Trabalho Prático 4 Grupo 24

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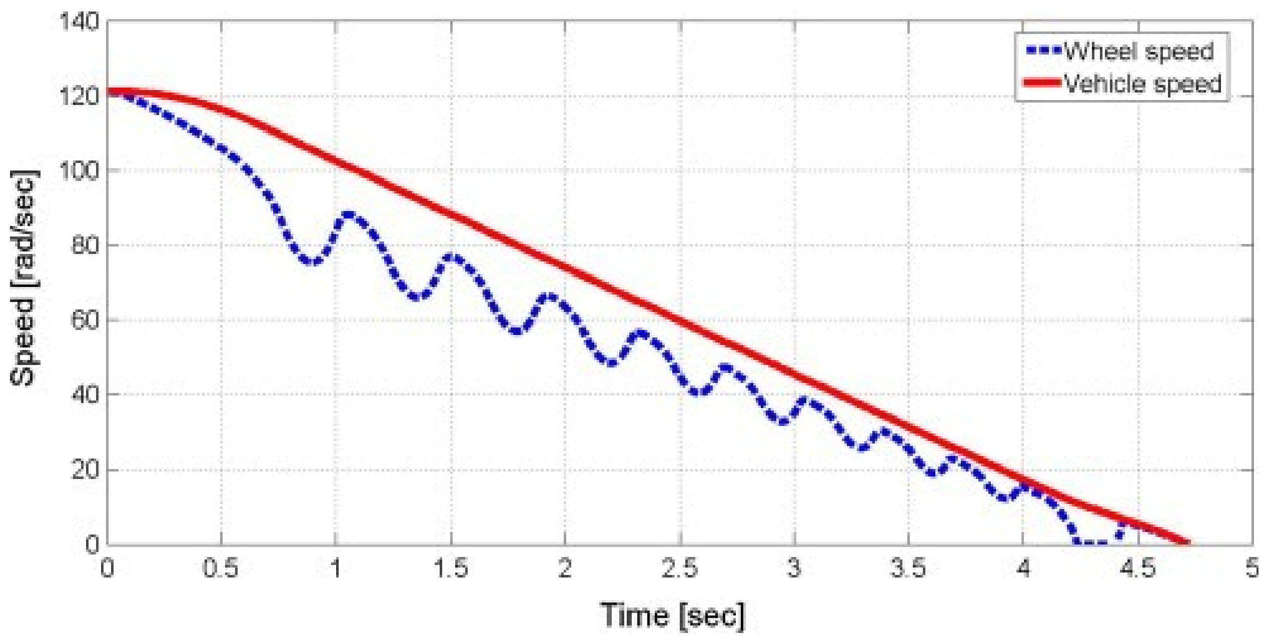
Problema 1

No contexto do sistema de travagem ABS (“Anti-Lock Breaking System”), pretende-se construir um autómato híbrido que descreva o sistema e que possa ser usado para verificar as suas propriedades dinâmicas.

Objetivo:



Velocidade do veículo deve ser estritamente decrescente

```
from pysmt.shortcuts import *
from pysmt.typing import *
import matplotlib.pyplot as plt

# Os estados possíveis que temos de considerar são:
# Start - 0
# Free - 1
# Stopping - 2
# Blocked - 3
# Stopped - 4

MODE = {0 : "Start", 1 : "Free", 2 : "Stopping", 3 : "Blocked", 4 :
"Stopped"}

def declare(i):
```

```
    state = {}
    state["state"] = Symbol("state"+str(i), types.INT)   # Representa
o estado em que o autómato está num determinado momento
    state["V"] = Symbol("V"+str(i), types.REAL)          # Representa
a velocidade do corpo do veículo em relação ao solo
    state["v"] = Symbol("v"+str(i), types.REAL)          # Representa
a velocidade das rodas em relação ao solo
    state["t"] = Symbol("t"+str(i), types.REAL)          # Representa
a contagem do tempo durante toda a operação do sistema
    state["timer"] = Symbol("timer"+str(i), types.REAL)   # Representa
a contagem do tempo num determinado estado que tenha um limite de
tempo de operação
    return state

def init(state, V0):
    return And (Equals(state["t"], Real(0)),
                Equals(state["V"], Real(V0)),        # V0 é o input do
problema e representa a velocidade do veículo no momento da ativação
dos sistema
                Equals(state["v"], Real(V0)),
                Equals(state["state"], Int(0)),      # O estado 0
representa o estado "Start"
                Equals(state["timer"], Real(0))
                )

def trans(curr, prox, VelPrecision, intervaloTempoB, intervaloTempoF,
fTravagemAlta, fTravagemBaixa, atrito, atritoAr, peso, V0, delta_t):

   equalstt = Equals(curr["t"], prox["t"])
   equalsVV = Equals(curr["V"], prox["V"])
   equalsvv = Equals(curr["v"], prox["v"])

   # Untimed (reprentam as transições da parte discreta do autómato
híbrido)

   TStartFree = And(Equals(curr["state"], Int(0)),
                    Equals(prox["state"], Int(1)),
                    Equals(curr["timer"], Real(0)),
                    Equals(prox["timer"], Real(0)),
                    equalstt,
                    equalsVV,
                    equalsvv)

   TFreeStopping = And(Equals(curr["state"], Int(1)),
                       Equals(prox["state"], Int(2)),
                       GT(curr["V"], Real(VelPrecision)),
                       GT(curr["v"], Real(VelPrecision)),
                       Or(GE(curr["timer"], Real(intervaloTempoF)),
                          GE(curr["V"] - curr["v"],
Real(VelPrecision))),
```

```
                       equalstt,
                       equalsVV,
                       equalsvv)

    TStoppingBlocked = And(Equals(curr["state"], Int(2)),
                           Equals(prox["state"], Int(3)),
                           LT(curr["V"] - curr["v"], Real(0.2)), #
                           Equals(curr["timer"], Real(0)),
                           Equals(prox["timer"], Real(0)),
                           GT(curr["v"], Real(VelPrecision)),
                           equalstt,
                           equalsVV,
                           equalsvv)

    TBlockedFree = And(Equals(curr["state"], Int(3)),
                        Equals(prox["state"], Int(1)),
                        GT(curr["V"], Real(VelPrecision)),
                        GT(curr["v"], Real(VelPrecision)),
                        GE(curr["timer"], Real(intervaloTempoB)), #
                        Equals(prox["timer"], Real(0)),
                        equalstt,
                        equalsVV,
                        equalsvv)

    TAnyStopped = And(Or(Equals(curr["state"], Int(1)),
                         Equals(curr["state"], Int(2)),
                         Equals(curr["state"], Int(3))),
                      Equals(prox["state"], Int(4)),
                      LE(curr["V"], Real(VelPrecision)),
                      LE(curr["v"], Real(VelPrecision)),
                      GE(prox["V"], Real(0)),
                      GE(prox["v"], Real(0)),
                      equalstt,
                      equalsVV,
                      equalsvv)

    untimed = Or(TStartFree, TFreeStopping, TStoppingBlocked,
TBlockedFree, TAnyStopped)

    # Timed (representam as transições da parte contínua do autómato
híbrido)

    TFree = And(Equals(curr["state"], Int(1)),
                Equals(prox["state"], Int(1)),
                Equals(prox["timer"], curr["timer"] + delta_t),
                Equals(prox["t"], curr["t"] + delta_t),
                LT(prox["timer"], Real(intervaloTempoF) + delta_t),
                GT(curr["v"], Real(VelPrecision)),

                Equals(prox["V"], curr["V"] + (Real(-fTravagemBaixa) *
```

```
(curr["V"] - curr["v"]) - Real(atritoAr)) * delta_t),
               Equals(prox["v"], curr["v"] + (Real(-atrito) *
Real(peso) + Real(fTravagemBaixa) * (curr["V"] - curr["v"])) *
delta_t))

   TStopping = And(Equals(curr["state"], Int(2)),
                  Equals(prox["state"], Int(2)),
                  GT(curr["V"] - curr["v"], Real(0.2)),
                  Equals(prox["timer"], curr["timer"] + delta_t),
                  Equals(prox["t"], curr["t"] + delta_t),
                  LT(prox["v"], prox["V"]),

                  Equals(prox["V"], Max(Real(0), curr["V"] + (Real(-
fTravagemAlta) * (curr["V"] - curr["v"]) - Real(atritoAr)) *
delta_t)),
                  Equals(prox["v"], Max(Real(0), curr["v"] + (Real(-
atrito) * Real(peso) + Real(fTravagemAlta) * (curr["V"] - curr["v"]))
* delta_t)))

   TBlocked = And(Equals(curr["state"], Int(3)),
                  Equals(prox["state"], Int(3)),
                  Equals(prox["timer"], curr["timer"] + delta_t),
                  Equals(prox["t"], curr["t"] + delta_t),
                  LT(prox["timer"], Real(intervaloTempoB) + delta_t),

                  Equals(prox["V"], Max(Real(0), curr["V"] + (-
Real(atrito) * Real(peso) - Real(atritoAr)) * delta_t)),
                  Equals(prox["v"], Max(Real(0), curr["v"] + (-
Real(atrito) * Real(peso) - Real(atritoAr)) * delta_t)))

   timed = Or(TFree, TStopping, TBlocked)

   end = And(Equals(curr["state"], Int(4)),
            Equals(prox["state"], Int(4)),
            Equals(prox["t"], curr["t"] + delta_t),
            equalsVV,
            equalsvv)

   return Or(untimed, timed, end)

def print_vars(state, solver):
    for var in state:
        if state[var].get_type() == REAL:
            print(f"  {var} =
{float(solver.get_py_value(state[var]))}")
        if var == "state":
            print(f"  {var} = {MODE[solver.get_py_value(state[var])]}
({solver.get_py_value(state[var])})")

def geraTraco(declare, init, trans, atrito, atritoAr, fTravagemAlta,
```

```python
fTravagemBaixa, intervaloTempoB, intervaloTempoF, peso, VelPrecision,
V0, k, delta_t):

    states = [declare(i) for i in range(k)]

    t=[]
    V=[]
    vr=[]

    with Solver() as solver:
        solver.add_assertion(init(states[0], V0))
        for i in range(k - 1):
            solver.add_assertion(trans(states[i], states[i+1],
VelPrecision, intervaloTempoB, intervaloTempoF, fTravagemAlta,
fTravagemBaixa, atrito, atritoAr, peso, V0, delta_t))

        if solver.solve():
            print("Solver found a solution")
            for i,s in enumerate(states):
                print(f"> State {i}:")
                print_vars(s, solver)

                t.append(float(solver.get_py_value(s["t"])))
                V.append(float(solver.get_py_value(s["V"])))
                vr.append(float(solver.get_py_value(s["v"])))

        else:
            print("Solver failed to find a solution")
            print("> Not feasible.")

    fig ,ax = plt.subplots()

    line1, = ax.plot(t, V, color="red", linestyle="solid",
label="VVeiculo")
    line2, = ax.plot(t, vr, color="blue", linestyle="dashed",
label="vRodas")

    ax.set_xlabel("Tempo (t)")
    ax.set_ylabel("velocidade (v)")

    ax.legend([line1, line2], ["VVeiculo", "vRodas"])

    plt.show()

# Parâmetros do problema

atrito = 0.02
atritoAr = 0.001

peso = 2000
```

```
fTravagemAlta = atrito * peso
fTravagemBaixa = atrito * peso * 0.1

intervaloTempoB = 0.5
intervaloTempoF = 0.5
delta_t = 0.25

V0 = 200

VelPrecision = 0.1
k = 49

geraTraco(declare, init, trans, atrito, atritoAr, fTravagemAlta,
fTravagemBaixa, intervaloTempoB, intervaloTempoF, peso, VelPrecision,
V0, k, delta_t)

Solver found a solution
> State 0:
  state = Start(0)
  V = 200.0
  v = 200.0
  t = 0.0
  timer = 0.0
> State 1:
  state = Free(1)
  V = 200.0
  v = 200.0
  t = 0.0
  timer = 0.0
> State 2:
  state = Free(1)
  V = 199.99975
  v = 190.0
  t = 0.25
  timer = 0.25
> State 3:
  state = Free(1)
  V = 189.99975
  v = 189.99975
  t = 0.5
  timer = 0.5
> State 4:
  state = Stopping(2)
  V = 189.99975
  v = 189.99975
  t = 0.5
  timer = 0.0
> State 5:
  state = Blocked(3)
  V = 189.99975
```

```
  v = 189.99975
  t = 0.5
  timer = 0.0
> State 6:
  state = Blocked(3)
  V = 179.9995
  v = 179.9995
  t = 0.75
  timer = 0.25
> State 7:
  state = Blocked(3)
  V = 169.99925
  v = 169.99925
  t = 1.0
  timer = 0.5
> State 8:
  state = Free(1)
  V = 169.99925
  v = 169.99925
  t = 1.0
  timer = 0.0
> State 9:
  state = Free(1)
  V = 169.999
  v = 159.99925
  t = 1.25
  timer = 0.25
> State 10:
  state = Free(1)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.5
> State 11:
  state = Stopping(2)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.0
> State 12:
  state = Blocked(3)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.0
> State 13:
  state = Blocked(3)
  V = 149.99875
  v = 149.99875
```

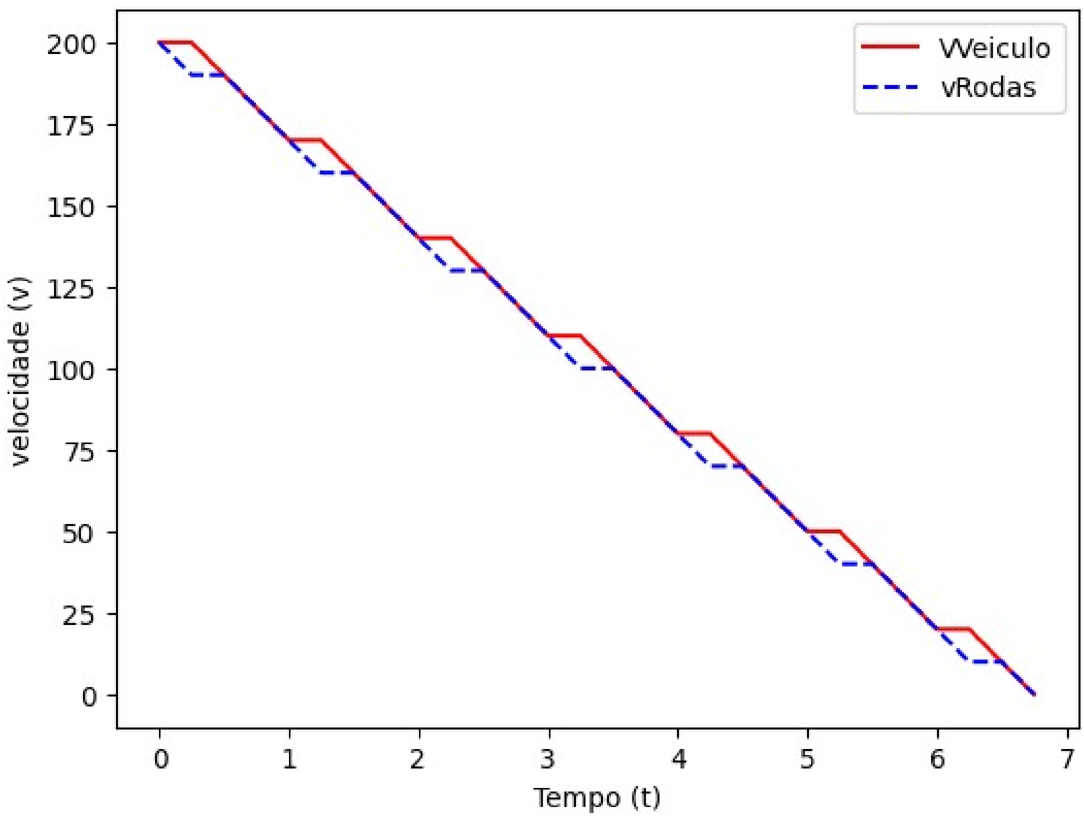
```
  t = 1.75
  timer = 0.25
> State 14:
  state = Blocked(3)
  V = 139.9985
  v = 139.9985
  t = 2.0
  timer = 0.5
> State 15:
  state = Free(1)
  V = 139.9985
  v = 139.9985
  t = 2.0
  timer = 0.0
> State 16:
  state = Free(1)
  V = 139.99825
  v = 129.9985
  t = 2.25
  timer = 0.25
> State 17:
  state = Free(1)
  V = 129.99824999999998
  v = 129.99824999999998
  t = 2.5
  timer = 0.5
> State 18:
  state = Stopping(2)
  V = 129.99824999999998
  v = 129.99824999999998
  t = 2.5
  timer = 0.0
> State 19:
  state = Blocked(3)
  V = 129.99824999999998
  v = 129.99824999999998
  t = 2.5
  timer = 0.0
> State 20:
  state = Blocked(3)
  V = 119.998
  v = 119.998
  t = 2.75
  timer = 0.25
> State 21:
  state = Blocked(3)
  V = 109.99775
  v = 109.99775
  t = 3.0
```

```
  timer = 0.5
> State 22:
  state = Free(1)
  V = 109.99775
  v = 109.99775
  t = 3.0
  timer = 0.0
> State 23:
  state = Free(1)
  V = 109.9975
  v = 99.99775
  t = 3.25
  timer = 0.25
> State 24:
  state = Free(1)
  V = 99.9975
  v = 99.9975
  t = 3.5
  timer = 0.5
> State 25:
  state = Stopping(2)
  V = 99.9975
  v = 99.9975
  t = 3.5
  timer = 0.0
> State 26:
  state = Blocked(3)
  V = 99.9975
  v = 99.9975
  t = 3.5
  timer = 0.0
> State 27:
  state = Blocked(3)
  V = 89.99725
  v = 89.99725
  t = 3.75
  timer = 0.25
> State 28:
  state = Blocked(3)
  V = 79.997
  v = 79.997
  t = 4.0
  timer = 0.5
> State 29:
  state = Free(1)
  V = 79.997
  v = 79.997
  t = 4.0
  timer = 0.0
```

```
> State 30:
  state = Free(1)
  V = 79.99674999999999
  v = 69.997
  t = 4.25
  timer = 0.25
> State 31:
  state = Free(1)
  V = 69.99674999999999
  v = 69.99674999999999
  t = 4.5
  timer = 0.5
> State 32:
  state = Stopping(2)
  V = 69.99674999999999
  v = 69.99674999999999
  t = 4.5
  timer = 0.0
> State 33:
  state = Blocked(3)
  V = 69.99674999999999
  v = 69.99674999999999
  t = 4.5
  timer = 0.0
> State 34:
  state = Blocked(3)
  V = 59.9965
  v = 59.9965
  t = 4.75
  timer = 0.25
> State 35:
  state = Blocked(3)
  V = 49.996249999999996
  v = 49.996249999999996
  t = 5.0
  timer = 0.5
> State 36:
  state = Free(1)
  V = 49.996249999999996
  v = 49.996249999999996
  t = 5.0
  timer = 0.0
> State 37:
  state = Free(1)
  V = 49.995999999999995
  v = 39.996249999999996
  t = 5.25
  timer = 0.25
> State 38:
```

```
  state = Free(1)
  V = 39.995999999999995
  v = 39.995999999999995
  t = 5.5
  timer = 0.5
> State 39:
  state = Stopping(2)
  V = 39.995999999999995
  v = 39.995999999999995
  t = 5.5
  timer = 0.0
> State 40:
  state = Blocked(3)
  V = 39.995999999999995
  v = 39.995999999999995
  t = 5.5
  timer = 0.0
> State 41:
  state = Blocked(3)
  V = 29.995749999999997
  v = 29.995749999999997
  t = 5.75
  timer = 0.25
> State 42:
  state = Blocked(3)
  V = 19.995499999999996
  v = 19.995499999999996
  t = 6.0
  timer = 0.5
> State 43:
  state = Free(1)
  V = 19.995499999999996
  v = 19.995499999999996
  t = 6.0
  timer = 0.0
> State 44:
  state = Free(1)
  V = 19.995249999999995
  v = 9.995499999999996
  t = 6.25
  timer = 0.25
> State 45:
  state = Free(1)
  V = 9.995249999999997
  v = 9.995249999999997
  t = 6.5
  timer = 0.5
> State 46:
  state = Stopping(2)
```

```
  V = 9.995249999999997
  v = 9.995249999999997
  t = 6.5
  timer = 0.0
> State 47:
  state = Blocked(3)
  V = 9.995249999999997
  v = 9.995249999999997
  t = 6.5
  timer = 0.0
> State 48:
  state = Blocked(3)
  V = 0.0
  v = 0.0
  t = 6.75
  timer = 0.25
```



Alinea d)

```
def bmc_alwaysi(declare,init,trans,inv, t, K):

    with Solver() as solver:
        states = [declare(i) for i in range(K)]
        solver.add_assertion(init(states[0], V0))
```

```
        for k in range(K):
            if k>0:
                solver.add_assertion(trans(states[k-1], states[k], 
VelPrecision, intervaloTempoB, intervaloTempoF, fTravagemAlta, 
fTravagemBaixa, atrito, atritoAr, peso, V0, delta_t))

            solver.push()
            solver.add_assertion(Not(inv(states[k], t)))

            if solver.solve():
                print(f"> Invariant does not hold for {k+1} first 
states. Counter-example:")
                for i,s in enumerate(states[:k+1]):
                    print(f"> State {i}:")
                    print_vars(s, solver)
                return
            else:
                if k==K-1:
                    print(f"> Invariant holds for the first {K} 
states.")
                else:
                    solver.pop()
```

Condição i:

```
"o veículo imobiliza-se completamente em menos de t segundos"

def beforeTSec(curr, t):
    return And(Implies(Equals(curr["state"], Int(4)), LE(curr["t"], 
Real(t))), # Verifica se quando chega ao estado 4 (Veículo 
imobilizado), o tempo decorrido é menor ou igual a t
              Implies(GE(curr["t"], Real(t)), Equals(curr["V"], 
Real(0)))) # Verifica se quando o tempo decorrido ultrapassou t, a 
velocidade do veículo é 0

t = 4
# t = 4 Por exemplo não resultaria

bmc_alwaysi(declare, init, trans, beforeTSec, t, 49)

> Invariant does not hold for 29 first states. Counter-example:
> State 0:
  state = Start(0)
  V = 200.0
  v = 200.0
  t = 0.0
  timer = 0.0
> State 1:
  state = Free(1)
```

```
  V = 200.0
  v = 200.0
  t = 0.0
  timer = 0.0
> State 2:
  state = Free(1)
  V = 199.99975
  v = 190.0
  t = 0.25
  timer = 0.25
> State 3:
  state = Free(1)
  V = 189.99975
  v = 189.99975
  t = 0.5
  timer = 0.5
> State 4:
  state = Stopping(2)
  V = 189.99975
  v = 189.99975
  t = 0.5
  timer = 0.0
> State 5:
  state = Blocked(3)
  V = 189.99975
  v = 189.99975
  t = 0.5
  timer = 0.0
> State 6:
  state = Blocked(3)
  V = 179.9995
  v = 179.9995
  t = 0.75
  timer = 0.25
> State 7:
  state = Blocked(3)
  V = 169.99925
  v = 169.99925
  t = 1.0
  timer = 0.5
> State 8:
  state = Free(1)
  V = 169.99925
  v = 169.99925
  t = 1.0
  timer = 0.0
> State 9:
  state = Free(1)
  V = 169.999
```

```
  v = 159.99925
  t = 1.25
  timer = 0.25
> State 10:
  state = Free(1)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.5
> State 11:
  state = Stopping(2)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.0
> State 12:
  state = Blocked(3)
  V = 159.999
  v = 159.999
  t = 1.5
  timer = 0.0
> State 13:
  state = Blocked(3)
  V = 149.99875
  v = 149.99875
  t = 1.75
  timer = 0.25
> State 14:
  state = Blocked(3)
  V = 139.9985
  v = 139.9985
  t = 2.0
  timer = 0.5
> State 15:
  state = Free(1)
  V = 139.9985
  v = 139.9985
  t = 2.0
  timer = 0.0
> State 16:
  state = Free(1)
  V = 139.99825
  v = 129.9985
  t = 2.25
  timer = 0.25
> State 17:
  state = Free(1)
  V = 129.99824999999998
  v = 129.99824999999998
```

```
  t = 2.5
  timer = 0.5
> State 18:
  state = Stopping(2)
  V = 129.99824999999998
  v = 129.99824999999998
  t = 2.5
  timer = 0.0
> State 19:
  state = Blocked(3)
  V = 129.99824999999998
  v = 129.99824999999998
  t = 2.5
  timer = 0.0
> State 20:
  state = Blocked(3)
  V = 119.998
  v = 119.998
  t = 2.75
  timer = 0.25
> State 21:
  state = Blocked(3)
  V = 109.99775
  v = 109.99775
  t = 3.0
  timer = 0.5
> State 22:
  state = Free(1)
  V = 109.99775
  v = 109.99775
  t = 3.0
  timer = 0.0
> State 23:
  state = Free(1)
  V = 109.9975
  v = 99.99775
  t = 3.25
  timer = 0.25
> State 24:
  state = Free(1)
  V = 99.9975
  v = 99.9975
  t = 3.5
  timer = 0.5
> State 25:
  state = Stopping(2)
  V = 99.9975
  v = 99.9975
  t = 3.5
```

```
  timer = 0.0
> State 26:
  state = Blocked(3)
  V = 99.9975
  v = 99.9975
  t = 3.5
  timer = 0.0
> State 27:
  state = Blocked(3)
  V = 89.99725
  v = 89.99725
  t = 3.75
  timer = 0.25
> State 28:
  state = Blocked(3)
  V = 79.997
  v = 79.997
  t = 4.0
  timer = 0.5
```

Condição ii:

```
“a velocidade V diminui sempre com o tempo”.

def bmc_alwaysii(declare,init,trans,inv, K):

    with Solver() as solver:
        states = [declare(i) for i in range(K)]
        solver.add_assertion(init(states[0], V0))

        for k in range(K-1):

            solver.add_assertion(trans(states[k], states[k+1],
VelPrecision, intervaloTempoB, intervaloTempoF, fTravagemAlta,
fTravagemBaixa, atrito, atritoAr, peso, V0, delta_t))

            solver.push()
            solver.add_assertion(Not(inv(states[k], states[k+1])))

            if solver.solve():
                print(f"> Invariant does not hold for {k+1} first
states. Counter-example:")
                for i,s in enumerate(states[:k+1]):
                    print(f"> State {i}:")
                    print_vars(s, solver)
                return
            else:
                if k==K - 2:
                    print(f"> Invariant holds for the first {K}
```

```
states.")
                else:
                    solver.pop()

def VStrictlyDcr(curr, prox):
    return Implies(LE(curr["t"], prox["t"]), GE(curr["V"], prox["V"]))

bmc_alwaysii(declare, init, trans, VStrictlyDcr, 50)

> Invariant holds for the first 50 states.
```