Algumas Primitivas

$$\int \frac{dx}{(x^2+b)^{3/2}} = \frac{1}{b} \frac{x}{\sqrt{x^2+b}}$$

$$\int \frac{xdx}{(x^2+b)^{3/2}} = -\frac{1}{\sqrt{x^2+b}}$$

$$\int \frac{xdx}{\sqrt{x^2+b}} = \sqrt{x^2+b}$$

$$\int \frac{dx}{\sqrt{x^2+b}} = \ln\left(x+\sqrt{x^2+b}\right)$$

$$\int \frac{dx}{\sqrt{x^2+b}} = \ln\left(x+\sqrt{x^2+b}\right)$$

Para o cálculo analítico de integrais pode ser consultado o endereço web: http://integrals.wolfram.com

[t] Coordenadas cartesianas (x, y, z)

$$\begin{split} d\vec{\ell} &= dx \ \vec{u}_x \ + \ dy \ \vec{u}_y + \ dz \ \vec{u}_z \\ dS &= dx \ dy \\ dv &= dx \ dy \ dz \end{split} \qquad \qquad \text{Coordenadas polares } (r,\theta) \\ \vec{\nabla} F &= \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \\ \vec{\nabla} \cdot \vec{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ \vec{\nabla} \times \vec{A} &= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \times (A_x, A_y, A_z) \end{split}$$

Coordenadas cilíndricas (r, θ , z)

$$\begin{split} d\vec{\ell} &= dr \; \vec{u}_r + r \; d\theta \; \vec{u}_\theta + dz \; \vec{u}_z \\ dv &= r \; dr \; d\theta \; dz \\ \vec{\nabla} F &= \left(\frac{\partial F}{\partial r}, \frac{1}{r} \frac{\partial F}{\partial \theta}, \frac{\partial F}{\partial z} \right) \\ \vec{\nabla} \cdot \vec{A} &= \frac{1}{r} \frac{\partial (r \; A_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \\ \vec{\nabla} \times \vec{A} &= \left(\frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \vec{u}_\theta + \left(\frac{1}{r} \frac{\partial (r \; A_\theta)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) \vec{u}_z \end{split}$$

Coordenadas esféricas (r, θ, ϕ)

$$\begin{split} d\vec{\ell} &= dr \; \vec{u}_r + r \; d\theta \; \vec{u}_\theta + r \; sen\theta \; d\phi \; \vec{u}_\phi \\ dv &= r^2 \; dr \; sen\theta \; d\theta \; d\phi \\ \vec{\nabla} F &= \left(\frac{\partial F}{\partial r}, \frac{1}{r} \frac{\partial F}{\partial \theta}, \frac{1}{r sen\theta} \frac{\partial F}{\partial \phi} \right) \\ \vec{\nabla} \cdot \vec{A} &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A_r \right) + \frac{1}{r sen\theta} \frac{\partial}{\partial \theta} \left(sen\theta A_\theta \right) + \frac{1}{r sen\theta} \frac{\partial}{\partial \phi} \left(A_\phi \right) \\ \vec{\nabla} \times \vec{A} &= \left[\frac{1}{r sen\theta} \frac{\partial (sen\theta A_\phi)}{\partial \theta} - \frac{\partial (sen\theta A_\theta)}{\partial \phi} \right] \vec{u}_r + \frac{1}{r} \left[\frac{1}{sen\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right] \vec{u}_\theta + \frac{1}{r} \left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \vec{u}_\phi \end{split}$$

Teorema da Divergência

$$\int_{v} \vec{\nabla} \cdot \vec{A} \ dv = \oint_{S} \vec{A} \cdot \vec{n} \ dS$$

Teorema da Stokes

$$\int_S ec{
abla} imes ec{A} \cdot dec{S} = \oint_\Gamma ec{A} \cdot dec{\ell}$$

Identidades vectoriais

Electrostática

$$\bullet \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{u}_r$$

$$\bullet \quad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N.m^2.C^{-2}$$

$$\bullet \quad \oint_{\Gamma} \vec{E} \cdot d\vec{\ell} = 0$$

$$oldsymbol{
abla} imesec{E}=0$$

$$ullet \int_S ec D \cdot ec n \; dS = \int_v
ho_{liv} \; dv$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{liv}$$

$$ullet \oint_S ec{P} \cdot ec{n} \; dS = - \int_v
ho_{pol} dv$$

$$\rho_{pol} = -\vec{\nabla} \cdot \vec{P}$$

$$\sigma_{pol} = \vec{P} \cdot \vec{n}_{ext}$$

$$ullet V_P = \int_P^{Ref} ec{E} \cdot dec{\ell}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\bullet \quad \vec{D} = \vec{P} + \varepsilon_0 \vec{E}$$

$$ec{D} = arepsilon_0 (1 + \chi_E) ec{E} = arepsilon ec{E}$$

$$\bullet$$
 $Q = CV$

$$ullet \ U_E = \left[rac{1}{2}
ight] \sum_i q_i \ V_i$$

$$\bullet \quad u_E = \frac{1}{2} \varepsilon E^2$$

$$U_E = \int_{r} u_E dv$$

$$\bullet \quad \vec{F}_s = \pm \frac{dU_E}{ds} \vec{u}_s$$

Corrente eléctrica estacionária

$$ullet$$
 $ec{J}=Nqec{v}$

$$ullet$$
 $ec{J}=\sigma_cec{E}$

$$\bullet \quad I = \int_S \vec{J} \cdot \vec{n} \ dS$$

$$ullet p = ec{J} \cdot ec{E}$$

$$\oint_S \vec{J} \cdot \vec{n} \ dS = -\frac{d}{dt} \int_v \rho dv$$

$$ec{
abla}\cdotec{J}=-rac{d
ho}{dt}$$

Ondas electromagnéticas

$$ullet$$
 $ec{S}=ec{E} imesec{H}$

$$\bullet \quad \vec{n} = \frac{\vec{\kappa}}{\kappa} = \frac{\vec{E}}{E} \times \frac{\vec{B}}{B}$$

$$\bullet \quad \frac{E}{B} = v$$

$$\bullet \quad u = u_E + u_M$$

$$ullet$$
 $I=\left\langle ec{S}\cdotec{n}
ight
angle$

Magnetostática

$$\vec{B} = \int_{\Gamma} \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \vec{u}_r}{r^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} H/m$$

$$\bullet \quad d\vec{F} = Id\vec{\ell} \times \vec{B}$$

$$\bullet \quad \oint_S \vec{B} \cdot \vec{n} \ dS = 0$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\oint_{\Gamma} \vec{H} \cdot d\vec{\ell} = \int_{S} \vec{J} \cdot \vec{n} \ dS$$

$$ec{
abla} imesec{H}=ec{J}$$

•
$$\vec{B} = \mu_0(\vec{M} + \vec{H})$$

 $\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$

$$igoplus \int_{\Gamma} ec{M} \cdot dec{\ell} = \int_{S} ec{J}_{M} \cdot ec{n} \ dS$$

$$ec{J}_M = ec{
abla} imes ec{M}$$

$${ec J}_M^{\;\prime} = {ec M} imes {ec n}_{ext}$$

Interacção de partículas e campos

$$ullet \ ec F = q \left(ec E + ec v imes ec B
ight)$$

Campos variáveis e indução

$$ec{
abla} imesec{E}=-rac{\partialec{B}}{\partial t}$$

$$\bullet \quad \Phi_i = L_i I_i + M_{ij} I_j$$

$$ullet \ U_M = \left[rac{1}{2}
ight] \sum_i \Phi_i I_i$$

$$\bullet \quad u_M = \frac{1}{2} \frac{B^2}{\mu}$$

$$U_M = \int_v u_M dv$$

$$\bullet \quad \vec{F}_s = \pm \frac{dU_M}{ds} \vec{u}_s$$

$$\bullet \quad \oint_{\Gamma} \vec{H} \cdot d\vec{\ell} = \int_{S} \vec{J} \cdot \vec{n} \ dS + \frac{d}{dt} \int_{S} \vec{D} \cdot \vec{n} \ dS$$

$$ec{
abla} imesec{H}=ec{J}+rac{\partialec{D}}{\partial t}$$

Óptica

•
$$n_1 \operatorname{sen} \theta_1 = n_2 \operatorname{sen} \theta_2$$

•
$$tg\theta_B = \frac{n_2}{n_1}$$

interferência entre fendas

•
$$dsen\theta_{max} = m\lambda$$

$$ullet \ dsen heta_{min} = m \lambda + rac{\lambda}{m'} \ \ (extsf{m}' \leq N ext{ e par})$$
difracção

•
$$asen\theta_{min} = m\lambda$$