Atividades Vetores

terça-feira, 13 de setembro de 2022 06:48

- Atttividade III - Produto Misto
  Questões 6(a), 7,10,11

6) Determinar o valor de k para que sejam coplanares os vetores

a) $\vec{u} = (2, -1, k)$, $\vec{v} = (1, 0, 2)$ e $\vec{w} = (k, 3, k)$

- Os Vetores são coplanares quando
  a determinante de suas coordenadas é nula;

$$\left|\begin{array}{ccc|cc} 2 & -1 & k & 2 & -1 \\ 1 & 0 & 2 & 1 & 0 \\ k & 3 & k & k & 3 \end{array}\right.$$

0 −12 k 0 −2k 3k

$$|D| = 0 \rightarrow \underbrace{-2k + 3k - 12}\ + \underbrace{k} = 0$$

$$2k = 12$$

$$\boxed{K = 6}$$

7) Verificar se são coplanares os pontos

a) A(1, 1, 0), B(-2, 1, -6), C(-1, 2, -1) e D(2, -1, -4)

b) A(2, 1, 2), B(0, 1, -2), C(1, 0, -3) e D(3, 1, -2)

A) $\vec{AB} = B - A = (-2,1,-6) - (1,1,0) = (-3,0,-6)$

$\vec{AC} = C - A = (-1,2,-1) - (1,1,0) = (-2,1,-1)$

$\vec{AD} = D - A = (2,-1,-4) - (1,1,0) = (1,-2,-4)$

$$\left|\begin{array}{ccc|cc} -3 & 0 & -6 & -3 & 0 \\ -2 & 1 & -1 & -2 & 1 \\ 1 & -2 & -4 & 1 & -2 \end{array}\right.$$

+6 +6 0 12 0 −24

$$|D| = 0 \Rightarrow \underbrace{12 + 0 - 24}\ + \underbrace{6 + 6}\ + 0 = 0$$

$$-12 + 12 = 0$$

$$0 = 0 \longrightarrow \text{SÃO COPLANARES!}$$

$$-12 + 12 = 0$$
$$0 = 0 \longrightarrow \text{SÃO COPLANARES!}$$

B) $\overrightarrow{AB} = B - A = (0, 1, -2) - (2, 1, 2) = (-2, 0, -4)$

$\overrightarrow{AC} = C - A = (1, 0, -3) - (2, 1, 2) = (-1, -1, -5)$

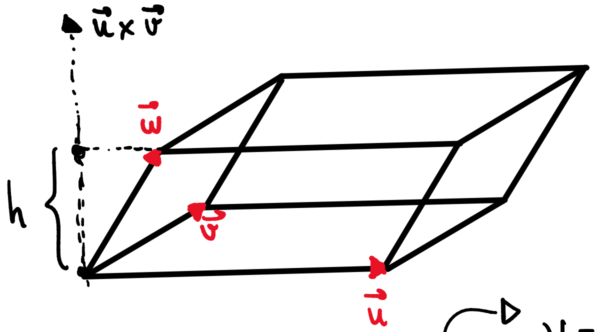
$\overrightarrow{AD} = D - A = (3, 1, -2) - (2, 1, 2) = (1, 0, -4)$

$$\begin{array}{ccc|cc} -2 & 0 & -4 & -2 & 0 \\ -1 & -1 & -5 & -1 & -1 \\ 1 & 0 & -4 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} -4 & 0 & -4 & -8 & 0 & 0 \end{array}$$

$$|D| = 0 \Rightarrow -8 + 0 + 0 - 4 + 0 - 4 = 0$$
$$-8 - 8 = 0$$
$$-16 \neq 0 \Rightarrow \bar{N} \text{ SÃO COPLANARES!}$$

10) Um paralelepípedo é determinado pelos vetores $\vec{u} = (3, -1, 4)$, $\vec{v} = (2, 0, 1)$ e $\vec{w} = (-2, 1, 5)$. Calcular seu volume e a altura relativa à base definida pelos vetores $\vec{u}$ e $\vec{v}$.



$$h = \left|\text{proj}_{\vec{u} \times \vec{v}} \vec{w}\right|$$

$$h = \frac{|\vec{w} \cdot (\vec{u} \times \vec{v})|}{|\vec{u} \times \vec{v}|}$$

$$V = |\vec{u} \times \vec{v}| \cdot h$$
$$V = |\vec{u} \times \vec{v} \cdot \vec{w}|$$

$$V = |(-1, 5, 2) \cdot (-2, 1, 5)|$$
$$V = |2 + 5 + 10|$$
$$\underline{V = 17 \text{ u.v}}$$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 3 & -1 & 4 \\ 2 & 0 & 1 \end{vmatrix}$$

$$\vec{u}\times\vec{v} = (-1)^{1+1}\cdot\begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix}\cdot\vec{i} + (-1)^{1+2}\cdot\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix}\cdot\vec{j} + (-1)^{1+3}\cdot\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}\cdot\vec{k}$$

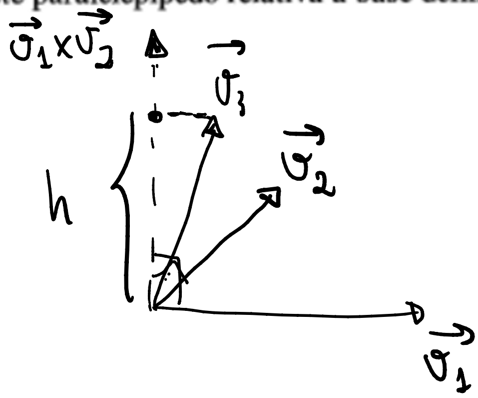
$$\vec{u}\times\vec{v} = (-1-0)\cdot\vec{i} - (3-8)\cdot\vec{j} + (0+2)\cdot\vec{k}$$

$$\vec{u}\times\vec{v} = -\vec{i} + 5\vec{j} + 2\vec{k} \Rightarrow (-1, 5, 2) \rightarrow |\vec{u}\times\vec{v}| = \sqrt{1+25+4}$$

$$= \sqrt{30}$$

$$h = \frac{|\vec{w}\cdot(\vec{u}\times\vec{v})|}{|\vec{u}\times\vec{v}|} \rightsquigarrow h = \frac{17}{\sqrt{30}} \text{ u.c.}$$

11) Calcular o valor de m para que o volume do paralelepípedo determinado pelos vetores $\vec{v}_1 = (0, -1, 2)$, $\vec{v}_2 = (-4, 2, -1)$ e $\vec{v}_3 = (3, m, -2)$ seja igual a 33. Calcular a altura deste paralelepípedo relativa à base definida por $\vec{v}_1$ e $\vec{v}_2$.



$$h = \frac{V}{|\vec{v}_1 \times \vec{v}_2|}$$

$$\boxed{h = \frac{33}{\sqrt{89}} \text{ u.v}}$$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ 0 & -1 & 2 \\ -4 & 2 & -1 \end{vmatrix} \Rightarrow (1-4)\cdot\vec{i} - (0+8)\cdot\vec{k} + (0-4)\cdot\vec{j}$$

$$\Rightarrow -3\vec{i} - 8\vec{k} - 4\vec{j} = (-3, -8, -4)$$

$$|\vec{v}_1 \times \vec{v}_2| = \sqrt{9+64+16} = \sqrt{89}$$

$$V = |\vec{v}_1 \times \vec{v}_2 \cdot \vec{v}_3| \Rightarrow V = |(-3, -8, -4)\cdot(3, m, -2)|$$

$$33 = |-9 - 8m + 8| \longrightarrow -9 - 8m + 8 = 33$$

$$-8m = 34$$

$$\boxed{m = -\frac{17}{4}}$$

$$9 + 8m - 8 = 33$$

$$8m + 1 = 33$$

$8m + 1 = 33$

$$\boxed{m = \frac{16}{4}}$$