Atividade 3 – Integral Indefinida

GABARITO

1) Calcular as integrais seguintes usando o método da substituição:

$a)\ \int (2x^2+2x-3)^{10}(2x+1)dx$

$Fazendo-se:$

$u = 2x^2+2x-3$

$du = (4x+2)dx = 2(2x+1)dx$

$Temos:$

$$\int (2x^2+2x-3)^{10}(2x+1)\,dx = \frac{1}{2}\frac{(2x^2+2x-3)^{11}}{11} + c.$$

$b)\ \int (x^3-2)^{1/7}x^2dx$

$Fazendo-se:$

$u = x^3-2$

$du = 3x^2dx$

$Temos:$

$$\int (x^3-2)^{1/7}x^2dx = \frac{1}{3}\frac{(x^3-2)^{\frac{8}{7}}}{\frac{8}{7}} + c = \frac{7}{24}(x^3-2)^{\frac{8}{7}} + c.$$

$c)\ \int \frac{xdx}{\sqrt[5]{x^2-1}}$

$$\int (x^2-1)^{-\frac{1}{5}}\, x\,dx$$

$Fazendo-se:$

$u = x^2-1$

$du = 2x\,dx$

$Temos:$

$$\int \frac{x\,dx}{\sqrt[5]{x^2-1}} = \frac{1}{2}\frac{\left(x^2-1\right)^{4/5}}{\frac{4}{5}} + c = \frac{5}{8}(x^2-1)^{\frac{4}{5}} + c$$

$d)\ \int 5x\sqrt{4-3x^2}dx$

$$= \int 5x(4-3x^2)^{\frac{1}{2}}dx$$

$Fazendo-se:$

$u = 4-3x^2$

$du = -6x\,dx$

$Temos:$

$$\int 5x\sqrt{4-3x^2}dx = 5.\frac{-1}{6}\frac{\left(4-3x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-5}{9}\left(4-3x^2\right)^{\frac{3}{2}} + c.$$

$e)\ \int \sqrt{x^2+2x^4}dx$

$$= \int x\left(1+2x^2\right)^{\frac{1}{2}}dx$$

$$= \frac{1}{4}\frac{\left(1+2x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

Fazendo: $u = 1+2x^2$
$du = 4x\,dx$

$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + c$$

$f)\ \int (e^{2t}+2)^{\frac{1}{3}}e^{2t}dt$

$$Fazendo - se:$$
$$u = e^{2t} + 2$$
$$du = 2e^{2t}\, dt$$
$$Temos:$$

$$\int (e^{2t}+2)^{1/3} e^{2t} dt = \frac{1}{2}\frac{\left(e^{2t}+2\right)^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3}{8}\left(e^{2t}+2\right)^{\frac{4}{3}} + c.$$

$g)\ \int \frac{e^t dt}{e^t + 4}$

$$= \int \frac{du}{u} = \ln\left|e^t + 4\right| + c\text{, sendo que } u = e^t + 4 \text{ e } du = e^t\, dt.$$

$h)\ \int \frac{e^{\frac{1}{x}} + 2}{x^2} dx = \int \frac{e^{\frac{1}{x}}}{x^2} dx + \int \frac{2}{x^2} dx$

$Fazendo - se$:

$u = \frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2} \leftrightarrow -du = \frac{1}{x^2} dx$

Substituindo:

$$\int \frac{e^{\frac{1}{x}} + 2}{x^2} dx = \int e^u(-du) = -\, e^u = -e^{\frac{1}{x}} + c$$

Portanto: $\int \frac{e^{\frac{1}{x}}+2}{x^2} dx = \int \frac{e^{\frac{1}{x}}}{x^2} dx + \int \frac{2}{x^2} dx = -e^{\frac{1}{x}} + \frac{2}{x} + C$

$i) \int tgx\ sec^2x\ dx$

considerando-se: $u = tg\ x$

$du = \sec^2 x\ dx$

$$\int udu = \frac{u^2}{2} + C = \frac{tg^2x}{2} + C$$

Outra solução:

$u = \sec^2 x$

$$\frac{du}{dx} = 2\sec^2 x\, tgx \leftrightarrow \frac{du}{2} = \sec^2 x tgx$$

Substituindo:

$$\int tgx\ sec^2x\ dx = \int \frac{du}{2} = \frac{1}{2}\int du = \frac{1}{2}u + C = \frac{1}{2}\sec^2 x + C$$

$j) \int sen^4x\ cosx\ dx$

considerando-se: $u = sen\ x$

$du = \cos x\, dx$

$$\int sen^4x\ cosx\ dx = \int u^4du = \frac{u^5}{5} + C = \frac{(senx)^5}{5} + C$$

2) Resolver as seguintes integrais usando a técnica de integração por partes.

$a) \int x\ sen\ 5x\ dx$

$$u = x \Rightarrow du = dx$$

$$dv = sen5x\,dx \quad \Rightarrow \quad v = \int sen5x\,dx = \frac{-1}{5}\cos 5x$$

$$I = x\frac{-1}{5}\cos 5x - \int \frac{-1}{5}\cos 5x\,dx$$
$$= \frac{-x}{5}\cos 5x + \frac{1}{5}\cdot\frac{1}{5}sen5x + c$$
$$= \frac{-x}{5}\cos 5x + \frac{1}{25}sen5x + c$$

$b)$ $\int \ln(1-x)dx$

$$u = \ln(1-x) \quad \Rightarrow \quad du = \frac{-1}{1-x}dx$$
$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = \ln(1-x)x - \int x\frac{-1}{1-x}dx$$
$$I = x\ln(1-x) + \int\left(-1 + \frac{1}{1-x}\right)dx$$
$$I = x\ln(1-x) - x - \ln(1-x) + c$$
$$I = (x-1)\ln(1-x) - x + c$$

Outra solução:

$u = 1 - x$

$\frac{du}{dx} = -1 \leftrightarrow -du = dx$

Substituindo:

$$\int \ln(1-x)dx = \int \ln(u)\,(-du) = -\int \ln(u)\,du = -(uln(u) - u) + C$$
$$= -\big((1-x)\ln(1-x) - (1-x)\big) + C = (x-1)\ln(1-x) - x + C$$

$c) \int te^{4t}dt$

$$u = t \quad \Rightarrow \quad du = dt$$

$$dv = e^{4t}dt \quad \Rightarrow \quad v = \int e^{4t}dt = \frac{1}{4}e^{4t}$$

$$I = t\,\frac{1}{4}\,e^{4t} - \int \frac{1}{4}\,e^{4t}\,dt$$

$$= \frac{t}{4}\,e^{4t} - \frac{1}{4}.\frac{1}{4}\,e^{4t} + c$$

$$= e^{4t}\left(\frac{t}{4} - \frac{1}{16}\right) + c$$

$d) \int (x+1)\cos 2x\,dx$

$$u = x+1 \quad \Rightarrow \quad du = dx$$

$$dv = \cos 2x\,dx \quad \Rightarrow \quad v = \int \cos 2x\,dx = \frac{1}{2}sen\,2x$$

$$I = (x+1)\frac{1}{2}sen\,2x - \int \frac{1}{2}sen\,2x\,dx$$

$$= \frac{x+1}{2}sen\,2x + \frac{1}{4}\cos 2x + c$$

$e)\ \int x \ln 3x\, dx$

$$u = \ln 3x \quad \Rightarrow \quad du = \frac{3}{3x} dx$$

$$dv = x dx \quad \Rightarrow \quad v = \int x\, dx = \frac{x^2}{2}$$

$$I = (\ln 3x)\frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$= \frac{x^2}{2} \ln 3x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$= \frac{x^2}{2}\left(\ln 3x - \frac{1}{2} \right) + c$$