Lista 2 - Guilherme Amarilla

1 a) $\int x e^{-x^2} dx$

$u = -x^2$

$\frac{du}{dx} = -2x \leftrightarrow \frac{du}{-2} = x\,dx$

$= \int e^u \frac{du}{-2} = -\frac{1}{2}\int e^u du$

$= -\frac{1}{2} e^u + C$

$= -\frac{1}{2} e^{-x^2} + C$

b) $\int 2x(1+x^2)^{-1} dx = \int \frac{2x}{(1+x^2)} = 2\int \frac{1}{2u} du$

$\frac{2}{2}\int \frac{1}{u} du = \int \ln|u| = \ln(1+x^2) + C$

$u = 1+x^2$

$du = 0 + 2x$

$\int (u)^{-1} du = \ln(u) + C$

$\frac{du}{dx} = 2x$

$= \ln(1+x^2) + C$

$du = 2x\,dx$

$\frac{1}{u} * \frac{dx}{x}$

c) $\int (x \ln x)^{-1} dx = \int \left(\frac{1}{x \ln x}\right) dx = \int \frac{1}{u} \cdot du$

$u = \ln x \quad \frac{1}{x}$

$= \ln u + C$

$v = \ln x \quad \frac{dv}{dx} = \frac{1}{x}$

$= \ln(\ln x) + C$

$\frac{du}{dx} = \frac{1}{x} \Leftrightarrow du = \frac{1}{x} dx$

d) $\int (5x+7)^{-1} = \int \frac{1}{5x+7} dx = \int \frac{v^{-1} dv}{5} \int \frac{1}{v} \times \frac{dv}{5} = \frac{1}{5} \int \frac{dv}{v}$

$v = 5x+7$

$\frac{dv}{dx} = 5$

$\frac{\ln(5x+7) + C}{5}$

e) $\int (5x-7)^{-3} dx = \int \frac{v^{-3} dx}{(5x-7)} = \int v^{-3} \times \frac{du}{5} = \frac{1}{5} \int v^{-3} du$

$v = 5x-7$

$\frac{1}{5} \times \frac{v^{-3+1}}{-3+1} = \frac{1}{5} \times \frac{v^{-2}}{-2} = \frac{v^{-2}}{-10}$

$\frac{dv}{dx} = 5$

$\frac{(5x-7)^{-2}}{-10} + C = \frac{1}{-10 \times (5x-7)^2} + C$

$dv = 5 dx$

$dx = \frac{dv}{5}$

f) $\int \sin(x+9)\,dx$ $\int \sin(v)\,dv$

$-\cos(v)+C$

$v = x+9$

$\frac{dv}{dx} = 1$

$dv = dx$

$-\cos(x+9)+C$

g) $\int \sin^2 x \cos x\,dx$ $\int v^2 \cancel{\cos x} \cdot \frac{1}{\cos x}\,dv$

$v = \sin x$

$dv = \cos(x)\,dx$

$dx = \frac{1}{\cos x}\,dv$

$\int v^2\,dv = \frac{v^3}{3}$

$\frac{\sin^3 x}{3} + C$

h) $\int x + \int \sec^2 x 3x = \frac{x^2}{2} +$ $\int \sec^2 x \cdot x\,dx$

$\int \sec^2 v\,dv \frac{1}{3}$

$\frac{1}{3} tg\, v$

$\frac{tg(3x)}{3}$

$v = 3x$

$dv = 3dx$

$dx = dv = \frac{1}{3}$

$\frac{x^2}{2} + \frac{tg(3x)}{3} + C$

i) $\int \frac{1}{\sqrt{(9-4x^2)}} = \int \frac{1}{\sqrt{(3-2x)(3+2x)}} = \int \frac{1}{\sqrt{(3-v)(3+v)}} \frac{dv}{2}$

$v = 2x$

$\frac{dv}{dx} = 2dx$

$dx = \frac{dv}{2}$

$\frac{1}{2}\int \frac{1}{\sqrt{9-v^2}}\,dv = \frac{1}{2} \arcsen \frac{v}{3} \times \frac{1}{2}$

$\arcsen\left(\frac{u}{3}\right) \cdot \frac{1}{2} = \frac{\arcsen\left(\frac{2x}{3}\right)}{2} + C$

j) $\int (16 + x^2)^{-1} dx = \int \frac{1}{16+x^2} dx = \frac{1}{4} \arctg\left(\frac{x}{4}\right) + C$

k) $\int x(9 - 4x^2)^{-\frac{1}{2}} dx = \int x * \frac{1}{\sqrt{(3+2x)(3-2x)}} dx$

não deu pois o x não deixa para sair arcseno

l) $\int \frac{(x-2)^{\frac{1}{2}}}{(x+1)} dx = \int \frac{v}{x+1} 2v\, dv = \int \frac{2v^2}{x+1} dv$

$v = (x-2)^{\frac{1}{2}}$

$dx = dv\, 2(x-2)^{\frac{1}{2}}$

$dx = \frac{dv\, 2v}{}$

$\int \frac{2v^2}{v^2+3} dv = 2\int \frac{v^2}{v^2+3} dv$

$dv = \frac{1}{2(x-2)^{\frac{1}{2}}} dx$

$2\int -\frac{3}{v^2+3} + 1\, dv$

?

não vou tentar

$v^2 = x - 2$

$x = v^2 + 2$

m) $\int 2x(x^2+1)^{1/2}\,dx = \int x\,u^{1/2}\,\frac{du}{2x} = \int u^{1/2}\,du$ /

o) $\int x(x^2+1)^{1/2}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\frac{u^{3/2}}{3/2} = \frac{2u^{3/2}}{3} = \frac{2(x^2+1)^{3/2}}{3} + C$

n) $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$

o) $\int \frac{(5x^4-1)}{(x^5-x-7)}\,dx = \int \frac{(5x^4-1)}{u}\,\frac{du}{(5x^4-1)} = \int \frac{du}{u}$

$u = x^5-x-7$

$du = 5x^4-1\,dx$

$dx = \frac{du}{5x^4-1}$

$\ln(u) = \ln(x^5-x-7) + C$

p) $\int x^2 e^{x^3}$

$u = x^3$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int x^2 e^u \frac{1}{3x^2}\,du \quad \int \frac{e^u}{3}\,du = \frac{1}{3}\int e^u\,du$

$\frac{1}{3}e^{x^3} + C$

Q) $\int \sin 5x$

$u = 5x$

$du = 5\,dx$

$dx = \frac{du}{5}$

$\int \sin u\,\frac{du}{5} = \frac{1}{5}(-\cos u) = \frac{-\cos(5x)+C}{5}$

/ /

m) $\int \frac{(2x+1)}{(x^2+x+1)}$ $\int \frac{(2x+1)}{u} \cdot \frac{du}{(2x+1)} = \int \frac{du}{u}$

$u = x^2+x+1$ $\ln u = \ln(x^2+x+1)$

$du = 2x+1\,dx$

$dx = \frac{du}{2x+1}$

n) $\int \cos^3 x \sin x$ $\int u^3 \sin x \times \left(\frac{-1}{\sin x}\,du\right) = \int -u^3\,du$

$u = \cos x$

$du = -\sin x$ $\frac{-u^4}{4} = \frac{-\cos^4 x + C}{4}$

$dx = \frac{du}{-\sin x}$

t) $\int e^x (1+e^x)^{\frac{1}{2}}$ $\int e^x u^{\frac{1}{2}} \frac{du}{e^x} = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2u^{\frac{3}{2}}}{3}$

$u = 1+e^x$

$du = e^x dx$

$dx = \frac{du}{e}$ $\frac{2}{3}(1+e^x)^{\frac{3}{2}} + C$