

$$\underbrace{\sum_{i=1}^{N-1}}_{C^A} \underbrace{\sum_{j=i+1}^N}_{B^A} \underbrace{\sum_{k=1}^j}_{A^A} 1$$

$$\sum = \lim \sup - \lim \inf + 1$$

$$(A) \sum_{k=1}^j 1 = j - 1 + 1$$

$$(B) \sum_{j=i+1}^N j = \sum_{j=1}^{N-1} j + i \Rightarrow \underbrace{\sum_{j=1}^{N-1} j}_{B^1} + \underbrace{\sum_{j=1}^{N-1} i}_{B^2}$$

$$(B^1) \sum_{j=1}^{N-1} j = \frac{N-1(N-1+1)}{2} \Rightarrow \frac{N^2 - N}{2}$$

$$(B^2) \sum_{j=1}^{N-1} i = i \sum_{j=1}^{N-1} 1 = i(N-1-1+1) \Rightarrow iN - i$$

$$(B) \frac{N^2 - N}{2} + iN - i$$

$$(C) \sum_{i=1}^{N-1} \left(\frac{N^2 - N}{2} + \underbrace{iN}_{N \sum i} - i \right) \Rightarrow \underbrace{\sum_{i=1}^{N-1} \left(\frac{N^2 - N}{2} \right)}_{C^3} + N \underbrace{\sum_{i=1}^{N-1} i}_{C^2} - \underbrace{\sum_{i=1}^{N-1} i}_{C^1}$$

$$(C^1) \sum_{i=1}^{N-1} i = \frac{N-1(N-1+1)}{2} \Rightarrow \frac{N^2 - N}{2}$$

$$(C^2) N \sum_{i=1}^{N-1} i = N \left(\frac{N-1(N-1+1)}{2} \right) = N \left(\frac{N^2 - N}{2} \right) \Rightarrow \frac{N^3 - N^2}{2}$$

$$\textcircled{C} \sum_{i=1}^{N-1} \left(\frac{N^2 - N}{2} \right) = \frac{N^2 - N}{2} \left(\sum_{i=1}^{N-1} 1 \right) \Rightarrow \frac{N^2 - N}{2} (N-1-1+1) \quad \leftarrow$$

$$\frac{(N^2 - N) \cdot (N-1)}{2} = \frac{N^3 - N^2 - N^2 + N}{2}$$

$$\textcircled{C} \frac{N^3 - 2N^2 + N}{2} + \frac{N^3 - N}{2} - \left(\frac{N^2 - N}{2} \right) \Rightarrow \frac{N^3 - 2N^2 + N}{2} + \frac{N^3 - N}{2} + \frac{N - N^2}{2}$$

$$\frac{N^3 - 2N^2 + N + N^3 - N + N - N^2}{2} = \frac{2N^3 - 3N^2 + N}{2}$$

$$f(N) = \frac{2N^3 - 3N^2 + N}{2} \therefore O(N^3)$$