Atividade de Grupo IV

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16. Verificar que são **válidos** os seguintes argumentos:

(a)
(1) $x = y \lor x > y$
(2) $x < 4 \lor x \nless z$
(3) $x = y \to x < z$
(4) $x > y \to x < z$
$\therefore x < 4$

(b)
(1) $2x + y = 5 \to 2x = 2$
(2) $2x + y = 5 \lor y = 3$
(3) $2x = 2 \to x = 1$
(4) $y = 3 \to 2x = 2$
$\therefore x = 1$

(c)
(1) $x < 3 \lor x > 4$
(2) $x < 3 \to x \neq y$
(3) $x > 4 \to x \neq y$
(4) $x < y \lor x \neq y \to x \neq 4 \land x = 2$
$\therefore x = 2$

(d)
(1) $x = 3 \to 2x^2 = 18$
(2) $x = 3 \lor x = -3$
(3) $x = -3 \to 2x^2 = 18$
(4) $2x^2 = 18 \to x^2 = 9$
$\therefore x^2 = 9$

(e)
(1) $z > x \to x \neq 7$
(2) $x < 6 \lor x = 3$
(3) $x = 3 \to z > x$
(4) $x < 6 \to z > x$
(5) $x = 7 \lor x = 5$
$\therefore x = 5$

(f)
(1) $x = 3 \lor x = 4$
(2) $x = 3 \to x^2 - 7x + 12 = 0$
(3) $x = 4 \to x^2 - 7x + 12 = 0$
(4) $x^2 - 7x + 12 = 0 \to x > 2$
(5) $x^2 < 9 \to x \ngtr 2$
(6) $x^2 \nless 9 \to x^2 = 9 \lor x^2 > 9$
$\therefore x^2 = 9 \lor x^2 > 9$

(g)
(1) $x > y \lor x < 4$
(2) $x < 4 \to x < y \land y \nless 4$
(3) $x > y \to x = 4$
(4) $x \neq 4$
$\therefore x < y$

(h)
(1) $x = \frac{5\pi}{6} \to \operatorname{sen} x = \frac{1}{2}$
(2) $x = \frac{\pi}{6} \lor x = \frac{5\pi}{6}$
(3) $\operatorname{sen} x = \frac{1}{2} \to \operatorname{csc} x = 2$
(4) $x = \frac{\pi}{6} \to \operatorname{sen} x = \frac{1}{2}$
$\therefore \cos x = \frac{\sqrt{3}}{2} \lor \operatorname{csc} x = 2$

(i)
(1) $x + 2y = 5 \lor 3x + 4y = 11$
(2) $x > y \lor x \nless 2 \to y < 2 \lor y < 1$
(3) $3x + 4y = 11 \to x = 1$
(4) $x > y \lor x \nless 2$
(5) $x + 2y = 5 \to x = 1$
$\therefore x = 1 \land (y < 2 \lor y < 1)$

A)
1) $x = y \lor x > y$
2) $x < 4 \lor x \nless z$
3) $x = y \to x < z$
4) $x > y \to x < z$

$x = y : p$
$x > y : q$
$x < 4 : t$
$x \nless z : \sim r$
$x < z : r$

1) $p \lor q \quad : \quad \sim p \to q$
2) $t \lor \sim r$
3) $p \to r$
4) $q \to r$
$\therefore t$

1) $\sim p \to q$
4) $q \to r$
$\sim p \to r$ ∴ Silogismo Hipotético.

1) $\sim p \to r \quad : \quad p \lor r$
2) $t \lor \sim r \qquad t \lor \sim r$
3) $p \to r \quad : \quad \sim p \lor r \quad : \quad r \lor \sim p$ (Comutativa)

1) $p \lor r$
3) $r \lor \sim p$

$(p \lor r) \land (r \lor \sim p) \Leftrightarrow$ Regra da Conjunção
$r \lor (p \land \sim p) \Leftrightarrow$
$r \lor C \Leftrightarrow$
$r \Leftrightarrow$ #

1) r
2) $t \lor \sim r \quad : \quad \sim r \lor t$ (Comutativa) $\quad : \quad r \to t$ | ...

$1)\ r$

comutativa

$2)\ t \vee \sim r \ :\ \sim r \vee t \ :\ r \to t$

$\quad r$

$\overline{\quad t \quad}$ $\quad t : x < 4$

Modus Ponens.

B)

(b) (1) $2x + y = 5 \to 2x = 2$
(2) $2x + y = 5 \vee y = 3$
(3) $2x = 2 \to x = 1$
(4) $y = 3 \to 2x = 2$
$\therefore\ x = 1$

$2x + y = 5 : p$
$2x = 2 : q$
$y = 3 : r$
$x = 1 : t$

(1) $p \to q$
(2) $p \vee r$
(3) $q \to t$
(4) $r \to q$
$\overline{\quad t \quad}$

$\Rightarrow$

(1) $p \to q$
(3) $q \to t$
$\overline{\quad p \to t \quad}$ $\therefore$ Silogismo Hipotético

$\Rightarrow$

(1) $p \to t$
(2) $p \vee r$
(3) $r \to q$
$\overline{\quad t \vee q \quad}$ $\therefore$ Dilema construtivo

$\boxed{t = q}$ ✓

$t \vee t \Rightarrow t$: Adição!

(c)

(c) (1) $x<3 \vee x>4$
(2) $x<3 \to x \neq y$
(3) $x>4 \to x \neq y$
(4) $x<y \vee x \neq y \to x \neq 4 \wedge x = 2$
$\therefore\ x = 2$

$x < 3 : p$
$x > 4 : q$
$x \neq y : r$
$x < y : s$
$x \neq 4 : w$
$x = 2 : t$

Comutativa

(1) $p \vee q : q \vee p$
(2) $p \to r$
(3) $q \to r$
(4) $(s \vee r) \to w \wedge t$
$\overline{\quad t \quad}$

$\Rightarrow$

$\sim q \to p$
$p \to r$
$\overline{\quad \sim q \to r \quad}$ $\therefore$ Silogismo Hipotético

$\Rightarrow$

(1) $\sim q \to r$
(2) $q \to r$
(3) $(s \vee r) \to (w \wedge t)$

$\Rightarrow$ $q \vee r \Leftrightarrow q \vee r$
$\sim q \vee r \Leftrightarrow \underline{r \vee \sim q}$

$(q \vee r) \wedge (r \vee \sim q) \Leftrightarrow$ $\therefore$ Regra da conjunção!

$r \vee (q \wedge \sim q) \Leftrightarrow$

$r \vee C \Leftrightarrow$

$r \Leftrightarrow$

$\Rightarrow$

...ções simplificações!

$r \Leftrightarrow$

$\#$

(1) r

(2) $(s \vee r) \rightarrow (w \wedge t)$

$\Rightarrow$ (1) r → Adição Simplificação!

(2) $r \rightarrow t$

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t ∴ Modus Ponens

$\Rightarrow$ $r \vdash s \vee r$: Adição

$w \wedge t \vdash t$: Simplificação

D

(d) (1) $x = 3 \rightarrow 2x^2 = 18$

(2) $x = 3 \vee x = -3$

(3) $x = -3 \rightarrow 2x^2 = 18$

(4) $2x^2 = 18 \rightarrow x^2 = 9$

∴ $x^2 = 9$

$x = 3 : p$

$2x^2 = 18 : q$

$x = -3 : r$

$x^2 = 9 : s$

(1) $p \rightarrow q$

(2) $p \vee r$

(3) $r \rightarrow q$

(4) $q \rightarrow s$

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s

$\Rightarrow$ (1) $p \rightarrow q$

(4) $q \rightarrow s$

$p \rightarrow s$ ∴ Silogismo Hipotético

$\Rightarrow$ (1) $p \rightarrow s$

(2) $p \vee r$

(3) $r \rightarrow q$

$\Rightarrow$

$\Rightarrow$ (2) $\sim p \rightarrow r$

(3) $r \rightarrow q$

$\sim p \rightarrow q$ ∴ Silogismo Hipotético

$\Rightarrow$ (1) $p \rightarrow s \Leftrightarrow$ $p \rightarrow s$

(2) $\sim p \rightarrow q \Leftrightarrow$ p

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s ∴ Modus ponens

$p \vdash p \vee q$: Adição

?

E

(e) (1) $z > x \rightarrow x \neq 7$

(2) $x < 6 \vee x = 3$

(3) $x = 3 \rightarrow z > x$

(4) $x < 6 \rightarrow z > x$

(5) $x = 7 \vee x = 5$

∴ $x = 5$

$z > x : p$

$x \neq 7 : q$

$x < 6 : r$

$x = 3 : s$

$x = 7 : t$

$x = 5 : w$

(1) $p \rightarrow q$

(2) $r \vee s$

(3) $s \rightarrow p$

(4) $r \rightarrow p$

(5) $t \vee w$

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w

$\Rightarrow$ (3) $s \rightarrow p$

(4) $r \rightarrow p$

(2) $r \vee s$

$p \vee p \Leftrightarrow p$ ∴ Dilema Construtivo

$\Rightarrow$ (1) $p \rightarrow q$

(2) p

(3) $t \vee w$

$\Rightarrow$ (1) $p \rightarrow q$

(2) p

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q ∴ Modus ponens

$\Rightarrow$ (1) q

(2) $t \vee w$:

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w

q

$\sim t \rightarrow w$

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w , $q = \sim t$ ∴ Modus ponens

F

(f) (1) $x = 3 \vee x = 4$
(2) $x = 3 \rightarrow x^2 - 7x + 12 = 0$
(3) $x = 4 \rightarrow x^2 - 7x + 12 = 0$
(4) $x^2 - 7x + 12 = 0 \rightarrow x > 2$
(5) $x^2 < 9 \rightarrow x \not> 2$
(6) $x^2 \not< 9 \rightarrow x^2 = 9 \vee x^2 > 9$
$\therefore\ x^2 = 9 \vee x^2 > 9$

$x = 3 : p$
$x = 4 : q$
$x^2 - 7x + 12 : r$
$x > 2 : s$

$x^2 < 9 : t$
$x \not> 2 : \sim s$
$x^2 \not< 9 : \sim t$

$x^2 = 9 : w$
$x^2 > 9 : u$

(1) $p \vee q$
(2) $p \rightarrow r$
(3) $q \rightarrow r$
(4) $r \rightarrow s$
(5) $t \rightarrow \sim s$
(6) $\sim t \rightarrow w \vee u$

$\Rightarrow$ (1) $p \vee q$
(2) $p \rightarrow r$
(3) $q \rightarrow r$
$r \vee r : r$ ∴ Dilema Construtivo

$\Rightarrow$ (1) r
(2) $r \rightarrow s$
(3) $t \rightarrow \sim s$
(4) $\sim t \rightarrow w \vee u$

$\Rightarrow$ (1) r
(2) $r \rightarrow s$
s ∴ Modus ponens

$\Rightarrow$ (1) s
(2) $t \rightarrow \sim s$
(3) $\sim t \rightarrow w \vee u$ $\Rightarrow$

$\Rightarrow$ (1) s
(2) $s \rightarrow \sim t$
$\sim t$: Modus Ponens

$\Rightarrow$ (1) $\sim t$
(2) $\sim t \rightarrow w \vee u$
$w \vee u$ ∴ Modus ponens

G

(g) (1) $x > y \vee x < 4$
(2) $x < 4 \rightarrow x < y \wedge y \not< 4$
(3) $x > y \rightarrow x = 4$
(4) $x \neq 4$
$\therefore\ x < y$

$x > y : p$
$x < 4 : q$
$x < y : r$
$y \not< 4 : s$

$x = 4 : t$
$x \neq 4 : w$

(1) $p \vee q$
(2) $q \rightarrow r \wedge s$
(3) $p \rightarrow t$
(4) $w = \sim t$

$\Rightarrow$ (1) $\sim p \rightarrow q$
(2) $q \rightarrow r \wedge s$
$\sim p \rightarrow r \wedge s$ ∴ Silogismo Hipotético

$\Rightarrow$ (1) $\sim p \rightarrow r \wedge s$
(2) $p \rightarrow t$
(3) $\sim t$ $\Rightarrow$

$\Rightarrow$ (2) $p \rightarrow t$
(3) $\sim t$
$\sim p$ ∴ Modus Tollens

$\Rightarrow$ (1) $\sim p \rightarrow r \wedge s$
$\sim p$
$r \wedge s$ ∴ Modus Ponens

Simplificação
$\Rightarrow r \wedge s \vdash r$

H

(h) (1) $\overset{p}{x = \frac{5\pi}{6}} \rightarrow \overset{q}{\operatorname{sen} x = \frac{1}{2}}$
(2) $\overset{r}{x = \frac{\pi}{6}} \vee \overset{p}{x = \frac{5\pi}{6}}$
(3) $\overset{q}{\operatorname{sen} x = \frac{1}{2}} \rightarrow \overset{s}{\operatorname{csc} x = 2}$
(4) $\overset{r}{x = \frac{\pi}{6}} \rightarrow \overset{q}{\operatorname{sen} x = \frac{1}{2}}$
$\therefore\ \cos x = \frac{\sqrt{3}}{2} \vee \operatorname{csc} x = 2$

(1) $p \rightarrow q$
(2) $r \vee p$
(3) $q \rightarrow s$
(4) $r \rightarrow q$

$\Rightarrow$ (1) $p \rightarrow q$
(3) $q \rightarrow s$
$p \rightarrow s$ ∴ Silogismo Hipotético

$\Rightarrow$

(1) $p \to s$
(2) $r \vee p$
(3) $r \to q$

$s \vee r$ ∴ Dilema Construtivo

(i) (1) $x + 2y = 5 \vee 3x + 4y = 11$
(2) $x > y \vee x \nless 2 \to y < 2 \vee y < 1$
(3) $3x + 4y = 11 \to x = 1$
(4) $x > y \vee x \nless 2$
(5) $x + 2y = 5 \to x = 1$

∴ $x = 1 \wedge (y < 2 \vee y < 1)$

$x + 2y = 5 : p$
$3x + 4y = 11 : q$
$x > y : r$
$x \nless 2 : s$
$y < 2 : t$
$y < 1 : u$
$x = 1 : w$

(1) $p \vee q$
(2) $r \vee s \to t \vee u$
(3) $q \to w$
(4) $r \vee s$
(5) $p \to w$

$w \wedge (t \vee u)$

$\Rightarrow$

(1) $p \vee q$
(2) $q \to w$
(3) $p \to w$

$w \vee w$ ∴ Dilema Construtivo
w

$\Rightarrow$

(1) w
(2) $r \vee s \to t \vee u$
(3) $r \vee s$

$\Rightarrow$

(2) $r \vee s \to t \vee u$
(3) $r \vee s$

$t \vee u$ ∴ Modus Ponens

$\Rightarrow$

(1) $t \vee u$
(2) w

$w \wedge (t \vee u)$

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17. Usar as **Regras de Inferência** para mostrar que são **válidos** os seguintes argumentos:

(a) $p \vee q \to \sim r,\quad p,\quad s \to r \vdash \sim s$
(b) $p \wedge (q \vee r),\quad q \vee r \to \sim s,\quad s \vee t \vdash t$
(c) $p \vee q \to \sim r,\quad q,\quad s \wedge t \to r \vdash \sim (s \wedge t)$
(d) $p \to q,\quad \sim q,\quad \sim p \vee \sim r \to s \vdash s$
(e) $p \vee (q \wedge r),\quad q \to s,\quad r \to t,\quad s \wedge t \to p \vee r,\quad \sim p \vdash r$
(f) $q \vee (r \to t),\quad q \to s,\quad \sim s \to (t \to p),\quad \sim s \vdash r \to p$
(g) $p \vee q \to (p \to s \wedge t),\quad p \wedge r \vdash t \vee u$

(A) (1) $p \vee q \to \sim r$
(2) p
(3) $s \to r$

$\sim s$

$\Rightarrow$

$p \vdash p \vee q$ ∴ Adição

$\Rightarrow$

(1) $p \vee q \to \sim r$
(2) $p \vee q$

$\Rightarrow$

(1) $p \vee q \to \sim r$
$p \vee q$

$\Rightarrow$

$(1)\ p \vee q \rightarrow \sim r$

$(2)\ p \vee q$

$(3)\ s \rightarrow r$

$\Rightarrow$

$(1)\ p \vee q \rightarrow$

$$\frac{p \vee q}{\sim r}\ \therefore \text{Modus Ponens}$$

$\Rightarrow$

$(1)\ \sim r$

$$\frac{(2)\ s \rightarrow r}{\sim s}\ \therefore \text{Modus Tollens} ↯$$

Ⓑ $(1)\ p \wedge (q \vee r)$

$(2)\ q \vee r \rightarrow \sim s$

$$\frac{(3)\ s \vee t}{t}$$

$\Rightarrow$ $p \wedge (q \vee r) \vdash q \vee r \ \therefore$ Simplificação

$\Rightarrow$ $(1)\ q \vee r$

$$\frac{(2)\ q \vee r \rightarrow \sim s}{\sim s}\ \therefore \text{Modus Ponens}$$

$\Rightarrow$ $(1)\ \sim s$

$$\frac{(2)\ \sim s \rightarrow t}{t}\ \therefore \text{Modus Ponens}$$

Ⓒ $(1)\ p \vee q \rightarrow \sim r$

$(2)\ q$

$$\frac{(3)\ s \wedge t \rightarrow r}{\sim (s \wedge t)}$$

$\Rightarrow$ $q \vdash p \vee q \ \therefore$ Adição

$\Rightarrow$ $(1)\ p \vee q \rightarrow \sim r$

$$\frac{(2)\ p \vee q}{\sim r}\ \therefore \text{Modus Ponens}$$

$\Rightarrow$ $(1)\ \sim r$

$(2)\ s \wedge t \rightarrow r$

$\Rightarrow$ $(1)\ \sim r$

$$\frac{(2)\ \sim r \rightarrow \sim (s \vee t)}{\sim (s \vee t)}\ \therefore \text{Modus Ponens}$$

Ⓓ $(1)\ p \rightarrow q$

$(2)\ \sim q$

$$\frac{(3)\ \sim p \vee \sim r \rightarrow s}{s}$$

$\Rightarrow$ $(1)\ \sim q \rightarrow \sim p$

$$\frac{(2)\ \sim q}{\sim p}\ \therefore \text{Modus Ponens}$$

$\Rightarrow$

$\Rightarrow$ (1) $\sim p$

(2) $\sim p \vee \sim r \to s$

$\Rightarrow$ (1) $\sim p \vdash \sim p \vee \sim r$ $\therefore$ Adição

$\Rightarrow$ (1) $\sim p \vee \sim r$

(2) $\sim p \vee \sim r \to s$

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s $\therefore$ Modus ponens

(E) (1) $p \vee (q \wedge r)$

(2) $q \to s$

(3) $r \to t$

(4) $s \wedge t \to p \vee r$

(5) $\sim p$

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r

(6) $(q \wedge r)$ 1,5 : SD

$\Rightarrow$ (7) r : SIMP

(F) (1) $q \vee (r \to t)$

(2) $q \to s$

(3) $\sim s \to (t \to p)$

(4) $\sim s$

$r \to p$

(5) $t \to p$: 3,4 : MP

(6) $\sim s \to \sim q$: 2 : CP

(7) $\sim q$: 4,6 : MP

(8) $r \to t$: 1,7 : SD

(9) $r \to p$: 5,8 : SH

(G) (1) $p \vee q \to (p \to s \wedge t)$

(2) $p \wedge r$

$t \vee u$

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(3) p : 2 : SIMP

(4) $p \vee q$: 3 : AD

(5) $p \to s \wedge t$: 1,4 : MP

(6) $s \wedge t$: 3,5 : MP

(7) t : 6 : SIMP

(8) $t \vee u$: 7 : AD