Alg. Otimizado

$$\underbrace{\sum_{i=1}^{N-1}}_{B}\underbrace{\sum_{J=i+1}^{N} 1}_{A}$$

(A) $$\sum_{J=1}^{N-1} 1+i = \underbrace{\sum_{J=1}^{N-1} 1}_{A2} + \underbrace{\sum_{J=1}^{N-1} i}_{A1}$$

(A1) $$\sum_{J=1}^{N-1} i \Rightarrow i\sum_{J=1}^{N-1} 1 = i(N-1) = iN-1$$

(A2) $$\sum_{J=1}^{N-1} 1 \Rightarrow N-1 \cancel{-1} \cancel{+1} = N-1$$

(B) $$\sum_{i=1}^{N-1} iN-1+N-1 = \underbrace{\sum_{i=1}^{N-1} i.N}_{B4} - \underbrace{\sum_{i=1}^{N-1} 1}_{B3} + \underbrace{\sum_{i=1}^{N-1} N}_{B2} - \underbrace{\sum_{i=1}^{N-1} 1}_{B1}$$

(B1) $$= \sum_{i=1}^{N-1} 1 = N-1\cancel{-1}\cancel{+1} = N-1$$

(B2) $$\sum_{i=1}^{N-1} N = N\sum_{i=1}^{N-1} 1 = N(N-1\cancel{-1}\cancel{+1}) = N^2-N$$

(B3) $$\sum_{i=1}^{N-1} 1 = N-1\cancel{-1}\cancel{+1} = N-1$$

(B4) $$\sum_{i=1}^{N-1} i\cdot N = N\sum_{i=1}^{N-1} i = N\left(\frac{N-1(N\cancel{-1}\cancel{+1})}{2}\right) = N\left(\frac{N^2-N}{2}\right) = \frac{N^3-N^2}{2}$$

(B) $$\sum_{i=1}^{N-1} i.N - \sum_{i=1}^{N-1} 1 + \sum_{i=1}^{N-1} N - \sum_{i=1}^{N-1} 1 = \frac{N^3-N^2}{2} - N+1 + N^2-N-N+1$$

$$= \frac{N^3-N^2}{2} - N^2 - 3N + 2$$