Lista 3

1 a) $\int (2x^2+2x-3)^{10}(2x+1)dx \Rightarrow \int U^{10} \cancel{(2x+1)} \frac{du}{\cancel{4x+2}\ 2}$ $(2x+1)2$

$U = 2x^2+2x-3dx$

$\frac{du}{dx} = 4x+2$

$dx = \frac{du}{4x+2}$

$\frac{1}{2}\int U^{10} du = \frac{1}{2}\frac{U^{11}}{11}$

$\boxed{\frac{2x^2+2x-3}{22} + C}$

b) $\int (x^3-2)^{1/7} x^2 dx \Rightarrow \int U^{\frac{1}{7}} \cancel{x^2} \frac{du}{3\cancel{x^2}} \Rightarrow \frac{1}{3}\int U^{\frac{1}{7}} du$

$U = x^3-2$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\frac{1}{3}\frac{U^{\frac{8}{7}}}{\frac{8}{7}} = \frac{7(x^3-2)^{\frac{8}{7}}}{24} + C$

$\frac{1}{7} + 1 = \frac{8}{7}$

c) $\int \frac{x\,dx}{\sqrt[5]{x^2-1}} = \int \frac{x\,dx}{(x^2-1)^{\frac{1}{5}}} = \int x\,dx\,(x^2-1)^{-1/5} \Rightarrow \int U^{-\frac{1}{5}} \cancel{x} \frac{du}{2\cancel{x}}$

$U = x^2-1$

$dx = \frac{du}{2x}$

$\frac{1}{2}\int U^{-\frac{1}{5}} du \Rightarrow \frac{1}{2}\frac{U^{\frac{4}{5}}}{4/5} = \frac{5U^{\frac{4}{5}}}{8}$

$\frac{5}{8}\sqrt[5]{(x^2-1)^4} + C$

d) $\int 5x\sqrt{4-3x^2}\,dx \Rightarrow \int 5\cancel{x} \cdot U^{1/2} \frac{du}{6\cancel{x}} = \frac{5}{6}\int U^{\frac{1}{2}} du$

$U = 4-3x^2$

$du = 6x$

$dx = \frac{du}{6x}$

$\frac{5}{6}\frac{U^{3/2}}{3/2} = \frac{10(4-3x^2)^{3/2}}{18} + C$

$x^2(2x^2+1)$

e) $\int \sqrt{x^2+2x^4} = \int \sqrt{x^2}\cdot\sqrt{1+2x^2} = \int x\sqrt{1+2x^2}$

$u = 1+2x^2$

$dx = \frac{du}{4x}$

$\int \cancel{x}\, u^{1/2} \frac{du}{4\cancel{x}} = \frac{1}{4}\int u^{1/2} du = \frac{1}{4} \frac{u^{3/2}}{3/2}$

$\frac{2}{12} u^{3/2} = \frac{1}{6}\sqrt{(2x^2+1)^3} + C$

f) $\int (e^{2t}+2)^{1/3} e^{2t} dt = \int u^{1/3} \cancel{e^{2t}} \frac{du}{2\cancel{e^{2t}}} = \frac{1}{2} \frac{u^{4/3}}{4/3}$

$u = e^{2t}+2\, dx$

$dt = \frac{du}{2e^{2t}}$

$\frac{3}{8}(e^{2t}+2)^{4/3} + C$

g) $\int \frac{e^t}{e^t+4} dt = \int \cancel{e^t} u^{-1} \frac{du}{\cancel{e^t}}$

$\frac{u^0}{0} \Rightarrow \ln|u|$

$u = e^t+4$

$dt = \frac{du}{e^t}$

$\ln(e^t+4) + C$

(pulei uma)

i) $\int tg\,x \sec^2 x\, dx = \int u \cancel{\sec^2 x} \frac{du}{\cancel{\sec^2 x}}$

$u = tg\,x$

$dx = \frac{du}{\sec^2 x}$

$\frac{u^2}{2} = \frac{tg^2 x}{2} + C$

① $\int \sin^4 x \cos x \, dx = \int u^4 \cancel{\cos x} \frac{du}{\cancel{\cos x}} = \frac{u^5}{5}$

$u = \sin x$

$dx = \frac{du}{\cos x}$

$\frac{\sin^5 x}{5} + C$

b) $\int \frac{e^{\frac{1}{x}} + 2}{x^2}$ $\quad \int \frac{e^u + 2}{\cancel{x^2}} du \cdot -\cancel{x^2} = \int -(e^u + 2) du$

$u = \frac{1}{x} = x^{-1}$

$du = \frac{-1}{x^2} dx$

$dx = -du \, x^2$

$-\int e^u du - \int 2 du = -e^u - 2u$

$-e^{\frac{1}{x}} - \frac{2}{x} + C$

② a) $\int x \sin 5x \, dx$ $\quad \boxed{u \cdot v - \int v \cdot du}$

$u = x$

$du = 1 \, dx$ $\quad du = dx$

$dv = \sin 5x \, dx$

$\int \sin(5x) dx$

$w = 5x$

$dw = 5dx$

$dx = \frac{dw}{5}$

$\int \sin(w) \frac{dw}{5}$

$\boxed{-\frac{1}{5} \cos 5x}$

$v = \frac{-1}{5} \cos 5x$

$\frac{x \cdot \cos 5x}{5} - \int \frac{-1}{5} \cos 5x \, du$

$-\frac{x \cos 5x}{5} + \frac{\sin 5x}{25} + C$

$-\frac{1}{5} \int \cos 5x \, dx$

$z = 5x$

$dz = 5$

$dx = \frac{dz}{5}$

$-\frac{1}{5} \int \cos z \frac{dz}{5}$

$\frac{-1}{25} \sin 5x$

b) $\int \ln(1-x)\,dx = \int \ln(u) \cdot -du = -\int \ln u \, du$

$u = 1-x$

$du = -1dx$

$dx = -du$

$v = \ln(u)$: $dv = \frac{1}{u}$

$dw = 1$

$w = \int 1 du = u$

$(-1)\; u \times \ln(u) - \int 1 du$

$-(u\ln u - u)$

$-(1-x)\ln(1-x) + (1-x)$

$-\ln(1-x) + x\ln x(1-x) + 1 - x + C$

c) $\int t\, e^{4t}\, dt = \frac{1}{4}\int t e^{u}\, du = \frac{1}{16}\int u e^{u}\, du$ $\quad uv - \int v\,du$

$u = 4t$

$du = 4dx$

$dx = \frac{du}{4}$

$t = \frac{u}{4}$

$u = u$

$du = 1$

$dv = e^{u}du$

$v = e^{u}$

$\frac{1}{16}(ue^{u} - \int e^{u} du)$

$\frac{1}{16}(ue^{u} - e^{u})$

$\frac{1}{16}(4te^{4t} - e^{4t}) + C$

d) $\int (x+1)\cos 2x\, dx$

$u = (x+1$

$du = 1\, dx$

$dv = \int \cos 2x\, dx$

$w = 2x$

$dx = \frac{dw}{2}$

$\int \cos w \frac{dw}{2}$

$\frac{1}{2}\sin 2x$

$v = \frac{1}{2}\sin 2x$

$uv - \int v\, du$

$\frac{(x+1)\sin 2x}{2} - \int \frac{1}{2}\sin 2x\, dx$

$\frac{(x+1)\sin 2x}{2} + \frac{\cos 2x}{4} + C$

$\frac{1}{4} \times \cos 2x$.

$-\frac{\cos 2x}{4}$

$\frac{(x+1)\sin 2x}{2} + \frac{\cos 2x}{4} + C$

e) $\int x \ln 3x \, dx$

$u = \ln 3x$

$du = \frac{1}{x} dx$

$dv = x$

$v = \frac{x^2}{2}$

$\ln 3x \, \frac{x^2}{2} - \int \frac{x^2}{2} \frac{dx}{x}$

$\frac{x^2 \ln 3x}{2} - \int \frac{x}{2} dx$

$\int \frac{1}{2} x \, dx = \frac{1}{2} \times \frac{x^2}{2} = \frac{x^2}{4}$

$\frac{1}{2} x^2 \ln 3x - \frac{x^2}{4} + C$