

# ***Mars Entry Phase Trajectory Tracking Controller using Dynamic Inversion***

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**Abstract**— Typical Mars Lander mission involves entry phase, parachute descent and the terminal powered descent. Reference trajectory for the entry phase will be planned on ground considering the initial and target states along with satisfying the aerodynamic constraints. This paper proposes to use a controller designed on the basis of Dynamic Inversion for tracking the reference state trajectory along with a novel strategy to reduce cross-range error by partial bank angle control. Nominal two-dimensional trajectories are planned with lift and drag as the control parameters which inadvertently points out to required angle-of-attack and zero bank angle. Under perturbations in initial states, reference state trajectory will be tracked through dynamic inversion. New control solution would finally come out in terms of new angle-of-attack and bank angle manoeuvres. Through considering cross-range and cross velocities as additional states, bank angle solution of one axis is imposed onto the other alternatively at regular intervals thus reducing the cross-range error. The resultant tracking law has been observed to track even large dispersions especially in initial velocity and even under constraints in aerodynamic angles.

**Index Terms**—*Dynamic Inversion; Entry Corridor; Lift; Drag; Angle-of-Attack; Lander.*

## I. INTRODUCTION

Upon arrival on Mars from an interplanetary cruise, the Lander finally has to traverse through the thin Martian atmosphere and land at a designated site. Typical entry interface happens at 125km with a large velocity between 3km/s and 7.5km/s. In the initial phase of aero braking it is required to ensure that the spacecraft is not structurally stressed or gets too much heated up. When the Lander reaches an altitude between 10 to 15km, parachute deployment will take place for further braking. By this time the vehicle velocity will come around Mach 2 due to aero braking.

Nominal trajectory planning for the entry phase will be

done before the flight through ground simulations [8]. This can be done through numerical optimization techniques while also including the entry constraints. Typical performance index may be to minimize the terminal state errors having given the initial states at entry and aerodynamic constraints.

Actual flight conditions at entry may be slightly different. Strategy would be to track the reference nominal trajectory planned on ground under perturbations in initial states. This paper discusses about the use of Dynamic Inversion based controller for tracking the nominal state trajectory within the tolerable entry corridor. Nominal trajectory has been defined in a two-dimensional plane with no bank angle manoeuvre. Drag and Lift forces are considered as the control parameters. Under perturbations, the required angle-of-attack and bank angle can be computed from these forces. However these angles may also be constrained and hence during the initial transient phase of trajectory tracking, lift and drag forces are not expected to meet the required value. Nevertheless, satisfactory tracking is demonstrated.

Shuttle entry guidance is discussed in [1]. In order to have trajectory tracking, the required ratio of vertical component of lift and drag is obtained by adding to the reference ratio of these components, terms involving error in current drag and reference drag and error in current vertical velocity and reference vertical velocity. The error terms are multiplied by suitable gains. Bank angle is computed as inverse cosine of the ratio of reference vertical lift-drag ratio to the required vertical lift-drag ratio. Similar philosophy is mentioned in [2]. However an additional integral term is included in computing the reference lift-drag ratio. This prevents steady-state trajectory deviations due to error in determining altitude rate. A slightly different approach is mentioned in reference [3] & [4]. The predicted range-to-go is calculated as a function of drag and altitude rate errors with respect to the nominal reference trajectory profile. The desired vertical component of lift to drag ratio is calculated as a function of actual and predicted range-to-go. Bank angle reversal logic is also mentioned in [3] &

[4] whenever cross-range crosses a threshold. A numerical Roll Reversal Predictor-Corrector guidance algorithm for the atmospheric flight portion is introduced in [5]. Linear Quadratic Regulator based trajectory tracking controller was designed in [6]. Reference [7] discusses about the pre-flight aerodynamics data for the Mars phoenix entry capsule and has been taken as reference for the aerodynamic data considered for analysis results put in this paper. First part of the analysis through Dynamic Inversion [9] in this paper has been carried out to track the reference profile along the two-dimensional plane. This will result in bank angle manoeuvre also which would contribute to cross direction movement. An intelligent adaptive predictor-corrector guidance method with improvements over the conventional predictor-corrector method is brought out in [10]. A guidance based on adaptive reference drag profile is discussed in [11] where reference drag profile is tracked through feedback linearization. A convex optimization based guidance approach is proposed in [12].

The second part of this paper introduces a strategy to plan bank reversals so as to make cross-range error to minimum. The approach is to consider the cross-range and cross velocity as additional states in state space formulation. Since bank angle influences dynamics along cross axis direction as well as an in-plane axis, bank angle solution that satisfies one axis at a time and perturbs the other and vice versa is adopted. Simulation results show satisfactory trajectory tracking along in-plane direction and minimized cross-range error. Section II introduces typical entry dynamics and entry constraints. Section III discusses about the reference trajectory planning. Dynamic Inversion controller formulation is mentioned in section IV along with simulation results. Section V discusses about the strategy to reduce cross-range errors with simulation results.

## II. ENTRY DYNAMICS AND CONSTRAINTS

The in-plane equations of motion [3] associated with atmospheric flight can be expressed in state space form as:

$$\dot{x}_1 = x_3 \sin(x_4) \quad (1)$$

$$\dot{x}_2 = x_3 \cos(x_4)/x_1 \quad (2)$$

$$\dot{x}_3 = -g \times \sin(x_4) - \text{drag} \quad (3)$$

$$\dot{x}_4 = (\text{lift} \times \cos(\sigma)/\text{mass} \times x_3) + ((x_3^2/x_1 - g) \times \cos(x_4)/x_3) \quad (4)$$

Where,

$x_1$  is the position along local vertical,  $x_2$  is the polar angle from the spin axis of mars to the local vertical vector,  $x_3$  is the velocity vector,  $x_4$  is the flight path angle,  $\sigma$  is the bank angle, control  $u_1$  is lift  $\times \cos(\sigma)$  and  $u_2$  is drag acceleration.

$$g = g_0 \times R^2/x_1^2 \quad (5)$$

$$u_1 = (0.5 \times c_l \times S \times \rho \times x_3^2) \times \cos(\sigma) / \text{mass} \quad (6)$$

$$u_2 = (0.5 \times c_d \times S \times \rho \times x_3^2) / \text{mass} \quad (7)$$

Here,  $c_l$  is the coefficient of lift;  $c_d$  is the coefficient of drag. Both of them are assumed as functions of angle-of-attack ( $\alpha$ ).  $g_0$  is the acceleration due to gravity on mars surface.  $R$  is the radius of mars.  $S$  is the reference area of the vehicle.

Entry constraints considered are expressed as:

$$0.5 \times \rho \times v^3 \leq H_{\max} \quad (8)$$

$$0.5 \times \rho \times v^2 \leq P_{\max} \quad (9)$$

$$\sqrt{\text{lift}^2 + \text{drag}^2} / g_0 \leq LF_{\max} \quad (10)$$

Where,

$H_{\max}$  is the maximum allowable heat load,  $P_{\max}$  is the maximum allowable dynamic pressure and  $LF_{\max}$  is the maximum allowable load factor.

Cross-range position and velocity are expressed in state form as:

$$\dot{x}_5 = x_6 \quad (11)$$

$$\dot{x}_6 = \text{lift} \times \sin(\sigma) / \text{mass} \quad (12)$$

Where,  $x_5$  is the cross-range and  $x_6$  is the cross velocity.

## III. REFERENCE TRAJECTORY PLANNING

The entry dynamics can be represented as

$$\dot{x} = Ax + Gu \quad (13)$$

Where,

$$A = \begin{bmatrix} 0 & 0 & \sin(x_4) & 0 \\ 0 & 0 & \cos(x_4)/x_1 & 0 \\ g_0 \times R^2 \times \sin(x_4)/x_1^3 & 0 & 0 & 0 \\ g_0 \times R^2 \times \cos(x_4)/x_1^3 \times x_3 & 0 & \cos(x_4)/x_1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 1/x_3 & 0 \end{bmatrix}$$

Lift and drag coefficients are assumed as function of angle of attack as follows:

$$c_l = 0.0162 \times \alpha + 0.4062 \quad (14)$$

$$c_d = 0.0162 \times \alpha - 0.234 \quad (15)$$

The simulation data based on online survey of past mars missions is as follows

Flight time = 145 seconds  
 Lander mass = 800kg  
 Reference area (S) = 3.0124 m<sup>2</sup>  
 Initial altitude = 125km,  
 Initial Velocity = 5000 m/s  
 Initial Flight path angle = -54 deg,  
 Initial downrange = 0 km  
 Maximum heat load = 5.665\*10<sup>7</sup> kg/s<sup>3</sup>,  
 Maximum Dynamic pressure = 14000Pa  
 Maximum load factor = 4g  
 Simulation step size=1second

For reference trajectory generation, angle-of-attack is constantly maintained at 50 degrees and bank angle at 0 degrees for the entire simulation duration.

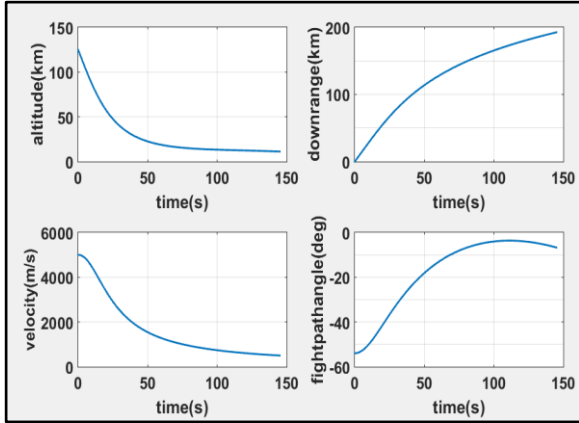


Fig.1: Reference state trajectory

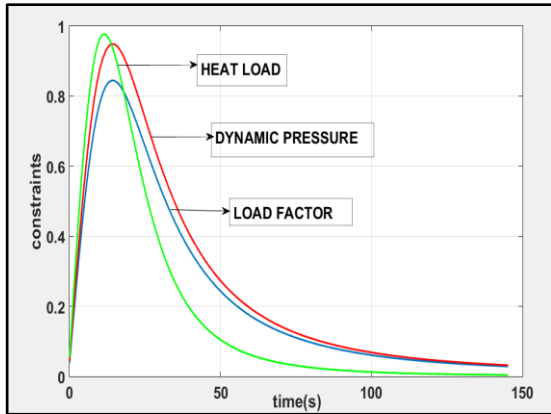


Fig.2: Normalized reference constraints

Fig. 1 shows the variation in Lander states. After 145 seconds altitude reaches 11.2km and velocity is brought down to 502 m/s. Total downrange travelled is 192.6km and terminal flight path angle is -6.7 degrees. Fig.2 shows the normalized constraints plot.

#### IV. TRACKING THROUGH DYNAMIC INVERSION CONTROLLER

Dynamic Inversion Controller is applicable to systems that can be represented in the state form as shown below,

$$\dot{x} = f_y(x) + [g_y(x)] \quad (16)$$

If the error tracking is defined as

$$E(t) = x(t) - x^*(t) \quad (17)$$

Where,  $x(t)$  is the current state and  $x^*(t)$  is the reference state.

Then, a gain K can be selected such that

$$\dot{E} + KE = 0 \quad (18)$$

Usually,  $K = \text{diag}(1/\tau_i)$

Carrying out the algebra,

$$(\dot{x} - \dot{x}^*) + K(x - x^*) = 0 \quad (19)$$

$$f_y(x) + [g_y(x)]U = \dot{x}^* - K(x - x^*) \quad (20)$$

Solving for the controller,

$$U = g_y(x)^{-1} \{ \dot{x}^* - K(x - x^*) - f_y(x) \} \quad (21)$$

Simulation studies were carried out by perturbing the initial altitude and velocity.

Gain matrix k is selected as 0.5\*I, where I is the identity matrix. Fig.3. shows the tracking of the states. The altitude and the velocity were perturbed while the flight path angle and the downrange were kept at the same value as that of the reference. All the states were found to track the state reference trajectory. The perturbed values taken are as follows:

Altitude=128 km,  
 Velocity=5300 m/s.

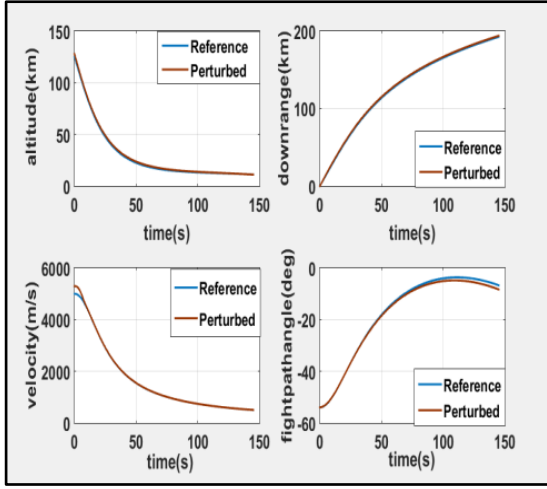


Fig.3: Reference and perturbed states

Fig.4. shows the normalized constraints under nominal and perturbed conditions. The constraints were not violated and were found to track the nominal constraint trajectory profile.

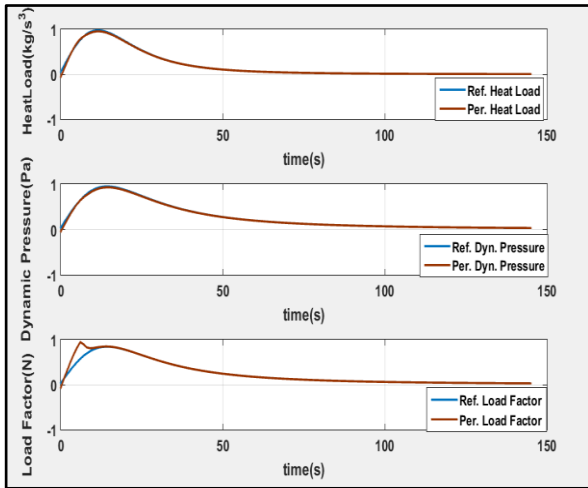


Fig.4: Aerodynamic constraints profile

Fig.5. shows the angle-of-attack and the bank angle profile. It is required to saturate angle of attack and bank angle saturated at 87 degrees. Angle-of- attack is found to be within saturation for 11 seconds. This would mean that for a brief moment drag and lift forces would not be the required one as the solution of Eq. 21 demands. However, the perturbed trajectory tracks the reference satisfactorily.

Due to the presence of banking there will be out-of-plane parameter error. Fig. 6 shows the cross- range position and velocity errors. It can be understood from the plot that there is a high error of cross-range (around 62 km) and cross velocity (550m/s).

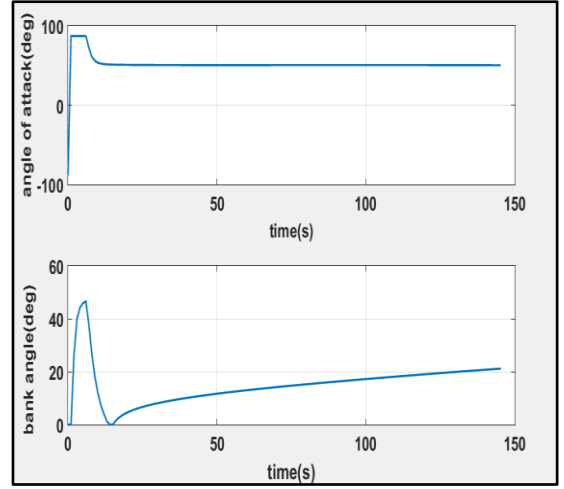


Fig.5: Angle-of-attack and bank angle

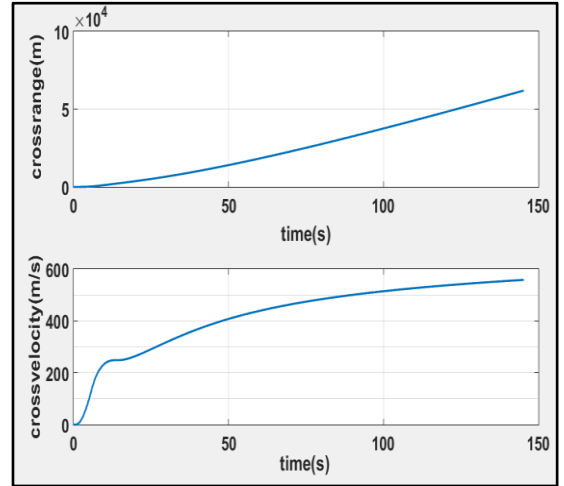


Fig.6: Out-of-plane profile

## V. STRATEGY FOR CROSS-RANGE MINIMIZATION

In order to minimize the out-of-plane errors, the corresponding states and additional control is introduced in the Dynamic Inversion Formulation. The additional control is as

$$u_3 = (0.5 \times c_l \times S \times \rho \times x_3^2) \times \sin(\sigma) / \text{mass} \quad (22)$$

In order to do perturbation study, the reference trajectory is generated with the cross-range position kept at 0 m, cross-range velocity at 0 m/s along with the states discussed in the section III of this paper. The dynamic inversion strategy is then applied with the altitude and the velocity perturbation same as that in section IV. Once the solution is obtained by the controller, it needs to be said that control  $u_1$  and  $u_3$  will not have solution satisfying both of them simultaneously. In order to deal with this, strategy adopted is to take bank angle solution that satisfies one axis at the current step time and perturbs the other and vice versa. So the bank angle solution

alternatively satisfies one axis and perturbs the other. Fig.7 shows satisfactory trajectory tracking under perturbations. Fig.8 shows the normalized constraints under nominal and perturbed condition. It is found that the constraints are not violated even under perturbation. Like two-dimensional formulation in previous section, angle-of-attack and bank angle are saturated at 87 degrees if the requirement is more. Subsequently, lift and drag forces actually influencing the dynamics is re-calculated. Angle-of-attack and bank angle solutions are given in Fig.9. Bank angle completes a cycle in 2 seconds. A suitable attitude controller having bandwidth more than 0.5 Hz would track this command. It needs to be emphasised here that the cross-range error is only 1.08km and cross velocity has converged near to zero (Fig. 10). From Fig. 9 it can be inferred that constraints are also satisfactory under perturbations.

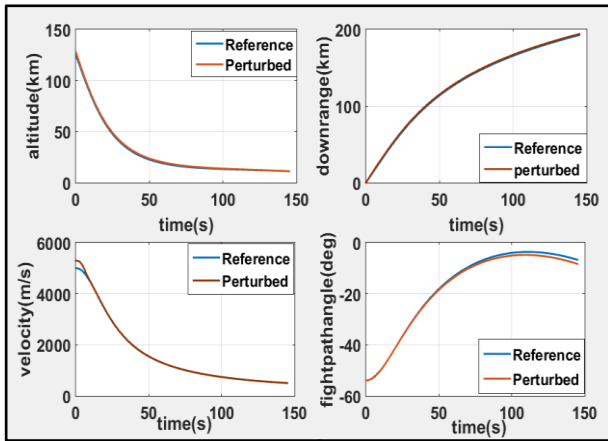


Fig.7: Reference and perturbed states

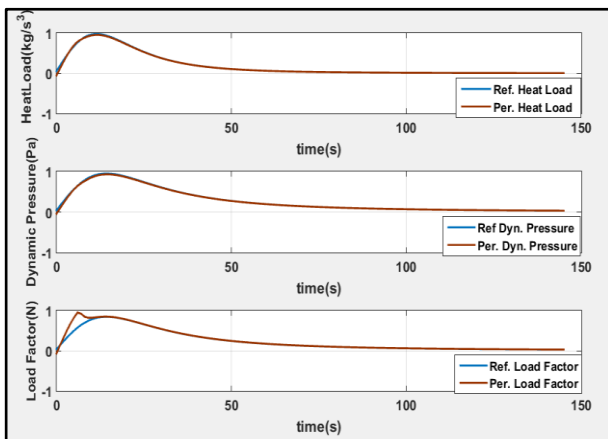


Fig.8: Aerodynamic constraints profile

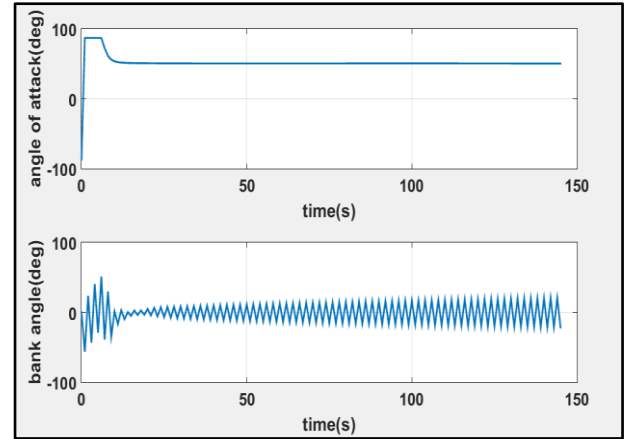


Fig.9: Angle-of-attack and bank angle

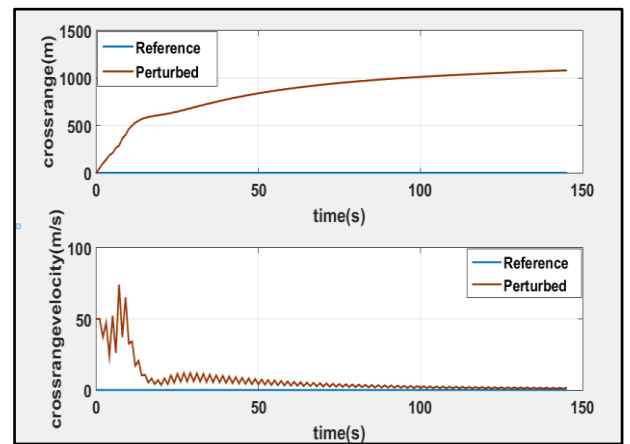


Fig.10: Cross-range position and velocity

## VI. CONCLUSIONS

A reference trajectory planned for the entry phase of mars landing mission has been successfully tracked under initial perturbations using a Dynamic Inversion controller. Initial trajectory tracking has been demonstrated for two-dimensional motion with significant out-of-plane errors and constraints in angle-of-attack and bank angle. Subsequently out-of-plane states were also considered in the controller formulation. A novel strategy was adopted to take bank angle solution that satisfies one axis at the current step time and perturbs the other and vice versa. Simulation results show satisfactory trajectory tracking under initial perturbation in position and velocity and even under large dispersions in velocity.

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