Degree in Informatics Engineering

Worksheet 3

Subject(s):

- Simplex method

Exercises from: F. S. Hillier and G. L. J., Introduction to Operations Research. McGraw-Hill Education, 2015.

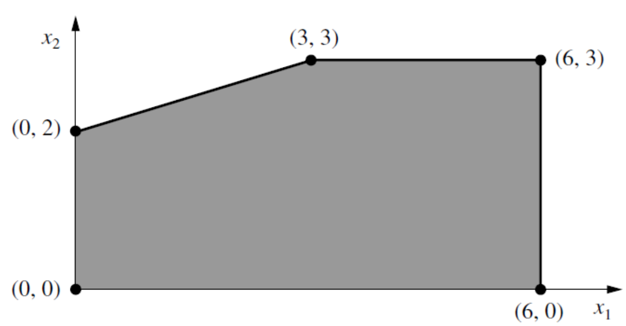
Part 1

4.5-2. Suppose that the following constraints have been provided for a linear programming model with decision variables x_1 and x_2.

$$-x_1 + 3x_2 \leq 30$$
$$-3x_1 + x_2 \leq 30$$
$$x_1 \geq 0, x_2 \geq 0$$

a) Demonstrate graphically that the feasible region is unbounded.
b) If the objective is to maximize $Z = -x_1 + x_2$, does the model have an optimal solution? If so, find it. If not, explain why not.
c) Repeat part (b) when the objective is to maximize $Z = x_1 - x_2$.
d) For objective functions where this model has no optimal solution, does this mean that there are no good solutions according to the model? Explain. What probably went wrong when formulating the model?
e) Select an objective function for which this model has optimal solution. Then work through the simplex method step by step to achieve the optimal Z.

4.5-7. Consider a two-variable linear programming problem whose CPF solutions are $(0,0)$, $(6,0)$, $(6,3)$, $(3,3)$, and $(0,2)$. See the next figure for a graph of the feasible region.



a) Use the graph of the feasible region to identify all the constraints for the model.
b) For each pair of adjacent CPF solutions, give an example of an objective function such that all the points on the line segment between these two corner points are optimal solutions.
c) Now suppose that the objective function is $Z = -x_1 + 2x_2$. Use the graphical method to find all the optimal solutions.
d) For the objective function in part (c), work through the simplex method step by step to find all the optimal BF solutions. Then write an algebraic expression that identifies all the optimal solutions.

Part 2

4.5-8. Consider the following problem.

$$Maximize\ Z = x_1 + x_2 + x_3 + x_4$$

subject to

$$x_1 + x_2 \leq 3$$
$$x_3 + x_4 \leq 2$$

and

$$x_j \geq 0, for\ j = 1, 2, 3, 4.$$

Work through the simplex method step by step to find all the optimal BF solutions.