sexta-feira, 19 de agosto de 2022 13:45

Gui - 23, 28, 29, 30, 32, 33, 35, 36 e 38 (A)

$$\vec{a}+\vec{b} = (x_a+x_b),(y_a+y_b)$$

$$\vec{a}-\vec{b} = (x_a-x_b),(y_a-y_b)$$

23) Sendo $\vec{a} \perp \vec{b}$, $|\vec{a}| = 6$ e $|\vec{b}| = 8$, calcular $|\vec{a}+\vec{b}|$ e $|\vec{a}-\vec{b}|$.

$\vec{a} = (x_a, y_a)$

$\vec{b} = (x_b, y_b)$

$\vec{a} \cdot \vec{b} = 0$

$$|\vec{a}| = \sqrt{(x_a)^2+(y_a)^2}$$

$$6^2 = x_a^2 + y_a^2$$

$$\boxed{x_a^2+y_a^2 = 36}$$

$$|\vec{b}|^2 = (x_b)^2+(y_b)^2$$

$$\boxed{x_b^2+y_b^2 = 64}$$

$$(x_a \cdot x_b)+(y_a \cdot y_b) = 0$$

$$|\vec{a}+\vec{b}| = \sqrt{(x_a+x_b)^2+(y_a+y_b)^2}$$

$$|\vec{a}+\vec{b}| = \sqrt{x_a^2+2x_ax_b+x_b^2+y_a^2+2y_ay_b+y_b^2}$$

$$|\vec{a}+\vec{b}| = \sqrt{\underbrace{x_a^2+y_a^2}_{36}+\underbrace{x_b^2+y_b^2}_{64}+2.\underbrace{(x_a \cdot x_b+y_a \cdot y_b)}_{0}}$$

$$|\vec{a}+\vec{b}| = \sqrt{100} \Rightarrow \boxed{|\vec{a}+\vec{b}| = 10 \text{ u.c}}$$

$$|\vec{a}-\vec{b}| = \sqrt{(x_a-x_b)^2+(y_a-y_b)^2}$$

$$|\vec{a}-\vec{b}| = \sqrt{\underbrace{x_a^2+y_a^2}_{36}+\underbrace{x_b^2+y_b^2}_{64}-2\underbrace{(x_a \cdot x_b+y_a \cdot y_b)}_{0}}$$

$$|\vec{a}-\vec{b}| = \sqrt{100} = 10 \text{ u.c.}$$

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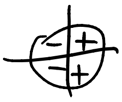
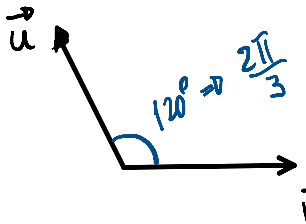
28) Calcular o valor de m de modo que seja 120º o ângulo entre os vetores $\vec{u} = (1, -2, 1)$ e $\vec{v} = (-2, 1, \underbrace{m+1}_{t})$.





$\vec{u} = (1,-2,1)$

$\vec{v} = (-2,1,t)$

$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$$

$$-\frac{1}{2} = \frac{(1,-2,1).(-2,1,t)}{\sqrt{1+4+1}\cdot\sqrt{4+1+t^2}} \quad (*)$$

$$(*) \quad -\frac{1}{2} = \frac{-2-2+t}{\sqrt{30+6t^2}} \Rightarrow -\frac{1}{2} = \frac{t-4}{\sqrt{30+6t^2}}$$

(*) $-\frac{1}{2} = \frac{\quad}{\sqrt{30+6t^2}} \Rightarrow -\frac{1}{2} = \frac{\quad}{\sqrt{30+6t^2}}$

$\Rightarrow (2t-8)^2 = (-1)^2 \cdot \left(\sqrt{30+6t^2}\right)^2 \Rightarrow 4t^2 - 32t + 64 = 30 + 6t^2$

$\Rightarrow 2t^2 + 32t - 34 = 0 \ (\div 2) \Rightarrow t^2 + 16t - 17 = 0$ → $t' = -17$; $t'' = 1$

$t' = m' + 1 \Rightarrow -17 = m' + 1 \Rightarrow \underline{m' = -18}$

$t'' = m'' + 1 \Rightarrow 1 = m'' + 1 \Rightarrow \underline{m'' = 0}$

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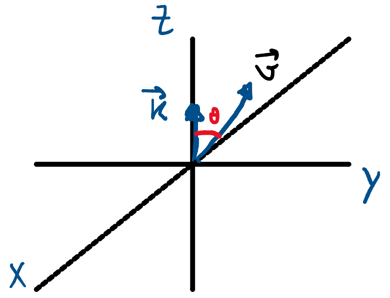
29) Calcular n para que seja de 30º o ângulo entre os vetores $\vec{v} = (-3, 1, n)$ e $\vec{k}$.



$\vec{k} = (0, 0, 1)$

$\vec{v} = (-3, 1, n)$

$\cos 30° = \frac{\vec{k} \cdot \vec{v}}{|\vec{k}| \cdot |\vec{v}|}$

$\frac{\sqrt{3}}{2} = \frac{0 + 0 + n}{\sqrt{1} \cdot \sqrt{9+1+n^2}}$ → $\frac{\sqrt{3}}{2} = \frac{n}{\sqrt{10+n^2}}$ (*)

(*) $(2n)^2 = \left(\sqrt{30+3n^2}\right)^2 \Rightarrow 4n^2 = 30 + 3n^2 \Rightarrow n^2 = 30$

$\boxed{n = \sqrt{30}}$

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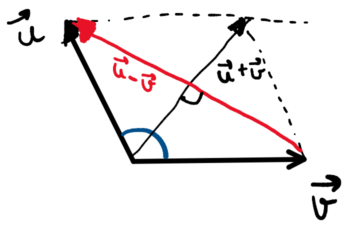
30) Se $|\vec{u}| = 4$, $|\vec{v}| = 2$ e 120º o ângulo entre os vetores $\vec{u}$ e $\vec{v}$, determinar o ângulo entre $\vec{u} + \vec{v}$ e $\vec{u} - \vec{v}$ e construir uma figura correspondente a estes dados.



$\cos 120° = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} \Rightarrow -\frac{1}{2} = \frac{\vec{u} \cdot \vec{v}}{4 \cdot 2}$

$\Rightarrow -8 = 2 \cdot (\vec{u} \cdot \vec{v}) \Rightarrow \boxed{\vec{u} \cdot \vec{v} = -4}$

$\vec{u} + \vec{v} = (x_u + x_v), (y_u + y_v)$ / $\vec{u} - \vec{v} = (x_u - x_v), (y_u - y_v)$

$|\vec{u} + \vec{v}| = \sqrt{\underline{x_u^2} + 2 \cdot x_u \cdot x_v + x_v^2 + \underline{y_u^2} + 2 \cdot y_u \cdot y_v + y_v^2}$

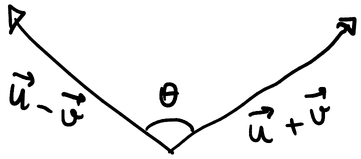
$|\vec{u} + \vec{v}| = \sqrt{\underbrace{x_u^2 + y_u^2}_{16} + \underbrace{x_v^2 + y_v^2}_{4} + 2\underbrace{(x_u \cdot x_v + y_u \cdot y_v)}_{-4}}$

$|\vec{u} + \vec{v}|^2 = 20 - 8$

$\underbrace{\quad}_{16} \qquad \underbrace{\quad}_{4} \qquad \underbrace{\quad}_{-4}$

$|\vec u+\vec v|^2 = 20 - 8$

$|\vec u+\vec v| = 2\sqrt{3}$



$|\vec u-\vec v|^2 = 20 + 8$

$|\vec u-\vec v|^2 = 28$

$|\vec u-\vec v| = 2\sqrt{7}$

$$\cos\theta = \frac{(\vec u+\vec v)\cdot(\vec u-\vec v)}{|\vec u+\vec v|\cdot|\vec u-\vec v|} \Rightarrow \cos\theta = \frac{(x_u^2-x_v^2)+(y_u^2-y_v^2)}{(2\sqrt{3})\cdot(2\sqrt{7})}$$

$$\Rightarrow \cos\theta = \frac{\overbrace{x_u^2+y_u^2}^{16}-\overbrace{x_v^2-y_v^2}^{4}}{4\sqrt{21}} \Rightarrow \cos\theta = \frac{16-4}{4\sqrt{21}}$$

$$\Rightarrow \cos\theta = \frac{3\,\cancel{12}}{{}_1\cancel{4}\sqrt{21}} \Rightarrow \cos\theta = \frac{3\sqrt{21}}{21} \Rightarrow \cos\theta = \frac{\sqrt{21}}{7}$$

$$\theta = \arccos\left(\frac{\sqrt{21}}{7}\right)$$

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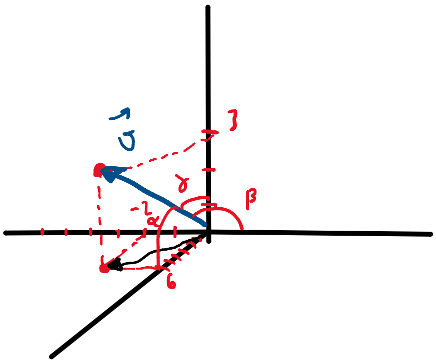
32) Calcular os ângulos diretores do vetor $\vec v = (6, -2, 3)$.

$\vec v \cdot \vec i = |\vec v| \cdot |\vec i| \cdot \cos\alpha$

$\vec v \cdot (1, 0, 0) = |\vec v| \cdot \cos\alpha$

$\frac{v_x}{|\vec v|} = \cos\alpha$



$$\cos\alpha = \frac{6}{\sqrt{36+4+9}} \Rightarrow \cos\alpha = \frac{6}{7} \Rightarrow \alpha = \arccos(6/7)$$

$$\cos\beta = \frac{v_y}{|\vec v|} \Rightarrow \cos\beta = \frac{-2}{7} \Rightarrow \beta = \arccos(-2/7)$$

$$\cos\gamma = \frac{v_z}{|\vec v|} \Rightarrow \cos\gamma = \frac{3}{7} \Rightarrow \beta = \arccos(3/7)$$

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33) Os ângulos diretores de um vetor $\vec a$ são 45°, 60° e 120° e $|\vec a| = 2$. Determinar $\vec a$.

$$\cos\alpha = \frac{a_x}{\ } \Rightarrow \cos 45^\circ = \frac{a_x}{2} \Rightarrow \frac{\sqrt{2}}{\cancel{2}} = \frac{a_x}{\cancel{2}} \Rightarrow \boxed{a_x = \sqrt{2}}$$

$$\cos\alpha = \frac{a_x}{|\vec{a}|} \Rightarrow \cos 45^\circ = \frac{a_x}{2} \Rightarrow \frac{\sqrt{2}}{\cancel{2}} = \frac{a_x}{\cancel{2}} \Rightarrow \boxed{a_x = \sqrt{2}}$$

$$\cos\beta = \frac{a_y}{|\vec{a}|} \Rightarrow \cos 60^\circ = \frac{a_y}{2} \Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \frac{a_y}{\cancel{2}} \Rightarrow \boxed{a_y = \sqrt{3}}$$

$$\cos\gamma = \frac{a_z}{|\vec{a}|} \Rightarrow \cos 120^\circ = \frac{a_z}{2} \Rightarrow -\frac{1}{\cancel{2}} = \frac{a_z}{\cancel{2}} \Rightarrow \boxed{a_z = -1}$$

$$\vec{a} = (\sqrt{2}, \sqrt{3}, -1)$$

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35) Mostrar que existe um vetor cujos ângulos diretores são 30°, 90° e 60°, respectivamente, e determinar aquele que tem módulo 10.

① EXISTÊNCIA DO VETOR COM Ângulos $30^\circ, 90^\circ, 60^\circ$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 30 + \cos^2 90 + \cos^2 60 = 1$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + 0^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$\frac{3}{4} + \frac{1}{4} = 1 \Leftrightarrow 1 = 1 \checkmark$$

$$\vec{v} = (5\sqrt{3}, 0, 5)$$

② $\cos\alpha = \dfrac{v_x}{|\vec{v}|}$

$\cos 30^\circ = \dfrac{v_x}{10} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{v_x}{10}$

$\boxed{v_x = 5\sqrt{3}}$

$\cos\beta = \dfrac{v_y}{|\vec{v}|}$

$\cos 90^\circ = \dfrac{v_y}{10} \Rightarrow \boxed{v_y = 0}$

$\cos\gamma = \dfrac{v_z}{|\vec{v}|}$

$\cos 60^\circ = \dfrac{v_z}{10}$

$\dfrac{1}{2} = \dfrac{v_z}{10}$

$\boxed{v_z = 5}$

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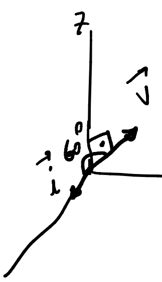
36) Determinar um vetor unitário ortogonal ao eixo Oz e que forme 60° com o vetor $\vec{i}$.



$|\vec{v}| = 1$

$\vec{i} = (1, 0, 0)$

$\vec{v} \cdot \vec{i} = |\vec{v}| \cdot |\vec{i}| \cdot \cos 60^\circ$

$\vec{v} \cdot (1, 0, 0) = \dfrac{1}{2}$

$\alpha = 60^\circ$ e $\gamma = 90^\circ$ e $\beta = 30^\circ$ ou $\beta = 150^\circ$

$\boxed{v_x = \dfrac{1}{2}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ | $\cos 90^\circ = \dfrac{v_z}{1}$

$\cos^2 \ldots + \cos^2 30^\circ + \cos^2 90^\circ = 1$ | v_y

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 60 + \cos^2 30^\circ + \cos^2 90^\circ = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 0 = 1$

$\frac{1}{4} + \frac{3}{4} = 1 \Leftrightarrow \overline{\underline{1 = 1}}$

$\cos 30^\circ = \frac{v_y}{1}$

ou $\cos 150^\circ$

$v_y = \pm\frac{\sqrt{3}}{2}$

$\cos 90^\circ = \frac{v_z}{1}$

$\overline{\underline{v_z = 0}}$

$\vec{v} = \left(\frac{1}{2}, \pm\sqrt{3}/2, 0\right)$

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38) Determinar o vetor $\vec{v}$ nos casos

a) $\vec{v}$ é ortogonal ao eixo Oz, $|\vec{v}| = 8$, forma ângulo de 30° com o vetor $\vec{i}$ e ângulo obtuso com $\vec{j}$;

$\gamma = 90^\circ$

$\alpha = 30^\circ$

$\beta = 120^\circ$

$\vec{v} = (4\sqrt{3}, -4, 0)$

$\cos\alpha = \frac{v_x}{|\vec{v}|}$

$\frac{\sqrt{3}}{\cancel{2}} = \frac{v_x}{\cancel{8}\,4}$

$\boxed{v_x = 4\sqrt{3}}$

$\cos\beta = \frac{v_y}{|\vec{v}|}$

$-\frac{1}{2} = \frac{v_y}{8}$

$\boxed{v_y = -4}$

$\cos\gamma = \frac{v_z}{|\vec{v}|}$

$0 = \frac{v_z}{8}$

$\boxed{v_z = 0}$
