1.

a. $2x + x^2y + y^2 = 5x^4 + 1$

$$2 + 2xy + x^2y' + 2yy' = 20x^3$$

$$\frac{dy}{dx} = y' = \frac{20x^3 - 2xy - 2}{(x^2 + 2y)}$$

b. $sen(y)\cos(x) = 1$

$$-\mathrm{sen(x)}sen(y) + \cos(x)\cos(y)\,y' = 0$$

$$\frac{dy}{dx} = y' = \frac{sen(x)sen(y)}{\cos(x)\cos(y)} = tg(x)tg(y)$$

2. $y = \sqrt[3]{1 + tan^2(x)}$

$$\frac{dy}{dx} = y' = \frac{1}{3}(1 + tan^2(x))^{-\frac{2}{3}} = \frac{1}{3\left(\sqrt[3]{1 + tan^2(x)}\right)^2}$$

3. $x^2y^2 - 4 = 0$

$$2xy^2 + x^2 2yy' = 0$$

$$y' = -\frac{xy^2}{x^2y} = -\frac{y}{x}$$

$$\frac{d^2y}{dx^2} = y'' = -\frac{xy' - y}{x^2} = -\frac{x\left(-\frac{y}{x}\right) - y}{x^2} = \frac{2y}{x^2}$$

4. Eq. da reta tg de $y^2 - 2x + 1 = 0$ em (5, 3).

$$2yy' - 2 = 0$$

$$y' = \frac{2}{2y} = \frac{1}{y}$$

Inclinação da reta tg = $y' = \frac{1}{y}$ no ponto (5,3) = $\frac{1}{3}$.

$$y = \frac{1}{3}x + b$$

$$3 = \frac{5}{3} + b$$

$$b = \frac{4}{3}$$

Eq. da reta tg:

$$y = \frac{x+4}{3}$$

5. Aproximação linear de x^{11} em $x_0 = 1$.

$$y' = \frac{d}{dx}x^{11} = 11x^{10}$$

Aproximação linear:
$y = x_0^{11} + y'(x - x_0)$ com $x_0 = 1$

$$y = 1 + 11x^{10}(x-1)$$

$$(1{,}0003)^{11} \approx 1 + 11(1{,}0003)^{10}(1{,}0003 - 1) \approx 1{,}00331$$

6. $y = \sqrt{9x-2}$; $x_0 = 2$; $x = 2{,}01$; $dx = \Delta x = 0{,}01$

$$\Delta y = f(x_0 + \Delta x) - f(x_0) = f(2{,}01) - f(2)$$
$$\Delta y = \sqrt{9(2{,}01) - 2} - \sqrt{9(2) - 2} = \sqrt{16{,}09} - 4 \approx 0{,}01123$$

$$dy = \left(\frac{1}{2}(9x-2)^{-\frac{1}{2}}\right)dx = \left(\frac{1}{2\sqrt{9(2)-2}}\right)dx = \frac{1}{8}dx = \frac{0{,}01}{8} = 0{,}00125$$

A seguinte diferença pequena indica que dy pode ser usado para aproximar o valor de Δy

$$e = \Delta y - dy \approx 0{,}01123 - 0{,}00125 \approx 0{,}00998$$

7.

a. $\sqrt{25{,}04}$

$$f(x) = \sqrt{x};\ x_0 = 25; x = 25{,}04$$

$$f'(x_0) = \frac{1}{2}x_0^{-\frac{1}{2}} = \frac{1}{2\sqrt{x_0}} = \frac{1}{2\sqrt{25}} = \frac{1}{10} = 0{,}1$$

$$f(x) \approx f(x_0) + f'(x_0).(x - x_0)$$

$$\sqrt{25{,}04} \approx \sqrt{25} + 0{,}1(25{,}04 - 25)$$

$$\sqrt{25{,}04} \approx 5{,}004$$

b. $(2{,}98)^4$

$$f(x) = x^4;\ x_0 = 2{,}97; x = 2{,}98$$

$$f'(x_0) = 4x_0^3 = 4(2{,}97)^3 = 104{,}7923$$

$$f(x) \approx f(x_0) + f'(x_0).(x - x_0)$$

$$(2{,}98)^4 \approx (2{,}97)^4 + 104{,}7923.(2{,}98 - 2{,}97)$$

$$(2{,}98)^4 \approx 78{,}8562$$

c. $sen(31°)$

$$f(x) = sen(x);\ x_0 = 30°; x = 31°$$

$$f'(x_0) = \cos(30°) = \frac{\sqrt{3}}{2}$$

$$f(x) \approx f(x_0) + f'(x_0).(x - x_0)$$

$$sen(31°) \approx sen(30°) + \frac{\sqrt{3}}{2}.(31° - 30°)$$

$$sen(31°) \approx \frac{1}{2} + \frac{\pi\sqrt{3}}{360}$$