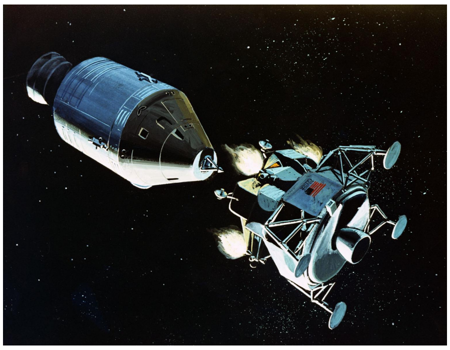

Guidance Navigation and Control System for a Rendezvous and Docking Mission

Design Report

Group Number 2

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Report elaborated for the Curricular Unit

Guidance Navigation and Control

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Contents

Chapter 1

Introduction

One of the most complex and demanding maneuvers in an Apollo mission was the rendezvous and docking of the Lunar Module (LM) ascent stage with the Command and Service Module (CSM). The mission design principles were largely shaped by experience from the Gemini program, which not only refined rendezvous techniques but also influenced the design and capabilities of the Lunar Module. Two primary methods emerged from this experience: the coelliptic and direct rendezvous approaches. Both relied on the same fundamental principles, assuming that a single "active" vehicle would perform all the necessary maneuvers. This vehicle would be placed in a lower orbit to catch up with its target, executing precise engine burns to adjust its orbital plane (if needed) and then raising its orbit to intercept the passive vehicle.

During the development of the Gemini rendezvous strategy, it became clear that standardizing the terminal phase was crucial for mission success. Defining specific vehicle positions, closure rates, and lighting conditions helped simplify the most critical phases of rendezvous and docking, significantly reducing both crew training requirements and in-flight workload. Mathematical models, simulations, and Gemini flight data established that the optimal altitude difference between the two spacecraft should be 15 miles, with the active vehicle positioned below the passive one. A transfer angle of 130 degrees was chosen to ensure that the active vehicle would traverse the required distance while ascending to the target orbit. Additionally, lighting conditions were carefully planned so that the Sun would remain behind the active vehicle during the braking phase. With these parameters in place, mission planners worked backward to design ascent trajectories and orbital profiles that would precisely set up the final approach phase.

For Apollo 11, the coelliptic rendezvous method was selected. To simplify the simulation and maintain a two-dimensional approach, the rendezvous maneuvers in this project assume that the LM starts in an 83 km circular orbit, while the passive CSM is positioned in a 110 km coplanar circular orbit. This setup ensures a planar rendezvous, where the LM performs ascent burns and orbital adjustments to match the CSM's position and velocity. By focusing on a 2D model, the analysis remains centered on radial and along-track dynamics in lunar orbit, avoiding the complexities of simulating plane changes [1].

1.1 Document Outline

The following chapters will outline the project's objectives, starting with the minimum required goal and progressing toward the ideal outcome. This will involve transitioning from a simplified, constrained version of the previously introduced problem to a more general case that better reflects real-world conditions. Next, an overview will be provided on the approach taken to achieve these objectives, beginning with the fundamental equations that define the problem and the methodology used to solve them. Finally, a detailed examination of the vehicles involved in the mission will be presented, with particular emphasis on the active element—the Lunar Module—as well as its sensors and actuators, which play a crucial role in the mission's success.

Chapter 2

Goal

2.1 General Idea

Within this project the general goal is to design, implement and simulate a GNC system for a rendezvous and docking mission of a chasing spacecraft to a target spacecraft. In the following, these spacecrafts are referred to as chaser and target. To be able to accomplish the desired goal, which is described in section 2.5, the problem will be approached from a very simple scenario which will then be extended incrementally towards a 2D orbit docking scenario. In the following, the minimum, the intermediate and the desired goal are presented in more detail after the constraints are listed.

2.2 Constraints

Everything is done in 2D - what else??

Point masses with inertia

tolerance range for docking with respect to speed, position, and angle.

For the minimum and intermediate goals, first no disturbance forces and torques are considered. After successful implementation the simulation might be extended by adding disturbances. For the desired goal, no orbit disturbances are considered first but might be considered later as well.

2.3 Minimum Goal

The starting scenario is depicted in fig. 2.1 consists of a the target located at a fixed position in space $p_{target} = (0, 0)$ and of the chaser initially located at a certain distance d with respect to the target $p_{chaser} = (0, -d)$. No forces are acting on the two spacecrafts except of the ones generated by the actuators of the chaser. The initial attitude of the chaser is chosen so that the docking port of the chaser and of the target are aligned. Using the actuators (specified in section 4), the chasers velocity will be controlled so that a successful docking will take place. The Apollo 11 used a probe and drogue docking system, and more modern spacecraft use the soft dock hard dock system, these systems allow minor alignment errors, which will be

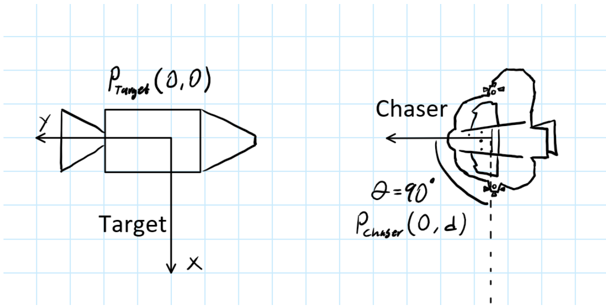

Figure 2.1: Minimum goal

included in this simulation as tolerances in the angle and position when docking. The docking speed will also have a tolerance of 0.03-0.06 m/s and will be regarded as being handled by the docking system, after which the simulation will end. To implement and validate a GNC system for this scenario is considered as the minimum goal of this project.

TO DO: Specify criteria for successful docking (velocity, position);

Regarding velocity: in matlab simulation, they used for distances $> 10m$ 0.3 m/s and reduce the speed to 0.03 m/s if they get closer [2]

2.4 Intermediate Goal

The first extension consists of a random initialization of the chaser's attitude and position $p_{chaser} = (x_{c,0}, y_{c,0})$ awsell as an initial angel θ_0 (see fig. 2.2). For this scenario the attitude control must be taken into account and will be implemented within the GNC system. Initially, no forces are acting on the two spacecrafts except of the ones generated by the actuators of the chaser. The same success criteria as defined in section 2.3 are used to validate the GNC system. One idea of approaching this problem is to first set a target position and attitude of $p_1 = (0, -d_1)$ and $\theta_1 = 90\,\mathrm{deg}$. After successful moving the chaser to that position and attitude, the controller developed in the first task can be used.

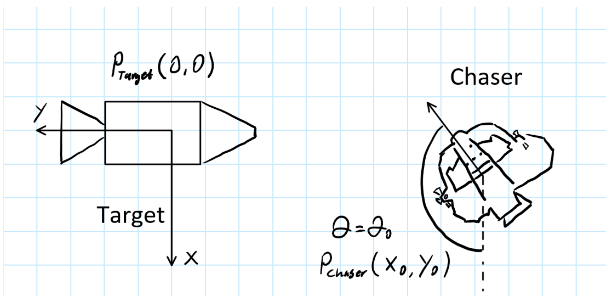

Figure 2.2: Intermediet goal

2.5 Desired Goal

The desired goal of this project is to implement a GNC system for a 2D orbit docking. In this scenario, the target is in a circular lunar orbit with a certain radius r_{target}. The chaser is in the same orbit plane with a different true anomaly and initially has a random attitude. The task of the GNC system is to rendezvous and dock to the target spacecraft performing the v-bar approach, meaning that the chaser approaches the target from behind. For that the actuators are used to implement a position and attitude control.

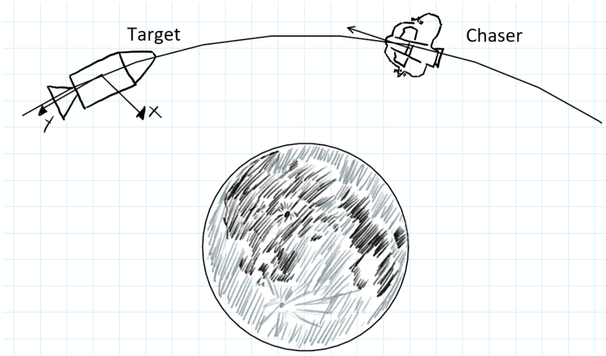

Figure 2.3: Desierd goal

2.6 Optional Extension

If the desired goal can be achieved early and additional time is available, the scenario mentioned in section 2.5 can be extended by starting the mission of the chaser in a lower circular lunar orbit with radius r_{chaser} (see fig. 2.4). The orbital plan is still the same as the one of the target. Then, first a Hohmann transfer must be performed to get the chaser into the same orbit as the target. Afterwards, a phasing manoeuvrer has to be performed to reduce the difference of the true anomaly of the chaser and the target. When these manoeuvrer are performed successfully by the GNC system, the remaining docking can be performed by the system developed in section 2.5.

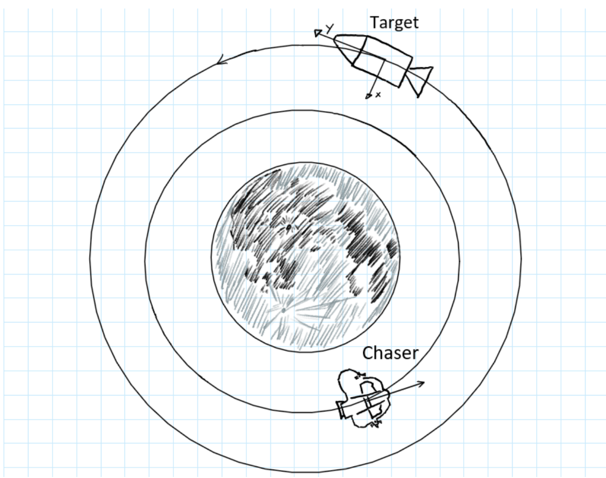

Figure 2.4: Optional extension

Chapter 3

Methodology

3.1 General Approach

To realise and validate the proposed problems, a closed-loop simulation will be developed. A block diagram of that simulation is shown in fig. 3.1.

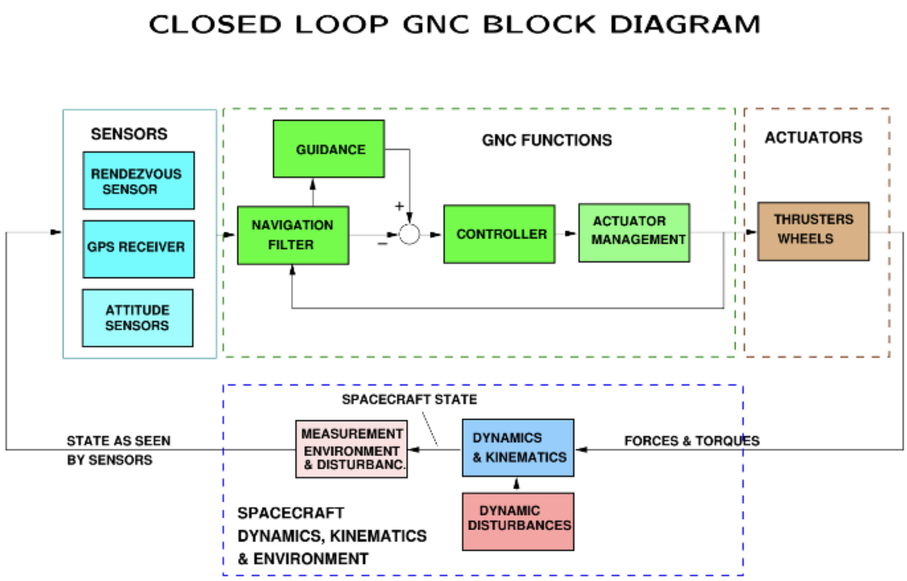

Figure 3.1: Block-diagram for the GNC system simulation

3.2 Non-Orbit Scenario

3.2.1 Translational Dynamics (Position and Velocity)

For the non-orbit scenario (referring to section 2.3 and 2.4), an inertial 2D coordinate frame is defined where the target is fixed at the origin and the position of the chaser is given by $p_{chaser}(t) = \begin{bmatrix} x_c(t) \; y_c(t) \end{bmatrix}^T$.

The motion of the chaser relative to the target can be described by Newton's law:

$$F_x = m_c \cdot \ddot{x}_c \tag{3.1}$$

$$F_y = m_c \cdot \ddot{y}_c \tag{3.2}$$

where:

- $\ddot{x}, \ddot{y}$: Relative velocity of the chaser.
- F_x, F_y: Control forces from actuators.
- m_c: Mass of the chaser spacecraft.

A solution of these differential equations can be written in closed-form if the control forces are assumed to be constant within the considered time interval:

$$x_c(t) = x_{c,0} + v_{c,x_0} t + \frac{1}{2} a_x t^2, \tag{3.3}$$

$$y_c(t) = y_{c,0} + v_{c,y_0} t + \frac{1}{2} a_y t^2, \tag{3.4}$$

where:

- $x_{c,0}$, $y_{c,0}$: Initial position of chaser.
- v_{c,x_0}, v_{c,y_0}: Initial velocity of chaser.
- $a_x = \frac{F_x}{m_c}$, $a_y = \frac{F_y}{m_c}$: Acceleration generated by control force.

3.2.2 Rotational Dynamics (Attitude Motion)

The attitude dynamics of the chaser are described by Euler's equation in 2D:

$$I_c \ddot{\theta} = \tau, \tag{3.5}$$

where:

- I_c: Moment of inertia of the chaser about the z-axis.
- θ: Attitude angle relative to the LVLH frame.
- $\dot{\theta}$: Angular velocity.
- τ: Control torque applied to rotate the chaser.

A solution of this differential equations can be written in closed-form if the control torque is assumed to be constant within the considered time interval:

$$\theta(t) = \theta_{c,0} + \omega_{c,0} t + \frac{1}{2} \alpha t^2 \tag{3.6}$$

where:

- $\theta_{c,0}$: Initial orientation of chaser.
- $\omega_{c,0}$: Initial angular velocity of chaser.
- $\alpha = \frac{\tau}{I_c}$: Angular acceleration generated by control torque.

3.2.3 Control Inputs (Forces and Torques)

Control inputs are applied to maneuver the chaser:

- **Translational Control:** Adjusting thrusters to modify F_x and F_y.
- **Rotational Control:** Using reaction wheels or thrusters to control τ.

3.2.4 Full State Space Model

A combined state vector is defined that includes both translational and rotational dynamics:

$$\mathbf{x} = \begin{bmatrix} x \ y \ \dot{x} \ \dot{y} \ \theta \ \dot{\theta} \end{bmatrix}^T, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \ \dot{y} \ \ddot{x} \ \ddot{y} \ \dot{\theta} \ \ddot{\theta} \end{bmatrix}^T \tag{3.7}$$

With the following expressions for accelerations:

$$\ddot{x} = \frac{F_x}{m_c} \tag{3.8}$$

$$\ddot{y} = \frac{F_y}{m_c} \tag{3.9}$$

$$\ddot{\theta} = \frac{\tau}{I_c}. \tag{3.10}$$

The state evolution equations can be written in matrix form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3.11}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_c} & 0 & 0 \\ 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_c} \end{bmatrix} \tag{3.12}$$

and:

$$\mathbf{u}(t) = \begin{bmatrix} F_x(t) \\ F_y(t) \\ \tau(t) \end{bmatrix} \tag{3.13}$$

3.3 In-Orbit Scenario

3.3.1 Translational Dynamics (Position and Velocity)

For modelling the scenario in orbit, the 2D LVLH frame is considered which is centred on the target spacecraft, rotating with its orbit:

- x axis: Radial (towards the moon).
- y axis: Along-track (in the direction of velocity vector).

The motion of the chaser relative to the target in a circular orbit is governed by the 2D Hill's Equation equations [3]:

$$\ddot{x} - 3n^2 x + 2n\dot{y} = \frac{F_x}{m_c}, \tag{3.14}$$

$$\ddot{y} - 2n\dot{x} = \frac{F_y}{m_c}, \tag{3.15}$$

where:

- x, y: Relative position of the chaser in the LVLH frame.
- $\dot{x}, \dot{y}$: Relative velocity.
- F_x, F_y: Control forces from actuators.
- m_c: Mass of the chaser spacecraft.
- $n = \sqrt{\mu/a^3}$: Mean motion of the target's orbit.

Assuming pulses (instantaneous changes of velocity) and for constant forces a closed-form solution can be derived by the Clohessy-Wiltshire equations [3]. However, as they are only approximations, a numerical integration of the Hill's equation could be required in order to obtain exact results.

3.3.2 Rotational Dynamics (Attitude Motion)

The attitude dynamics of the chaser are described by Euler's equation in 2D:

$$I_c\ddot{\theta} = \tau, \tag{3.16}$$

where:

- I_c: Moment of inertia of the chaser about the z-axis.
- θ: Attitude angle relative to the LVLH frame.
- $\dot{\theta}$: Angular velocity.
- τ: Control torque applied to rotate the chaser.

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A combined state vector is defined that includes both translational and rotational dynamics:

$$\mathbf{x} = \begin{bmatrix} x \ y \ \dot{x} \ \dot{y} \ \theta \ \dot{\theta} \end{bmatrix}^T, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \ \dot{y} \ \ddot{x} \ \ddot{y} \ \dot{\theta} \ \ddot{\theta} \end{bmatrix}^T \tag{3.17}$$

With the following expressions for accelerations:

$$\ddot{x} = 3n^2 x - 2n\dot{y} + \frac{F_x}{m_c}, \tag{3.18}$$

$$\ddot{y} = -2n\dot{x} + \frac{F_y}{m_c}, \tag{3.19}$$

$$\ddot{\theta} = \frac{\tau}{I_c}. \tag{3.20}$$

The state evolution equations can be written in matrix form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3.21}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -2n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_c} & 0 & 0 \\ 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_c} \end{bmatrix} \tag{3.22}$$

and:

$$\mathbf{u}(t) = \begin{bmatrix} F_x(t) \\ F_y(t) \\ \tau(t) \end{bmatrix} \tag{3.23}$$

Chapter 4

Vehicle Description

4.1 Actuators

4.2 Sensors

Chapter 5

Literature

For developing the GNC system for the rendezvous and docking, different material was found that could be helpful for the further development. They are shortly described in the following.

In [2], a detailed *Matlab* simulation is presented that covers the rendezvous and docking of a chaser and a target spacecraft in low earth orbit. Celestini [4] presents different algorithms for the navigation and guidance for docking missions in Earth orbit. Furthermore, general methods and concepts regarding all three system components - namely guidance, navigation and control subsystem - are presented in [3].

Bibliography

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