Algumas Primitivas

$$\int \frac{dx}{(x^2+b)^{3/2}} = \frac{1}{b}\frac{x}{\sqrt{x^2+b}} \qquad \int \frac{xdx}{(x^2+b)^{3/2}} = -\frac{1}{\sqrt{x^2+b}}$$

$$\int \frac{xdx}{\sqrt{x^2+b}} = \sqrt{x^2+b} \qquad \int \frac{dx}{\sqrt{x^2+b}} = \ln\left(x+\sqrt{x^2+b}\right)$$

$$\int \frac{dx}{x(x+a)} = \frac{1}{a}\ln(\frac{x}{x+a})$$

Para o cálculo analítico de integrais pode ser consultado o endereço web: http://integrals.wolfram.com

[t] **Coordenadas cartesianas (x, y, z)**

$$d\vec{\ell} = dx\ \vec{u}_x\ +\ dy\ \vec{u}_y +\ dz\ \vec{u}_z$$
$$dS = dx\ dy$$
$$dv\ = dx\ dy\ dz$$
$$\vec{\nabla} F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$$
$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\vec{\nabla}\times\vec{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (A_x, A_y, A_z)$$

Coordenadas polares (r, θ)

$$d\vec{\ell} = dr\ \vec{u}_r + r\ d\theta\ \vec{u}_\theta$$
$$dS = r\ dr\ d\theta$$

Coordenadas cilíndricas (r, θ, z)

$$d\vec{\ell} = dr\ \vec{u}_r + r\ d\theta\ \vec{u}_\theta + dz\ \vec{u}_z$$
$$dv\ = r\ dr\ d\theta\ dz$$
$$\vec{\nabla} F = \left(\frac{\partial F}{\partial r}, \frac{1}{r}\frac{\partial F}{\partial \theta}, \frac{\partial F}{\partial z}\right)$$
$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial(r\ A_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$
$$\vec{\nabla}\times\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u}_\theta + \left(\frac{1}{r}\frac{\partial(r\ A_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\vec{u}_z$$

Coordenadas esféricas (r, θ, ϕ)

$$d\vec{\ell} = dr\ \vec{u}_r + r\ d\theta\ \vec{u}_\theta + r\ sen\theta\ d\phi\ \vec{u}_\phi$$
$$dv\ = r^2\ dr\ sen\theta\ d\theta\ d\phi$$
$$\vec{\nabla} F = \left(\frac{\partial F}{\partial r}, \frac{1}{r}\frac{\partial F}{\partial \theta}, \frac{1}{rsen\theta}\frac{\partial F}{\partial \phi}\right)$$
$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{rsen\theta}\frac{\partial}{\partial \theta}\left(sen\theta A_\theta\right) + \frac{1}{rsen\theta}\frac{\partial}{\partial \phi}\left(A_\phi\right)$$
$$\vec{\nabla}\times\vec{A} = \left[\frac{1}{rsen\theta}\frac{\partial(sen\theta A_\phi)}{\partial \theta} - \frac{\partial(sen\theta A_\theta)}{\partial \phi}\right]\vec{u}_r + \frac{1}{r}\left[\frac{1}{sen\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(rA_\phi)}{\partial r}\right]\vec{u}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\vec{u}_\phi$$

Teorema da Divergência

$$\int_v \vec{\nabla}\cdot\vec{A}\ dv = \oint_S \vec{A}\cdot\vec{n}\ dS$$

Teorema da Stokes

$$\int_S \vec{\nabla}\times\vec{A}\ \cdot\ d\vec{S} = \oint_\Gamma \vec{A}\cdot d\vec{\ell}$$

Identidades vectoriais

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$$
$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$
$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

Electrostática

- $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{u}_r$
- $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N.m^2.C^{-2}$
- $\oint_\Gamma \vec{E} \cdot d\vec{\ell} = 0$
  $\nabla \times \vec{E} = 0$
- $\oint_S \vec{D} \cdot \vec{n}\, dS = \int_v \rho_{liv}\, dv$
  $\vec{\nabla} \cdot \vec{D} = \rho_{liv}$
- $\oint_S \vec{P} \cdot \vec{n}\, dS = -\int_v \rho_{pol} dv$
  $\rho_{pol} = -\vec{\nabla} \cdot \vec{P}$
  $\sigma_{pol} = \vec{P} \cdot \vec{n}_{ext}$
- $V_P = \int_P^{Ref} \vec{E} \cdot d\vec{\ell}$
  $\vec{E} = -\vec{\nabla} V$
- $\vec{D} = \vec{P} + \varepsilon_0 \vec{E}$
  $\vec{D} = \varepsilon_0(1 + \chi_E)\vec{E} = \varepsilon\vec{E}$
- $Q = CV$
- $U_E = \left[\frac{1}{2}\right] \sum_i q_i\, V_i$
- $u_E = \frac{1}{2}\varepsilon E^2$
  $U_E = \int_v u_E dv$
- $\vec{F}_s = \pm \frac{dU_E}{ds}\vec{u}_s$

Corrente eléctrica estacionária

- $\vec{J} = Nq\vec{v}$
- $\vec{J} = \sigma_c \vec{E}$
- $I = \int_S \vec{J} \cdot \vec{n}\, dS$
- $p = \vec{J} \cdot \vec{E}$
- $\oint_S \vec{J} \cdot \vec{n}\, dS = -\frac{d}{dt}\int_v \rho dv$
  $\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$

Ondas electromagnéticas

- $\vec{S} = \vec{E} \times \vec{H}$
- $\vec{n} = \frac{\vec{\kappa}}{\kappa} = \frac{\vec{E}}{E} \times \frac{\vec{B}}{B}$
- $\frac{E}{B} = v$
- $v = \frac{1}{\sqrt{\varepsilon\mu}}$
- $u = u_E + u_M$
- $I = \left\langle \vec{S} \cdot \vec{n} \right\rangle$

Magnetostática

- $\vec{B} = \int_\Gamma \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \times \vec{u}_r}{r^2}$
  $\frac{\mu_0}{4\pi} = 10^{-7} H/m$
- $d\vec{F} = Id\vec{\ell} \times \vec{B}$
- $\oint_S \vec{B} \cdot \vec{n}\, dS = 0$
  $\vec{\nabla} \cdot \vec{B} = 0$
- $\oint_\Gamma \vec{H} \cdot d\vec{\ell} = \int_S \vec{J} \cdot \vec{n}\, dS$
  $\vec{\nabla} \times \vec{H} = \vec{J}$
- $\vec{B} = \mu_0(\vec{M} + \vec{H})$
  $\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$
- $\oint_\Gamma \vec{M} \cdot d\vec{\ell} = \int_S \vec{J}_M \cdot \vec{n}\, dS$
  $\vec{J}_M = \vec{\nabla} \times \vec{M}$
  $\vec{J}'_M = \vec{M} \times \vec{n}_{ext}$

Interacção de partículas e campos

- $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Campos variáveis e indução

- $\oint_\Gamma \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{n}\, dS$
  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- $\Phi_i = L_i I_i + M_{ij} I_j$
- $U_M = \left[\frac{1}{2}\right] \sum_i \Phi_i I_i$
- $u_M = \frac{1}{2}\frac{B^2}{\mu}$
  $U_M = \int_v u_M dv$
- $\vec{F}_s = \pm \frac{dU_M}{ds}\vec{u}_s$
- $\oint_\Gamma \vec{H} \cdot d\vec{\ell} = \int_S \vec{J} \cdot \vec{n}\, dS + \frac{d}{dt}\int_S \vec{D} \cdot \vec{n}\, dS$
  $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Óptica

- $n_1 sen\theta_1 = n_2 sen\theta_2$
- $tg\theta_B = \frac{n_2}{n_1}$

  interferência entre fendas
- $dsen\theta_{max} = m\lambda$
- $dsen\theta_{min} = m\lambda + \frac{\lambda}{m'}$ ($m' \leq N$ e par)

  difracção
- $asen\theta_{min} = m\lambda$