Solutions to Problem Set 2 (6.042J)

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Problem 1

$$F_n = \frac{p^n - q^n}{\sqrt{5}} \tag{1}$$

$$F_{n+1} = \frac{p^{n+1} - q^{n+1}}{\sqrt{5}} \tag{2}$$

By the Fibonacci definition, $F_{n+1} = F_n + F_{n-1}$ for $n > 1$. Substitute this into (2), we have:

$$F_n = \frac{p^{n+1} - q^{n+1}}{\sqrt{5}} - F_{n-1} \ \forall n > 1 \tag{3}$$

Now we're ready to prove equation (1) is true for all n.
For $n = 0$, (1) gives $F_0 = \dfrac{p^0 - q^0}{\sqrt{5}} = 0$
For $n = 1$, (1) gives $F_1 = \dfrac{p^1 - q^1}{\sqrt{5}} = \dfrac{\sqrt{5}}{\sqrt{5}} = 1$
For $n = 2$, (1) gives $F_2 = \dfrac{p^2 + q^2}{\sqrt{5}} = \dfrac{p + 1 - q - 1}{\sqrt{5}} = F_1 = 1$ (which is true since by Fibonacci definition $F_2 = F_1 + F_0 = 1 + 0 = 1$).
For n =2, (3) gives $F_2 = \dfrac{p^3 + q^3}{\sqrt{5}} - F_1 = F_3 - F_1 \Rightarrow F_3 = F_2 + F_1$. Since F_2 and F_1 are true, F_3 is also true and so on for $F_4, F_5, ...$ We have used the strong induction to prove (1) in which $\forall n \in \mathbb{N}$, $F_0, F_1, ..., F(n)$ imply $F(n+1)$, then $F(m)$ is true $\forall m \in \mathbb{N}$.

Problem 3

a)

Since h is surjective, it has [≥ 1 arrows in] property. This means that every point in codomain C has at least one arrow pointing to it. h maps points in A into points in C, $h : A \to C$, which is the the same as going from $A \to B \to C$ by the composition $f \circ g$. If h is surjective, then f must be surjective.