Lista 12

① a) $f(x,y) = 5xy^2 - 4x^3y$ $\quad P=(1,2)$ $\quad U=\left(\frac{5}{13}, \frac{12}{13}\right)$

$f(x) = 5y^2 - 12x^2y = 20 - 24 = -4$ $\quad \begin{cases} -4 \cdot \frac{5}{13} + 16 \cdot \frac{12}{13} \end{cases}$

$f(y) = 10xy - 4x^3 = 20 - 4 = 16$

$$\frac{-20}{13} + \frac{192}{13} = \frac{172}{13}$$

b) $f(x,y,z) = \sqrt{x+yz}$ $\quad P=(1,3,0)$ $\quad U=\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

$f(x) = \sqrt{1} = 2/7$

$f(y) = \sqrt{1} = 3/7$

$f(z) = \sqrt{3} = 6\sqrt{3}/7$ $\quad \frac{6\sqrt{3}+5}{7}$

② a) $y = x^2 - z^2 \rightarrow x^2 - y - z^2 = 0$

$\pi = f_x(x_0 y_0 z_0)(x-x_0) + f_y(x_0 y_0 z_0)(y-y_0) + f_z(x_0 y_0 z_0)(z-z_0)$

$f(x) = 2x \Rightarrow 2 \times 4 = 8$

$f(y) = -1$

$f(z) = -2z = -2 \times 3 = -6$

$$8(x-4) - 1(y-7) - 6(z-3) = 0$$

$$8x - y - 6z = 7$$

$\begin{cases} x = 4 + 8t \\ y = 7 - t \\ z = 3 - 6t \end{cases}$

b) $xyz^2 = e$ $\quad P(3, 2, 1)$

$$\pi = p_x(x_0 y_0 z_0)(x - x_0) + p_y(x_0 y_0 z_0)(y - y_0) + p_z(x_0 y_0 z_0)(z - z_0) = 0$$

$p(x) = yz^2 = 2$
$p(y) = xz^2 = 3$
$p(z) = 2xyz = 12$

$\begin{cases} x = 3 + 2t \\ y = 2 + 3t \\ z = 1 + 12t \end{cases}$

$$2(x-3) + 3(x-2) + 12(z-1) = 0$$
$$2x + 3y + 12z = 24$$

c) $xy + yz + zx = 5$ $\quad P = (1, 2, 1)$

$$\pi = p_x(x_0 y_0 z_0)(x - x_0) + p_y(x_0 y_0 z_0)(y - y_0) + (x_0 y_0 z_0)(z - z_0) = 0$$

$p(x) = y + z = 3$
$p(y) = x + z = 2$
$p(z) = y + x = 3$

d) $p(x, y) = x^2 - xy + 3y$ $\quad P(1, 1, 3)$

$$\pi = p_x(x - x_0) + p_y(y - y_0) + p_z(z - z_0) = 0$$

$p(x) = 2x - y \rightarrow 1$
$p(x) = -x + 3 \rightarrow 2$
$p(z) = -1 \rightarrow -1$

$\begin{cases} x = 1 + t \\ y = 1 + 2t \\ z = 3 - t \end{cases}$

$$1(x-1) + 2(y-1) - 1(z-3) = 0$$
$$x + 2y - z = 0$$