Exercício 29

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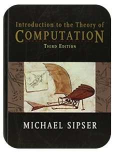

Introduction to the Theory of Computation

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Solução Certificado Solução fornecida há 2 anos

Passo 1 1 de 4

$\boxed{\textbf{Part a.}}$

Assume that the given language $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$ is regular. Then by pumping lemma there is pumping length $p \in \mathbb{N}$ such that any string w whose length is at least p can be divided up into three parts as $w = xyz$, for which the following conditions hold:

1. $xy^i z \in A_1$, for any $i \geq 0$,
2. y is not ε, and
3. length of initial part xy is not larger than pumping length p.

So let's take string $w = 0^p 1^p 2^p$ from language A_1. Its length is $3p$, which is certainly greater than p, so it can be pumped. This means that it can divided into three parts x, y and z, where $y \neq \varepsilon$, for which $w = xyz$ and any variant $xy^i z$ is also a member of A_1.

We also now that $|xy| \leq p$, which in this case implies that $xy = 0^k$, for some $k \leq p$ (because the length of first block of 0's is exactly p). Then y consists of only 0's, which means that pumping it up produces more 0's, while number of 1's and 2's remains unchanged. This new string then can not be member of language A_1, which is in contradiction with pumping lemma.

So we must conclude that our assumption about regularity of A_1 was completely wrong, i.e. language A_1 is **not regular**.

Passo 2

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$\boxed{\textbf{Part b.}}$

Consider string $\mathbf{a}^p\mathbf{b}\mathbf{a}^p\mathbf{b}\mathbf{a}^p\mathbf{b} \in A_2$ ($w = \mathbf{a}^p\mathbf{b}$, where p is the pumping length) and follow the reasoning from previous part.

Passo 3

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$\boxed{\textbf{Part c.}}$

Strings in language $A_3 = \{\mathbf{a}^{2^n} \mid n \geq 0\}$ consist of only $\mathbf{a}$'s and their length is equal to powers of 2. First we compute the difference between two consecutive powers of 2:

$$2^{n+1} - 2^n = 2 \cdot 2^n - 2^n = 2^n > n.$$

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Last inequality is obvious (and can be easily proved usi
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Let's now assume that $A_3$ is regular. Then there is pu
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$$2^p < 2^p + k = |xy^2z| \leq 2^p + p < 2^{p+1},$$

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i.e.\ it is strictly between two consecutive powers of $
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Resultado

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Try to discover which strings to pump.

‹ Exercício 28

Avaliar esta solução

☆ ☆ ☆ ☆ ☆

Exercício 30 ›