Folha para provas de avaliação Folha N.º ___ / ___

Faculdade de Engenharia da Universidade do Porto

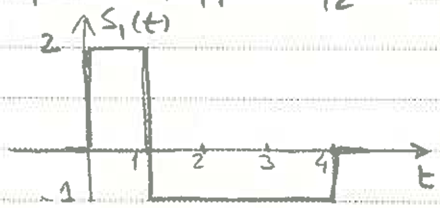
Curso ___ Data ___ / ___ / ___

Discipli ___ Ano ___ Semestre ___

Nome Problem Set #3

ESPAÇO RESERVADO PARA O AVALIADOR

1- a) e.g. $s_1(t) = 2\psi_1(t) - \psi_2(t) - \psi_3(t) - \psi_4(t)$



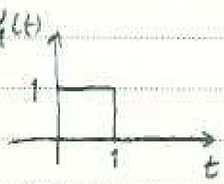
b) $d_{12} = \sqrt{\|s_1 - s_2\|} = \sqrt{4^2 + 2^2 + 2^2 + 1^2} = \sqrt{25} = 5$

c) $\cos\theta_{12} = \dfrac{s_1^T s_2}{\|s_1\| \cdot \|s_2\|} = \dfrac{-4-1-1}{\sqrt{4+1+1+1} \cdot \sqrt{4+1+1}} = -\dfrac{6}{\sqrt{7} \times \sqrt{6}} = -\sqrt{\dfrac{6}{7}}$

d) $E_3 = \|s_3\|^2 = 4$ $\quad E_4 = 1+4+4+4 = 13$

2- a) $E_1 = \int_0^1 4 \cdot dt = 4$

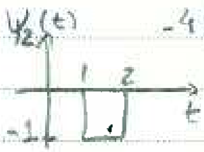
$\longrightarrow \psi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}} = \dfrac{s_1(t)}{2}$



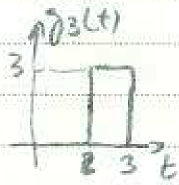
$s_{21}(t) = \int_0^1 s_2(t) \cdot \psi_1(t)\, dt = -4$

$g_2(t) = s_2(t) - 4 \cdot \psi_1(t)$

$\longrightarrow E_{g_2} = 16$



$\longrightarrow \psi_2(t) = \dfrac{g_2(t)}{\sqrt{E_{g_2}}} = \dfrac{g_2(t)}{4}$



$g_3(t) = s_3(t) - 3\psi_1(t) + 3\psi_2(t)$ $\quad E_{g_3} = 9$

$\longrightarrow \psi_3(t) = \dfrac{g_3(t)}{\sqrt{E_{g_3}}} = \dfrac{s_3(t)}{3}$

Usar frente e verso. Não escrever fora das margens.

b) $s_1 = [2\ 0\ 0]^T$; $s_2 = [-4\ 4\ 0]^T$; $s_3 = [3\ -3\ 3]^T$

$$d_{12} = \sqrt{(2+4)^2 + (-4)^2 + 0} = \sqrt{52} \qquad d_{13} = \sqrt{(2-3)^2 + 3^2 + 3^2} = \sqrt{19}$$

c) $$\cos\theta_{12} = \frac{-8+0+0}{\sqrt{4}\times\sqrt{32}} = -\frac{1}{\sqrt{2}}$$

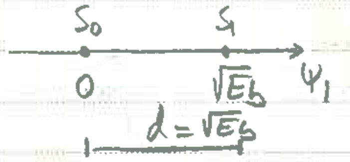
d) $$d_{23} = \sqrt{(-4-3)^2 + (4+3)^2 + 3^2} = \sqrt{107}$$

$$P_b = Q\left(\frac{d_{23}}{\sqrt{2N_0}}\right)$$

3 - $$s_1(t) = \sqrt{\frac{2E_b}{T}}\cdot\cos(\omega_c t) = \sqrt{E_b}\cdot\psi(t)$$ E_b - energy of symbol 1

$$s_0(t) = 0$$

Constellation:



$$P_b = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

4 - $$E_b = \frac{A^2}{2}\times T_b = \frac{A^2}{2R_b} = \frac{10^{-6}}{2\times5\times10^3} = 10^{-10}\,J \qquad N_0 = 10^{-11}$$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{20}) \simeq 3{,}87\times10^{-6}$$

from MATLAB or a table for $Q(x)$

There are $5\times10^3\times24\times3600$ bits received during one day from which we expect that
$5\times10^3\times24\times3600\times3{,}87\times10^{-6} \simeq 1671$ bits
will be in error.

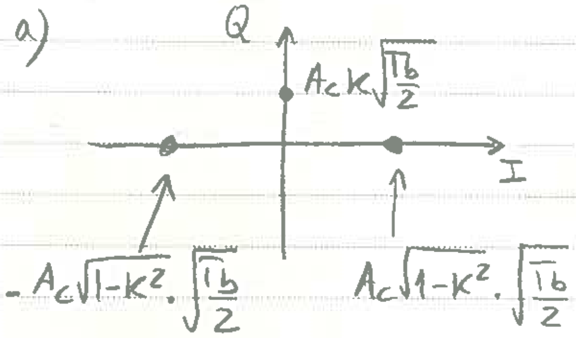
5 - $\psi(t) = \sqrt{\frac{2}{T_b}}\cos(\omega_c t)$; at the receiver $\psi_{rec}(t) = \sqrt{\frac{2}{T_b}}\cos(\omega_c t + \alpha) =$

$s_{1,2}(t) = \pm\sqrt{E_b}\cdot\psi(t)$ $\qquad = \sqrt{\frac{2}{T_b}}(\cos\omega_c t\cdot\cos\alpha - \sin\omega_c t\,\sin\alpha)$

$$\int_0^{T_b} s_{1,2}(t)\cdot\psi_{rec}(t)\cdot dt = \pm\sqrt{E_b}\cdot\cos\alpha \Rightarrow d = 2\sqrt{E_b}\cos\alpha$$

$$P_b = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b\cos^2\alpha}{N_0}}\right)$$

6 - a)



$$\psi_1(t) = \sqrt{\frac{2}{T_b}}\cos\omega_c t$$

$$\psi_2(t) = \sqrt{\frac{2}{T_b}}.\operatorname{sen}\omega_c t$$

b) for $K=0$ we have $d = 2\sqrt{E_b} \Rightarrow P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

for $K \neq 0$ we have $d = 2\sqrt{E_b}.\sqrt{1-K^2} \Rightarrow P_b = \left(\sqrt{\frac{2E_b}{N_0}(1-K^2)}\right)$

c) $Q(x) = 10^{-4} \Rightarrow x = 3{,}719$

↑ MATLAB

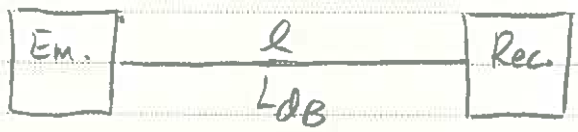
$K^2 = 0{,}1$

$$\frac{2E_b(1-0{,}1)}{N_0} = 3{,}719^2 \Rightarrow \frac{E_b}{N_0} = 7{,}7 \simeq 8{,}9\ dB$$

d) $\frac{2E_b}{N_0} = 3{,}719^2 \Rightarrow \frac{E_b}{N_0} = 6{,}9 \simeq 8{,}4\ dB$ $\quad \simeq 0{,}5\ dB$

↑ with $K^2=0$

7 -



$P_b = 10^{-6}$ $\quad Q(x) = 10^{-6} \Rightarrow x = 4{,}75$

↑ MATLAB

$$\frac{2E_{bR}}{N_0} = 4{,}75^2$$

$$E_{bR} = 11{,}28\ \mu J.$$

a) $E_{bE} = \frac{P_s.T_s}{2} = \frac{10\times10^{-3}}{2}\ J$

$$L_{dB} = 10\log\left(\frac{E_{bE}}{E_{bR}}\right) = 26{,}46\ dB$$

$$l = \frac{L_{dB}}{\alpha} = 2{,}646\ km.$$

b) E_{bR} has to be the same to get the same P_b. Using the same E_{bE} we got the same P_b

c) QPSK spends half bandwith of BPSK since we transmit symbols that correspond to dibits.

8 - For 16-QAM we have

$$P_e \simeq 4\left(1-\frac{1}{\sqrt{16}}\right) Q\left(\sqrt{\frac{3}{(16-1)} \cdot \frac{\langle E \rangle}{N_0}}\right) = 10^{-3} \Rightarrow \frac{\langle E \rangle}{N_0} \simeq 17{,}6\,dB$$

using MATLAB

with same N_0 16-QAM requires about 4dB less energy

For 16-PSK we have

$$P_e \simeq 2 \cdot Q\left(\sqrt{\frac{2E_s}{N_0}} \operatorname{sen}\frac{\pi}{16}\right) = 10^{-3} \Rightarrow \frac{\langle E \rangle}{N_0} \simeq 21{,}5\,dB$$

in this case $E_s = \langle E \rangle$

9 - $$B_{256\text{-QAM}} = \frac{2 \times R_b}{\log_2 256} = \frac{2R_b}{8}$$

$$B_{64\text{-QAM}} = \frac{2 \times R_b}{\log_2 64} = \frac{2R_b}{6}$$

$$\langle E_{256} \rangle = \frac{2(256-1)}{3} \cdot E_0 = 170\,E_0$$

$$\langle E_{64} \rangle = \frac{2(64-1)}{3} \cdot E_0 = 42\,E_0$$

10 - a) $B = \dfrac{2 \times R_b \times (1+\alpha)}{\log_2 M}$ and $B \leq 4\,kHz$ $\quad R = 9600\,bit/s$, $\alpha = 0{,}5$

$$\frac{2 \times (1+0{,}5) \cdot 9600}{\log_2 M} \leq 4000 \qquad \log_2 M \geq 7{,}2 \qquad M = 256$$

b) Value of $\frac{\langle E \rangle}{N_0}$ or $\frac{E_0}{N_0}$ or $\frac{E_b}{N_0}$ is missing in order to obtain

$$P_b = \frac{4\left(1-\frac{1}{\sqrt{M}}\right)}{\log_2 M} \cdot Q\left(\sqrt{\frac{3\log_2 M}{M-1} \cdot \frac{E_b}{N_0}}\right)$$

give an numerical value for $\frac{E_b}{N_0}$

Folha para provas de avaliação Folha N.º ___ / ___

Faculdade de Engenharia da Universidade do Porto

Curso ______ Data __ / __ / __

Disciplina ______ Ano ______ Semestre ______

Nome Problem Set #3

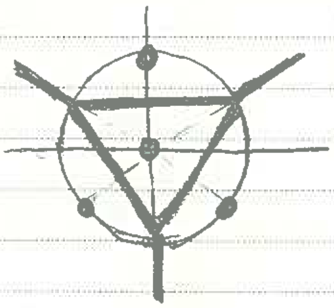
ESPAÇO RESERVADO PARA O AVALIADOR

11 – a) $E_2 = r^2 = 1{,}3^2 = 1{,}69\ V^2 s$

b) $\cos\theta_{12} = \cos 120^\circ = -0{,}5$

c) Yes.

d)



12 – $R = 2{,}5\ \text{kbit/s}$ $\quad N_0 = 10^{-20}\ \text{W/Hz}$ $\quad \underline{\underline{A = 1\mu V}}$ (instead of $1\ mV$)

$$E_b = \frac{A^2}{2} \cdot T_b = \frac{A^2}{2R_b} = \frac{(10^{-6})^2}{5\times10^{6}} = 2\times10^{-19}\ J$$

a) $P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2\times10^{-19}}{10^{-20}}}\right) = Q(\sqrt{20}) = 3{,}8\times10^{-6}$

b) $P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{40}) = 1{,}3\times10^{-10}$

c) $P_b = \frac{1}{2} e^{-\frac{E_b}{2N_0}} = \frac{1}{2} e^{-\frac{2\times10^{-19}}{2\times10^{-20}}} = \frac{1}{2} e^{-10} = 22{,}7\times10^{-6}$

13 - a) Solving the expression we get

$$\rho(t) = \frac{1}{2\pi T_b}\left[\frac{\text{sen}(2\pi\Delta f T_b)}{\Delta f} + \frac{\text{sen}(4\pi f_c T_b)}{2f_c}\right]$$

Since $f_c \gg \Delta f$ we may write

$$\rho(t) \simeq \frac{\text{Sen}(2\pi\Delta f T_b)}{2\pi\Delta f T_b} = \text{sinc}(2\Delta f T_b)$$

b) $\rho(t) = 0$ at $\Delta f_{min} = \dfrac{1}{2T_b}$

14 -