



Automâto com Pilha

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Automato com Pilha

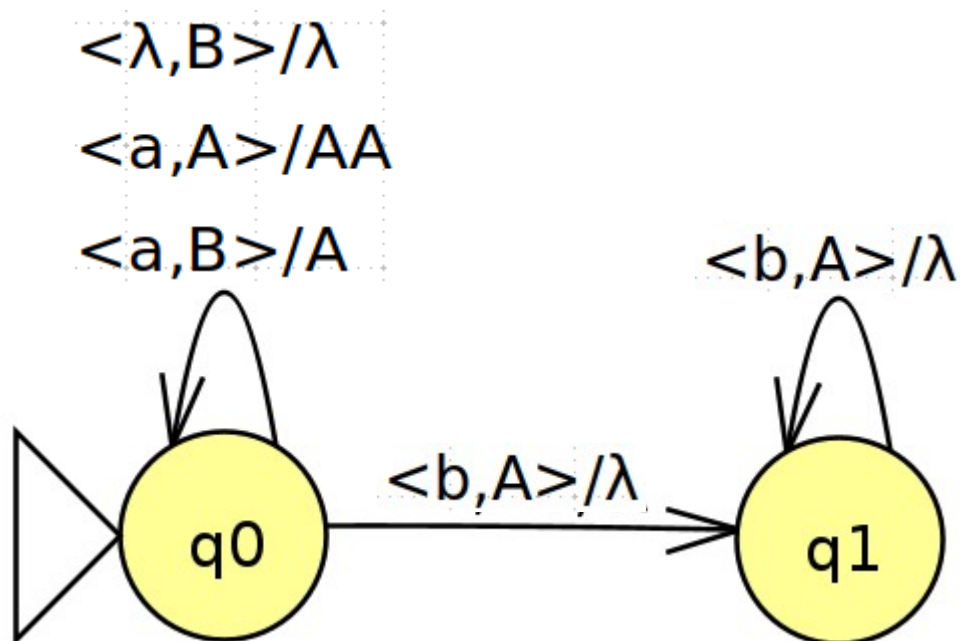
- Resolução de Exercícios;
 - Principais características;
 - Completar o automato;
 - Descrever o automato;
- Ferramenta JFLAP;
 - <http://www.jflap.org/>
 - Exemplos na ferramenta;

Automato com Pilha

- Dependência entre caracteres na cadeia.
- Exemplo: $\{a^n b^n \mid n \geq 0\}$

Automato com Pilha

- Dependência entre caracteres na cadeia;
- Exemplo: $\{a^n b^n \mid n \geq 0\}$
- Utiliza-se do mesmo elemento da pilha para esses caracteres.



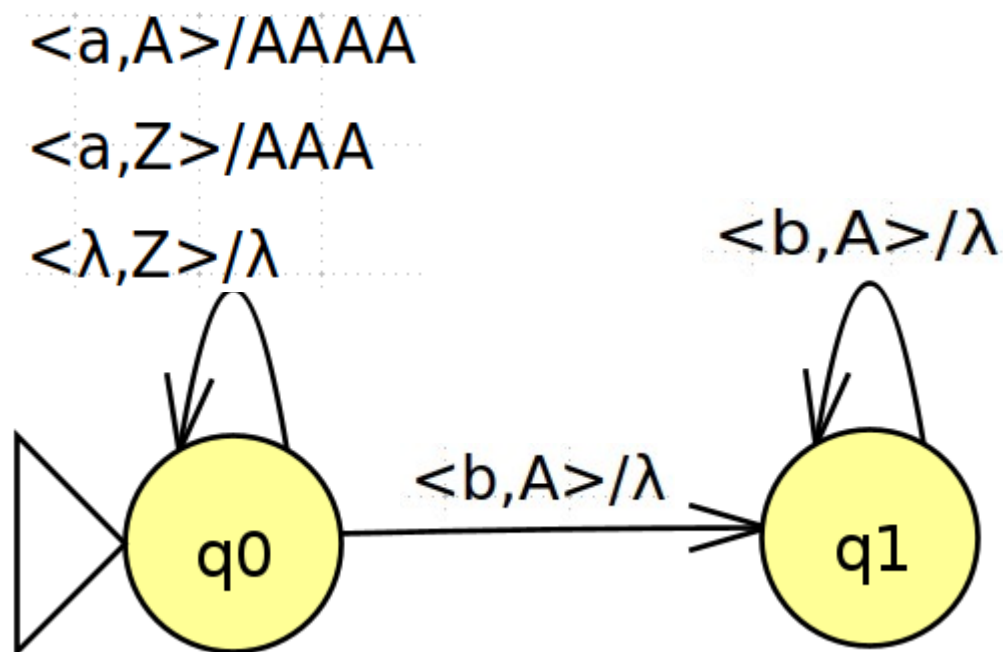
Automato com Pilha

- Dependência entre caracteres na cadeia.

- Exemplo: $\{a^n b^{3n} \mid n \geq 0\}$

Automôto com Pilha

- Dependência entre caracteres na cadeia;
- Exemplo: $\{a^n b^{3n} \mid n \geq 0\}$
- Utiliza-se do mesmo elemento da pilha para esses caracteres.



Automato com Pilha

- Não dependência entre caracteres na cadeia.
- Exemplo:

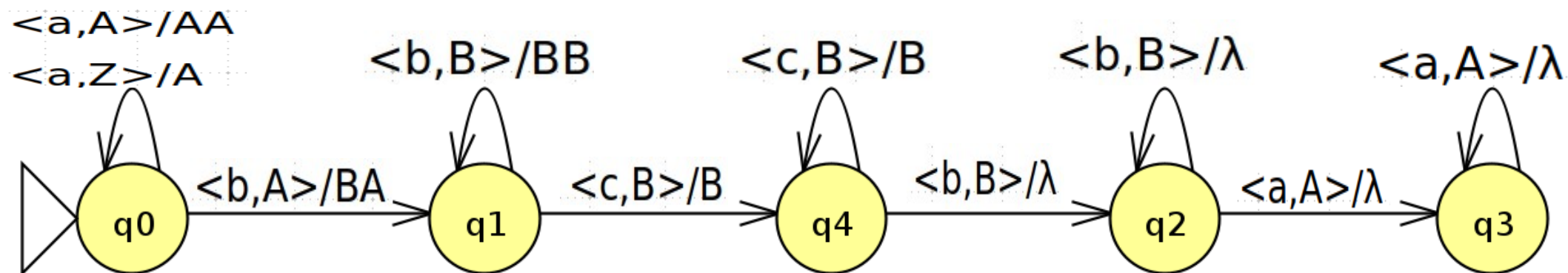
$$\{a^n b^m c^p b^m a^n \mid n > 0, m > 0, p > 0\}$$

Automato com Pilha

- Não dependência entre caracteres na cadeia;
- Exemplo:

$$\{a^n b^m c^p b^m a^n \mid n > 0, m > 0, p > 0\}$$

- Não se utiliza do mesmo elemento da pilha para esses caracteres.

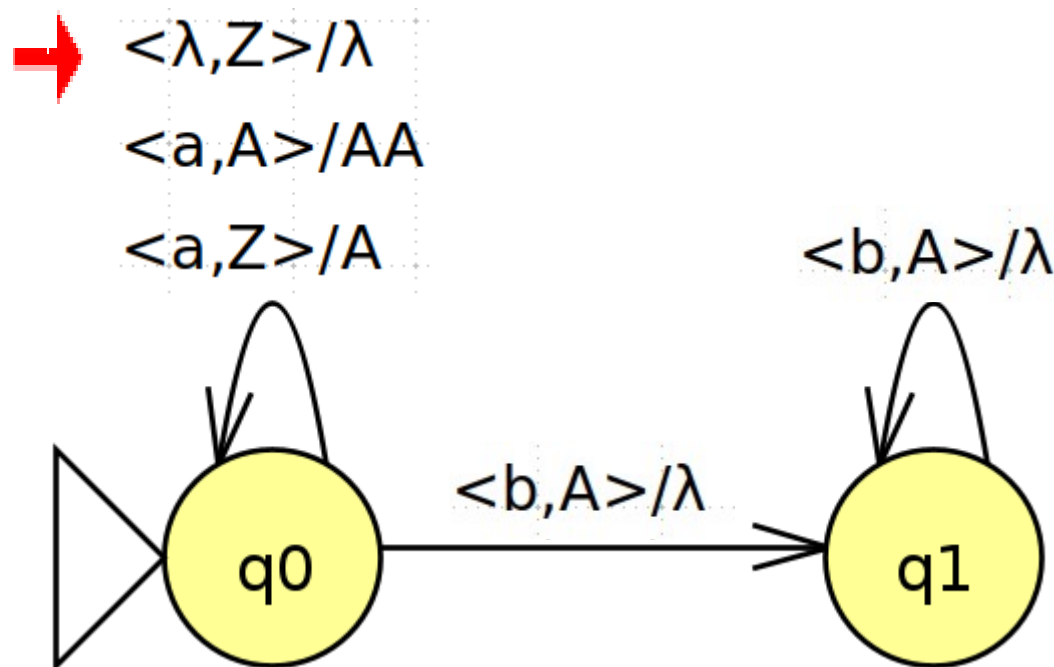


Automato com Pilha

- Aceitação da palavra λ .
- Exemplo: $\{a^n b^n \mid n \geq 0\}$

Automato com Pilha

- Aceitação da palavra λ ;
- Exemplo: $\{a^n b^n \mid n \geq 0\}$
- Realiza a transição retirando a Base e inserindo λ .

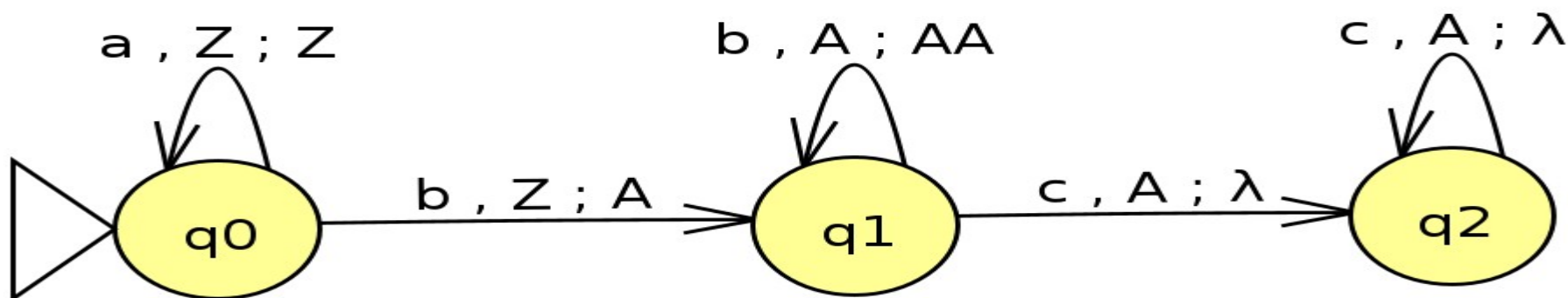


Automato com Pilha

- Caractere que não necessita de “armazenamento”;
- Exemplo: $\{a^n b^m c^m \mid n \geq 0, m > 0\}$

Automato com Pilha

- Caractere que não necessita de “armazenamento”;
- Exemplo: $\{a^n b^m c^m \mid n \geq 0, m > 0\}$
- Retira-se e coloca o mesmo elemento na pilha.

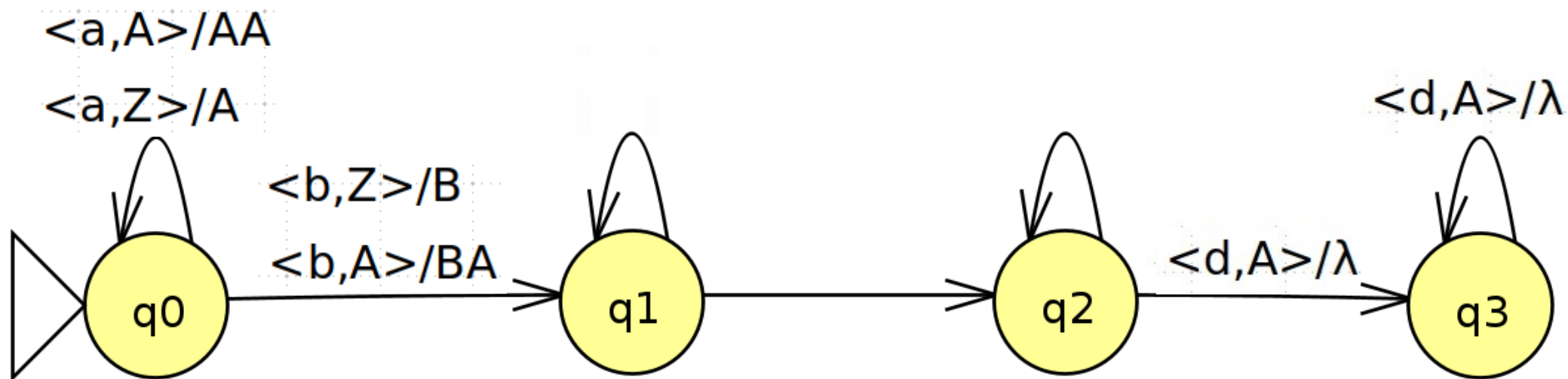


Automôto com Pilha

- Exercícios:
- Complete o automôto para a linguagem

$$\{a^n b^m c^m d^n \mid n \geq 0, m > 0\}$$

- Base: Z

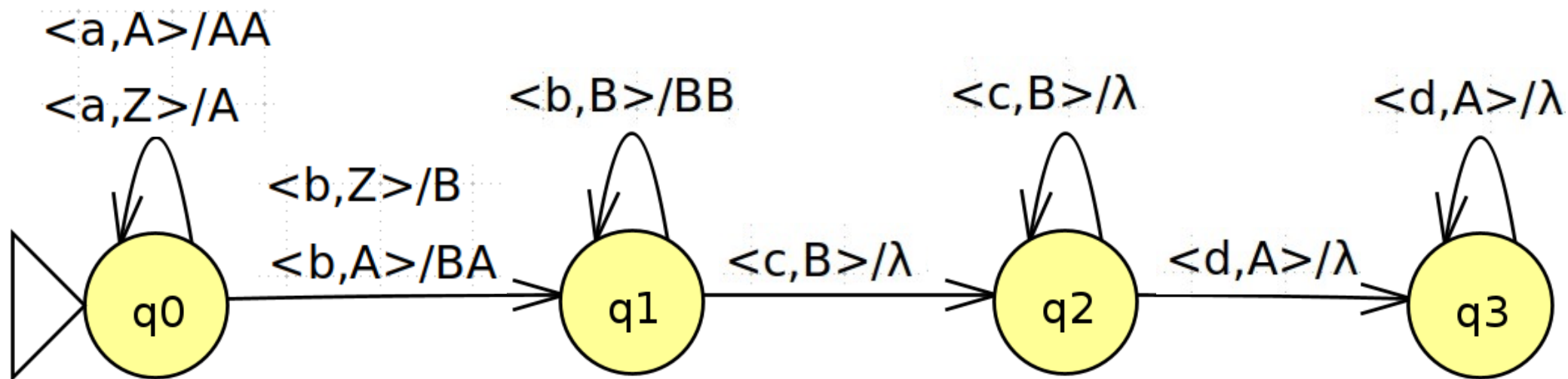


Automato com Pilha

- Exercícios:
- Complete o automato para a linguagem

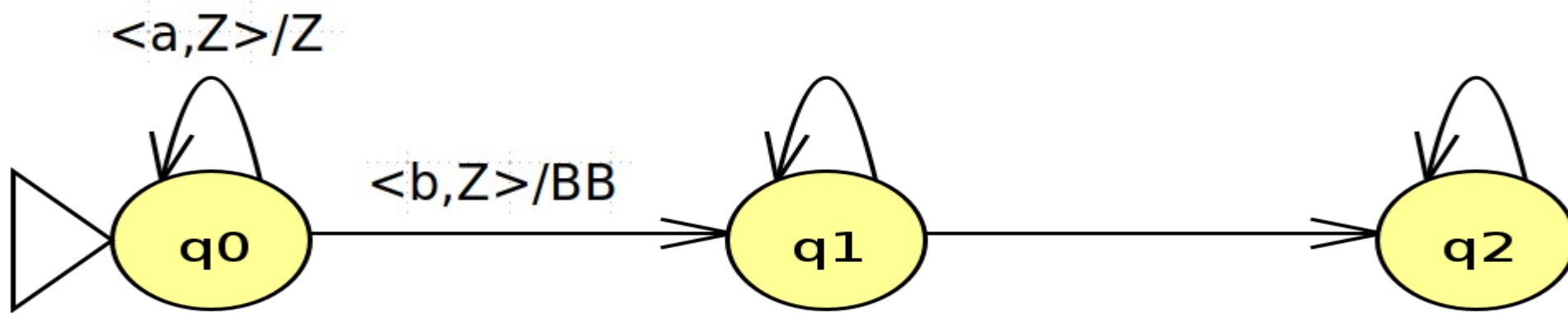
$$\{a^n b^m c^m d^n \mid n \geq 0, m > 0\}$$

- Base: Z



Automato com Pilha

- Exercícios:
- Complete o automato para a linguagem $\{a^n b^m c^{2m} \mid n \geq 0, m > 0\}$
- Base: Z

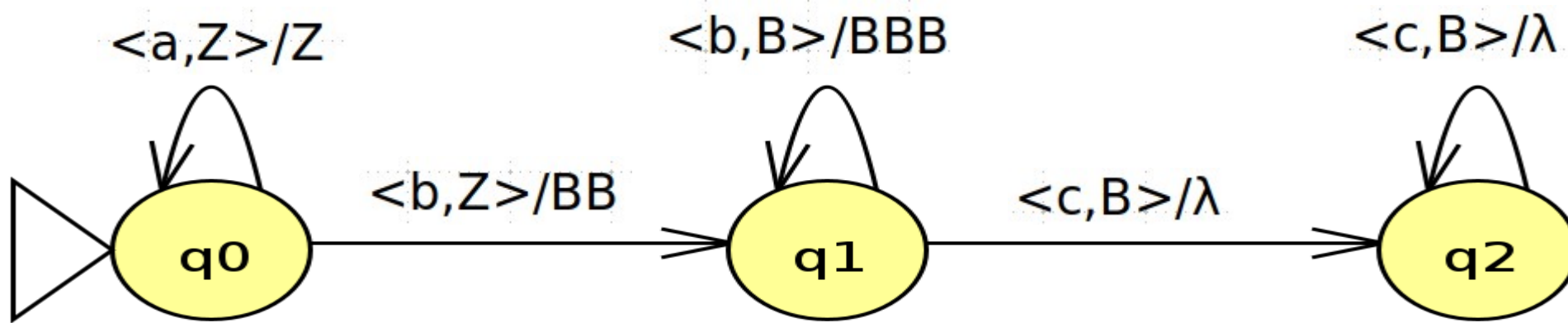


Automato com Pilha

- Exercícios:
- Complete o automato para a linguagem

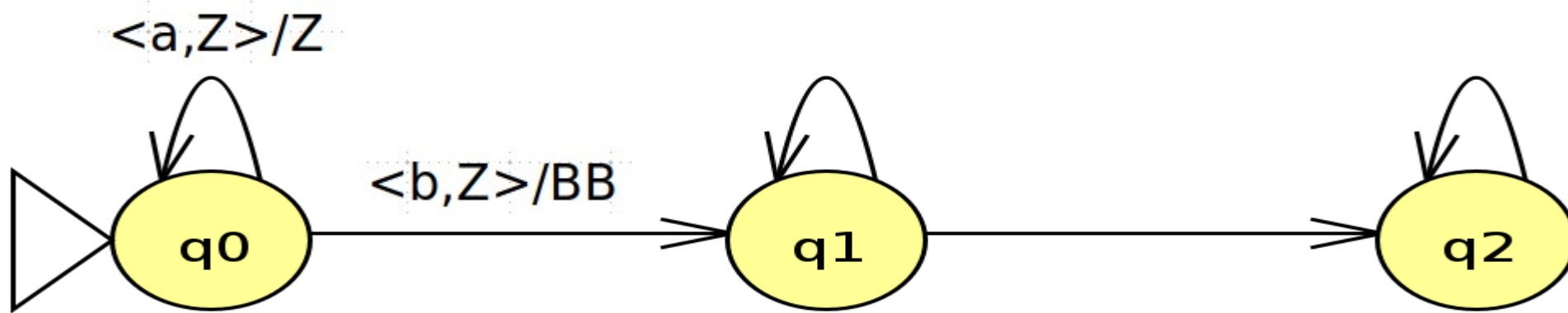
$$\{a^n b^m c^{2m} \mid n \geq 0, m > 0\}$$

- Base: Z



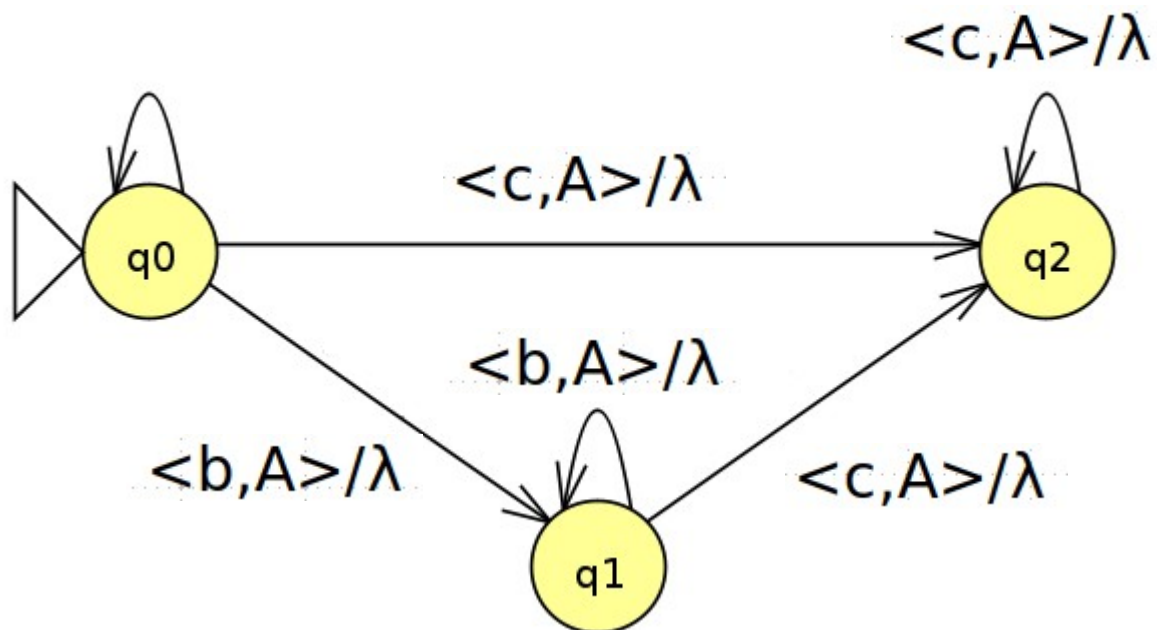
Automato com Pilha

- Exercícios:
- Complete o automato para a linguagem $\{a^n b^m c^{2m} \mid n \geq 0, m > 0\}$
- Base: Z



Automato com Pilha

- Exercícios:
- Complete o automato para a linguagem $\{a^i b^j c^k \mid i = j + k, j \geq 0, k \geq 0\}$
- Base: Z



Automato com Pilha

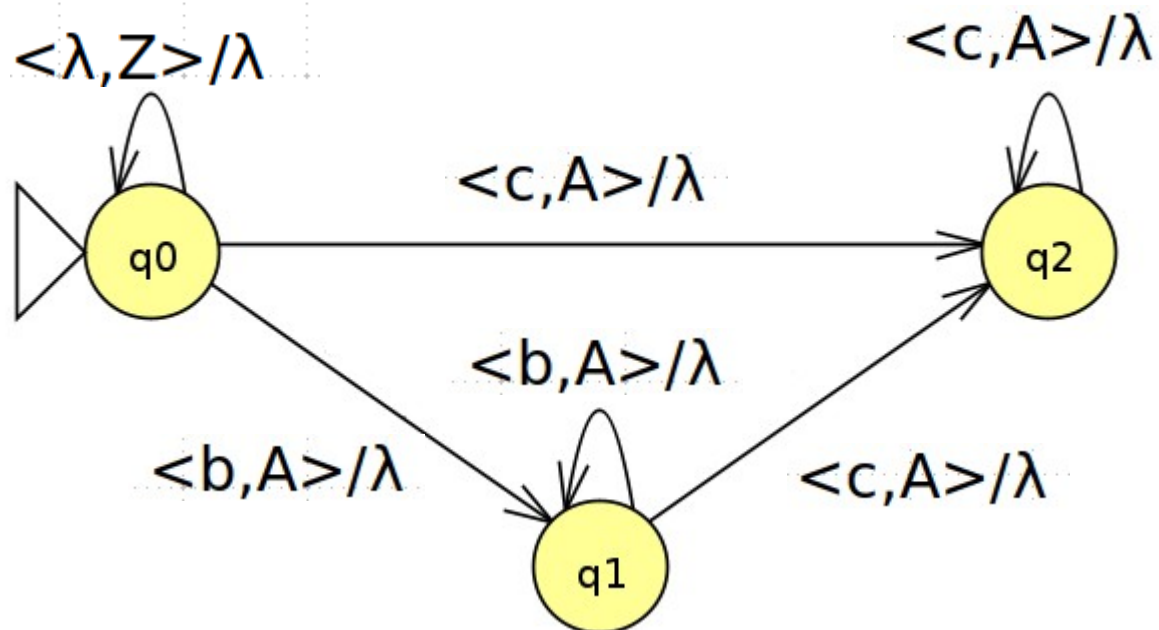
- Exercícios:
- Complete o automato para a linguagem

$$\{a^i b^j c^k \mid i = j + k, j \geq 0, k \geq 0\}$$

- Base: Z $\langle a, A \rangle / AA$

$\langle a, Z \rangle / A$

$\langle \lambda, Z \rangle / \lambda$



Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

$$\{a^i b^j c^k \mid k = i + j, j > 0, i > 0\}$$

- Base: \mathbb{Z}

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

$$\{a^i b^j c^k \mid k = i + j, j > 0, i > 0\}$$

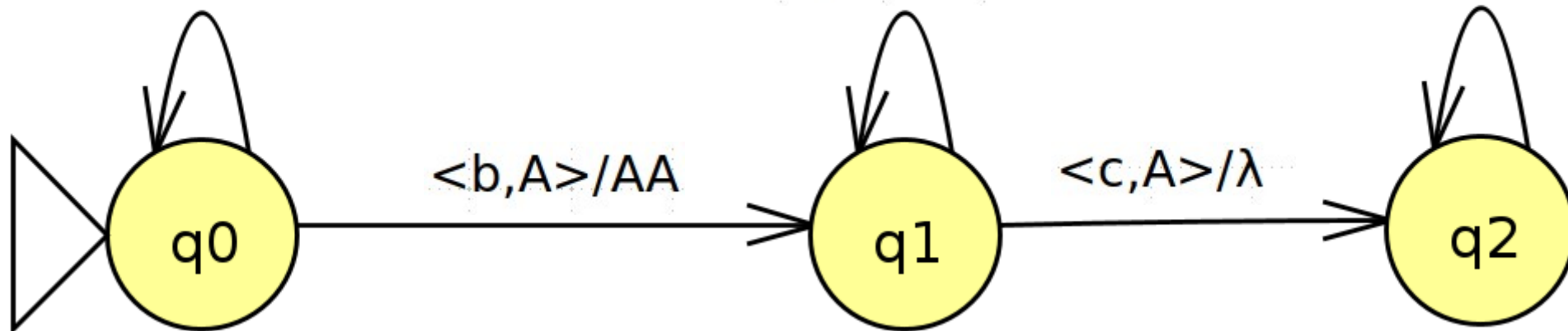
- Base: Z

$\langle a, A \rangle / AA$

$\langle a, Z \rangle / A$

$\langle b, A \rangle / AA$

$\langle c, A \rangle / \lambda$

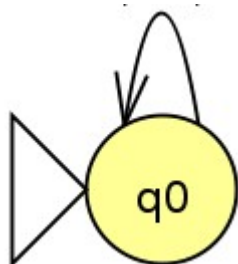


Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



$\langle a, B \rangle / \lambda$

$\langle b, A \rangle / \lambda$

$\langle b, B \rangle / BB$

$\langle a, A \rangle / AA$

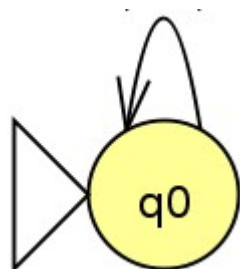
$\langle b, Z \rangle / BZ$

$\langle a, Z \rangle / AZ$

$\langle \lambda, Z \rangle / \lambda$

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem $\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$
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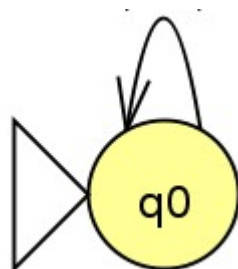
$\langle b, Z \rangle / BZ$

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Automato com Pilha

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- Base: Z



$\langle b, B \rangle / BB$

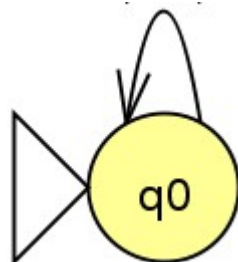
$\langle a, A \rangle / AA$

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Σ



$\langle a, B \rangle / \lambda$

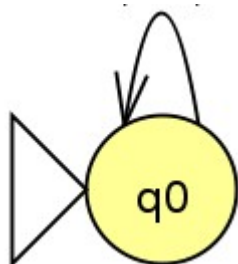
$\langle b, A \rangle / \lambda$

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



$\langle a, B \rangle / \lambda$

$\langle b, A \rangle / \lambda$

$\langle b, B \rangle / BB$

$\langle a, A \rangle / AA$

$\langle b, Z \rangle / BZ$

$\langle a, Z \rangle / AZ$

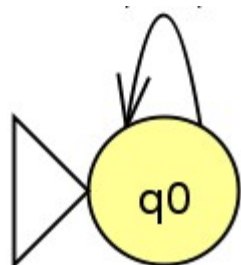
$\langle \lambda, Z \rangle / \lambda$

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

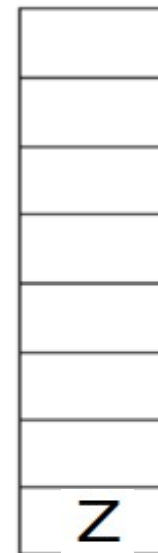
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



$\langle a, B \rangle / \lambda$
 $\langle b, A \rangle / \lambda$
 $\langle b, B \rangle / BB$
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 $\langle a, Z \rangle / AZ$
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aaaabbabbb
↑



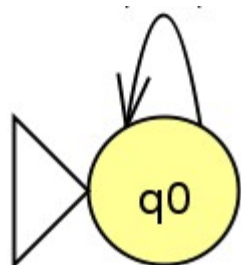
Pilha

Automato com Pilha

- Exercícios:
- Descreva o automato para a linguagem

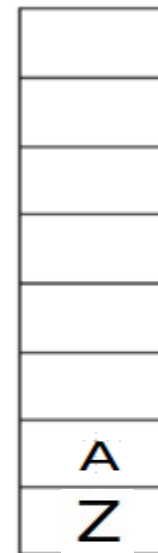
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

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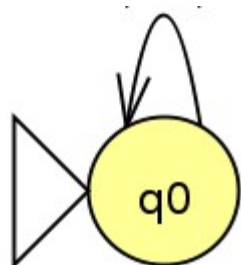
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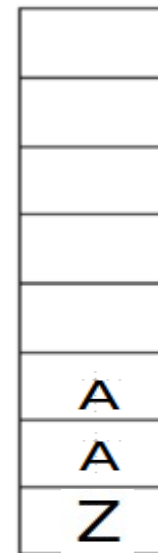
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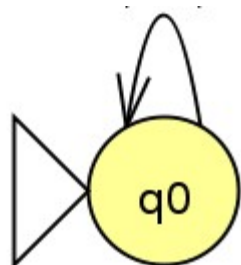
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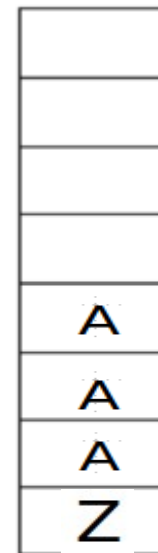
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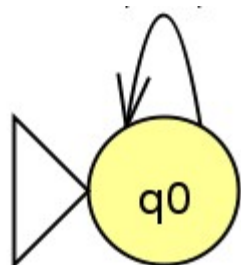
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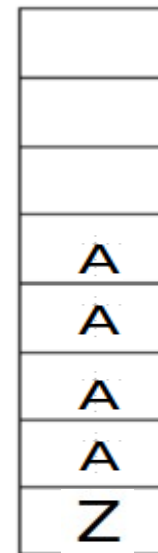
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- Base: Z



$\langle a, B \rangle / \lambda$
 $\langle b, A \rangle / \lambda$
 $\langle b, B \rangle / BB$
 $\langle a, A \rangle / AA$
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 $\langle a, Z \rangle / AZ$
 $\langle \lambda, Z \rangle / \lambda$

aaaabbbabbb
↑



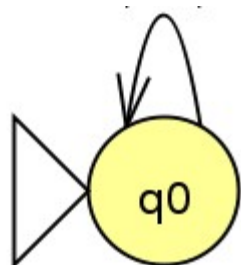
Pilha

Automato com Pilha

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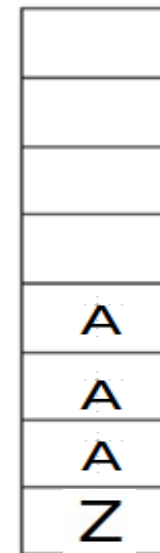
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$\langle a, B \rangle / \lambda$
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aaaabbbabbb
↑



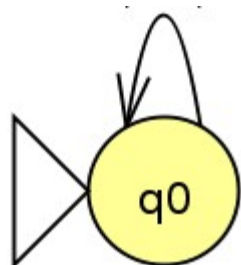
Pilha

Automôto com Pilha

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aaaabbabbb
↑



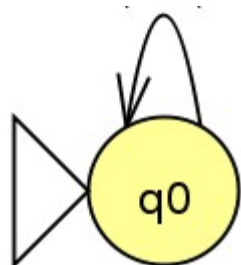
Pilha

Automato com Pilha

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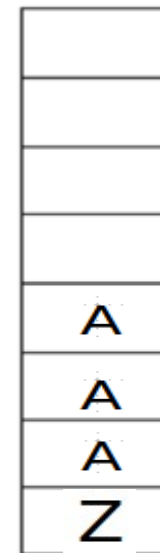
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

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$\langle a, B \rangle / \lambda$
 $\langle b, A \rangle / \lambda$
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aaaabbabbb
↑



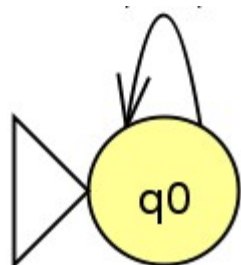
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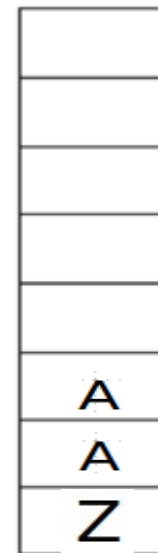
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



$\langle a, B \rangle / \lambda$
 $\langle b, A \rangle / \lambda$
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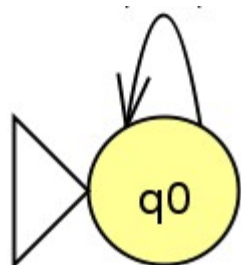
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Automato com Pilha

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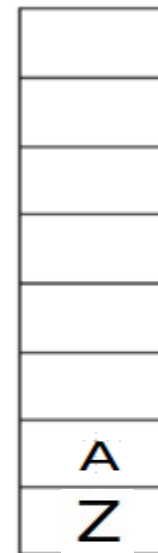
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

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aaaabbabbb
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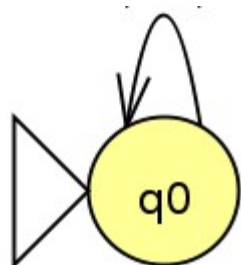
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Automato com Pilha

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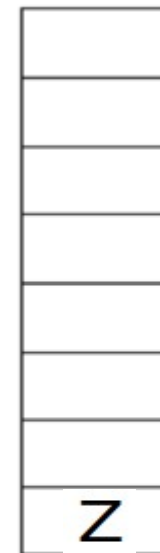
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



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 $\langle b, B \rangle / BB$
 $\langle a, A \rangle / AA$
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 $\langle a, Z \rangle / AZ$
 $\langle \lambda, Z \rangle / \lambda$

aaaabbabbb



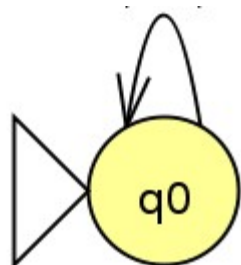
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Automôto com Pilha

- Exercícios:
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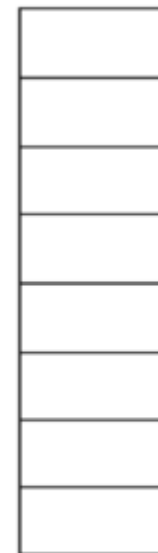
$$\{X \in \{a, b\}^* \mid |X|_a = |X|_b\}$$

- Base: Z



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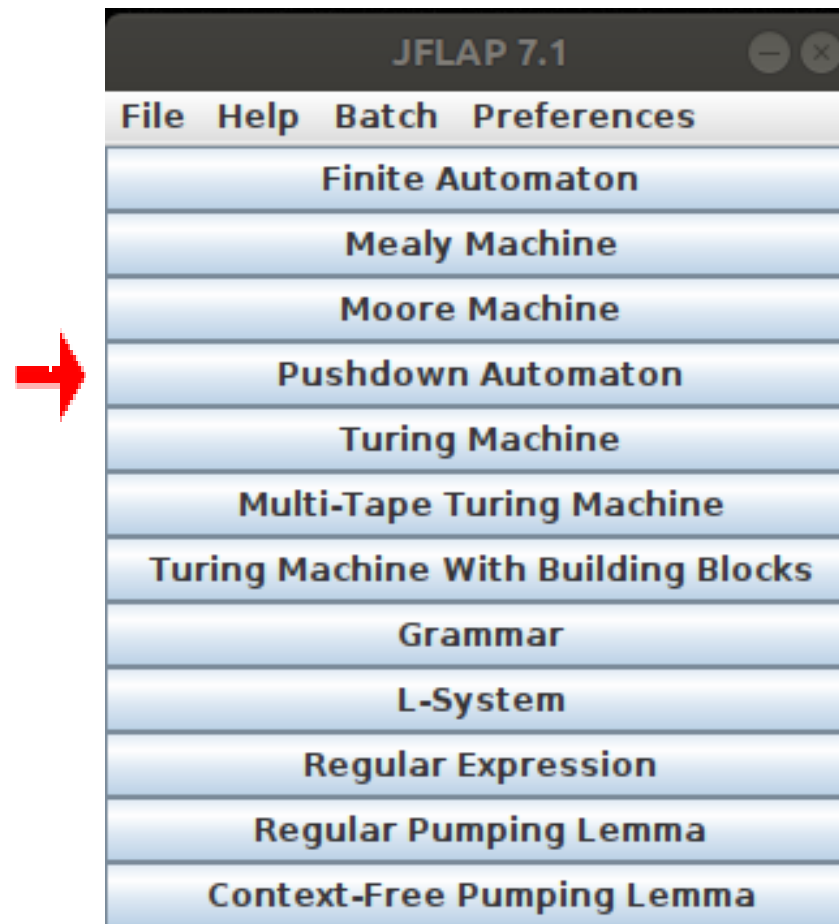
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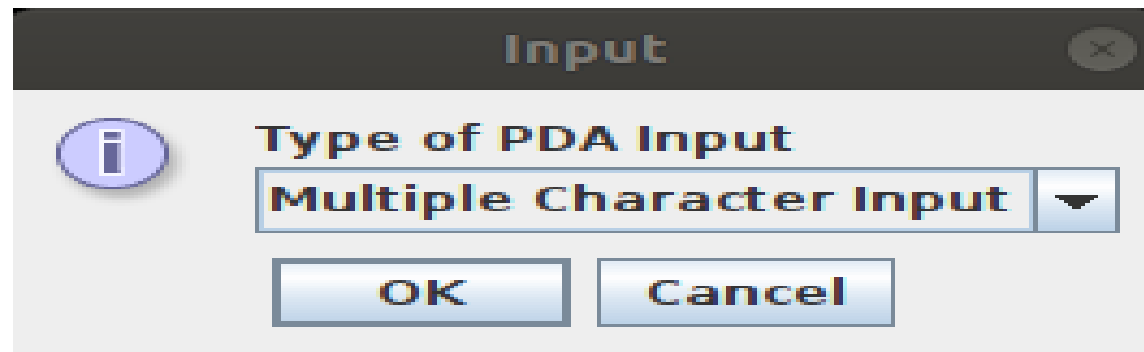
Pilha

Cadeia aceita

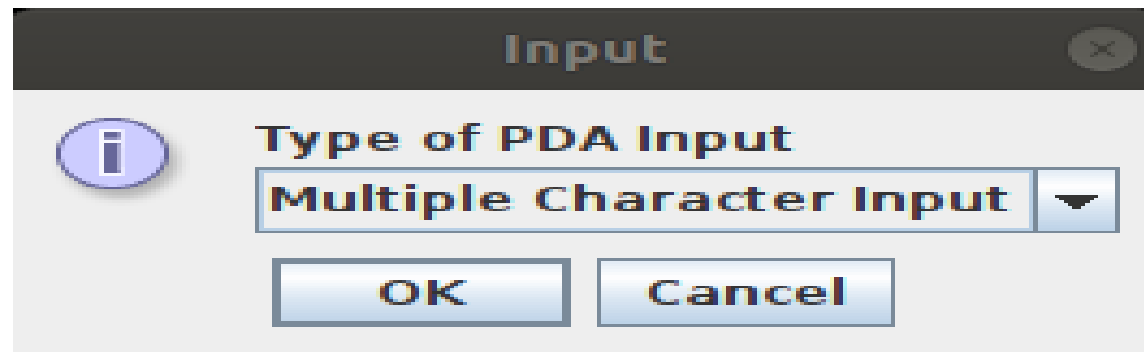
Ferramenta JFLAP



Ferramenta JFLAP



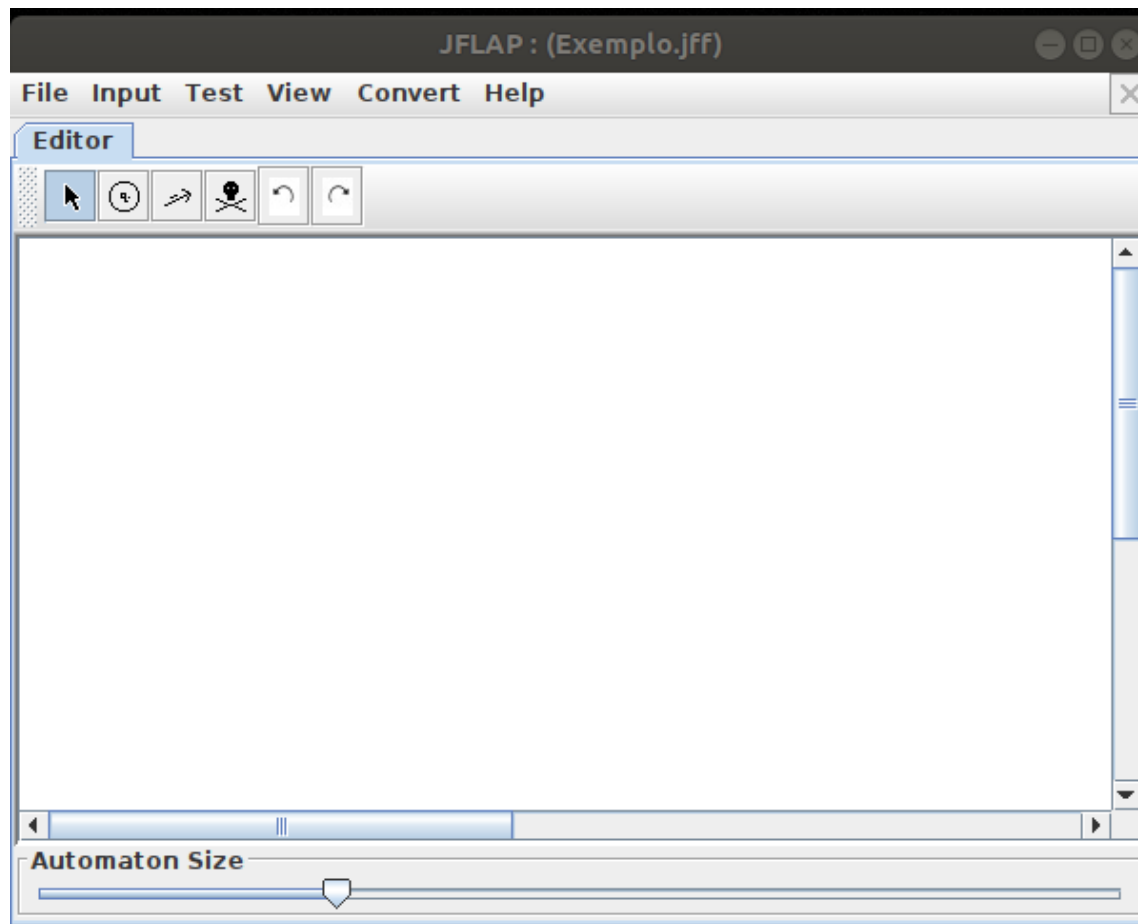
Ferramenta JFLAP



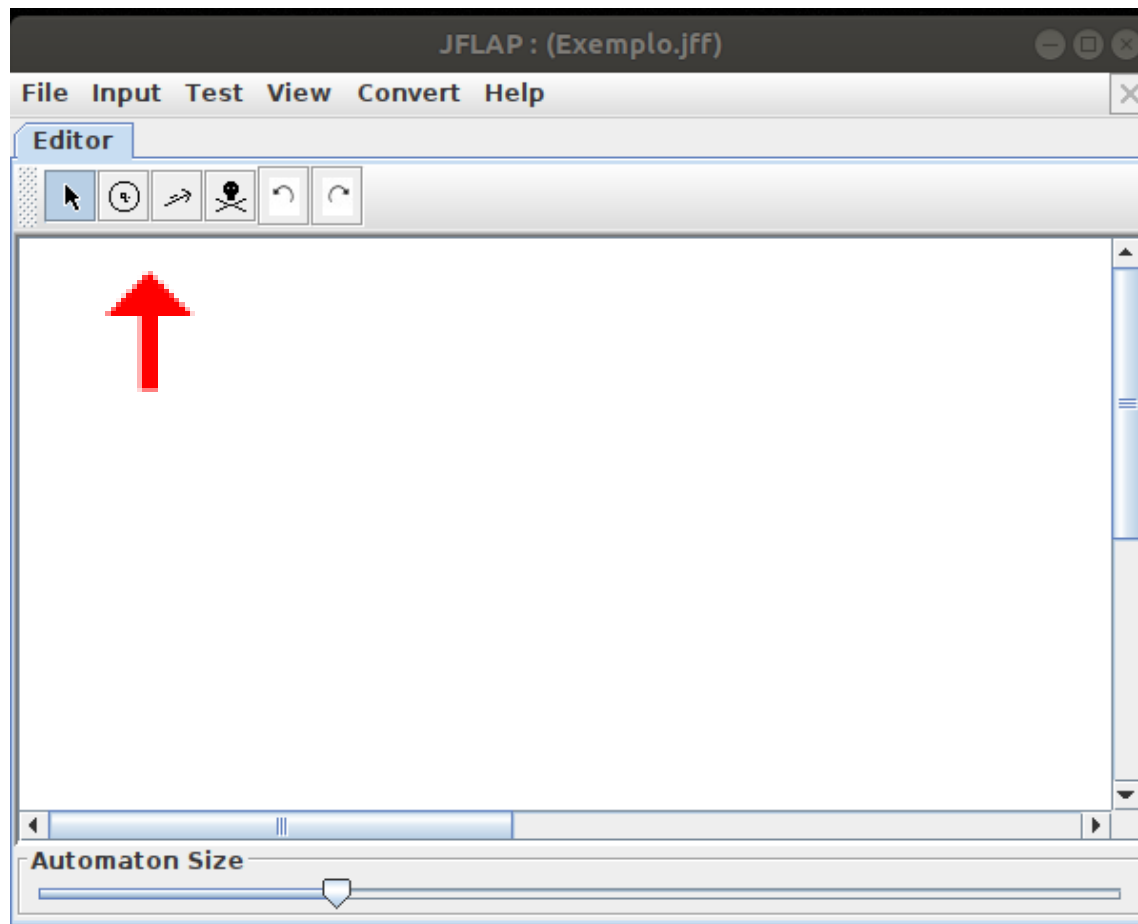
$\langle a, A \rangle / AA$



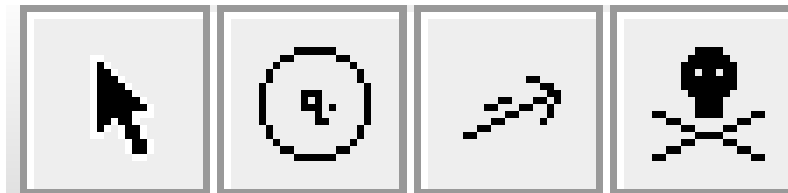
Ferramenta JFLAP



Ferramenta JFLAP

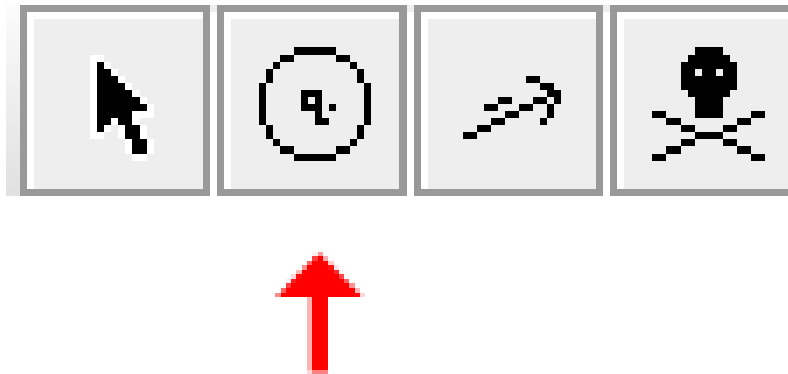


Ferramenta JFLAP



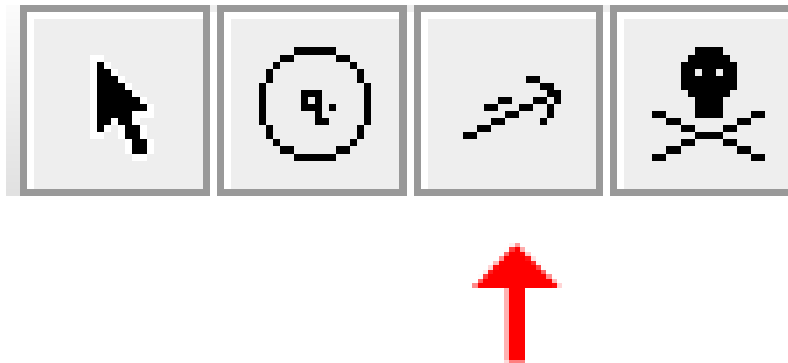
Utilizado para movimentar estados.

Ferramenta JFLAP



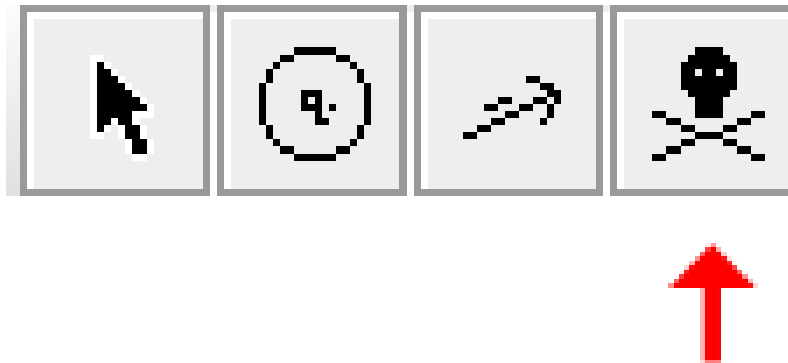
Utilizado para criar estados.

Ferramenta JFLAP



Utilizado para criar função de transições.

Ferramenta JFLAP



Utilizado para remover estados e funções de transições.

Ferramenta JFLAP



Multiple Run

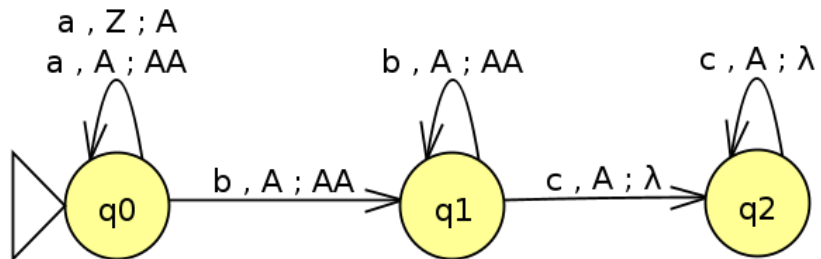
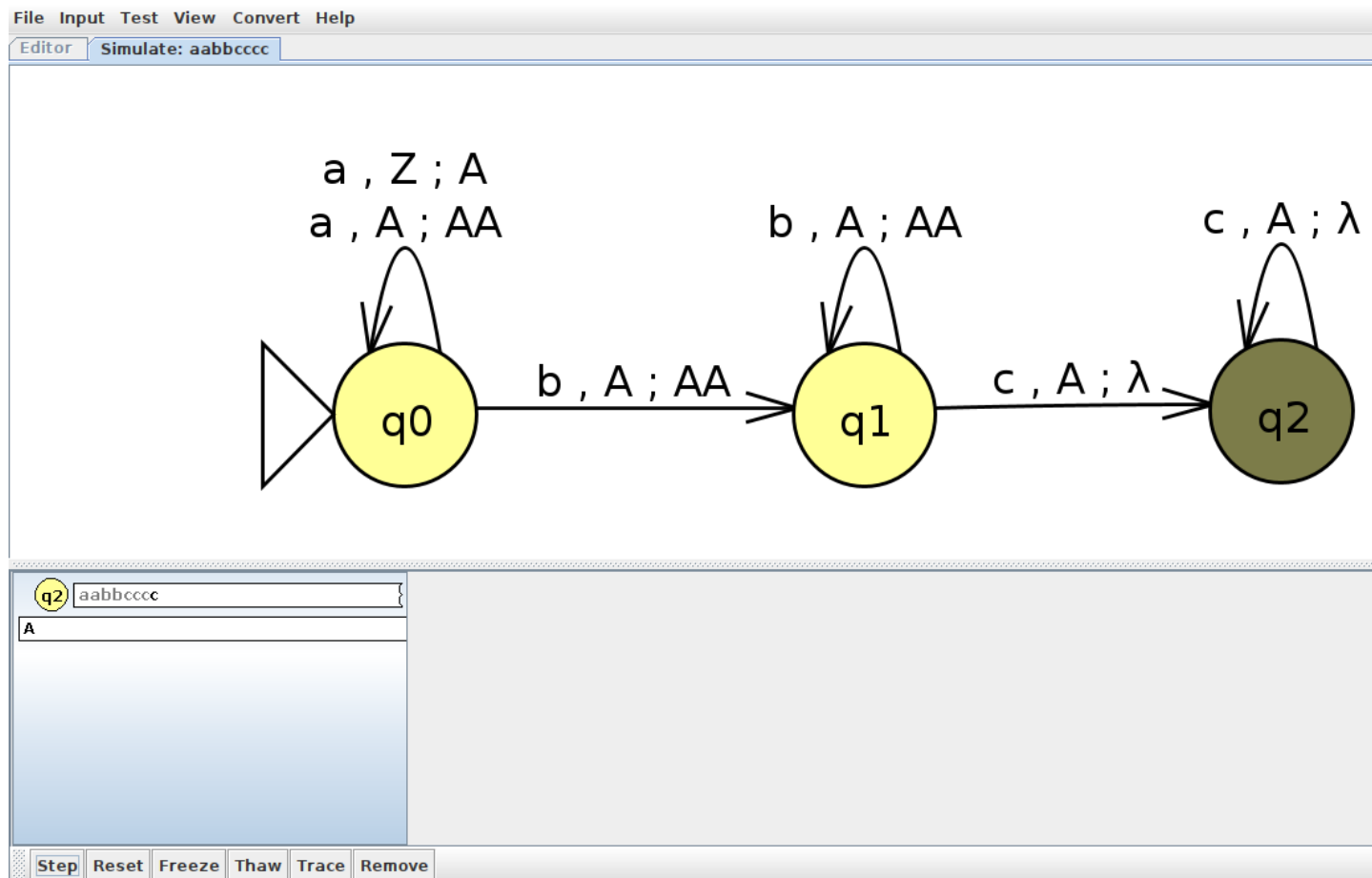


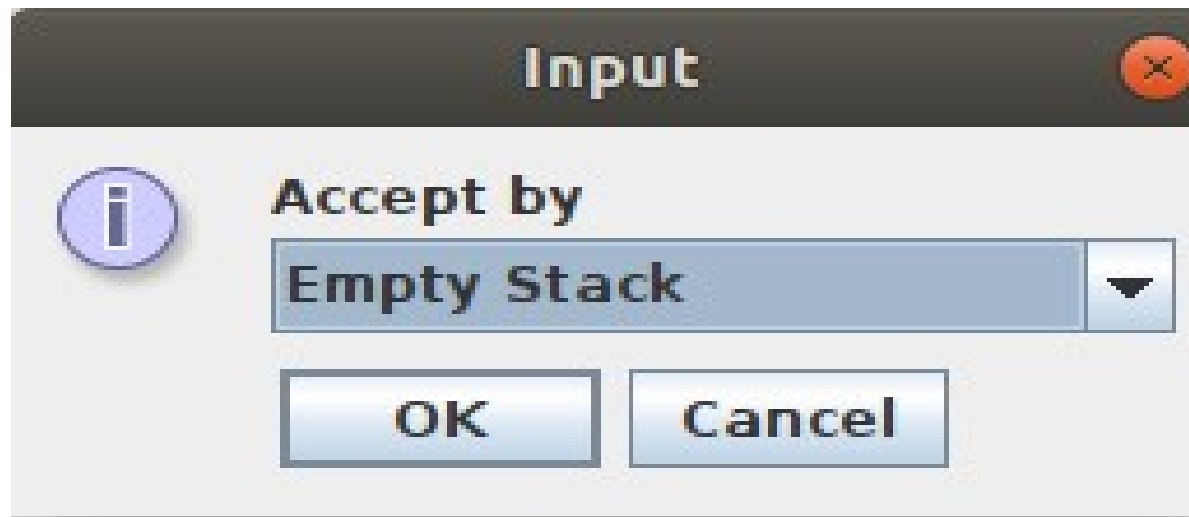
Table Text Size

Input	Result
abcc	Accept
abbbccccc	Reject
aaaabccccc	Accept
aaaccc	Reject
abc	Reject
abbbcccc	Accept

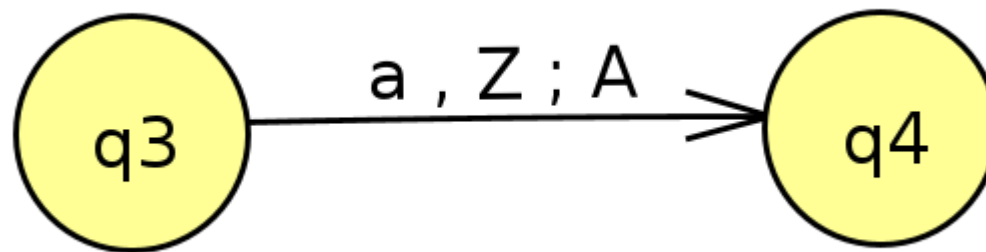
Step by Step



Ferramenta JFLAP



Função de transição - JFLAP



Transição $a, Z; A = \langle a, Z \rangle / A$
Base = Z;

Automato com Pilha

- Exercícios:
- Lista de exercícios nº 7 – Autômato com Pilha
 - <http://www.din.uem.br/yandre/TC/lista7.pdf>

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