

Lista 1

1) A) $f(x,y) = \sqrt{4-x^2-y^2}$ $P(0,1)$ $UF = (2,2)$

Se $a = \frac{2}{\sqrt{2^2+2^2}} = \frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}} \rightarrow \cos a = \frac{1}{\sqrt{2}}$

$\sin a = \frac{1}{\sqrt{2}}$

$\nabla f(x,y) = \frac{-x}{\sqrt{4-x^2-y^2}} \rightarrow \frac{-1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $(0,-1)$

$V = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$
 $\cos a = 0 + \sin a \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{6}}{6}$

b) $f(x,y) = e^{x^2-y^2}$ $P=(1,1)$ $U=(1,3)$

Se $a = \frac{1}{\sqrt{1+9}} = \frac{1}{\sqrt{10}} \rightarrow \cos \frac{1}{\sqrt{10}}$

$\sin a = \frac{3}{\sqrt{10}} \rightarrow \sin \frac{3}{\sqrt{10}}$

$f(x) = e^{x^2-y^2} \rightarrow 2xe^{x^2-y^2}$

$f(y) = e^{x^2-y^2} \rightarrow -2ye^{x^2-y^2}$

$\rightarrow \cos a \cdot 2xe^{x^2-y^2} + \sin a \cdot (-2ye^{x^2-y^2})$

$\frac{\sqrt{10}}{5} e^{x^2-y^2} (x-3y)$

5

$$c) f(x, y) = \sin(xy) + \ln(xy) \quad P=(1,0) \quad U=(1,2)$$

$$\cos \alpha = \frac{1}{\sqrt{5}}$$

$$\sin \alpha = \frac{2}{\sqrt{5}}$$

$$f(x) = Y \cos(xy) - Y \sin(xy) = 0 - 0 = 0$$

$$f(y) = X \cos(xy) - X \sin(xy) = 1 - 0 = 1$$

$$\cos 0 + \sin 1 = \frac{2}{\sqrt{5}}$$

$$d) f(x, y) = \frac{x}{y} \quad P=(1,1) \quad U=(2,1)$$

$$\cos \alpha = \frac{2}{\sqrt{5}}$$

$$\sin \alpha = \frac{1}{\sqrt{5}}$$

$$f(x) = -\frac{y}{x^2} \rightarrow -\frac{1}{1} = -1$$

$$f(y) = \frac{1}{x} \rightarrow \frac{1}{1} = 1$$

$$\cos(-1) + \sin 1 = -\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = -\frac{1}{\sqrt{5}}$$

$$e) f(x, y) = x^2 - xy + 3xy \quad P=(0,1) \quad U=(1,3)$$

$$\cos \alpha = \frac{1}{\sqrt{10}}$$

$$\sin \alpha = \frac{3}{\sqrt{10}}$$

$$f(x) = 2x - y = 0 + 1 = 1$$

$$f(y) = 9y^2 - x = 9 - 0 = 9$$

$$\cos 1 + \sin 9 = \frac{1}{\sqrt{10}} + \frac{27\sqrt{10}}{10} = \frac{14\sqrt{10}}{9}$$

$$\textcircled{2} \quad \left. \begin{array}{l} p(x) = 6 \\ p(xy) = 0 \end{array} \right\} \begin{array}{l} p(y) = 3 \\ p(xy) = 0 \end{array}$$

$$b) \quad \left. \begin{array}{l} p(y) = 2xy^2 + 5 \\ p(x) = 2y \end{array} \right\} \begin{array}{l} p(x) = y^2 \\ p(y) = 2y \end{array}$$

$$c) \quad \left. \begin{array}{l} p(y) = \frac{x(x-y)}{(x^2+y^2)^{3/2}} \\ p(x) = -\frac{x(x^2-2y^2+3xy)}{(x^2+y^2)^{5/2}} \end{array} \right\} \begin{array}{l} p(x) = \frac{y(y-x)}{(x^2+y^2)^{3/2}} \\ p(y) = -\frac{y(y^2-2x^2+3xy)}{(x^2+y^2)^{5/2}} \end{array}$$

$$d) \quad \left. \begin{array}{l} p(x) = ye^{xy} + 6x \\ p(y) = xye^{xy} + e^{xy} \end{array} \right\} \begin{array}{l} p(y) = xe^{xy} \\ p(x) = x^2e^{xy} \end{array}$$

$$e) \quad \begin{array}{l} p(x) = yx^2e^y + 12x^3 \\ p(x) = y^2x^2e^y + 36y^2 \\ p(x) = y^3x^2e^y + 72y^2 \end{array}$$