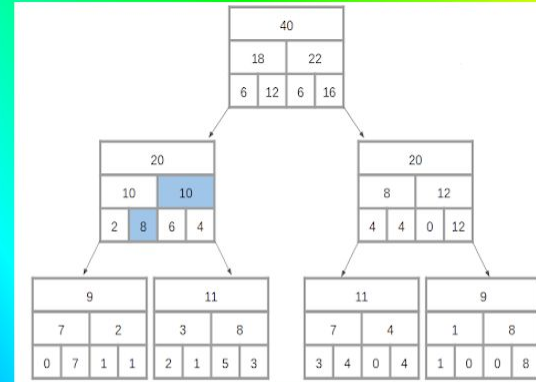


*Segment Tree is
my passion*



Segtree 2D - Dinâmica

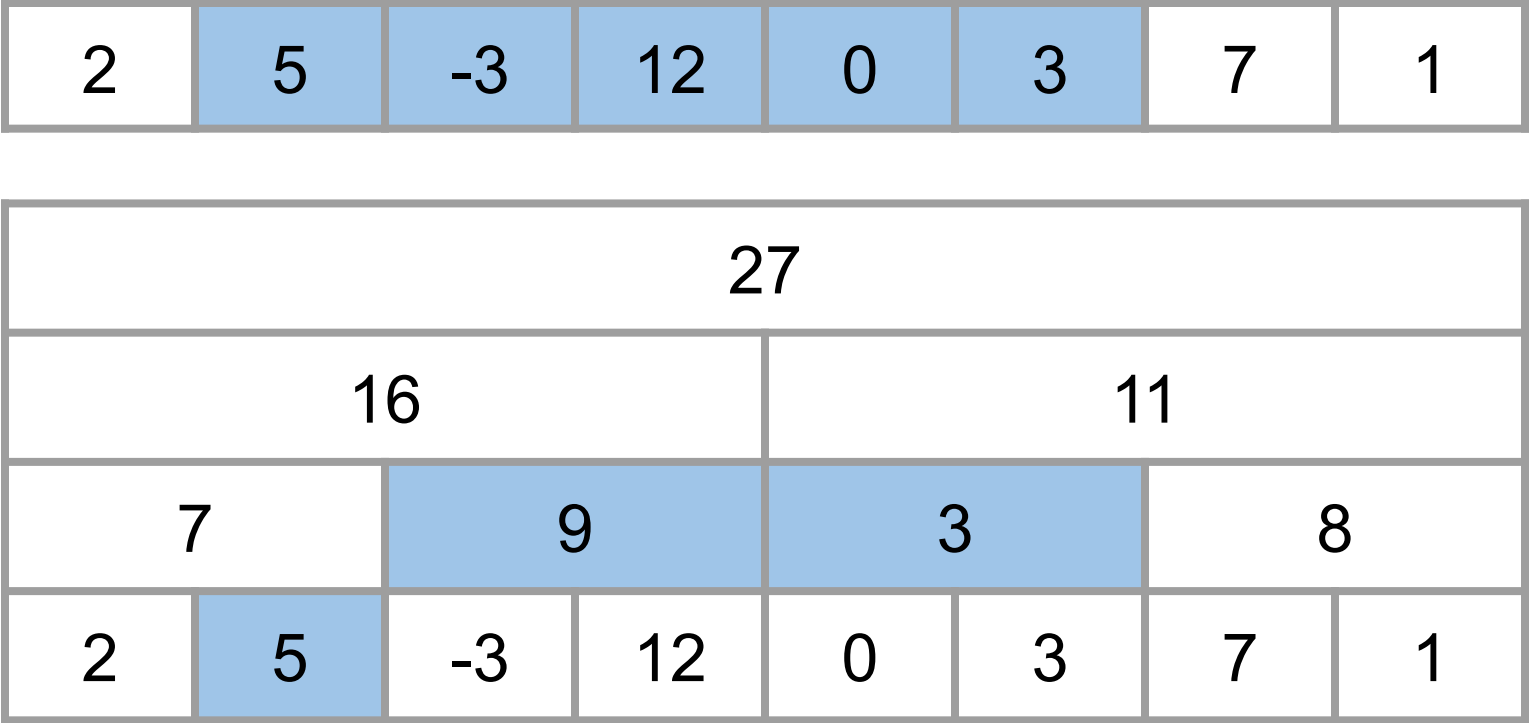
Segment Tree

2	5	-3	12	0	3	7	1
---	---	----	----	---	---	---	---

Segment Tree

2	5	-3	12	0	3	7	1
27							
16				11			
7		9		3		8	
2	5	-3	12	0	3	7	1

Segment Tree



Segment Tree

- Memória: $4N$
- Update/Query: $O(\log N)$
- Build: $O(N)$

Segment Tree 2D

Query em range:

X: (a, b)

Y: (c, d)

#	1	2	3	4
1	0	7	1	1
2	2	1	5	3
3	3	4	0	4
4	1	0	0	8

$X = (1, 2); Y = (2, 4)$

#	1	2	3	4
1	0	7	1	1
2	2	1	5	3
3	3	4	0	4
4	1	0	0	8

#	1	2	3	4
1	0	7	1	1
2	2	1	5	3
3	3	4	0	4
4	1	0	0	8

#	1	2	3	4
1	0	7	1	1

9			
7		2	
0	7	1	1

#	1	2	3	4
2	2	1	5	3

11			
3		8	
2	1	5	3

#	1	2	3	4
1	0	7	1	1

9			
7		2	
0	7	1	1

#	1	2	3	4
2	2	1	5	3

11			
3		8	
2	1	5	3

#	1	2	3	4
1	0	7	1	1
2	2	1	5	3



#	1	2	3	4
1	2	8	6	4
2				

#	1	2	3	4
1	2	8	6	4
2				

20			
10		10	
2	8	6	4

20			
10		10	
2	8	6	4



9			
7		2	
0	7	1	1

11			
3		8	
2	1	5	3

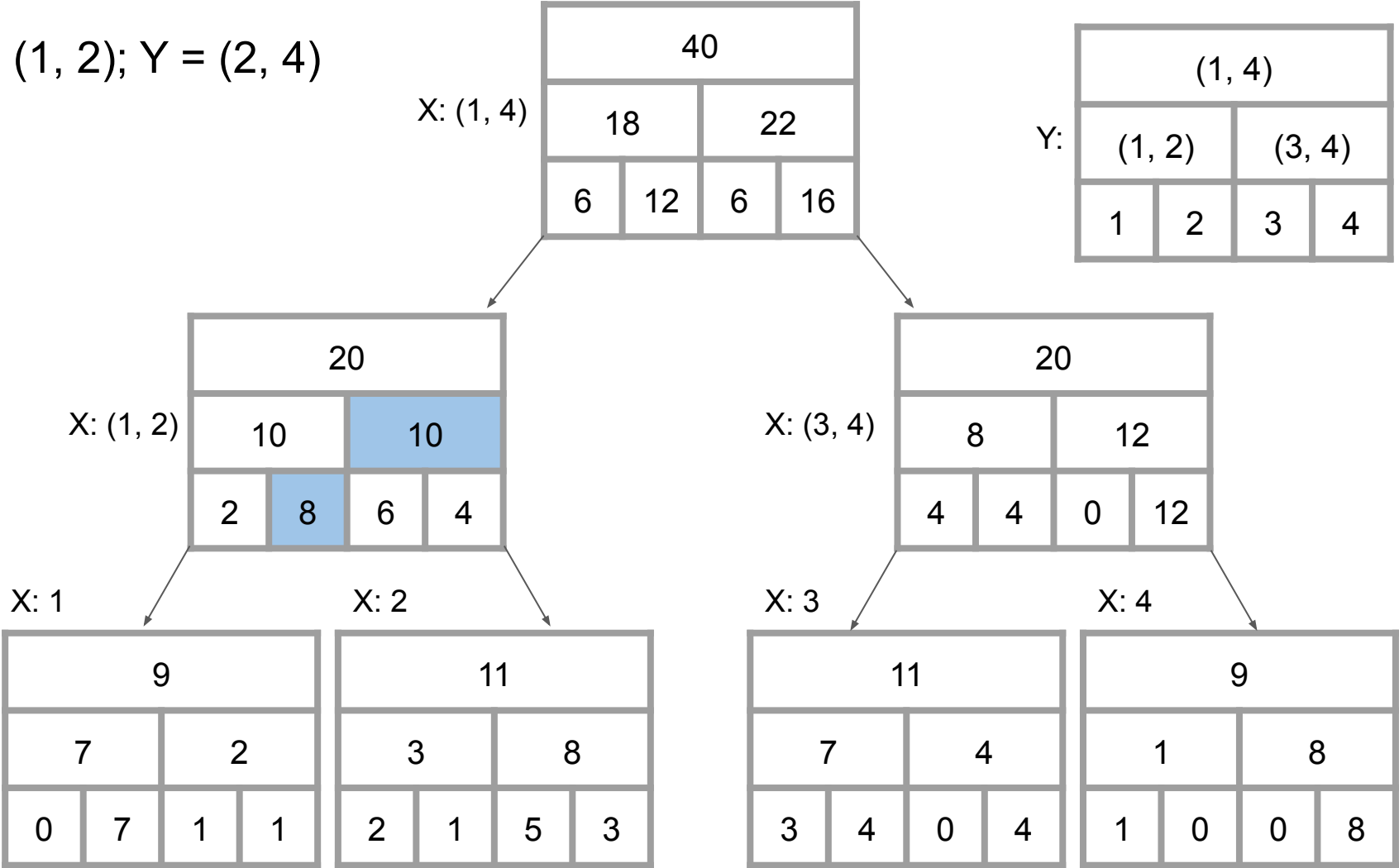
20			
10		10	
2	8	6	4



9			
7		2	
0	7	1	1

11			
3		8	
2	1	5	3

$X = (1, 2); Y = (2, 4)$



$X = (2, 4); Y = (1, 3)$

#	1	2	3	4
1	0	7	1	1
2	2	1	5	3
3	3	4	0	4
4	1	0	0	8

$X = (2, 4); Y = (1, 3)$

X: (1, 4)

40			
18		22	
6	12	6	16

Y:

(1, 4)			
(1, 2)		(3, 4)	
1	2	3	4

X: (1, 2)

20			
10		10	
2	8	6	4

X: (3, 4)

20			
8		12	
4	4	0	12

X: 1

9			
7		2	
0	7	1	1

X: 2

11			
3		8	
2	1	5	3

X: 3

11			
7		4	
3	4	0	4

X: 4

9			
1		8	
1	0	0	8

Como montar?

Como montar?

Segtree:

$$\text{Seg}[\text{no}] = \text{Seg}[2*\text{no}] + \text{Seg}[2*\text{no}+1]$$

Como montar?

Segtree 2D:

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[\text{noX}][2*\text{noY}] + \text{Seg}[\text{noX}][2*\text{noY}+1] \text{ ?}$$

Como montar?

Segtree 2D:

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[\text{noX}][2*\text{noY}] + \text{Seg}[\text{noX}][2*\text{noY}+1] \text{ ?}$$

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[2*\text{noX}][\text{noY}] + \text{Seg}[2*\text{noX}+1][\text{noY}] ??$$

Como montar?

Segtree 2D:

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[\text{noX}][2*\text{noY}] + \text{Seg}[\text{noX}][2*\text{noY}+1] \quad ?$$

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[2*\text{noX}][\text{noY}] + \text{Seg}[2*\text{noX}+1][\text{noY}] \quad ??$$

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[2*\text{noX}][2*\text{noY}] + \text{Seg}[2*\text{noX}+1][2*\text{noY}+1] \quad ???$$

Como montar?

20			
10		10	
2	8	6	4



9			
7		2	
0	7	1	1

11			
3		8	
2	1	5	3

Como montar?

20			
10		10	
2	8	6	4



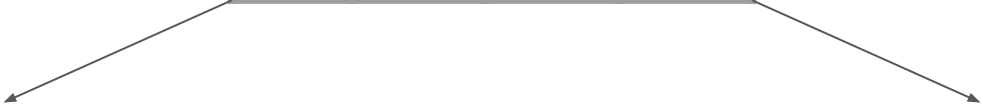
9			
7		2	
0	7	1	1

11			
3		8	
2	1	5	3

Como montar?

20			
10		10	
2	8	6	4

0	7
2	1



9			
7		2	
0	7	1	1

11			
3		8	
2	1	5	3

Como montar?

Segtree 2D:

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[2*\text{noX}][\text{noY}] + \text{Seg}[2*\text{noX}+1][\text{noY}] \quad ?$$

$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[\text{noX}][2*\text{noY}] + \text{Seg}[\text{noX}][2*\text{noY}+1] \quad ??$$

~~$$\text{Seg}[\text{noX}][\text{noY}] = \text{Seg}[2*\text{noX}][2*\text{noY}] + \text{Seg}[2*\text{noX}+1][2*\text{noY}+1] \quad ???$$~~

Segment Tree 2D

- Memória: $4N * 4M = 16MN$:
 - Seg externa(em X) - $4N$ nós
 - Cada nó é uma Seg(em Y) de tamanho $4M$
- Update/Query: $O(\log N * \log M)$
- Build: $O(NM)$

Build vs Update

- Build: $O(NM)$
- Update: $O(NM * \log N * \log M)$

Código

Memória

- 4N é muito.
- 16NM é demais.

Numeração dos nós

Filho esquerdo: $2i$ / Filho direito: $2i + 1$

1															
2								3							
4				5				6				7			
8		9		10		11		12		13		14		15	
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

N = 9:

Nós utilizados: 17

Maior índice: 31

1															
2								3							
4				5				6				7			
8		9		10		11		12		13		14		15	
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

Memória

$$2^x < N \leq 2^{x+1} :$$






$$1 + 2 + 4 + \dots + 2^x + 2^{x+1} = 2 \cdot 2^{x+1} - 1 \approx 2 \cdot 2^{x+1}$$

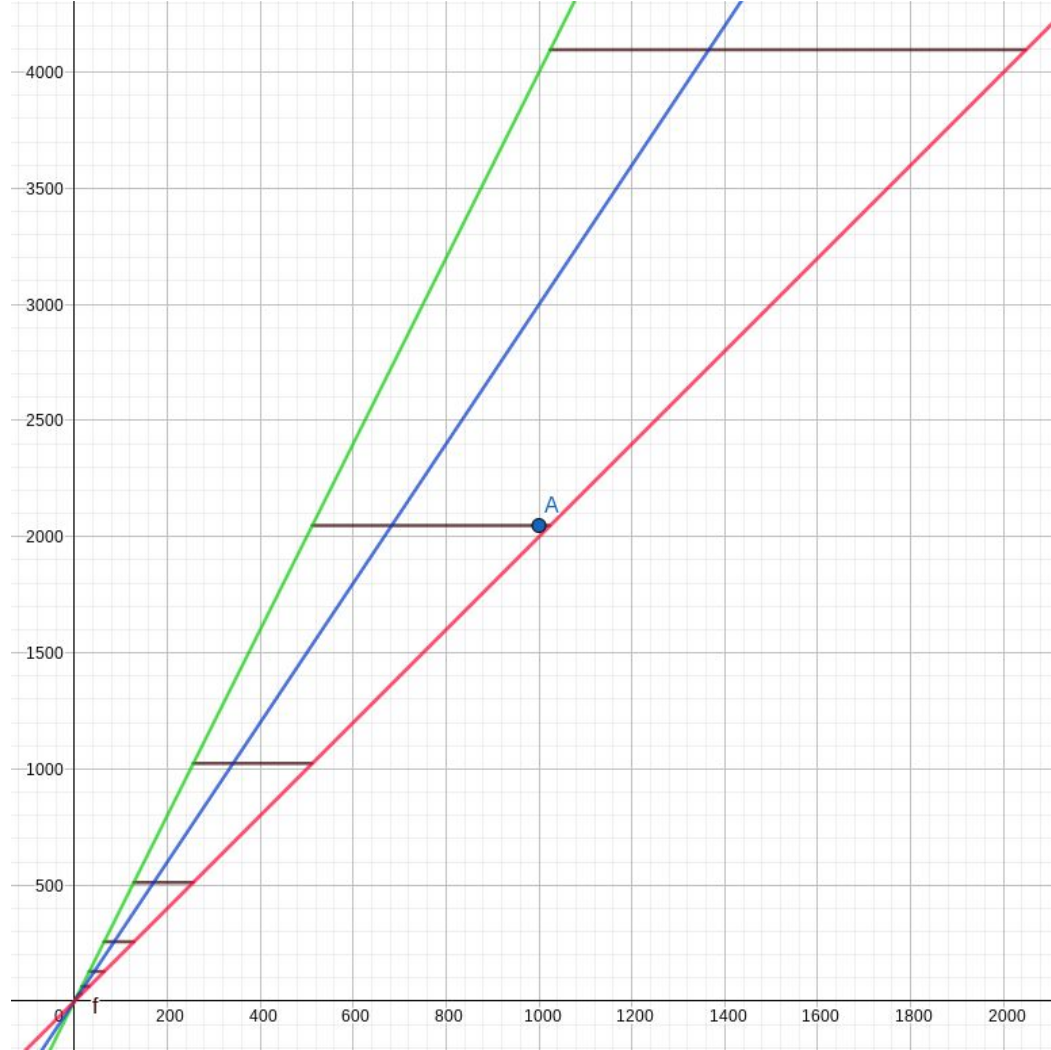
$$\text{Como: } x + 1 = \lceil \log_2 N \rceil$$

$$\text{Nós necessários: } 2 \cdot 2^{\lceil \log_2 N \rceil}$$

$$\text{Para } N \text{ próximo de } 2^x: 2 \cdot 2^{\lceil \log_2 N \rceil} \approx 4N$$

$$\text{Para } N \text{ próximo de } 2^{x+1}: 2 \cdot 2^{\lceil \log_2 N \rceil} \approx 2N$$

	$f : y = 2^{\lceil \log_2(x) \rceil + 1}$
	$g : y = 4x$
	$h : y = 3x$
	$p : y = 2x$
	$A = \text{Ponto}(f)$ $\rightarrow (1000, 2048)$



Em uma Seg 2D 1000 x 1000

$$16NM = 16 \cdot 1000 \cdot 1000$$

$$16NM = 16000000 = 1.6 \cdot 10^7$$

$$2 \cdot 2^{\lceil \log_2 N \rceil} \cdot 2 \cdot 2^{\lceil \log_2 M \rceil} = 2 \cdot 1024 \cdot 2 \cdot 1024$$

$$2 \cdot 2^{\lceil \log_2 N \rceil} \cdot 2 \cdot 2^{\lceil \log_2 M \rceil} = 4194304 = 4 \cdot 10^6$$

Segtree Dinâmica

Criamos os nós que vamos utilizando

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

*							
*				*			
*		*		*		*	
*	*	*	*	*	*	*	*

$S[5] = 4$

0	0	0	0	4	0	0	0
---	---	---	---	---	---	---	---

4							
*				4			
*		*		4	*		
*	*	*	*	4	*	*	*

$S[3] = -1$

0	0	-1	0	4	0	0	0
---	---	----	---	---	---	---	---

3							
-1				4			
*		-1		4		*	
*	*	-1	*	4	*	*	*

- Útil para N muito grande.
- Como a cada update criamos no máximo $\log N$ nós, em Q updates temos menos de $Q \log N$ nós.
- Ex:

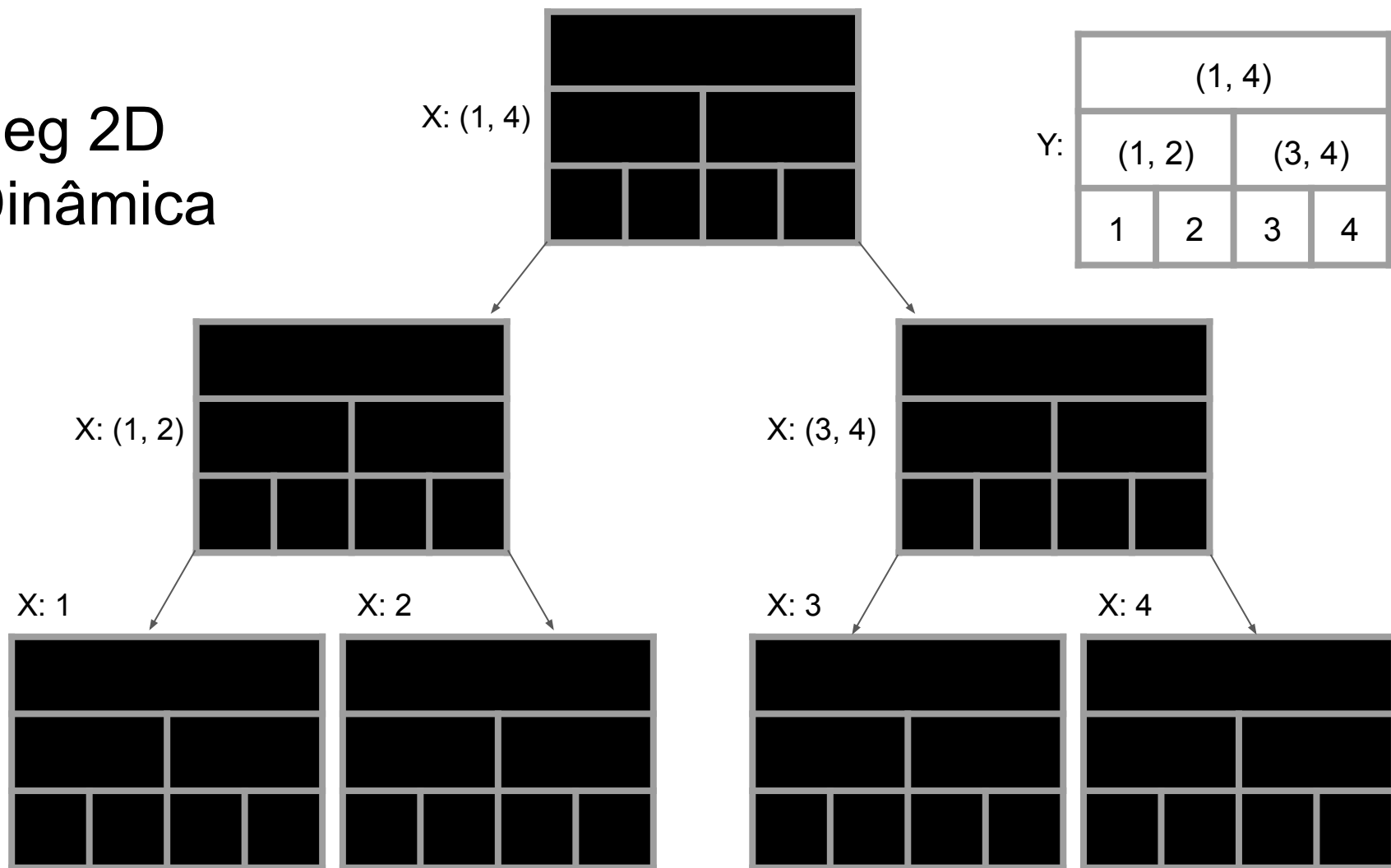
$$N = 10^9, Q = 10^5$$

$$\log N \approx 30$$

$$\text{Seg normal: } 4N = 4 \cdot 10^9$$

$$\text{Seg dinâmica: } Q \log N = 3 \cdot 10^6$$

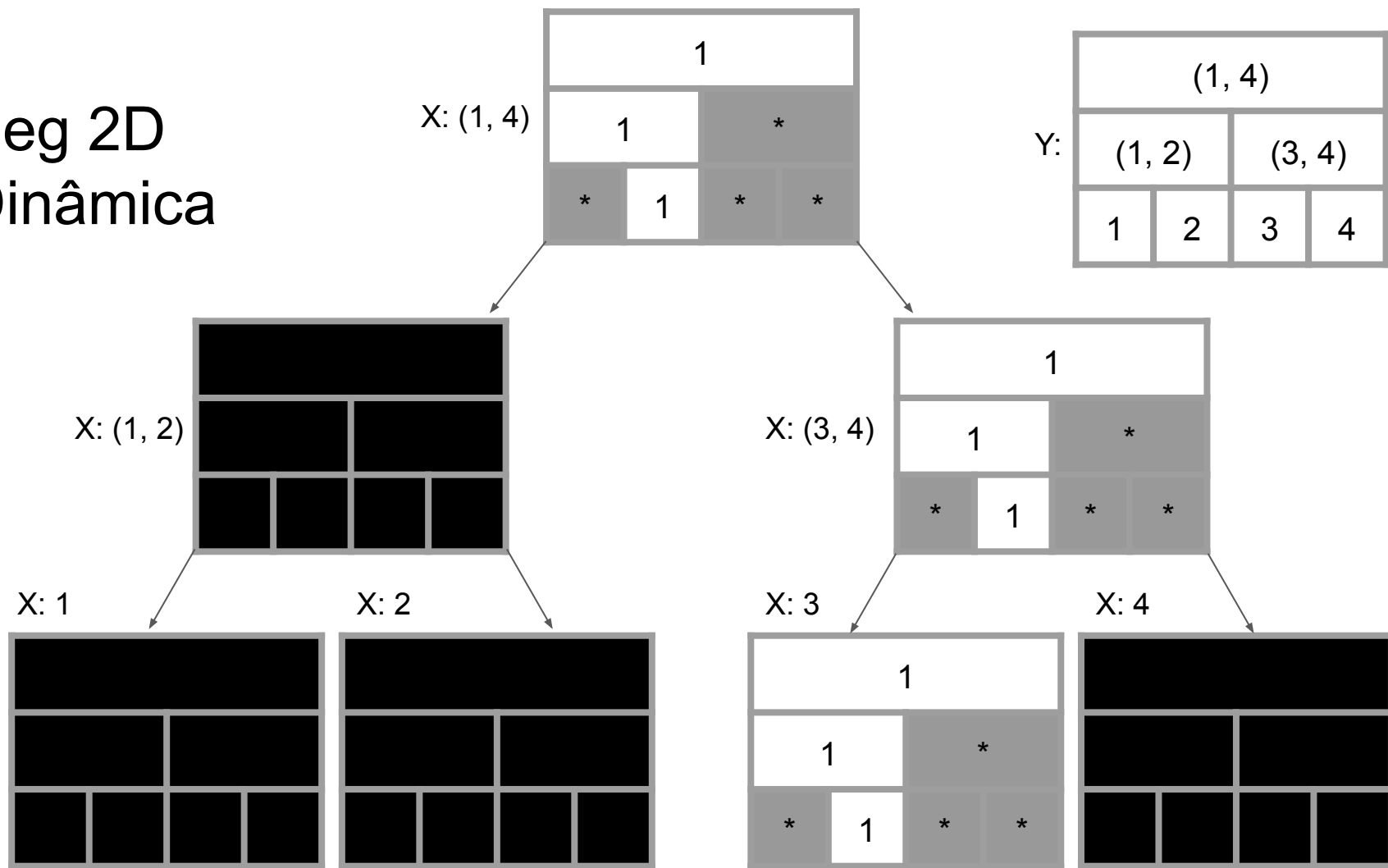
Seg 2D Dinâmica



$$S[3][2] = 1$$

#	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	0	1	0	0
4	0	0	0	0

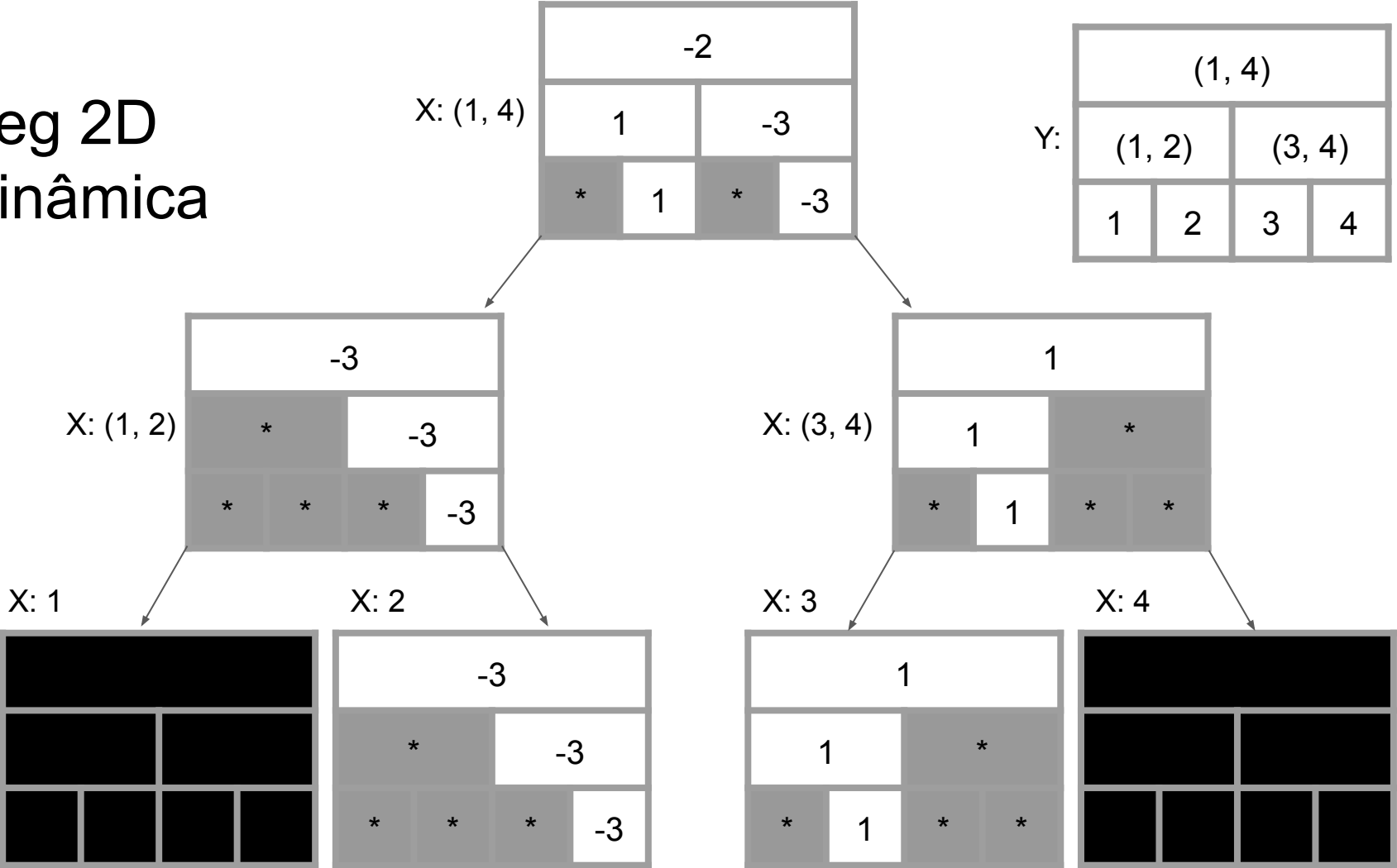
Seg 2D Dinâmica



$$S[2][4] = -3$$

#	1	2	3	4
1	0	0	0	0
2	0	0	0	-3
3	0	1	0	0
4	0	0	0	0

Seg 2D Dinâmica



- A cada update criamos $\log N$ Seg's, e em cada uma delas $\log M$ nós.
- $Q \log N \log M$.

Problemas:

- <https://vjudge.net/contest/404319>
- <https://neps.academy/problem/382>
- <https://dmoj.ca/problem/ioi13p6io>