Lista 06

① a) $\lim_{x\to 0}\left(\frac{\sin 2x}{x}\right) = \frac{\cos 2x \cdot 2}{x} = 2$ | b) $\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{3}{5}\times\frac{\lim x}{x} = \frac{3}{5}$

b) $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x} = \frac{\cos(3x)}{\cos(2x)}\cdot\frac{3}{2} = \frac{3}{2}$ | d) $\lim_{x\to 0}\frac{tg\,x}{x} = \frac{\sec^2 x}{1} = \frac{\sec^2 0}{1} = 1$

e) $\lim_{x\to 0}\frac{\cos x - 1}{x} = \frac{-\sin(x)}{1} = 0$ | g) $\lim_{x\to 0}\frac{\sin 3x}{x} = \frac{3\sin x}{x} = 3$

g) $\lim_{x\to 0}\frac{tg\,2x}{x} = \frac{\sec^2(2\cdot 0)\times 2}{1} = 2$ | h) $\lim_{x\to 0}\frac{tg\,5x}{tg\,x} = \frac{\sec^2(5\times 0)\times 5}{\sec^2(0)} = 5$

i) $\lim_{x\to 0}\frac{\sin^2 x}{x} = \frac{1-\cos^2 x}{x} = \frac{\sin 2x}{1} = 0$ | j) $\lim_{x\to 0}\frac{\sin x}{tg\,2x} = \sin(x)\cot(x) = \frac{\cos(x)}{\sec^2(2x)\cdot 2} = \frac{1}{2}$

② a) $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{2x} = e^{2x\ln(1+\frac{1}{x})} = e^2$ | b) $\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^{x} = e^{x\ln(1+\frac{2}{x})} = e^2$

c) $\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^{3x} = e^{3x\ln(1+\frac{2}{x})} = e^6$ | d) $\lim_{x\to-\infty}\left(1-\frac{2}{x}\right)^{3x} = e^{2x\ln(1-\frac{2}{x})} = \frac{1}{e^6} = e^{-6}$

e) $\lim_{x\to 0}(1+4x)^{\frac{1}{4x}} = e^{\frac{1}{4}\ln(1+4x)} = e^1$ | f) $\lim_{x\to 0}(1-3x)^{\frac{2}{3x}} = e^{\frac{2}{3x}\ln(1-3x)} = \frac{1}{e^6} = e^{-6}$

g) $\lim_{x\to+\infty}\left(\frac{x-4}{x-1}\right)^{x+3} = e^{-3} = \frac{1}{e^3}$ | h) $\lim_{x\to+\infty}\left(\frac{2x+3}{2x+1}\right)^{x} = e^1 = e$

i) $\lim_{x\to 0}\frac{\ln(1+x)}{2x}\cdot\frac{1}{2}\ln(1+x) = \frac{1}{2}$ | j) $\lim_{x\to 0}\frac{\ln(1+2x)}{3x} = \frac{1}{3}\times\frac{\ln(1+2x)}{x} = \frac{1}{3}\times 2 = \frac{2}{3}$