Tuesday, 30 August 2022 08:23

Gui- 21, 22, 23, 25 (a) e 26

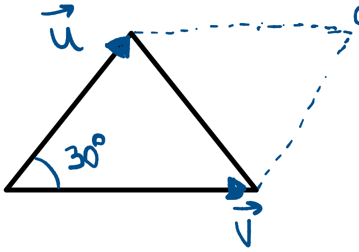
21) Sabendo que $|\vec{u}| = 6$, $|\vec{v}| = 4$ e $30°$ o ângulo entre $\vec{u}$ e $\vec{v}$, calcular

a) a área do triângulo determinado por $\vec{u}$ e $\vec{v}$;

b) a área do paralelogramo determinado por $\vec{u}$ e $(-\vec{v})$;

c) a área do paralelogramo determinado por $\vec{u} + \vec{v}$ e $\vec{u} - \vec{v}$.

$$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} 30°$$
$$= 6 \cdot 4 \cdot \frac{1}{2}$$
$$= 12$$

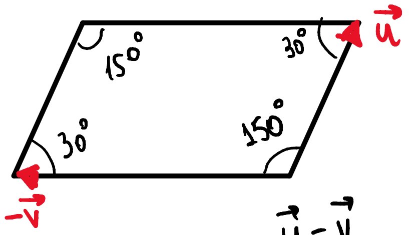


a) $$\text{Área} = \frac{|\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} 30°}{2}$$

$$\text{Área} = \frac{6 \cdot 4 \cdot 1/2}{2}$$

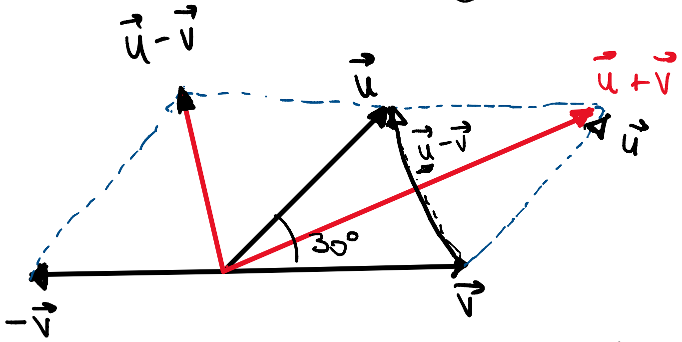
$$\boxed{\text{Área} = 6 \text{ u.a.}}$$

b)



$$\text{Área} = |\vec{u} \times (-\vec{v})| = |\vec{u}| \cdot |-\vec{v}| \cdot \operatorname{sen} 150°$$
$$\text{Área} = |\vec{u} \times (-\vec{v})| = 6 \cdot 4 \cdot \frac{1}{2}$$
$$\text{Área} = 12 \text{ u.a.}$$

c)



$$|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2 \cdot |\vec{u}| \cdot |\vec{v}| \cdot \cos 30°$$
$$|\vec{u} - \vec{v}|^2 = 6^2 + 4^2 - \cancel{2} \cdot 6 \cdot 4 \cdot \frac{\sqrt{3}}{\cancel{2}}$$
$$|\vec{u} - \vec{v}|^2 = 36 + 16 - 24\sqrt{3}$$
$$|\vec{u} - \vec{v}|^2 = 52 - 24\sqrt{3}$$
$$|\vec{u} - \vec{v}| = \sqrt{52 - 24\sqrt{3}}$$

$$|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2 \cdot |\vec{u}| \cdot |\vec{v}| \cdot \cos 150°$$
$$|\vec{u} + \vec{v}|^2 = 6^2 + 4^2 - \cancel{2} \cdot 6 \cdot 4 \cdot \left(-\frac{\sqrt{3}}{\cancel{2}}\right)$$
$$|\vec{u} + \vec{v}|^2 = 36 + 16 + 24\sqrt{3}$$
$$|\vec{u} + \vec{v}| = \sqrt{52 + 24\sqrt{3}}$$

$$(\vec{u} - \vec{v}) \cdot (\vec{u} + \vec{v}) = |\vec{u}|^2 + \cancel{\vec{u} \cdot \vec{v}} - |\vec{v}|^2 - \cancel{\vec{u} \cdot \vec{v}} = |\vec{u}|^2 - |\vec{v}|^2$$

(*) $$\cos\theta = \frac{(\vec{u} - \vec{v}) \cdot (\vec{u} + \vec{v})}{|\vec{u} - \vec{v}| \cdot |\vec{u} + \vec{v}|}$$

$$\cos\theta = \frac{|\vec{u}|^2 - |\vec{v}|^2}{\left(\sqrt{52 - 24\sqrt{3}}\right) \cdot \left(\sqrt{52 + 24\sqrt{3}}\right)}$$

$$\cos\theta = \frac{6^2 - 4^2}{\sqrt{52^2 - (24\sqrt{3})^2}} \ (*)$$

(*) $$\cos\theta = \frac{20}{\sqrt{976}}$$

$$\cos\theta = \frac{20}{4\sqrt{61}}$$

$$\boxed{\cos\theta = \frac{5}{\sqrt{61}}}$$

$$sen^2\theta + \cos^2\theta = 1 \Rightarrow sen^2\theta = 1 - \cos^2\theta \Rightarrow sen^2\theta = 1 - \left(\frac{5}{\sqrt{61}}\right)^2$$

$$sen^2\theta = 1 - \frac{25}{61} \rightarrow sen^2\theta = \frac{36}{61} \Rightarrow \boxed{sen\theta = \frac{6}{\sqrt{61}}}$$

$$\text{Área} = \left|(\vec{u}+\vec{v}) \times (\vec{u}-\vec{v})\right| = |\vec{u}+\vec{v}| \cdot |\vec{u}-\vec{v}| \cdot sen\theta$$

$$= \left(\sqrt{52 - 24\sqrt{3}}\right) \cdot \left(\sqrt{52 + 24\sqrt{3}}\right) \cdot \frac{6}{\sqrt{61}}$$

$$= \sqrt{52^2 - (24\sqrt{3})^2} \cdot \frac{6}{\sqrt{61}}$$

$$= \sqrt{976} \cdot \frac{6}{\sqrt{61}} \Rightarrow \frac{4 \cdot \cancel{\sqrt{61}} \cdot 6}{\cancel{\sqrt{61}}} = \underline{24 \text{ u.a.}}$$

Resp.: A área do Paralelogramo determinado por $(\vec{u}+\vec{v})$ e $(\vec{u}-\vec{v})$ é igual a <u>24 u.a.</u>

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22) Calcular a área do paralelogramo determinado pelos vetores $\vec{u}$ e $\vec{v}$, sabendo que suas diagonais são $\vec{u} + \vec{v} = (-1, 3, 4)$ e $\vec{u} - \vec{v} = (1, -1, 2)$.

$$\begin{cases} \vec{u}+\vec{v} = (-1,3,4) \Rightarrow \vec{u} = (-1,3,4) - \vec{v} \\ \vec{u}-\vec{v} = (1,-1,2) \end{cases}$$

$$(-1,3,4) - \vec{v} - \vec{v} = (1,-1,2)$$

$$(-1,3,4) - 2\vec{v} = (1,-1,2)$$

$$-2\vec{v} = (1,-1,2) - (-1,3,4)$$

$$-2\vec{v} = (2,-4,-2)$$

$$\vec{v} = -\frac{1}{2} \cdot (2,-4,-2)$$

$$\vec{v} = (-1,2,1)$$

$$\vec{u} = (-1,3,4) - (-1,2,1)$$

$$\vec{u} = (0,1,3)$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ -1 & 2 & 1 \end{vmatrix} =$$

$$(-1)^{1+1} \cdot \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \cdot \vec{i} + (-1)^{1+2} \cdot \begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} \cdot \vec{j} + (-1)^{1+3} \cdot \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} \cdot \vec{k}$$

$$= (1-6) \cdot \vec{i} - (0+3) \cdot \vec{j} + (0+1) \cdot \vec{k}$$

$$= -5 \cdot \vec{i} - 3\vec{j} + 1\vec{k}$$

$$|\vec{u} \times \vec{v}| = \sqrt{(-5)^2 + (-3)^2 + (1)^2} = \sqrt{25+9+1} = \sqrt{35} \text{ u.a.}$$

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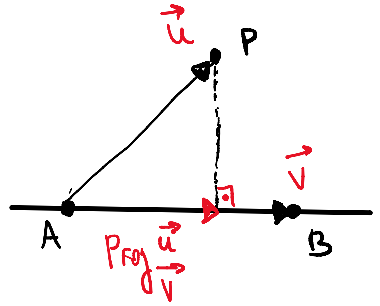
23) Calcular a distância do ponto P(4, 3, 3) à reta que passa por A(1, 2, -1) e B(3, 1, 1).



$$\vec{AP} = \vec{u} = P - A = (4,3,3) - (1,2,-1) = (3,1,4)$$

$$\vec{AB} = \vec{v} = B - A = (3,1,1) - (1,2,-1) = (2,-1,2)$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 3 & 1 & 4 \\ 2 & -1 & 2 \end{vmatrix} = (2+4) \cdot \vec{i} - (6-8)\vec{j} + (-3-2)\vec{k}$$

$$= 6\vec{i} + 2\vec{j} - 5\vec{k}$$

$$|\vec{u} \times \vec{v}| = \sqrt{6^2 + 2^2 + (-5)^2} \Rightarrow \sqrt{36+4+25}$$

$$\Rightarrow \sqrt{65}$$

$$|\vec{v}| = \sqrt{2^2 + (-1)^2 + 2^2} \Rightarrow \sqrt{9} = 3$$

$$D = \frac{|\vec{u} \times \vec{v}|}{|\vec{v}|}$$

$$D = \frac{\sqrt{65}}{3} \text{ u.c.}$$

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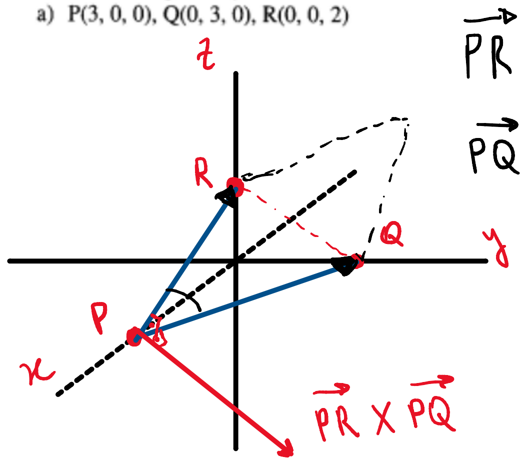
25) Encontrar um vetor ortogonal ao plano determinado pelos pontos P, Q e R e calcular a área do triângulo PQR.

a) P(3, 0, 0), Q(0, 3, 0), R(0, 0, 2)



$$\vec{PR} = R - P = (0,0,2) - (3,0,0) = (-3,0,2)$$

$$\vec{PQ} = Q - P = (0,3,0) - (3,0,0) = (-3,3,0)$$

$$\vec{PR} \times \vec{PQ} = \begin{vmatrix} i & j & k \\ -3 & 0 & 2 \\ -3 & 3 & 0 \end{vmatrix}$$

$$= (0-6)\vec{i} - (0+6)\vec{j} + (-9-0)\vec{k}$$

$A = \frac{1}{2} \cdot |\vec{PR} \times \vec{PQ}| \qquad = -6\vec{i} - 6\vec{j} - 9\vec{k}$

$A = \frac{1}{2} \cdot |-6\vec{i} - 6\vec{j} - 9\vec{k}|$

$A = 3\vec{i} + 3\vec{j} + \frac{9}{2}\vec{k}$

$\longrightarrow |A| = \sqrt{3^2 + 3^2 + (9/2)^2}$

$|A| = \sqrt{9 + 9 + \frac{81}{4}}$

$|A| = \frac{\sqrt{153}}{2}$ u.a

$|A| = \frac{3\sqrt{17}}{2}$ u.a

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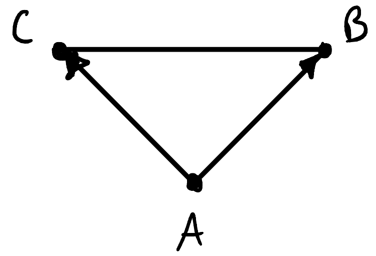
26) Calcular z, sabendo-se que A (2, 0, 0), B(0, 2, 0) e C(0, 0, z) são vértices de um triângulo de área 6.



$\vec{AC} = C - A = (0, 0, z) - (2, 0, 0) = (-2, 0, z)$

$\vec{AB} = B - A = (0, 2, 0) - (2, 0, 0) = (-2, 2, 0)$

$\vec{AC} \times \vec{AB} = \begin{vmatrix} i & j & k \\ -2 & 0 & z \\ -2 & 2 & 0 \end{vmatrix}$

$= (0 - 2z)\vec{i} - (0 + 2z)\vec{j} + (-4 - 0)\vec{k}$

$= -2z\vec{i} - 2z\vec{j} - 4\vec{k}$

$|\vec{AC} \times \vec{AB}| = \sqrt{4z^2 + 4z^2 + 16}$

$A = \frac{1}{2} \cdot |\vec{AC} \times \vec{AB}| \longrightarrow 6 = \frac{1}{2} \cdot \sqrt{8z^2 + 16}$

$12^2 = 8z^2 + 16$

$8z^2 = 144 - 16$

$8z^2 = 128$

$z^2 = 16 \Longleftrightarrow \boxed{z = 4 \text{ ou } z = -4}$