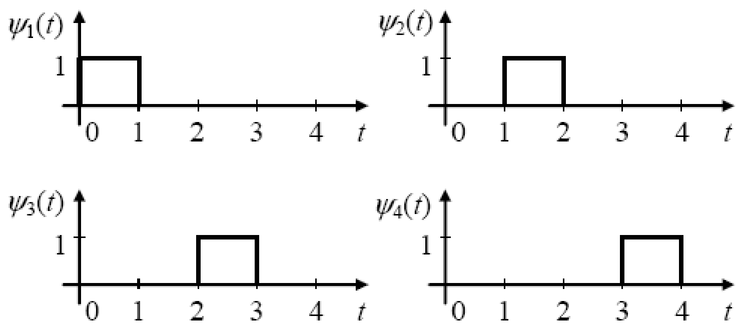
Digital Communication Systems

2019/2020

Problem Set #3

1. Four waveforms are represented in the signal space, with basis functions shown below, by the following vectors:

$$\mathbf{s}_1 = \begin{bmatrix} 2 & -1 & -1 & -1 \end{bmatrix}^T \quad \mathbf{s}_2 = \begin{bmatrix} -2 & 1 & 1 & 0 \end{bmatrix}^T \quad \mathbf{s}_3 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T \quad \mathbf{s}_4 = \begin{bmatrix} 1 & -2 & -2 & 2 \end{bmatrix}^T$$



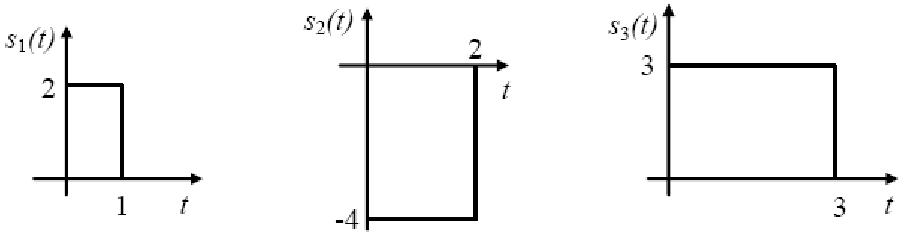
   a) Sketch the waveforms.

   b) Determine the euclidean distance between vectors $\mathbf{s}_1$ and $\mathbf{s}_2$.

   c) Determine the correlation coefficient between waveforms $s_1(t)$ and $s_2(t)$.

   d) Determine the energy of $s_3(t)$ and $s_4(t)$.

2. Consider the following waveforms.



   a) Determine a complete orthonormal set of basis functions using Gram-Schmidt procedure.

   b) Express each waveform in terms of the basis functions. What is the euclidean distance between $\mathbf{s}_1$ and $\mathbf{s}_2$?

   c) Determine the correlation coefficient between $s_1(t)$ and $s_2(t)$.

d) Determine the probability of bit error if functions $s_2(t)$ and $s_3(t)$ are used to represent the binary symbols and the transmission is through an AWGN channel with $G_n(f) = N_o/2$.

3. In the on-off keying (OOK) version of an ASK system, symbol 1 is represented by the sinusoidal carrier for a duration T_b while symbol zero corresponds to switching off the carrier during T_b. Assume symbols are equiprobable and consider transmission through an AWGN channel.

   For a coherent detector show that $P_b = Q\left(\sqrt{E_b/2N_o}\right)$, where E_b is the energy of symbol 1.

   Note: It can be shown that for non-coherent detection we have $P_b = \frac{1}{2}e^{\frac{-4E_b}{N_o}}$

4. Determine the expected number of bits in error for a BPSK receiver operating continuously during one day. The bit rate is 5 kbit/s and the received signal is $s_1(t) = A\cos(2\pi f_c t) = -s_0(t)$ with A = 1 mV. The noise power is No = 10^{-11} W/Hz.

5. A BPSK signal is supplied to a correlator that has a phase error that lies within α radians from the exact phase. Show that the average probability of error is given by $P_b = Q\left(\sqrt{2E_b \cos^2\alpha/N_o}\right)$.

6. The signal component of a BPSK system is defined by:

$$s(t) = A_c K \sin(2\pi f_c t) \pm A_c\sqrt{1-k^2}\cos(2\pi f_c t), \quad 0 \le t \le T_b$$

   where the plus sign corresponds to symbol 1 and the minus sign to symbol 0. The first term represents a pilot carrier component included for the purpose of synchronizing the receiver to the transmitter.

   a) Draw the signal-space diagram for the scheme described.

   b) For an AWGN channel with power spectral density $N_o/2$ show that the probability of bit error is given by $P_b = Q\left(\sqrt{\frac{2E_b}{N_o}\left(1-k^2\right)}\right)$.

c) Suppose that 10% of the transmitted signal power is allocated to the pilot carrier and determine the value of E_b/N_o to get $P_b = 10^{-4}$.

d) Compare with the value of E_b/N_o required by a conventional BPSK system with the same P_b.

7. Consider a QPSK system for which the symbol duration is 1 ms and the symbol transmitted power is 10 W. This signal goes through an AWGN channel, with $N_o = 10^{-6}$ W/Hz and attenuation $\alpha = 10$ db/km.

   a) Determine the maximum distance that still guarantees a probability of bit error of 10^{-6}.

   b) What is the distance if we use BPSK instead?

   c) Compare the necessary channel bandwidths for the two modulation types.

8. Two passband data transmission systems are to be compared. One uses 16-PSK while the other uses 16-QAM. Both systems are required to produce the same average probability of symbol error equal to 10^{-3}. Compare the signal-to-noise ratio requirements for the two cases.

9. Determine the transmission bandwidth reduction of a 256-QAM system compared to a 64-QAM. Obtain also the average signal energy relation between the two systems (consider the same value E_o for both cases).

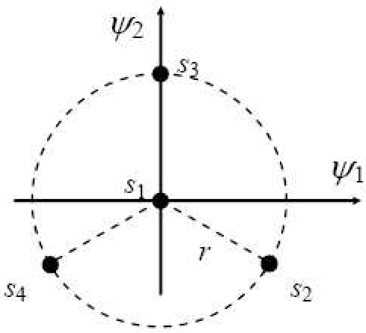
10. Consider that a 9600 bit/s signal must be sent through a passband channel with 4 kHz bandwidth. We are considering the use of an M-QAM system, with gray encoding, and we want to use raised-cosine impulses, with roll-off factor 0.5.

    a) Determine the minimum value for M.

    b) Give the expression for the probability of bit error for the case of 64-QAM.

11. Consider the signal constellation shown on the right, where $r = 1.3$.

    a) Determine the energy of $\mathbf{s}_2$.



b) Determine the correlation coefficient between $\mathbf{s}_2$ and $\mathbf{s}_3$.

c) Does this signal have minimum energy? Justify.

d) Draw the decision boundaries.

12. An FSK system transmits data at a rate of 2.5 Mbit/s. The channel is AWGN with $N_o = 10^{-20}$ W/Hz. In the absence of noise the amplitude of the received sinusoidal wave is 1 μV. Calculate the average probability of bit error for the following cases:

    a) Coherent FSK.

    b) Coherent MSK.

    c) Non-coherent FSK.

13. In a coherent FSK system we have signals

$$s_{1,2}(t) = A_c \cos\left[2\pi\left(fc \pm {}^{\Delta f}\!/_{2}\right)t\right], \quad 0 \le t \le T_b .$$

    a) Assuming that $f_c >> \Delta f$, show that the correlation coefficient of the two signals is given by:

$$\rho(t) = \frac{\int_0^{T_b} s_1(t)s_2(t)dt}{\int_0^{T_b} s_1(t)^2\,dt} \approx \mathrm{sinc}(2\Delta f T_b)$$

    b) What is the minimum frequency shift Δf for which the two signals are orthogonal?

14. Sketch the phase trellis of an MSK signal in response to the binary input sequence 1100100010.

15. MATLAB/SIMULINK – Continue the simulations of 8PSK case by implementing a detector and estimating the probability of bit error. Make similar studies for the cases of QPSK and MSK and GMSK (use the baseband modulators available in the Communications Blockset of SIMULINK).