Subject(s):

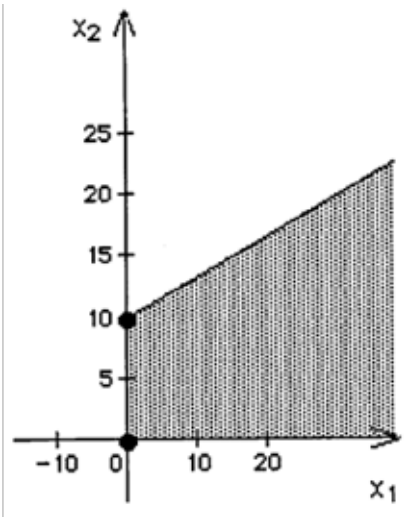
- Simplex Method

Exercises from: F. S. Hillier and G. L. J., Introduction to Operations Research. McGraw-Hill Education, 2015.

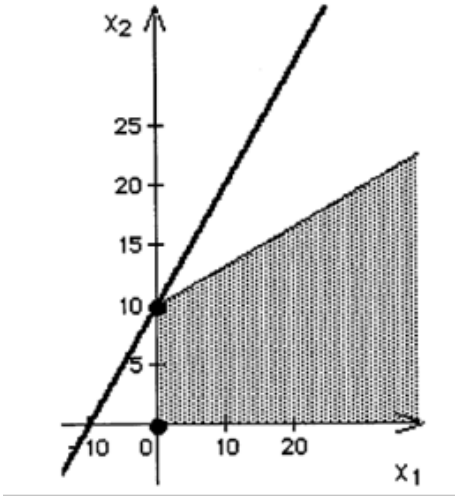
Part 1

4.5-2.

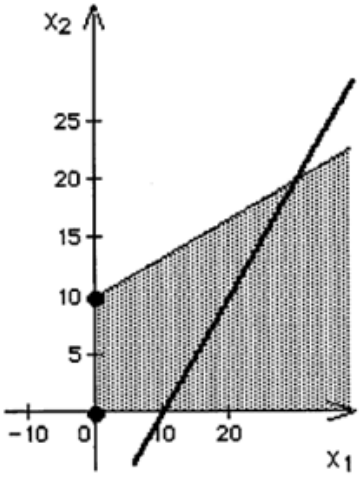
a)



b) Yes. Optimal solution is: $(x_1, x_2) = (0, 10)$ e $Z^* = 10$.



c) No. The objective function value is maximized by sliding the objective function line to the right. This can be done forever, so there is no optimal solution.



d) No, solutions exist that will make the profit arbitrarily large. This usually occurs when a constraint is left out of the model.

e)

Iteration 0	x_1	x_2	x_3	x_4	Right side
Z	1	-1	0	0	0
x_3	-1	3	1	0	30
x_4	-3	1	0	1	30

Iteration 1	x_1	x_2	x_3	x_4	Right side
Z	2/3	0	1/3	0	10
x_2	-1/3	1	1/3	0	10
x_4	-8/3	0	-1/3	1	20

$Z^*_{(0,10)} = 10$.

4.5-7.

a)

s.t. $x_1 \leq 6$

$x_2 \leq 3$

$-x_1 + 3x_2 \leq 6$

b)

Objective function (Z)	Multiple optimal solutions
$Z = -x_1 + 3x_2$	Line segment between (0, 2) e (3, 3)
$Z = x_2$	Line segment between (3, 3) e (6, 3)
$Z = x_1$	Line segment between (6, 3) e (6, 0)
$Z = -x_2$	Line segment between (0, 0) e (6, 0)
$Z = -x_1$	Line segment between (0, 0) e (0, 2)

c) Optimal solution: $Z^*_{(0,\,2)} = 4$.

Corner point (x_1, x_2)	$Z = -x_1 + 2x_2$
(0, 0)	Z = 0
(0, 2)	Z = 4
(3, 3)	Z = 3
(6, 3)	Z = 0
(6, 0)	Z = -6

d)

max. $Z = -x_1 + 2x_2$

s.t. $x_1 \le 6$

$x_2 \le 3$

$-x_1 + 3x_2 \le 6$

Iteration 0	x_1	x_2	x_3	x_4	x_5	Right side
Z	1	-2	0	0	0	0
x_3	1	0	1	0	0	6
x_4	0	1	0	1	0	3
x_5	-1	3	0	0	1	6

Iteration 1	x_1	x_2	x_3	x_4	x_5	Right side
Z	1/3	0	0	0	2/3	4
x_3	1	0	1	0	0	6
x_4	1/3	0	0	1	-1/3	1
x_2	-1/3	1	0	0	1/3	2

Optimal solution: $Z^*_{(0,\,2)} = 4$.

Part 2

4.5-8.

Iteration 0:

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Rignt Side
Z	0	1	-1	-1	-1	-1	0	0	0
X_5	1	0	1	1	0	0	1	0	3
X_6	2	0	0	0	1	1	0	1	2

Iteration 1:

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	0	-1	-1	1	0	3
X_1	1	0	1	1	0	0	1	0	3
X_6	2	0	0	0	1	1	0	1	2

Iteration 2:

Bas Var	Eq No	Coefficient of Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	0	0	0	1	1	5
X_1	1	0	1	1	0	0	1	0	3
X_3	2	0	0	0	1	1	0	1	2

Optimal solution: $Z^*_{(3, 0, 2, 0)} = 5$.