly insignificant offset in pitch, the capacitance is momentously affected, as is the resulting resonant frequency. Given the proper offset to correct for variations in helix pitch, the model produces the correct result. Nevertheless, a method for building SCMR coils that will remain consistent with the theoretical model for every new coil made, needed to be developed. We also realized that the progression of [3], which maximizes the frequency in order to maximize Q, is not practical until r_c becomes fairly big (>1/4"), otherwise the required coil pitch to produce the desired capacitance is infinitely large. The coils in Design II were already too heavy to be used for a practically sized embedded system (i.e. sustainable flight drone), thus attempting a design with even larger coils was not a sensible solution. When r_c is considerably small, it becomes much more practical to maximize the design for inductance as opposed to maximizing for frequency.

4.1.3 Design III

Cross-sectional radius = 0.57477 mm (17 AWG)
Loop radius = 4.12750 cm (1.625001 in)
Loop diameter = 8.25500 cm (3.250002 in)
Pitch = 0.12070 cm (0.047520 in)
N = 7.00 turns
Length of wire/tube = 181.537 cm (5.955936 ft)
Estimated L = 11.064472 uH
Estimated C = 22.905348 pF
Estimated Q = 3397.202663

Calculated resonant frequency: **9.997 MHz**
True resonant frequency: **10.2 MHz**

Figure 11. Design III using 17 AWG solid copper

Based on the results of Design III, our model is shown to be accurate to well within the desired range of 5% error. To verify the consistency of the model across coils, we built several other variations, each with a unique radius and gauge of wire (not shown), all of which were within the range of 5% error. It is worth noting that Figures 9-11 do not show the SCMR system in use – they only show the design of the coils – the coils would be positioned with a common center axis when in use, as in Figure B-1.

The demos available on our website use design III coils. We were able to transfer 6V peak to peak with 2mA current. The source coil operated at 25V peak to peak at 10.2MHz. Figure 12 shows a wirelessly powered LED using design III.

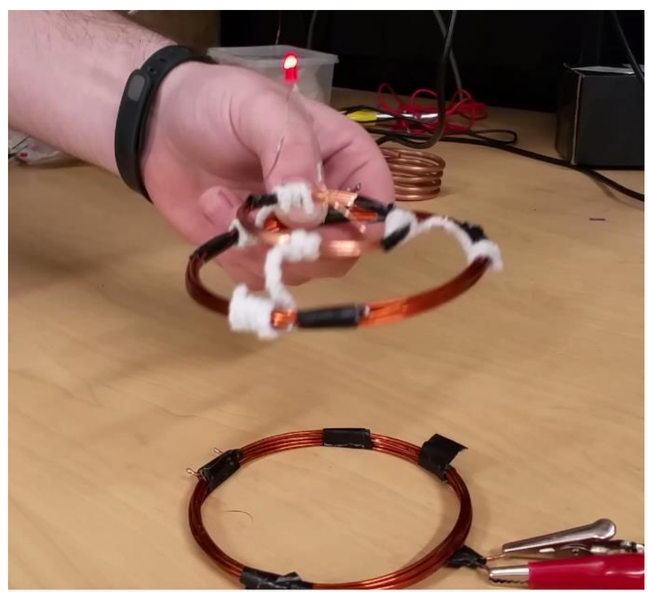

Figure12. Wireless Power Transfer via SCMR lighting up an LED

The second video in our presentation (available on our webpage) introduces the effects of external capacitance on the resonant frequency of design III coils. The 9pF capacitance from the oscilloscope adds to the capacitance of the coils and brings down the resonant frequency from 10.2MHz to roughly 4.4MHz. Disconnecting the oscilloscope leads brings the resonant frequency back up to 10.2MHz. This posed as a huge problem initially as we never considered the external capacitance from a measurement device to add to the capacitance of the coil. Therefore, to experimentally determine the resonant frequency of a coil without using an oscilloscope, we decided to simply light up our load LED.

4.2 Challenges

As one may expect, the first challenge confronted was developing a method to properly shape thick copper coils. Without the proper equipment to reshape the solid copper, an approach of sheer brute force, time, and some applied heat was the best option – surprisingly, this resulted in very closely match coils (within 0.2 MHz). However, maintaining the exact pitch between successive rungs of the helix is not possible when shaping by hand and ultimately lead to a high discrepancy between our coils and our model. Additionally, copper oxidizes very quickly– after a long process of shaping and molding our copper coils, we were dismayed to find that what once was a majestic golden hue had changed overnight to a muddy dirt-brown. More importantly than aesthetics though – the Q had also significantly depreciated.

Additionally, humidity has a great impact on the achievable efficiency of the SCMR system. This can be seen in Table 2 from [3]

Table 2. Humidity level vs. Efficiency table for SCMR

Humidity Level (%)	RX loop's a (mm)	TX loop Q-factor	Rx loop Q-factor	Efficiency (%)
0.2	2.2	3650	2010	59.0
2.8	2.2	920	1269	18.5
5.5	2.2	900	915	14.5
6.2	2.2	890	830	13
9.3	2.2	750	479	9.8
12	2.2	300	432	8.7

In our final demonstration, it's often perceived that design 3, (which provided our best results) is quite easy to make. As with any good demonstration, we've done our best to make it appear as simple and relaxed as possible. Despite the fact it can be built for less than the cost of a full-spectrum 40W light bulb (excluding test equipment), without experience (as was our case initially), building a working version is about as far from "flicking a light-switch" as one can get. Aside from correctly wrapping both coils identically to meet the "goldilocks" requirements, maintaining the coils' radial tautness and pitch after it's been wrapped presents a challenge of its own – in future versions we plan to coat the wrappings in a quartz-based silica to reliably maintain the static geometry without interfering with the magnetic coupling and simultaneously allowing any heat that may build up from high current to dissipate.

5. CONCLUSION & FUTURE WORK

In conclusion, we successfully developed a scaled-down version of an SCMR system intended for use in future embedded systems. It is lightweight, economical, consistent with the model, relatively easy to maintain, and can be built in just a few hours with proper training. The MATLAB model is accurate within 10% of the experimental values. While our SCMR system is indeed a viable solution, the next challenge to tackle is that of efficiently driving the resonators at higher power. The alternative of using a Colpitts oscillator reliant on large 500W tube amplifier (as performed in [1]) is not nearly efficient or small enough to be used in a personal router-like form factor. Finally, experimentation with a phased-array of multi-harmonic resonators, the effects of superconductive temperatures on SCMR, cylindrical waveguides as field directing mechanism, and of course, a sustainable flight drone, are all untouched areas of research that we plan to explore.

The original team from MIT [1] went on to form a company called WiTricity. Their products, wireless cell phone charger and wireless electric car battery charger are commercially available but significantly scaled down and therefore, shorter range than 2 meters.

We picture a future where wireless power is available in homes, cafes and offices just like wireless internet. Devices ship with RX resonating coils built-in and a standardized SCMR system with dedicated resonant frequencies to operate on. One would simply walk into a building and their devices would start charging in its wireless power cloud.

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