GABARITO ATIVIDADE 2

Método Substituição

a) $\int f(x)dx$, sendo f(x) = x e^{-x^2}

$$u = -x^2$$

$$\frac{du}{dx} = -2x \rightarrow \frac{du}{-2} = xdx$$

Substituindo:

$$\int xe^{-x^2}dx = \int \frac{e^u du}{-2} = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C$$

Voltando:

$$\int xe^{-x^2}dx = -\frac{1}{2}e^{-x^2} + C$$

b) $\int f(x)dx$, sendo f(x) = $2x(1+x^2)^{-1}$

$$u = 1 + x^2$$

$$\frac{du}{dx} = 2x \rightarrow du = 2xdx$$

Substituindo:

$$\int 2x(1+x^2)^{-1}dx = \int u^{-1}du = \ln|u| + C$$

Voltando:

$$\int 2x(1+x^2)^{-1}dx = \ln|1+x^2| + C$$

c) $\int f(x)dx$, sendo $f(x) = (xlnx)^{-1} = \frac{1}{x}.\frac{1}{lnx}$

$$u = lnx$$

$$\frac{du}{dx} = \frac{1}{x} \rightarrow du = \frac{dx}{x}$$

Substituindo:

$$\int \frac{1}{x}.\frac{1}{lnx}dx = \int \frac{1}{u}du = \ln|u| + C$$

Voltando:

$$\int \frac{1}{x}.\frac{1}{lnx}dx = \ln|lnx| + C$$

d) $\int f(x)dx$, sendo $f(x) = (5x+7)^{-1}$

$$u = 5x + 7$$

$$\frac{du}{dx} = 5 \to \frac{du}{5} = dx$$

Substituindo:

$$\int (5x+7)^{-1}dx = \int \frac{u^{-1}du}{5} = \frac{1}{5}\int \frac{1}{u}du = \frac{1}{5}\ln|u| + C$$

Voltando:

$$\int (5x+7)^{-1}dx = \frac{1}{5}\ln|5x+7| + C$$

e) $\int f(x)dx$, sendo $f(x) = (5x-7)^{-3}$

$$u = 5x - 7$$

$$\frac{du}{dx} = 5 \to \frac{du}{5} = dx$$

Substituindo:

$$\int (5x-7)^{-3}dx = \int \frac{u^{-3}du}{5} = \frac{1}{5}\int u^{-3}du = \frac{1}{5}.\frac{u^{-2}}{-2} + C = -\frac{1}{10u^2} + C$$

Voltando:

$$\int (5x+7)^{-3}dx = -\frac{1}{10(5x-7)^2} + C$$

f) $\int f(x)dx$, sendo $f(x) = sen(x+9)$

$$u = x + 9$$

$$\frac{du}{dx} = 1 \to du = dx$$

Substituindo:

$$\int sen(x+9)dx = \int sen(u)du = -\cos(u) + C$$

Voltando:

$$\int sen(x+9)dx = -\cos(x+9) + C$$

g) $\int f(x)dx$, sendo $f(x) = \mathrm{sen}^2\mathrm{x}\cos\mathrm{x}$

$$u = senx$$

$$\frac{du}{dx} = cosx \to du = cosxdx$$

Substituindo:

$$\int \mathrm{sen}^2 \mathrm{x} \cos \mathrm{x}\, dx = \int u^2 du = \frac{u^3}{3} + C$$

Voltando:

$$\int \mathrm{sen}^2 \mathrm{x} \cos \mathrm{x}\, dx = \frac{sen^3 x}{3} + C$$

h) $\int f(x)dx$, sendo $f(x) = \mathrm{x} \ + \sec^2 3x$

$$\int f(x)dx = \int xdx + \int \sec^2 3x\, dx$$

$$\int xdx = \frac{x^2}{2} + C$$

$$\int \sec^2 3x\, dx = ?$$

Fazendo $u = 3x$

$$\frac{du}{dx} = 3 \rightarrow \frac{du}{3} = dx$$

Substituindo:

$$\int \sec^2(3x)\, dx = \int \frac{\sec^2 u\, du}{3} = \frac{tgu}{3} + C$$

Voltando:

$$\int \sec^2 3x\, dx = \frac{tg(3x)}{3} + C$$

Portanto:

$$\int (\mathrm{x} \ + \sec^2 3x)\, dx = \frac{x^2}{2} + \frac{tg(3x)}{3} + C$$

i) $\int f(x)dx$, sendo $f(x) = (9 - 4x^2)^{-\frac{1}{2}} = \frac{1}{\sqrt{(3-2x)(3+2x)}}$

Fazendo $u = 2x$

$$\frac{du}{dx} = 2 \rightarrow \frac{du}{2} = dx$$

Substituindo:

$$\int \frac{1}{\sqrt{(3-2x)(3+2x)}} \quad dx$$

$$= \int \frac{1}{\sqrt{(3-u)(3+u)}} \frac{du}{2} \ = \frac{1}{2} \int \frac{1}{\sqrt{3^2 - u^2}} du = \frac{1}{2} arcsen\left(\frac{u}{3}\right) + C$$

Voltando:

$$\int (9-4x^2)^{-\frac{1}{2}}dx = \frac{1}{2}arcsen\left(\frac{2x}{3}\right)+C$$

j) $\int f(x)dx$, sendo $f(x) = (16 + x^2)^{-1}$
Fazendo $u = x$

$$\frac{du}{dx} = 1 \rightarrow du = dx$$

Substituindo:

$$\int (16 + x^2)^{-1}dx = \int \frac{1}{(16+u^2)}du = \int \frac{1}{4^2+u^2}du = \frac{1}{4}arctg\left(\frac{u}{4}\right)+C$$

Voltando:

$$\int (16 + x^2)^{-1}dx = \frac{1}{4}arctg\left(\frac{x}{4}\right)+C$$

k) $\int f(x)dx$, sendo $f(x) = x.(9 - 4x^2)^{-\frac{1}{2}} = \frac{x}{(9-4x^2)^{\frac{1}{2}}}$
Fazendo $u = 9-4x^2$

$$\frac{du}{dx} = -8x \rightarrow -\frac{du}{8} = xdx$$

Substituindo:

$$\int \frac{x}{(9-4x^2)^{\frac{1}{2}}}dx = \int \frac{1}{(u)^{\frac{1}{2}}}\left(-\frac{du}{8}\right) = -\frac{1}{8}\int u^{-\frac{1}{2}}du = -\frac{1}{8}.\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)+C = -\frac{1}{4}u^{\frac{1}{2}}+C$$

Voltando:

$$\int \frac{x}{(9-4x^2)^{\frac{1}{2}}}\,dx = -\frac{1}{4}(9-4x^2)^{\frac{1}{2}}+C$$

l) $\int f(x)dx$, sendo $f(x) = \frac{(x-2)^{\frac{1}{2}}}{x+1}$

Fazendo $u = (x-2)^{\frac{1}{2}}$

$$\frac{du}{dx} = \frac{1}{2}(x-2)^{-\frac{1}{2}}.1 \rightarrow du = \frac{1}{2\sqrt{x-2}}dx \rightarrow 2udu = dx$$
$$u^2 = x-2 \rightarrow u^2+3 = x+1$$

Substituindo:

$\int \frac{(x-2)^{\frac{1}{2}}}{x+1}dx = \int \frac{u}{u^2+3}2u\,du = 2\int \frac{u^2}{u^2+3}du = 2\int \left(1-\frac{3}{u^2+3}\right)du = 2\int 1du -$
$6\int \frac{1}{u^2+3}du = 2u - 6\int \frac{1}{u^2+\sqrt{3}^2}du = 2u - \frac{6.1}{\sqrt{3}}arctg\left(\frac{u}{\sqrt{3}}\right)+C = 2u -$
$2\sqrt{3}arctg\left(\frac{u}{\sqrt{3}}\right)+C$

Voltando:

$$\int \frac{(\mathrm{x}-2)^{\frac{1}{2}}}{\mathrm{x}+1}\, dx = 2\sqrt{x-2} - 2\sqrt{3}arctg\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right) + C$$

m) $\int f(x)dx$, sendo f(x) $=2x(x^2+1)^{\frac{1}{2}}$

Fazendo $u = x^2 + 1$

$$\frac{du}{dx} = 2x \rightarrow du = 2xdx$$

Substituindo:

$\int 2x(x^2+1)^{\frac{1}{2}}dx = \int u^{\frac{1}{2}}\, du \ = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}u^{\frac{3}{2}} + C$

Voltando:

$$\int 2x(x^2+1)^{\frac{1}{2}}dx = \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$$

n) $\int f(x)dx$, sendo $f(x) = e^{-3x}$

Fazendo $u = -3x$

$$\frac{du}{dx} = -3 \rightarrow -\frac{1}{3}du = dx$$

Substituindo:

$\int e^{-3x}dx = \int e^{u}\left(-\frac{1}{3}\right)du \ = -\frac{1}{3}\int e^{u}du = -\frac{1}{3}e^{u} \ + C$

Voltando:

$$\int e^{-3x}dx = -\frac{1}{3}e^{-3x} + C$$

o) $\int f(x)dx$, sendo $f(x) = \frac{(5x^4-1)}{x^5-x-7}$

Fazendo $u = x^5 - x - 7$

$$\frac{du}{dx} = 5x^4 - 1 \rightarrow du = (5x^4-1)dx$$

Substituindo:

$\int \frac{(5x^4-1)}{x^5-x-7}\ \ dx = \int \frac{1}{u}du \ = \ln u \ + C$

Voltando:

$$\int \frac{(5x^4-1)}{x^5-x-7}\ \ dx = \ln(x^5-x-7) + C$$

p) $\int f(x)dx$, sendo $f(x) = x^2 e^{x^3}$

Fazendo $u = x^3$

$$\frac{du}{dx} = 3x^2 \rightarrow \frac{du}{3} = x^2 dx$$

Substituindo:

$\int x^2 e^{x^3} dx = \int \frac{e^u du}{3} \;=\; 1/3 \int e^u du = \frac{1}{3} e^u + C$

Voltando:

$$\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + C$$

q) $\int f(x)dx$, sendo $f(x) = \mathrm{sen}\ 5\mathrm{x}$

Fazendo $u = 5x$

$$\frac{du}{dx} = 5 \rightarrow \frac{du}{5} = dx$$

Substituindo:

$\int \mathrm{sen}\ 5\mathrm{x}\, dx = \int sen\, u \frac{du}{5} \;= \frac{1}{5} \int sen\, u\, du = \frac{1}{5}\ (-cosu) + C$

Voltando:

$$\int \mathrm{sen}\ 5\mathrm{x}\, dx = -\frac{1}{5} \cos{(5x)} + C$$

r) $\int f(x)dx$, sendo $f(x) = \frac{2x+1}{x^2+x+1}$

Fazendo $u = x^2 + x + 1$

$$\frac{du}{dx} = 2x + 1 \rightarrow du = (2x + 1)dx$$

Substituindo:

$\int \frac{2x+1}{x^2+x+1}\, dx = \int \frac{du}{u} \;= \ln u + C$

Voltando:

$\int \frac{2x+1}{x^2+x+1}\, dx = \ln(x^2 + x + 1) + C$

s) $\int f(x)dx$, sendo $f(x) = \cos^3 x\, \mathrm{sen\ x}$

Fazendo $u = cos\ x$

$$\frac{du}{dx} = -sen(x) \rightarrow -du = sen(x)dx$$

Substituindo:

$\int \cos^3 x \operatorname{sen} x \ dx = \int u^3(-du) \ = -\int u^3 du = -\frac{u^4}{4} + C$

Voltando:

$\int \cos^3 x \operatorname{sen} x \ dx = -\frac{1}{4}\cos^4 x\, C$

t) $\int f(x)dx$, sendo $f(x) = e^x(1+e^x)^{\frac{1}{2}}$

Fazendo $u = 1 + e^x$

$$\frac{du}{dx} = e^x \rightarrow du = e^x dx$$

Substituindo:

$\int e^x(1+e^x)^{\frac{1}{2}} dx = \int u^{\frac{1}{2}} du) \ = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}u^{\frac{3}{2}} + C$

Voltando:

$\int e^x(1+e^x)^{\frac{1}{2}} dx = \frac{2}{3}(1+e^x)^{\frac{3}{2}} + C$