

Eigen-Value: Efficient Domain-Robust Data Valuation via Eigenvalue-Based Approach

Youngjun Choi¹

Joonseong Kang¹

Sungjun Lim¹

Kyungwoo Song^{1*}

¹Dept. of Statistics and Data Science, Yonsei University, Seoul, Korea

Abstract

Data valuation has become central in the era of data-centric AI. It drives efficient training pipelines and enables objective pricing in data markets by assigning a numeric value to each data point. Most existing data valuation methods estimate the effect of removing individual data points by evaluating changes in model validation performance under in-distribution (ID) settings, as opposed to out-of-distribution (OOD) scenarios where data follow different patterns. Since ID and OOD data behave differently, data valuation methods based on ID loss often fail to generalize to OOD settings, particularly when the validation set contains no OOD data. Furthermore, although OOD-aware methods exist, they involve heavy computational costs, which hinder practical deployment. To address these challenges, we introduce *Eigen-Value* (EV), a plug-and-play data valuation framework for OOD robustness that uses only an ID data subset, including during validation. EV provides a new spectral approximation of domain discrepancy, which is the gap of loss between ID and OOD using ratios of eigenvalues of ID data's covariance matrix. EV then estimates the marginal contribution of each data point to this discrepancy via perturbation theory, alleviating the computational burden. Subsequently, EV plugs into ID loss-based methods by adding an EV term without any additional training loop. We demonstrate that EV achieves improved OOD robustness and stable value rankings across real-world datasets, while remaining computationally lightweight. These results indicate that EV is practical for large-scale settings with domain shift, offering an efficient path to OOD-robust data valuation.

1 Introduction

Machine learning has achieved strong performance in image recognition, autonomous driving, and conversational systems. Yet domain shifts between in-distribution (ID) training data and out-of-distribution (OOD) deployment data can sharply reduce accuracy. Robustness to such shifts is essential, especially in safety-critical applications. Most work addresses this with model-centric strategies such as distributionally robust optimization [Hu et al. \[2018\]](#), [Staib and Jegelka \[2019\]](#), [Rahimian and Mehrotra \[2022\]](#) and domain-invariant representation learning, which often require specialized architectures and complex training pipelines. However, even with state-of-the-art architectures and training algorithms, performance remains bounded by data quality and composition. A complementary data-centric approach called data valuation evaluates the training set itself and identifies informative examples [Sim et al. \[2022\]](#), [Tian et al. \[2022\]](#), [Agarwal et al. \[2019\]](#). Curating data, rather than continually updating models, offers multiple benefits. It reduces computation,

*Corresponding author

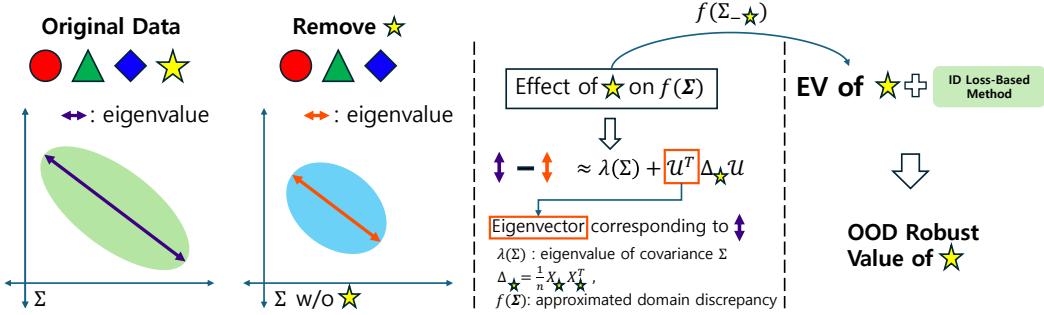


Figure 1: Overview of EV. Estimating the change in covariance eigenvalues induced by removing a single normalized embedding to quantify domain discrepancy, which is then integrated into ID loss-based data valuation for improved OOD robustness.

accelerates learning, and improves model performance [Xu et al. \[2025\]](#). This view also aligns with pricing in data marketplaces.

Data valuation measures the change in model performance induced by including or excluding each training example. Most existing methods estimate this effect under the training distribution [Shapley et al. \[1953\]](#), [Roth \[1988\]](#), [Ghorbani and Zou \[2019\]](#), [Alvarez-Melis and Fusi \[2020\]](#). However, validation and OOD distributions often diverge, so values inferred from validation-based performance changes do not hold under OOD. This gap in distributional alignment makes OOD-robust data valuation necessary. However, existing shift-aware methods are computationally prohibitive [Lin et al. \[2024\]](#), limiting their practicality in data marketplaces that require reliable metrics computed without OOD data [Sim et al. \[2022\]](#), [Tian et al. \[2022\]](#).

We address this gap with *Eigen-Value* (EV), a data valuation framework that targets robustness under domain shift. First, EV establishes a connection between domain discrepancy and the Hessian of the loss function under distribution shift. This relies on the observation that, in logistic regression, the Hessian approximates the data covariance [Le Cun et al. \[1991\]](#), [Lin et al. \[2007\]](#), [Hazan et al. \[2014\]](#). Building on this observation, EV introduces a novel formulation that relates domain discrepancy to the eigenvalues of the covariance structure, as shown in Figure 1. Second, to address the computational burden of repeated eigendecomposition, EV employs perturbation theory [Kato \[2013\]](#), which approximates the effect of removing a single data point without requiring a new decomposition. Lastly, EV augments existing ID loss-based data valuation methods with a marginal value for the domain discrepancy term that reflects the induced eigenvalue shifts. Importantly, EV operates solely on ID data, requiring no OOD samples for training or validation. It estimates the potential impact of domain shifts by quantifying how each data point perturbs the largest and smallest eigenvalues in the ID setting. In experiments across diverse datasets, augmenting baselines with EV improves OOD performance over the baselines alone, while maintaining efficiency and stability.

In summary, our contributions are:

- We relate domain discrepancy to covariance eigenvalues, which enables data valuation without OOD samples.
- We introduce EV, a scalable and easy-to-combine term that upgrades ID-based methods via perturbation theory.
- We present evidence on real-world datasets that EV improves OOD robustness, stability, and efficiency, indicating readiness for practical use.

2 Related Work

2.1 Data Valuation

Data valuation measures how each training example changes a model’s performance [Sim et al. \[2022\]](#), [Sidi et al. \[2012\]](#). Its importance grows with large-scale datasets and retrieval augmented generation [Lewis et al. \[2020\]](#), where careful curation improves efficiency and interpretability [Koh and Liang \[2017\]](#). Data Shapley [Ghorbani and Zou \[2019\]](#) estimates a point’s value by retraining across many subsets, which is expensive. KNN Shapley [Jia et al. \[2019\]](#) and Data-OOB [Kwon and Zou \[2023\]](#) reduce cost using local neighbors and out-of-bag scores, but they face limits in scalability or accuracy. LAVA [Just et al. \[2023\]](#) removes the retraining bottleneck by measuring Wasserstein distances and gives fast estimates. However, its effectiveness is limited to data sampled from the same distribution as the training set, leading to degraded utility under domain shift. Deviation [Lin et al. \[2024\]](#) formulates a worst-case distributional shift objective via the neural tangent kernel (NTK), enabling applicability beyond the training distribution. However, it requires n times of inverting an $n \times n$ kernel matrix (with n training samples), incurring huge computation, which hampers practical deployment at scale. Even on reduced subsets, the method can be unstable due to its worst-case formulation [Zhai et al. \[2021\]](#), where small changes in the dataset lead to significant fluctuations in value rankings, limiting its suitability for data-centric AI at scale. Our work departs from these existing methods and proposes an eigenvalue-based scheme that remains accurate under domain shift and is accelerated by perturbation theory.

2.2 OOD Robustness

OOD robustness seeks models that remain reliable when the test distribution differs from that of the training and validation data [Yadav et al. \[2023\]](#), [Oh et al. \[2024\]](#), [Wortsman et al. \[2022\]](#). Distribution shifts, from small corruptions to large domain gaps, can reduce accuracy even for state-of-the-art networks, so training and evaluation must anticipate such changes. Recent advances include ensembling and spectral criteria. WiSE-FT [Wortsman et al. \[2022\]](#) ensembles pre- and post-fine-tuned weights to improve both ID and OOD performance. RankMe [Garrido et al. \[2023\]](#) and CaRoT [Oh et al. \[2024\]](#) relate generalization to spectra of weight or feature matrices and guide optimization toward high rank or large minimum singular values. These results suggest that spectral or weight space signals are useful proxies for OOD risk. However, most of this literature on OOD robustness has been model-centric, and there has been a lack of an efficient data valuation method for assessing OOD loss. To enable efficient data pipeline management and provide objective pricing in data marketplaces, we propose an OOD-robust data valuation method. This data-centric approach allows us to evaluate OOD robustness from the perspective of data itself.

2.3 Eigenvalue Methods

Eigenvalue analysis has guided machine learning since classical PCA [Hotelling \[1933\]](#). Its view of covariance spectra also influenced kernel PCA, spectral clustering, and diffusion maps [Mackiewicz and Ratajczak \[1993\]](#), [Schölkopf et al. \[1997\]](#), [Von Luxburg \[2007\]](#), [Lafon \[2004\]](#). Recent works use spectral information to improve robustness. CaRoT [Oh et al. \[2024\]](#) steers parameter updates using Hessian eigenvalues, and Eigen-SAM [Luo et al. \[2024\]](#) discourages sharp directions to find flatter minima. We extend this spectral line to data valuation. By linking covariance eigenvalues to OOD generalization error and using perturbation theory to approximate each sample’s spectral influence, our method, EV provides a scalable valuation approach aware of domain shift.

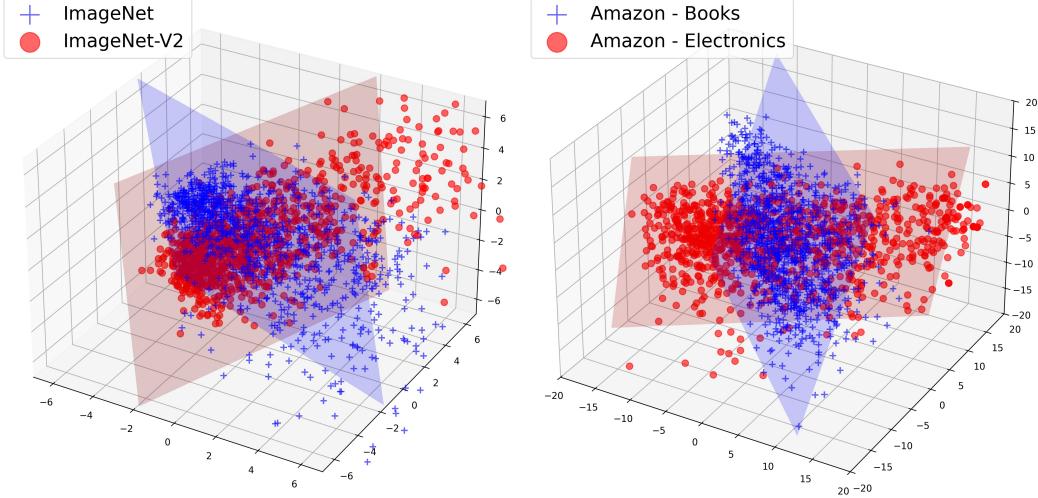


Figure 2: PCA visualization of normalized embeddings sampled (1K each) from different domain sources. The two distributions, corresponding to different domains, partially overlap due to normalization, illustrating that the matching marginal assumption remains applicable in real-world scenarios.

3 Setting and Preliminaries

Data Valuation. Let $S = \{(x_i, y_i)\}_{i=1}^N$ be a dataset of N normalized (embedding, label) pairs with $x_i \in \mathcal{X} \subseteq \mathbb{R}^d$ and $y_i \in \mathcal{Y}$. For any subset $\mathcal{D} \subseteq S$ of size n , a utility function $U : 2^n \rightarrow \mathbb{R}$ maps the subset to a scalar score. We define the marginal data value $V : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$ of a point (x_k, y_k) as the drop in utility when that point is removed, i.e., $V(x_k, y_k) = U(\mathcal{D}) - U(\mathcal{D} \setminus \{(x_k, y_k)\})$. Thus $V(x_k, y_k) > 0$ indicates that deleting the point hurts performance, implying high importance. Existing methods share this core definition and differ only in how they approximate U and compute V efficiently.

Matching Marginal. Let $P_{\text{ID}}(x)$ denote the in-distribution (ID) used for training, and $P_{\text{OOD}}(x)$ an out-of-distribution (OOD) that differs only in the marginal $P(x)$ but shares the same conditional $P(y | x)$ Shimodaira [2000], Moreno-Torres et al. [2012]. In other words, the input distribution shifts between ID and OOD, yet the relationship between input and label remains identical Sugiyama et al. [2007]. To further analyze this OOD scenario, we consider the matching marginal condition. This condition implies that the covariance matrices of ID and OOD share the same diagonal entries, meaning that each feature has identical variance across domains. In contrast, the off-diagonal elements may differ Huang et al. [2006], reflecting possible shifts in cross-feature correlations. Although normalization is not inherent to the definition, in our setting, it makes the condition easier to satisfy by aligning the diagonal entries. As illustrated in Figure 2, PCA visualizations of 1K embeddings sampled from different domains show partial overlap after normalization, supporting that the matching marginal assumption is both empirically plausible and practically valid.

Perturbation Theory. Perturbation theory offers a mathematical framework for approximating how a matrix's eigenvalues and eigenvectors shift under a perturbation. For instance, if a matrix A is modified by a small term ϵ such that $B = A + \epsilon$, one can express the eigenvalues and eigenvectors of B as expansions in ϵ Moro and Dopico [2003], Greenbaum et al. [2020]. In the context of covariance matrices, this technique elucidates how adjustments to the underlying data affect eigenvalue structures Sugiyama et al. [2020], Mohammed et al. [2017]. Consequently, removing a single data point k from a dataset can be viewed as applying a perturbation to the covariance matrix, thereby enabling an analysis

of how individual samples influence eigenvalue shifts, as shown in Section 4.2. This perspective aids in evaluating dataset stability and identifying observations that exert outsized influence on the learned model.

4 Efficient Domain Robust Data Valuation

We propose an eigenvalue-based framework for OOD-robust data valuation, ensuring reliability under distribution shifts. First, we characterize domain discrepancy using eigenvalues of covariance matrix derived from normalized training dataset with zero mean $\mathcal{D}_{\text{ID}} = \{z_i = (x_i, y_i)\}_{i=1}^n$, which is i.i.d. sampled from the ID distribution to quantify shifts between ID and OOD data (Section 4.1). After that, we develop an efficient perturbation-based method to compute marginal valuations directly from eigenvalue terms, reducing computational overhead (Section 4.2). Finally, we integrate these components into a unified valuation framework, enabling scalable and robust data valuation without requiring explicit OOD samples (Section 4.3).

4.1 Utility Function Based on OOD Loss

In the Shapley value framework, the utility function U , which forms the basis for data valuation, is typically defined as the model’s performance on a validation set, with $U(\mathcal{D})$ denoting validation performance on dataset \mathcal{D} . Similarly, we adopt $\mathcal{L}_{\text{OOD}}(\theta)$ as our utility function, which is the loss function of a model with parameter θ trained on the ID set \mathcal{D}_{ID} and evaluated on OOD set \mathcal{D}_{OOD} . The OOD loss $\mathcal{L}_{\text{OOD}}(\theta)$ can be upper-bounded using the ID loss $\mathcal{L}_{\text{ID}}(\theta)$ and a measure of domain discrepancy $\Gamma(\mathcal{D}_{\text{OOD}}, \mathcal{D}_{\text{ID}}) := \sup_{\theta} |\mathcal{L}_{\text{OOD}}(\theta) - \mathcal{L}_{\text{ID}}(\theta)|$. For notational simplicity, we denote $\mathcal{L}_{\text{OOD}}(\theta)$ and $\mathcal{L}_{\text{ID}}(\theta)$ as \mathcal{L}_{OOD} and \mathcal{L}_{ID} .

$$\mathcal{L}_{\text{OOD}} \leq \mathcal{L}_{\text{ID}} + \Gamma(\mathcal{D}_{\text{OOD}}, \mathcal{D}_{\text{ID}}) \quad (1)$$

To quantify domain discrepancy, we first define it in terms of the distributional shift between ID and OOD data. A well-established approach is to approximate domain shift using a measure derived from the model’s sensitivity to input variations. Notably, when the loss function is formulated as Normalized Cross Entropy (NCE) Ma et al. [2020], domain discrepancy can be related to the spectral properties of the Hessian matrices $H_{\text{ID}} = \nabla_{\theta}^2 \mathcal{L}_{\text{ID}}$ and $H_{\text{OOD}} = \nabla_{\theta}^2 \mathcal{L}_{\text{OOD}}$, corresponding to the ID and OOD distributions, respectively.

Proposition 1. We assume $\mathcal{L}_{\text{ID}} \leq \mathcal{L}_{\text{OOD}}$ under the NCE loss function. Then, the domain discrepancy $\Gamma(\mathcal{D}_{\text{OOD}}, \mathcal{D}_{\text{ID}})$ is bounded as follows:

$$\Gamma(\mathcal{D}_{\text{OOD}}, \mathcal{D}_{\text{ID}}) \leq \frac{\lambda_{\max}(H_{\text{OOD}})}{\lambda_{\min}(H_{\text{ID}})} \quad (2)$$

where λ_{\min} and λ_{\max} stand for minimum eigenvalue and maximum eigenvalue. The derivation of Proposition 1 is provided in Appendix A.1. By representing domain discrepancy as the ratio of eigenvalues of the model, it becomes possible to leverage the characteristics of the logistic regression task to interpret information about the model’s loss in terms of data under matching marginal assumptions.

In Proposition 1, we represent domain discrepancy as a ratio of the eigenvalues $\frac{\lambda_{\max}(H_{\text{OOD}})}{\lambda_{\min}(H_{\text{ID}})}$. Furthermore, we express the ratio of a model’s eigenvalues in terms of the eigenvalues of the data. We can exploit the properties of logistic regression, where the Hessian of the loss corresponds to the data’s covariance matrix. As a result, we can formulate the notion of domain discrepancy not in terms of the model but rather using the eigenvalues of the data’s covariance matrix, allowing us to assess OOD robustness without relying on model-specific loss functions.

However, in practice, we usually lack information about OOD data. So we have no direct way of computing the eigenvalue for the OOD covariance matrix, which is used to approximate the numerator of the eigenvalue ratio in Eq. 2. In this context, the matching marginal assumption implies that the OOD data's covariance matrix Σ_{OOD} can be modeled as the ID data's covariance matrix Σ_{ID} with a perturbation. By taking the Frobenius norm of both sides of this relationship and then applying the triangle inequality, we can leverage standard properties of the Frobenius norm to upper-bound the eigenvalue of Σ_{OOD} by that of Σ_{ID} .

Theorem 1. We assume $\Sigma_{\text{OOD}} = \Sigma_{\text{ID}} + E$, where $E \in \mathbb{R}^{d \times d}$ has zero diagonal entries and non-zero off-diagonal elements representing domain discrepancies. Based on this assumption, we derive the following bound on \mathcal{L}_{OOD} in terms of λ_{\max} , λ_{\min} , and the dimensionality d of Σ_{ID} .

$$\mathcal{L}_{\text{OOD}} \leq \mathcal{L}_{\text{ID}} + \frac{\lambda_{\max}(\Sigma_{\text{ID}}) \times \sqrt{d} + \sqrt{d^2 - d}}{\lambda_{\min}(\Sigma_{\text{ID}})} \quad (3)$$

Through this formulation, the bound on \mathcal{L}_{OOD} of Eq. 1 is established in Theorem 1, with its derivation given in Appendix A.2.

4.2 Marginal Calculation of Eigenvalue term

The change in the eigenvalue term from Eq. 3, arising when the k -th data point x_k is absent, serves as our target measure for data valuation. However, directly computing the eigenvalue for every single data point to capture this change becomes highly inefficient. To streamline the process, we first calculate Σ_{-k} , the covariance matrix without data point x_k , by perturbing the covariance matrix with the ID data subset $\Sigma_{\text{ID}} = \frac{1}{n} \sum_i x_i x_i^\top$ with $\Delta_k = -\frac{1}{n} x_k x_k^\top$. Specifically, we can express it as:

$$\Sigma_{-k} = \frac{1}{n-1} \sum_{i \neq k} x_i x_i^\top \approx \Sigma_{\text{ID}} + \Delta_k \quad (4)$$

Drawing on perturbation theory, we then approximate the eigenvalues of Eq. 4 using the corresponding eigenvector u as follows:

$$\begin{aligned} \lambda_{\max}(\Sigma_{-k}) &\approx \lambda_{\max}(\Sigma_{\text{ID}} + \Delta_k) \approx \lambda_{\max}(\Sigma_{\text{ID}}) + u_{\max}^\top \Delta_k u_{\max}, \\ \lambda_{\min}(\Sigma_{-k}) &\approx \lambda_{\min}(\Sigma_{\text{ID}} + \Delta_k) \approx \lambda_{\min}(\Sigma_{\text{ID}}) + u_{\min}^\top \Delta_k u_{\min}. \end{aligned} \quad (5)$$

We denote $\delta_{\max}^{(k)} := u_{\max}^\top \Delta_k u_{\max}$, $\delta_{\min}^{(k)} := u_{\min}^\top \Delta_k u_{\min}$, capturing the sensitivity of the eigenvalues to the removal of data point x_k in Eq. 5. To evaluate how well this approximation estimates the eigenvalue shift, we conducted an experiment using each 1K normalized embedding data point. Specifically, we examined the actual difference in eigenvalues with and without x_k and assessed the linearity of our proposed approximation. Figure 3 demonstrates a consistent linear relationship between the actual eigenvalue difference and the estimated value. We observed that the difference in eigenvalues can be approximated using our proposed method.

Leveraging this insight, we approximate the difference in the eigenvalue term from Eq. 3 by directly substituting the approximation of the difference in eigenvalues. More concretely, when considering the covariance matrix without data instance k , we replace its eigenvalue with $\delta_{\max}^{(k)}, \delta_{\min}^{(k)}$. Then, by applying a Taylor expansion, we can use the resulting approximated terms, together with the eigenvalues of the full data covariance matrix, to approximate the marginal value of data point x_k to the domain discrepancy.

Theorem 2. Let $\lambda_{\max}(\Sigma_{\text{ID}})$ and $\lambda_{\min}(\Sigma_{\text{ID}})$ be the maximum and minimum eigenvalues of the covariance matrix Σ_{ID} , and let $\delta_{\max}^{(k)}$ and $\delta_{\min}^{(k)}$ be the changes in these eigenvalues due to the perturbation

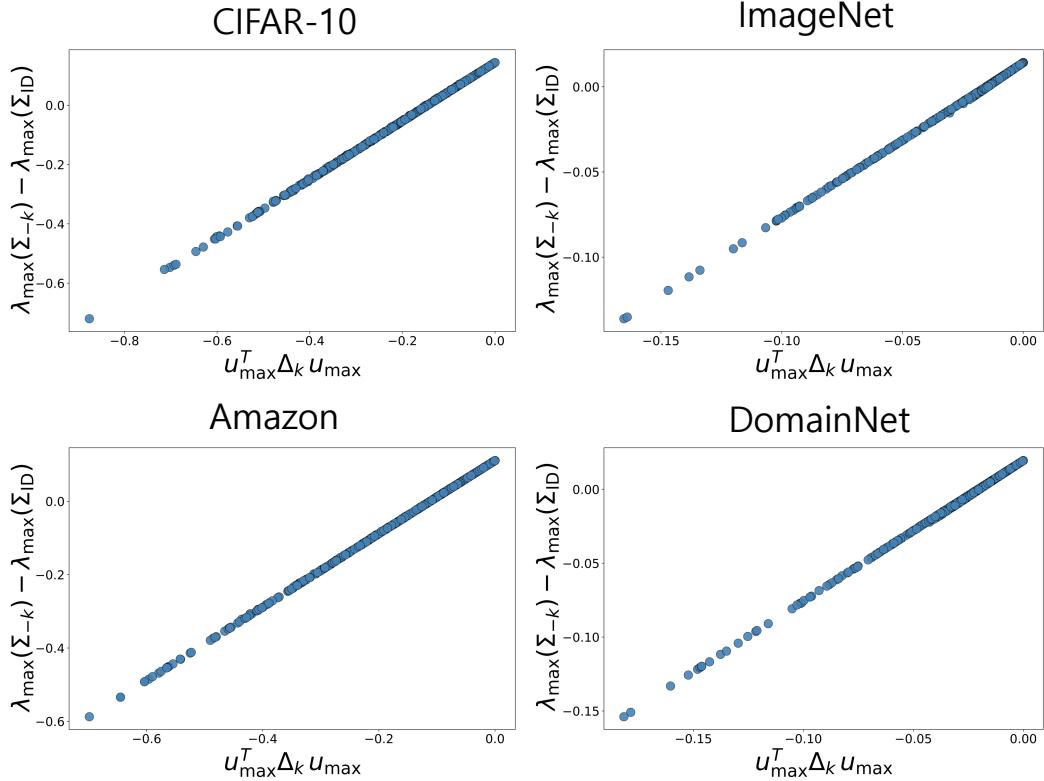


Figure 3: Relation between approximation values $u_{\max}^T \Delta_k u_{\max}$ in Eq. 5 and real values $\lambda_{\max}(\Sigma_{-k}) - \lambda_{\max}(\Sigma_{ID})$ for CIFAR-10, ImageNet, Amazon Reviews - Books and DomainNet - Real embedding datasets. This demonstrates that eigenvalue differences can be accurately approximated using our proposed method, highlighting its effectiveness in capturing spectral variations.

caused by removing data point x_k . The marginal value of data point x_k is then given by:

$$f(\Sigma_{-k}) - f(\Sigma_{ID}) \approx \frac{\sqrt{d} \times \delta_{\max}^{(k)}}{\lambda_{\min}(\Sigma_{ID})} - \frac{(\lambda_{\max}(\Sigma_{ID}) \times \sqrt{d} + \sqrt{d^2 - d}) \times \delta_{\min}^{(k)}}{\lambda_{\min}(\Sigma_{ID})^2}$$

where $f(\Sigma_{ID})$ is approximated domain discrepancy term in RHS of Eq. 3, with its derivation given in Appendix A.3 .

By leveraging this approximation, we can estimate the marginal value of each data point using perturbation theory without having to repeatedly perform costly eigendecompositions. This makes OOD-robust data valuation computationally efficient and feasible for practical deployment.

4.3 Eigen-Value: Plug and Play for ID Data Valuation Methodologies

Since the calculation in Section 4.2 pertains to the marginal value of domain discrepancy, it must be used in conjunction with the marginal value for ID loss. Leveraging this, we incorporated the eigenvalue-based term into existing data valuation methodologies for ID loss to perform marginal data value $V(x_k, y_k)$ for OOD loss,

$$\begin{aligned}
V(x_k, y_k) &= \mathcal{L}_{\text{ID}}[S_{-k}] + f(\Sigma_{-k}) - f(\Sigma_{\text{ID}}) \\
&\approx \mathcal{L}_{\text{ID}}[S_{-k}] - \frac{\sqrt{d} \times \lambda_{\max}(\Sigma_{\text{ID}}) + \sqrt{d^2 - d}}{\lambda_{\min}(\Sigma_{\text{ID}})^2} \delta_{\min}^{(k)} \\
&\quad + \frac{\sqrt{d}}{\lambda_{\min}(\Sigma_{\text{ID}})} \delta_{\max}^{(k)}
\end{aligned} \tag{6}$$

where $\mathcal{L}_{\text{ID}}[S_{-k}]$ is the marginal data value of other methods for ID loss.

5 Experiments

We evaluate EV in three parts. (1) **Cross-domain data removal and point addition.** We compute values on a source domain and measure performance on a different target domain to test whether the valuation is useful for selection. (2) **Stability and efficiency.** To assess stability, we repeatedly alter a small subset of training samples and measure the variance in the resulting value rankings. We also compare computation time across methods. (3) **Qualitative analysis.** Top-ranked samples capture semantically invariant features of each class as well as exhibit broader dispersion in the embedding space, as confirmed in the qualitative top-3 examples. In contrast, low-ranked samples form redundant clusters and often miss such robust cues, explaining why EV enhances generalization under domain shift. All valuations are computed using our implementation based on the OpenDataVal [Jiang et al. \[2023\]](#). Our code is available at <https://github.com/MLAI-Yonsei/Eigen-Value>.

5.1 Experimental Settings

Baselines. We compare our proposed method against several baseline approaches for data valuation: (a) **Random:** Assigns data values randomly from a uniform distribution $U(0, 1)$. (b) **InfluenceFunction** [Feldman and Zhang \[2020\]](#): Estimates the influence of an individual training example on the validation dataset by computing closely related sub-sampled influence. (c) **Deviation:** Compute data values using the distributionally robust generalization error (DRGE) based on NTK. (d) **LAVA:** A model-agnostic data valuation method that utilizes the class-wise Wasserstein distance. (e) **KNN Shapley:** Computes Shapley values using the K-Nearest Neighbors (KNN) approach. (f) **Data-OOB:** Employs the Out-of-Bag (OOB) technique to estimate data values. (g) **Eigen-Value:** Our proposed approach EV, we conducted experiments applying other methods (LAVA, KNN Shapley, Data-OOB).

Datasets. We conduct experiments on the following real-world datasets: (a) **CIFAR-10** [Krizhevsky et al. \[2009\]](#): A widely used image classification dataset consisting of 60K images across 10 classes. (b) **CIFAR-10 C** [Hendrycks and Dietterich \[2019\]](#): A variant of CIFAR-10 where common corruptions are applied, resulting in an image dataset with a distribution shift from the original data. (c) **VLCS** [Fang et al. \[2013\]](#): A dataset comprising images from four distinct domains (VOC2007, LabelMe, Caltech101, SUN09), all sharing the same label space. (d) **Amazon Reviews** [Hou et al. \[2024\]](#): Amazon user product review data, which is organized into domains by product category. We convert the original 5-point ratings into three sentiment classes. In this work, we focus on the Books, Electronics, and Home and Kitchen domains. (e) **ImageNet** [Deng et al. \[2009\]](#): A large-scale dataset of labeled natural images spanning thousands of object categories, widely used for visual recognition research. In this work, we use several of its derived domains: V2 [Recht et al. \[2019\]](#), S [Gao et al. \[2022\]](#), R [Hendrycks et al. \[2021a\]](#), and A [Hendrycks et al. \[2021b\]](#). (f) **DomainNet** [Peng et al. \[2019\]](#): A benchmark dataset designed to evaluate cross-domain generalization, comprising images from six distinct domains covering the same set of object categories.

Setting. Our method views the upper bound of OOD loss as a utility to pick ID samples that best improve cross-domain generalization. We computed EV scores from normalized ID embeddings, combined them with ID loss-based valuations. Based on the integrated scores, we curated a subset of the ID data to train a logistic regression model evaluated on a target domain. For example, in VLCS, we designate SUN09 as the target domain. Data valuation and validation use the remaining three domains, and testing is performed solely on SUN09. Throughout this process, we use embeddings extracted from either ResNet50 He et al. [2016] and ViT-B/16 Dosovitskiy et al. [2020] for image-based datasets, and RoBERTa-base Liu et al. [2019] for text-based datasets such as Amazon Reviews.

Acc(%) (↓)	CIFAR-10 C	VLCS				Amazon Reviews		
		Caltech101	LabelMe	SUN09	VOC2007	Books	Electronics	H and K
Random	47.01	95.90	63.42	69.56	72.33	82.87	70.47	76.47
InfluenceFunction	47.84	96.96	62.36	70.81	67.97	82.27	70.30	76.17
Deviation	46.57	97.31	62.10	68.12	73.48	85.32	76.87	76.4
LAVA	46.30	97.17	63.23	75.68	72.15	82.37	70.55	76.50
KNN Shapley	38.84	92.57	59.04	55.85	52.39	65.55	60.92	75.17
Data-OOB	44.67	76.18	62.70	62.94	56.72	73.97	64.52	74.45
EV + LAVA	43.96	93.78	62.48	49.60	70.17	81.62	70.35	75.95
EV + KNN Shapley	38.68	85.86	58.14	49.08	52.19	55.85	47.92	69.67
EV + Data-OOB	43.65	75.40	62.48	62.79	56.54	56.15	48.12	67.85

Table 1: Data removal experiment. Train the model with 50% of the data, which is the lowest data value in the ID set, and evaluate the performance on different domain data. **Lower is better.** The proposed method, which integrates EV with an existing approach, demonstrates strong performance. These results suggest that augmenting ID data valuation methods with EV provides a clearer guarantee of OOD performance compared to the Deviation approach. (*H and K stands for Amazon Reviews Home and Kitchen domain.)

5.2 Cross Domain Experiment

Data Removal. In the data removal experiment, we evaluate whether a valuation method can correctly identify low utility samples. From a pool of 2K ID data points, we randomly sample 1K for training and compute the value of each sample using each method. For every valuation baseline, we discard 50% of samples assigned the highest values and train the model using only the remaining half. The model is then evaluated on OOD data. Since the retained training set is composed of data deemed less valuable, a larger accuracy drop indicates that the method was more effective at flagging uninformative samples. As shown in Table 1, augmenting each baseline with EV consistently reduces the performance drop. Notably, EV + KNN Shapley achieves the best performance in all but two domains, where EV + Data-OOB outperforms all other approaches. These results highlight that EV provides a clearer guarantee of OOD robustness than existing alternatives such as Deviation.

To examine scalability beyond controlled small-scale settings and to test performance on more realistic and complex distributions, we further apply the same protocol to two large-scale benchmarks, ImageNet and DomainNet. Specifically, we subsample 30K images from ImageNet and 10K from DomainNet, compute per-sample values with all baselines in Table 1, and repeat the data removal procedure. For scalability reasons, we omit LAVA and Deviation due to their prohibitive computational costs. On ImageNet, we evaluate across four domain shifts (V2, Sketch, Rendition, Real), while on DomainNet we report results averaged across six shifts (Clipart, Infograph, Painting, Quickdraw, Real, Sketch), with per-domain details provided in Appendix C.2. As summarized in Table 2, the EV-augmented methods once again yield the lowest error in almost every target domain, confirming that our approach scales effectively and delivers superior robustness under substantial distribution shifts.

Method	ImageNet				DomainNet
	V2	S	R	A	Avg.
Random	65.50	28.26	29.87	8.97	22.73
InfluenceFunction	65.60	28.36	29.54	8.72	22.04
KNN Shapley	40.39	18.03	16.95	7.82	17.76
Data-OOB	59.22	23.89	25.08	6.54	11.76
EV + KNN Shapley	40.34	17.97	16.91	7.81	17.04
EV + Data-OOB	54.76	21.82	22.75	5.37	11.01

Table 2: Data removal experiment. Train the model with 50% of the data, which is the lowest data value in the ID set, and evaluate performance on different domain data. **Lower is better.** EV augmented variants consistently achieve the lowest error, which means EV achieves stronger OOD robustness than other methods. Because of their prohibitive time complexity on large, high-cardinality datasets, Deviation and LAVA are omitted.

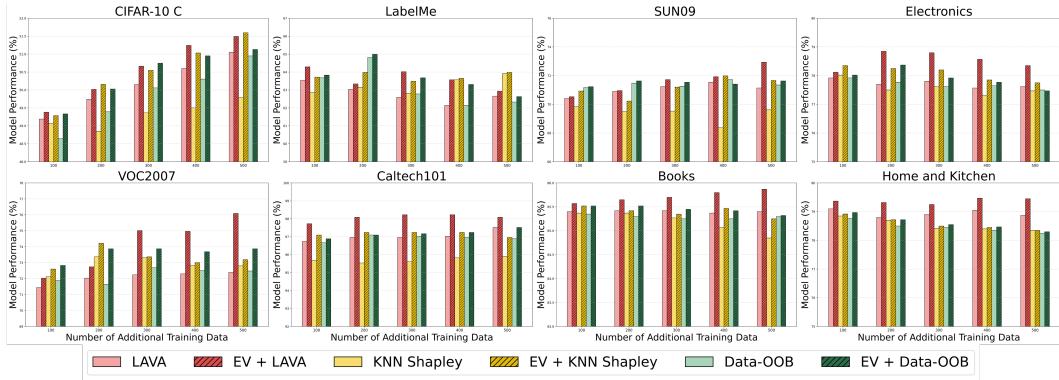


Figure 4: Performance comparison on OOD dataset, adding the highest data value of the remaining set. The hatched bars represent the performance of other methods when EV is applied. Results show that adding EV improves performance and enhances the robustness to OOD data. It highlights how selecting data based on our valuation approach can guide data inclusion in continual or online learning scenarios, where identifying the most beneficial data is crucial.

Point Addition. In the point addition experiment, we simulate a scenario where additional ID data is incrementally incorporated into training to assess its effect on OOD generalization. We begin by randomly sampling 2K ID data points and computing data values with each valuation method. From this pool, we construct an initial training set of 1K samples and then gradually expand it by adding the highest-valued samples from the remaining 1K points in descending order, retraining the model after each addition. This process is repeated on CIFAR-10 (evaluated on CIFAR-10 C), VLCS, and Amazon Reviews, thereby covering both vision and text domains. Figure 4 reports the performance after each addition step. Solid bars represent the baseline methods, and hatched bars indicate their results when EV is applied. Across domains, EV consistently yields higher accuracy and robustness to distribution shifts. These results show that incorporating EV into data valuation effectively guides data selection toward more informative and resilient samples. Overall, the point-addition experiment demonstrates that EV provides a principled criterion for selecting additional ID data that yields the greatest benefit under OOD evaluation. This makes EV particularly suitable for continual or online learning scenarios, where deciding which incoming samples to prioritize is a critical challenge.

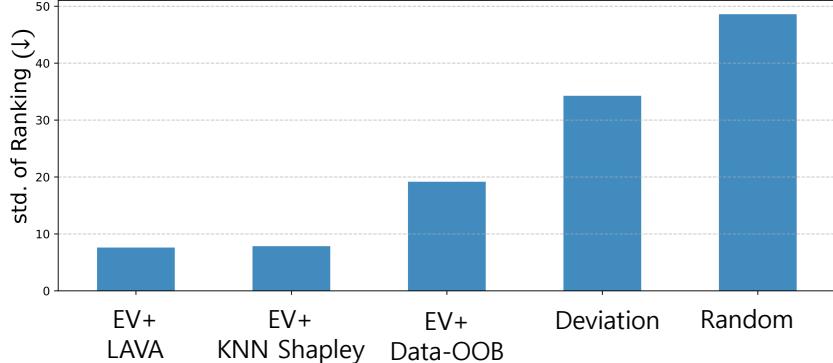


Figure 5: Stability under training-set perturbations. We conduct a valuation on 300 CIFAR-10 samples five times, keeping 290 fixed and resampling 10. Deviation’s data-value ranking exhibits a standard deviation comparable to random selection, whereas EV yields stable, efficient rankings while retaining ID-based valuation and strong OOD performance.

5.3 Stability and Efficiency of Eigen-Value

Instability Ranking. A good data valuation method must be stable to small changes in the subset. Deviation estimates worst-case distribution error using NTK analysis by constructing a separate leave-one-out NTK matrix for each sample, which amplifies sensitivity to dataset composition and leads to unstable rankings. To evaluate stability, we fixed 290 out of 300 training samples and randomly replaced the remaining 10, repeating this procedure five times to compute the standard deviation of rankings. Ideally, if only a small portion of the dataset changes, the rankings of the fixed samples should vary within that range. However, as shown in Figure 5, Deviation exhibits much larger fluctuations, with the standard deviation of rankings approaching that of random selection. EV quantifies each sample’s contribution to domain discrepancy through covariance eigenvalues and produces consistent rankings. This stability makes EV more reliable for real-world data markets.

Time Comparison. Another novelty of our approach is that it performs data valuation operations, yielding high performance within a short time. Although Deviation is a data valuation method that considers OOD data, it inverts an $n - 1 \times n - 1$ matrix for each data point, resulting in a prohibitive computational complexity. This excessive computational burden, which grows cubically with dataset size, makes the method practically infeasible and limits its scalability in real-world applications. In contrast, our method adds only a small amount of additional computation compared to existing methods while demonstrating superior OOD performance. As shown in Figure 6, the methods augmented with EV achieve better OOD performance with minimal overhead of computing approximate eigenvalues for 2K samples, and take less than 1 second. In contrast, Deviation requires nearly 30 minutes and lacks OOD robustness.

5.4 Qualitative Analysis

Top-3 Sample Analysis with and without EV. Previous quantitative experiments have demonstrated that EV improves domain robustness. To further investigate the reason, we conduct a qualitative analysis by examining which samples are assigned high data values. Specifically, we compare the three highest valued images selected by Data-OOB alone and by EV + Data-OOB. Since domain robustness requires capturing invariant features essential for stability under domain shift, this comparison directly reveals how the two approaches differ in practice. We focus on the dog sled class in ImageNet, corresponding to the results in Table 2. As shown in Figure 7, Data-OOB often fails to capture invariant features; some images show only dogs, while others include a sled

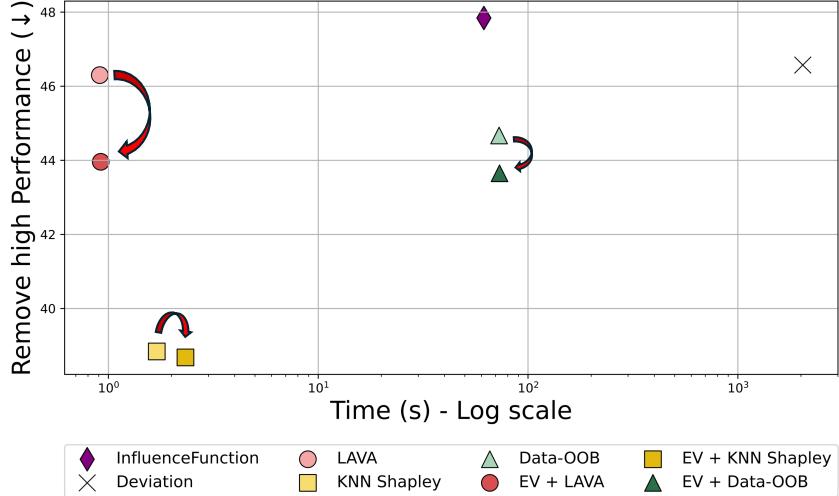


Figure 6: Time comparison on data valuation methods. Performance on CIFAR-10 C from Table 1, based on valuations and computation time using 2K samples of CIFAR-10. Despite its minimal overhead, EV outperforms Deviation in OOD robustness.



(a) Top-3 samples selected by Data-OOB. Some images fail to capture invariant structures (e.g., dogs without sleds or unclear pulling). (b) Top-3 samples selected by EV + Data-OOB. EV consistently highlights dogs visibly pulling a sled.

Figure 7: Qualitative comparison of the top-3 ranked images in the dog sled class, selected according to data values from (a) Data-OOB and (b) EV + Data-OOB.

without clear pulling. In contrast, when EV is incorporated, the top-3 samples consistently highlight the defining invariant feature of the class, dogs visibly pulling a sled. This observation provides an intuitive explanation for why EV enhances OOD robustness, and similar patterns were observed across other classes as well.

Impact of EV. In our PCA projection analysis, we visualize the top and bottom 1K samples of ImageNet as ranked by each valuation method, using the valuation results reported in Table 2. Figure 8 shows these samples, selected by Data-OOB and EV + Data-OOB, projected onto the top three principal components. For robust OOD performance, it is preferable to train on samples that are broadly distributed in the feature space, rather than narrowly clustered. To assess the impact of EV, we compared the variance of high- and low-value samples. Incorporating the EV term increased the variance of top-ranked as well as enlarged the gap between top- and bottom-ranked groups (Top: 1.09 vs. Bottom: 0.67 in EV + Data-OOB, compared to Top: 0.38 vs. Bottom: 0.60 in Data-OOB). In fact, Data-OOB even assigns lower variance to top-ranked samples than to bottom-ranked ones, indicating that it sometimes fails to prioritize diverse and informative examples. By contrast, EV consistently highlights widely dispersed, high-variance samples, while keeping low-value samples more concentrated. As a result, models trained on EV-selected data are exposed to richer and more representative features, which enhances their robustness under distribution shifts.

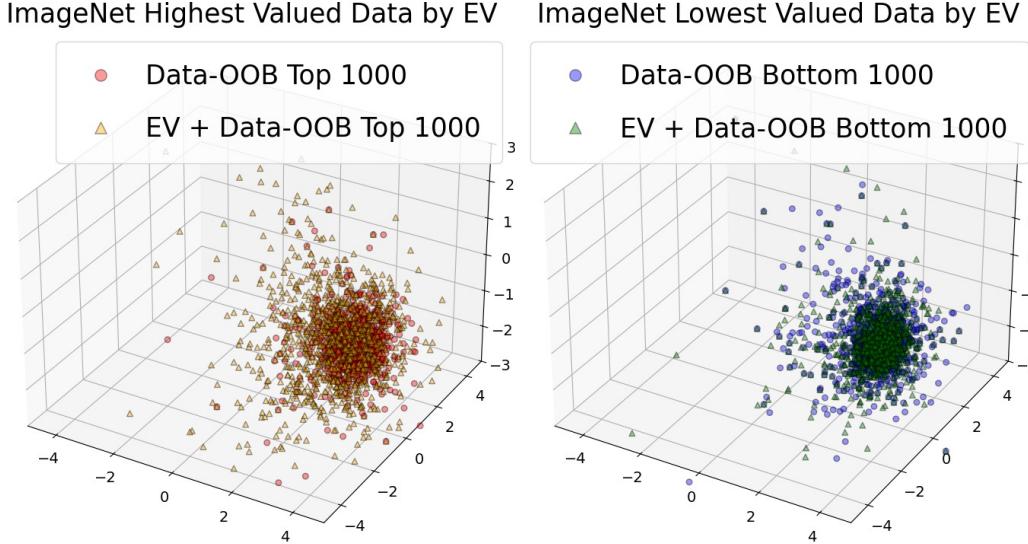


Figure 8: PCA projection of top-1K and bottom-1K CIFAR-10 samples selected by Data-OOB valuation with and without EV. Incorporating EV leads to a greater variance gap (Top: 1.09, Bottom: 0.67 in EV + Data-OOB vs. Top: 0.38, Bottom: 0.60 in Data-OOB), making it easier to identify widely dispersed, high-variance samples. Such samples better cover diverse features, which improves generalization to OOD data with different distributions.

6 Conclusion

In this paper, we propose *Eigen-Value* (EV), an efficient data valuation framework for OOD robustness. By approximating domain discrepancy via eigenvalues and perturbation theory, EV estimates the marginal contribution of each sample to OOD loss. Integrated with existing ID-based methods, it enables OOD-aware data selection without requiring any OOD data. Comprehensive cross-domain experiments on vision and text datasets demonstrate that EV consistently enhances domain robustness while maintaining stability and low computational cost. Qualitative analyses further reveal why EV improves robustness, showing that it prioritizes diverse, invariant features that are critical under distribution shifts. By shifting the focus from model- to data-centric OOD robustness, EV offers a scalable solution with theoretical guarantees linking spectral properties to OOD generalization. Together, these results establish EV as a practical and reliable tool for real-world applications, where robust and efficient data valuation is essential.

References

- Anish Agarwal, Munther Dahleh, and Tuhin Sarkar. A marketplace for data: An algorithmic solution. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, pages 701–726, 2019. [1](#)
- David Alvarez-Melis and Nicolo Fusi. Geometric dataset distances via optimal transport. *Advances in Neural Information Processing Systems*, 33:21428–21439, 2020. [1](#)
- Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In *2009 IEEE conference on computer vision and pattern recognition*, pages 248–255. Ieee, 2009. [5.1](#)
- Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, et al. An image is worth 16x16 words: Transformers for image recognition at scale. *arXiv preprint arXiv:2010.11929*, 2020. [5.1](#)
- Chen Fang, Ye Xu, and Daniel N Rockmore. Unbiased metric learning: On the utilization of multiple datasets and web images for softening bias. In *Proceedings of the IEEE international conference on computer vision*, pages 1657–1664, 2013. [5.1](#)
- Vitaly Feldman and Chiyuan Zhang. What neural networks memorize and why: Discovering the long tail via influence estimation. *Advances in Neural Information Processing Systems*, 33:2881–2891, 2020. [5.1](#)
- Shanghua Gao, Zhong-Yu Li, Ming-Hsuan Yang, Ming-Ming Cheng, Junwei Han, and Philip Torr. Large-scale unsupervised semantic segmentation. *IEEE transactions on pattern analysis and machine intelligence*, 45(6):7457–7476, 2022. [5.1](#)
- Quentin Garrido, Randall Balestrieri, Laurent Najman, and Yann Lecun. Rankme: Assessing the downstream performance of pretrained self-supervised representations by their rank. In *International conference on machine learning*, pages 10929–10974. PMLR, 2023. [2.2](#)
- Amirata Ghorbani and James Zou. Data shapley: Equitable valuation of data for machine learning. In *International conference on machine learning*, pages 2242–2251. PMLR, 2019. [1, 2.1](#)
- Anne Greenbaum, Ren-cang Li, and Michael L Overton. First-order perturbation theory for eigenvalues and eigenvectors. *SIAM review*, 62(2):463–482, 2020. [3](#)
- Elad Hazan, Tomer Koren, and Kfir Y Levy. Logistic regression: Tight bounds for stochastic and online optimization. In *Conference on Learning Theory*, pages 197–209. PMLR, 2014. [1](#)
- Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 770–778, 2016. [5.1](#)
- Dan Hendrycks and Thomas Dietterich. Benchmarking neural network robustness to common corruptions and perturbations. *arXiv preprint arXiv:1903.12261*, 2019. [5.1](#)
- Dan Hendrycks, Steven Basart, Norman Mu, Saurav Kadavath, Frank Wang, Evan Dorundo, Rahul Desai, Tyler Zhu, Samyak Parajuli, Mike Guo, et al. The many faces of robustness: A critical analysis of out-of-distribution generalization. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 8340–8349, 2021a. [5.1](#)
- Dan Hendrycks, Kevin Zhao, Steven Basart, Jacob Steinhardt, and Dawn Song. Natural adversarial examples. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 15262–15271, 2021b. [5.1](#)

- Harold Hotelling. Analysis of a complex of statistical variables into principal components. *Journal of educational psychology*, 24(6):417, 1933. [2.3](#)
- Yupeng Hou, Jiacheng Li, Zhankui He, An Yan, Xiusi Chen, and Julian McAuley. Bridging language and items for retrieval and recommendation. *arXiv preprint arXiv:2403.03952*, 2024. [5.1](#)
- Weihua Hu, Gang Niu, Issei Sato, and Masashi Sugiyama. Does distributionally robust supervised learning give robust classifiers? In *International Conference on Machine Learning*, pages 2029–2037. PMLR, 2018. [1](#)
- Jiayuan Huang, Arthur Gretton, Karsten Borgwardt, Bernhard Schölkopf, and Alex Smola. Correcting sample selection bias by unlabeled data. *Advances in neural information processing systems*, 19, 2006. [3](#)
- Ruoxi Jia, David Dao, Boxin Wang, Frances Ann Hubis, Nezihe Merve Gurel, Bo Li, Ce Zhang, Costas J Spanos, and Dawn Song. Efficient task-specific data valuation for nearest neighbor algorithms. *arXiv preprint arXiv:1908.08619*, 2019. [2.1](#)
- Kevin Jiang, Weixin Liang, James Y Zou, and Yongchan Kwon. Opendataval: a unified benchmark for data valuation. *Advances in Neural Information Processing Systems*, 36:28624–28647, 2023. [5](#)
- Hoang Anh Just, Feiyang Kang, Tianhao Wang, Yi Zeng, Myeongseob Ko, Ming Jin, and Ruoxi Jia. LAVA: Data valuation without pre-specified learning algorithms. In *The Eleventh International Conference on Learning Representations*, 2023. URL <https://openreview.net/forum?id=JJuP86nBl4q>. [2.1](#)
- Tosio Kato. *Perturbation theory for linear operators*, volume 132. Springer Science & Business Media, 2013. [1](#)
- Pang Wei Koh and Percy Liang. Understanding black-box predictions via influence functions. In *International conference on machine learning*, pages 1885–1894. PMLR, 2017. [2.1](#)
- Alex Krizhevsky, Geoffrey Hinton, et al. *Learning multiple layers of features from tiny images*. Toronto, ON, Canada, 2009. [5.1](#)
- Yongchan Kwon and James Zou. Data-oob: Out-of-bag estimate as a simple and efficient data value. In *International conference on machine learning*, pages 18135–18152. PMLR, 2023. [2.1](#)
- Stéphane S Lafon. *Diffusion maps and geometric harmonics*. Yale University, 2004. [2.3](#)
- Yann Le Cun, Ido Kanter, and Sara A Solla. Eigenvalues of covariance matrices: Application to neural-network learning. *Physical review letters*, 66(18):2396, 1991. [1](#)
- Patrick Lewis, Ethan Perez, Aleksandra Piktus, Fabio Petroni, Vladimir Karpukhin, Naman Goyal, Heinrich Kütller, Mike Lewis, Wen-tau Yih, Tim Rocktäschel, et al. Retrieval-augmented generation for knowledge-intensive nlp tasks. *Advances in neural information processing systems*, 33: 9459–9474, 2020. [2.1](#)
- Chih-Jen Lin, Ruby C Weng, and S Sathiya Keerthi. Trust region newton methods for large-scale logistic regression. In *Proceedings of the 24th international conference on Machine learning*, pages 561–568, 2007. [1](#)
- Xiaoqiang Lin, Xinyi Xu, Zhaoxuan Wu, See-Kiong Ng, and Bryan Kian Hsiang Low. Distributionally robust data valuation. In *Forty-first International Conference on Machine Learning*, 2024. [1, 2.1](#)
- Yinhan Liu, Myle Ott, Naman Goyal, Jingfei Du, Mandar Joshi, Danqi Chen, Omer Levy, Mike Lewis, Luke Zettlemoyer, and Veselin Stoyanov. Roberta: A robustly optimized bert pretraining approach. *arXiv preprint arXiv:1907.11692*, 2019. [5.1](#)

Haocheng Luo, Tuan Truong, Tung Pham, Mehrtash Harandi, Dinh Phung, and Trung Le. Explicit eigenvalue regularization improves sharpness-aware minimization. *Advances in Neural Information Processing Systems*, 37:4424–4453, 2024. [2.3](#)

Xingjun Ma, Hanxun Huang, Yisen Wang, Simone Romano, Sarah Erfani, and James Bailey. Normalized loss functions for deep learning with noisy labels. In Hal Daumé III and Aarti Singh, editors, *Proceedings of the 37th International Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pages 6543–6553. PMLR, 13–18 Jul 2020. URL <https://proceedings.mlr.press/v119/ma20c.html>. [4.1](#)

Andrzej Maćkiewicz and Waldemar Ratajczak. Principal components analysis (pca). *Computers & Geosciences*, 19(3):303–342, 1993. [2.3](#)

Irshad Mohammed, Uroš Seljak, and Zvonimir Vlah. Perturbative approach to covariance matrix of the matter power spectrum. *Monthly Notices of the Royal Astronomical Society*, 466(1):780–797, 2017. [3](#)

Jose G Moreno-Torres, Troy Raeder, Rocío Alaiz-Rodríguez, Nitesh V Chawla, and Francisco Herrera. A unifying view on dataset shift in classification. *Pattern recognition*, 45(1):521–530, 2012. [3](#)

Julio Moro and Froilán M Dopico. First order eigenvalue perturbation theory and the newton diagram. *Applied Mathematics and Scientific Computing*, pages 143–175, 2003. [3](#)

Changdae Oh, Hyesu Lim, Mijoo Kim, Dongyoon Han, Sangdoo Yun, Jaegul Choo, Alexander Hauptmann, Zhi-Qi Cheng, and Kyungwoo Song. Towards calibrated robust fine-tuning of vision-language models. *Advances in Neural Information Processing Systems*, 37:12677–12707, 2024. [2.2](#), [2.3](#)

Xingchao Peng, Qinxun Bai, Xide Xia, Zijun Huang, Kate Saenko, and Bo Wang. Moment matching for multi-source domain adaptation. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 1406–1415, 2019. [5.1](#)

Hamed Rahimian and Sanjay Mehrotra. Frameworks and results in distributionally robust optimization. *Open Journal of Mathematical Optimization*, 3:1–85, 2022. [1](#)

Benjamin Recht, Rebecca Roelofs, Ludwig Schmidt, and Vaishaal Shankar. Do imagenet classifiers generalize to imagenet? In *International conference on machine learning*, pages 5389–5400. PMLR, 2019. [5.1](#)

Alvin E Roth. *The Shapley value: essays in honor of Lloyd S. Shapley*. Cambridge University Press, 1988. [1](#)

Bernhard Schölkopf, Alexander Smola, and Klaus-Robert Müller. Kernel principal component analysis. In *International conference on artificial neural networks*, pages 583–588. Springer, 1997. [2.3](#)

Lloyd S Shapley et al. *A value for n-person games*. Princeton University Press Princeton, 1953. [1](#)

Hidetoshi Shimodaira. Improving predictive inference under covariate shift by weighting the log-likelihood function. *Journal of statistical planning and inference*, 90(2):227–244, 2000. [3](#)

Fatimah Sidi, Payam Hassany Shariat Panahy, Lilly Suriani Affendey, Marzanah A Jabar, Hamidah Ibrahim, and Aida Mustapha. Data quality: A survey of data quality dimensions. In *2012 International Conference on Information Retrieval & Knowledge Management*, pages 300–304. IEEE, 2012. [2.1](#)

Rachael Hwee Ling Sim, Xinyi Xu, and Bryan Kian Hsiang Low. Data valuation in machine learning: "ingredients", strategies, and open challenges. In *IJCAI*, pages 5607–5614, 2022. [1](#), [2.1](#)

- Matthew Staib and Stefanie Jegelka. Distributionally robust optimization and generalization in kernel methods. *Advances in Neural Information Processing Systems*, 32, 2019. 1
- Masashi Sugiyama, Matthias Krauledat, and Klaus-Robert Müller. Covariate shift adaptation by importance weighted cross validation. *Journal of Machine Learning Research*, 8(5), 2007. 3
- Naonori S Sugiyama, Shun Saito, Florian Beutler, and Hee-Jong Seo. Perturbation theory approach to predict the covariance matrices of the galaxy power spectrum and bispectrum in redshift space. *Monthly Notices of the Royal Astronomical Society*, 497(2):1684–1711, 2020. 3
- Zhihua Tian, Jian Liu, Jingyu Li, Xinle Cao, Ruoxi Jia, Jun Kong, Mengdi Liu, and Kui Ren. Private data valuation and fair payment in data marketplaces. *arXiv preprint arXiv:2210.08723*, 2022. 1
- Ulrike Von Luxburg. A tutorial on spectral clustering. *Statistics and computing*, 17:395–416, 2007. 2.3
- Mitchell Wortsman, Gabriel Ilharco, Jong Wook Kim, Mike Li, Simon Kornblith, Rebecca Roelofs, Raphael Gontijo Lopes, Hannaneh Hajishirzi, Ali Farhadi, Hongseok Namkoong, et al. Robust fine-tuning of zero-shot models. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 7959–7971, 2022. 2.2
- Jinda Xu, Yuhao Song, Daming Wang, Weiwei Zhao, Minghua Chen, Kangliang Chen, and Qinya Li. Quality over quantity: Boosting data efficiency through ensembled multimodal data curation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pages 21761–21769, 2025. 1
- Prateek Yadav, Derek Tam, Leshem Choshen, Colin A Raffel, and Mohit Bansal. Ties-merging: Resolving interference when merging models. *Advances in Neural Information Processing Systems*, 36:7093–7115, 2023. 2.2
- Runtian Zhai, Chen Dan, Zico Kolter, and Pradeep Ravikumar. Doro: Distributional and outlier robust optimization. In *International Conference on Machine Learning*, pages 12345–12355. PMLR, 2021. 2.1

A Theoretical Analysis

This section provides the theoretical proof of EV, as detailed in Section 4 Efficient Domain Robust Data Valuation.

A.1 Estimating Domain Discrepancy using Eigenvalue Shifts Induced by NCE

Normalized Cross-Entropy (NCE) is defined as

$$\text{NCE}(\theta) = \frac{-\sum_{k=1}^K q(y=k|x) \log p_\theta(k|x)}{-\sum_{j=1}^K \sum_{k=1}^K q(y=j|x) \log p_\theta(k|x)}, \quad (7)$$

with $0 \leq \text{NCE}(\theta) \leq 1$.

Since model parameter θ is trained on in-distribution (ID) data, it is assumed that NCE on OOD data (NCE_{OOD}) is larger than NCE on ID data (NCE_{ID})

$$0 < \text{NCE}_{\text{ID}}(\theta) \leq \text{NCE}_{\text{OOD}}(\theta) \leq 1.$$

Using the above relation, the domain discrepancy between OOD and ID can be defined as

$$\begin{aligned} \Gamma(\mathcal{D}_{\text{OOD}}, \mathcal{D}_{\text{ID}}) &= \sup_{\theta} \left(\text{NCE}_{\text{OOD}}(\theta) - \text{NCE}_{\text{ID}}(\theta) \right) \\ &\leq \sup_{\theta} \frac{\text{NCE}_{\text{OOD}}(\theta)}{\text{NCE}_{\text{ID}}(\theta)}. \end{aligned} \quad (8)$$

Assuming an optimal model θ_0 for both domains, a Taylor expansion around θ_0 yields

$$\begin{aligned} \text{NCE}(\theta) &\approx \text{NCE}(\theta_0) + (\theta - \theta_0)^\top \nabla_{\theta} \text{NCE}(\theta_0) \\ &\quad + \frac{1}{2} (\theta - \theta_0)^\top \nabla_{\theta}^2 \text{NCE}(\theta_0) (\theta - \theta_0). \end{aligned} \quad (9)$$

Since $\text{NCE}(\theta_0) \approx 0$ and $\nabla_{\theta} \text{NCE}(\theta_0) \approx 0$, it follows that

$$\text{NCE}(\theta) \approx \frac{1}{2} (\theta - \theta_0)^\top H (\theta - \theta_0),$$

where $H := \nabla_{\theta}^2 \text{NCE}(\theta_0)$.

Thus, the ratio can be approximated by Hessiansian of each distribution ($H_{\text{OOD}}, H_{\text{ID}}$)

$$\sup_{\theta} \frac{\text{NCE}_{\text{OOD}}(\theta)}{\text{NCE}_{\text{ID}}(\theta)} \approx \sup_{\theta} \frac{\frac{1}{2} (\theta - \theta_0)^\top H_{\text{OOD}} (\theta - \theta_0)}{\frac{1}{2} (\theta - \theta_0)^\top H_{\text{ID}} (\theta - \theta_0)}. \quad (10)$$

Using the Rayleigh quotient property, for any nonzero vector $v \in \mathbb{R}^d$

$$\lambda_{\min}(H) \leq \frac{v^\top H v}{v^\top v} \leq \lambda_{\max}(H).$$

Then, under the assumption that the Hessian is positive semi-definite, we approximate the ratio of NCE between distributions using the ratio of their maximum (λ_{\max}) and minimum (λ_{\min}) eigenvalues.

$$\frac{\text{NCE}_{\text{OOD}}(\theta)}{\text{NCE}_{\text{ID}}(\theta)} \leq \frac{\lambda_{\max}(H_{\text{OOD}})}{\lambda_{\min}(H_{\text{ID}})}. \quad (11)$$

A.2 Using Logistic Regression Hessian as a Covariance Approximation

In logistic regression, the negative log-likelihood is given by

$$-\ell(\theta) = -\sum_{i=1}^n \left[y_i \log \sigma(\theta^\top x_i) + (1 - y_i) \log(1 - \sigma(\theta^\top x_i)) \right].$$

Thus, the Hessian of the NCE (a variant of logistic regression) is upper bounded by the covariance matrix $\Sigma \in \mathbb{R}^{d \times d}$

$$\begin{aligned} H &= \sum_{i=1}^n \sigma(\theta_0^\top x_i) (1 - \sigma(\theta_0^\top x_i)) x_i x_i^\top \\ &\leq \frac{1}{4} \sum_{i=1}^n x_i x_i^\top = \frac{1}{4} \Sigma, \end{aligned} \tag{12}$$

since $\sigma(\theta_0^\top x_i)(1 - \sigma(\theta_0^\top x_i)) \leq \frac{1}{4}$.

Thus, it follows that

$$\frac{\text{NCE}_{\text{OOD}}}{\text{NCE}_{\text{ID}}} \leq \frac{\lambda_{\max}(H_{\text{OOD}})}{\lambda_{\min}(H_{\text{ID}})} \leq \frac{\lambda_{\max}(\Sigma_{\text{OOD}})}{\lambda_{\min}(\Sigma_{\text{ID}})}, \tag{13}$$

where Σ_{ID} and Σ_{OOD} are covariance matrices of data from ID and OOD, respectively.

With the eigendecomposition $\Sigma = Q \Lambda Q^\top$, the Frobenius norm is given by

$$\|\Sigma\|_F^2 = \text{tr}(\Sigma \Sigma^\top) = \text{tr}(\Lambda \Lambda^\top) = \sum \text{(eigenvalues)}^2. \tag{14}$$

In addition, we have the inequality

$$\sqrt{\lambda_{\max}^2(\Sigma)} \leq \|\Sigma\|_F \leq \sqrt{\text{rank}(\Sigma) \cdot \lambda_{\max}^2(\Sigma)}. \tag{15}$$

We assume that the ID and OOD covariance matrices satisfy the matching marginal condition, which means that the two distributions have identical marginal variances. In other words, the diagonal elements of their covariance matrices are the same, although the off-diagonal entries may differ. This condition preserves the variances of individual features across domains while allowing feature correlations to vary. In our study, this assumption is reasonable because we use normalized embeddings, which naturally align marginal variances. We empirically validate this condition on real datasets in Appendix C.1. This condition is formalized as:

$$\Sigma_{\text{OOD}} = \Sigma_{\text{ID}} + E,$$

where $E \in \mathbb{R}^{d \times d}$ is a matrix that captures domain-specific differences. By assumption, E has zero diagonal entries and non-zero off-diagonal entries, meaning it only affects feature correlations while preserving individual feature variances. By the triangle inequality,

$$\|\Sigma_{\text{OOD}}\|_F \leq \|\Sigma_{\text{ID}}\|_F + \|E\|_F,$$

and if $\|E\|_F \leq \sqrt{d^2 - d}$ (with $|E_{ij}| \leq 1$ for $i \neq j$), one can bound the maximum singular value of Σ_{OOD} . This leads to the bound with \mathcal{L}_{OOD} and \mathcal{L}_{ID} , which are losses of θ on OOD data and ID data, respectively.

$$\mathcal{L}_{\text{OOD}} \leq \mathcal{L}_{\text{ID}} + \frac{\lambda_{\max}(\Sigma_{\text{ID}}) \times \sqrt{d} + \sqrt{d^2 - d}}{\lambda_{\min}(\Sigma_{\text{ID}})}. \tag{16}$$

A.3 Approximating Marginal Contributions of the Eigenvalue Term

Problem Statement: How can we use perturbation to compute the marginal value of a data point?

Given a normalized embedding dataset $\{x_1, x_2, \dots, x_n\}$ of ID, the covariance matrix is defined as

$$\Sigma_{\text{ID}} = \frac{1}{n} \sum_{i=1}^n x_i x_i^\top. \quad (17)$$

When one data point x_k is removed, the new covariance matrix becomes

$$\Sigma_{-k} = \frac{1}{n-1} \sum_{i \neq k} x_i x_i^\top = \frac{n}{n-1} (\Sigma_{\text{ID}} + \Delta_k) \approx \Sigma_{\text{ID}} + \Delta_k, \quad (18)$$

where $\Delta_k = -\frac{1}{n} x_k x_k^\top$, $\frac{n}{n-1} \approx 1$.

Let $\lambda_{\max}(\Sigma_{\text{ID}})$ and $\lambda_{\min}(\Sigma_{\text{ID}})$ be the maximum and minimum eigenvalues of Σ_{ID} , with corresponding normalized eigenvectors u_{\max} and u_{\min} . A first-order perturbation yields

$$\begin{aligned} \lambda_{\max}(\Sigma_{-k}) &\approx \lambda_{\max}(\Sigma_{\text{ID}} + \Delta_k) \approx \lambda_{\max}(\Sigma_{\text{ID}}) + u_{\max}^\top \Delta_k u_{\max}, \\ \lambda_{\min}(\Sigma_{-k}) &\approx \lambda_{\min}(\Sigma_{\text{ID}} + \Delta_k) \approx \lambda_{\min}(\Sigma_{\text{ID}}) + u_{\min}^\top \Delta_k u_{\min}. \end{aligned}$$

Define

$$\delta_{\max}^{(k)} := u_{\max}^\top \Delta_k u_{\max}, \quad \delta_{\min}^{(k)} := u_{\min}^\top \Delta_k u_{\min}.$$

Let $f(\Sigma_{\text{ID}})$ denote the approximated domain discrepancy function from Eq. 16:

$$f(\Sigma_{\text{ID}}) = \frac{\lambda_{\max}(\Sigma_{\text{ID}}) \times \sqrt{d} + \sqrt{d^2 - d}}{\lambda_{\min}(\Sigma_{\text{ID}})}.$$

After removing x_k , we have

$$f(\Sigma_{-k}) \approx \frac{[\lambda_{\max}(\Sigma_{\text{ID}}) + \delta_{\max}^{(k)}] \times \sqrt{d} + \sqrt{d^2 - d}}{\lambda_{\min}(\Sigma_{\text{ID}}) + \delta_{\min}^{(k)}}. \quad (19)$$

Define

$$A = \lambda_{\max}(\Sigma_{\text{ID}}) \times \sqrt{d} + \sqrt{d^2 - d}, \quad B = \lambda_{\min}(\Sigma_{\text{ID}}).$$

A first-order expansion of the denominator gives:

$$\frac{1}{B + \delta_{\min}^{(k)}} \approx \frac{1}{B} \left(1 - \frac{\delta_{\min}^{(k)}}{B} \right).$$

Thus,

$$\begin{aligned} f(\Sigma_{-k}) &\approx \frac{A + \sqrt{d} \times \delta_{\max}^{(k)}}{B} \left(1 - \frac{\delta_{\min}^{(k)}}{B} \right) \\ &\approx \frac{A}{B} + \frac{\sqrt{d} \times \delta_{\max}^{(k)}}{B} - \frac{A \times \delta_{\min}^{(k)}}{B^2}. \end{aligned} \quad (20)$$

Therefore, the change in the function, which approximates the marginal OOD-robust data value of x_k , is

$$f(\Sigma_{-k}) - f(\Sigma_{\text{ID}}) \approx \frac{\sqrt{d} \times \delta_{\max}^{(k)}}{B} - \frac{A \times \delta_{\min}^{(k)}}{B^2} = \frac{\sqrt{d} \times \delta_{\max}^{(k)}}{\lambda_{\min}(\Sigma_{\text{ID}})} - \frac{(\lambda_{\max}(\Sigma_{\text{ID}}) \times \sqrt{d} + \sqrt{d^2 - d}) \times \delta_{\min}^{(k)}}{\lambda_{\min}(\Sigma_{\text{ID}})^2}. \quad (21)$$

The proposed term quantifies the marginal data value with respect to domain discrepancy, rather than the ID loss. Accordingly, it can be integrated into the marginal values derived from existing ID-based data valuation methods. Under the assumption that the OOD loss can be approximated by the sum of the ID loss and domain discrepancy, this enables principled data valuation in OOD settings.

Conclusion: The derivations above demonstrate how domain discrepancy can be bounded by the eigenvalue ratio of the Hessians, how the OOD covariance matrix can be related to the ID covariance matrix, and how perturbation analysis yields an approximation of the marginal data value.

B Additional Experiment Setting

B.1 Dataset

CIFAR-10. A widely used image classification dataset consisting of natural images from ten classes. We use it as the source domain for training.

CIFAR-10 C. A corrupted version of CIFAR-10 that introduces common distribution shifts through 15 corruption types, each with multiple severity levels. We use the 5 severity level. CIFAR-10 serves as the target domain for evaluating robustness under distribution shift.

VLCS. A domain generalization benchmark composed of four visual domains: VOC2007, LabelMe, Caltech101, and SUN09. In each evaluation setting, one domain is held out as the target while the model is trained on the remaining three. The target domain is rotated across all four domains.

Amazon Reviews. A sentiment classification dataset organized by product category, with each category treated as a separate domain. We convert the 5-point rating into three sentiment classes (negative: 1–2, neutral: 3, positive: 4–5) and perform 3-class classification. Models are trained on one or more source categories and evaluated on a disjoint target category to assess cross-domain generalization.

ImageNet. A large-scale image classification dataset with 1,000 classes. For scalability experiments, we use a subset of the training split. Robustness is measured under domain shifts (V2, Sketch, Rendition, Adversarial). For this benchmark, we performed the data valuation experiment using a subset of 30,000 samples.

DomainNet. A large-scale benchmark for multi-domain learning, containing six stylistically distinct domains: clipart, infograph, painting, quickdraw, real, and sketch. We evaluate generalization by holding out one domain as the target and training on the remaining five. For this benchmark, we performed the data valuation experiment using a subset of 2,000 samples for each domain.

B.2 Experiment setting

Evaluation protocols. We use three procedures.

- **Point addition.** We sample 2K in distribution examples. We compute values with each method. We form an initial training set of 1K and retrain while adding the highest value samples from the remaining pool. We evaluate on a different target domain.
- **Data removal.** We sample 1K of 2K in distribution points. We score them, remove the top 50 percent by value, and train on the rest. We evaluate on the target domain. A larger drop in accuracy indicates a better ability to identify low utility samples.
- **Instability.** We assess sensitivity to small changes in the training set. We fix 290 of 300 indices, resample the remaining 10, repeat valuation five times, and compute the standard deviation of value rankings on the fixed indices.

Baselines and parameters. For KNN Shapley, we use a validation set of 1K examples and set the

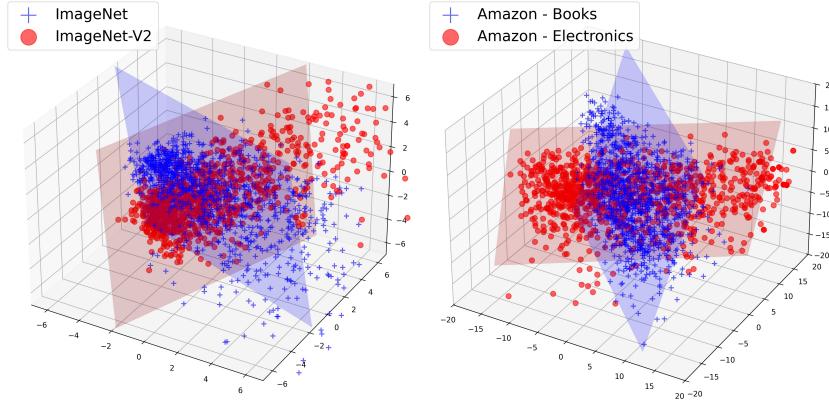


Figure 9: PCA visualization of normalized embeddings sampled (1K each) from different domain sources. The two distributions, corresponding to different domains, partially overlap due to normalization, illustrating that the matching marginal assumption remains applicable in real-world scenarios.

neighborhood size to 1K. For Data-OOB, we follow the original paper with num models = 800. We train a logistic regression classifier for 30 epochs with a learning rate of 0.01.

Hardware setting. We set seed 42 on a single RTX 4090 GPU and an Intel Xeon Gold 6426Y CPU with 32 cores.

B.3 B.2. Weight parameter

EV is combined with a baseline valuation score (V_{EV}). Since the EV term may have a different scale from other methods(V_{base}), we center and scale it using the baseline statistics to make the two terms comparable. Specifically, $\tilde{V}_{EV} = \frac{V_{EV} - \mu_{base}}{\sigma_{base}}$, where $\mu_{base} = \text{mean}(V_{base})$ and $\sigma_{base} = \text{std}(V_{base})$. The final score is $V_{final} = V_{base} + w \tilde{V}_{EV}$. In our experiments, we set $w \leq 1$.

C Supplementary Experiments

C.1 Empirical Validation of the Matching Marginal Assumption

Although the Matching Marginal assumption we used may appear impractical in real-world scenarios, prior work has shown that normalizing embeddings to have zero mean results in the covariance matrices of ID and OOD data sharing identical diagonal elements while differing in off-diagonal elements (Sun et al., Correlation Alignment for Unsupervised Domain Adaptation, in Domain Adaptation in Computer Vision Applications, Springer, 2017). Furthermore, we empirically verified that this condition holds on real-world datasets such as ImageNet and Amazon Reviews, as demonstrated in Figure 9. Specifically, we sampled 1K data points from each domain, computed their normalized embeddings, and projected them into three-dimensional space using PCA. The hyperplanes in the figure represent the PCA subspaces fitted independently to each domain’s embeddings. Despite differences in domain, we observe that normalized embeddings exhibit shared diagonal elements in their covariance matrices, while differing only in their off-diagonal structure. This supports the claim that the assumption employed in our method imposes no critical limitations when applied to real-world data distributions.

Acc (%) (↓)	ImageNet				DomainNet					
	V2	S	R	A	C	I	P	Q	R	S
Random	65.5	28.2	29.9	9.0	26.4	13.7	31.7	2.2	43.8	18.6
InfluenceFunction	65.6	28.3	29.5	8.7	25.6	12.6	30.9	2.2	41.2	19.7
KNN Shapley	40.3	18.0	17.0	7.8	21.5	9.5	24.7	2.7	34.0	14.2
Data-OOB	59.2	23.8	25.1	6.5	15.2	6.0	16.5	2.1	20.4	10.4
EV + KNN Shapley	40.3	17.9	16.9	7.8	20.2	9.2	23.7	2.6	33.0	13.5
EV + Data-OOB	54.7	21.8	22.8	5.4	14.5	5.8	15.1	2.0	18.6	10.2

Table 3: Data removal experiment. Train the model with 50% of the data, which is the lowest data value in the ID set, and evaluate performance on different domain data. **Lower is better.** Across both large and real benchmarks, EV augmented variants consistently achieve the lowest error, which means EV achieves stronger OOD robustness than other methods. Because of their prohibitive time complexity on large, high-cardinality datasets, LAVA and Deviation are omitted.

C.2 Experiment on Large and Difficult Domain Shift Benchmark

We extend the Data Removal experiment from the main paper to more challenging benchmarks. For ImageNet, data valuation was conducted on 30,000 training samples from the train split of ImageNet. For DomainNet, 2,000 samples were drawn from each domain, and data valuation was performed using the remaining 10,000 samples, excluding the target domain. The experimental setup follows that of Table 2 in the main paper. As shown in Table 3, EV continues to outperform other methods in OOD domains, and the performance gain from integrating EV is consistently observed compared to the base methods without EV. While Table 2 reports the averaged performance over DomainNet due to space constraints, per-domain results in the appendix also confirm that EV consistently improves performance across individual domains. Notably, on V2, EV + KNN Shapley slightly outperforms KNN Shapley alone, even at the second decimal place. Due to computational constraints, LAVA was excluded due to its sensitivity to the number of labels, and Deviation was excluded because it scales poorly with dataset size.