Lecture 18 - Policy Optimization

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DDA4230: Reinforcement Learning
Course Page: [Click]

Policy Gradient in Episodic MDP

Policy Gradient Methods:

- Let τ denote a state-action sequence $s_0, a_0, \ldots, s_T, a_T$.
- Let $r(\tau) = \sum_{t=0}^{T} r(s_t, a_t)$ denote trajectory reward .
- Let $P^{\pi_\theta}(\tau)$ denote the corresponding occupancy measure

Then for a policy π parameterized by θ, we desire to find

$$\max_\theta \mathbb{E}\left[P^{\pi_\theta}(\tau) r(\tau)\right] .$$

Policy Gradient in Episodic MDP

Policy Gradient Methods: Taking gradient (denoted as g) with respect to θ gives us

$$g = \mathbb{E}\left[r(\tau)\nabla_\theta \log\left(\sum_{t=1}^{T} P\left(s_{t+1} \mid s_t, a_t\right) \cdot \pi_\theta\left(a_t \mid s_t\right)\right)\right] = \mathbb{E}\left[r(\tau)\sum_{t=1}^{T} \nabla_\theta \log(\pi_\theta\left(a_t \mid s_t\right))\right].$$

This gradient is unbiased and we do not need access to the dynamic model to compute this.

Policy Gradient in Stationary MDP

There are several different related expressions for the policy gradient, which have the form

$$g = \mathbb{E}\left[\sum_{t=0}^{\infty} \Psi_t \nabla_\theta \log \pi_\theta (a_t \mid s_t)\right],$$

Policy Gradient in Stationary MDP

$\sum_{t=0}^{\infty} \Psi_t$ could be the following:

1. $\sum_{t=0}^{\infty} r_t$: the total reward of the trajectory Monta-Carlo;
2. $Q^\pi(s_t, a_t)$: the action value function Temporal Difference;
3. $\sum_{t'=t}^{\infty} r_{t'}$: the reward following action a_t Monta-Carlo;
4. $\sum_{t'=t}^{\infty} r_{t'} - b(s_t)$: the reward following action a_t with a baseline Monta-Carlo;
5. $\sum_{t'=t}^{\infty} A^\pi(s_t, a_t)$: the advantage function Temporal Difference;
6. $\sum_{t'=t}^{\infty} r_t + V^\pi(s_{t+1}) - V^\pi(s_t)$: the TD residual Temporal Difference.

The latter formulas use the definitions $A^\pi(s_t, a_t) := Q^\pi(s_t, a_t) - V^\pi(s_t)$, which is the advantage function.

Off Policy Optimization

- In policy optimization methods, the data samples we have collected may not correspond to the policy we wish to optimize.
- In this case, some special handling is needed so that the gradient estimates can be unbiased.

Let π_1 be the policy we are currently following and π_2 be the policy we want to optimize. Let them be parameterized by θ_1, θ_2, respectively. Then we can use importance sampling to re-weight our objective as

$$\max_{\theta_1} \mathbb{E}\left[\frac{P^{\pi_{\theta_2}}(\tau)}{P^{\pi_{\theta_1}}(\tau)} r(\tau)\right].$$

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Trust Region Policy Optimization (TRPO)

Motivation: The challenge of step size.

- In the classic supervised learning setting or in the optimization literature, having a bad step size may not be terrible. This is because the next update can partially correct the error in the previous steps.
- In policy optimization, when the step size is too far, we obtain a terrible policy. This indicates that the next batch of data will be collected under this terrible policy. Exploration could be exploratory, but updates should be more conservative.
- It becomes not clear how to recover short of going back and shrinking the step size.

Trust Region Policy Optimization (TRPO)

One method of choosing the step size is by **line search**. The procedure is:

1. Calculate the initial loss (e.g., with Monte-Carlo estimation) and initialize the step size to be a large value;
2. Update the parameter with the gradients under the current step size can calculate the new loss;
3. Decrease the value of step size until we have found a new loss that is less than the initial loss.

However, 1) it may be expensive to compute so many gradients, 2) this method ignores the quality of our gradients.

Trust Region Policy Optimization (TRPO)

An alternative method is the trust region method.

Let us first denotes $P(\tau \mid \theta) = P(s_0) \cdot \Pi_{t=1}^{T} P(s_{t+1} \mid s_t, a_t)\pi_\theta(a_t \mid s_t)$. Then the trust region method finds us the next parameter $\theta + \delta\theta$ by solving the following problem.

$$\max_{\delta\theta} \; g^\top \delta\theta$$

$$\text{subject to } d_{\mathrm{KL}}\left(P(\tau|\theta)||P(\tau|\theta+\delta\theta)\right) \leq \varepsilon,$$

where d_{KL} denotes the KL divergence, g is our gradient estimate, and ε is a parameter we can set. Here, the change in the objective function is estimated by assuming that the objective function in this neighboring area is linear.

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Trust Region Policy Optimization (TRPO)

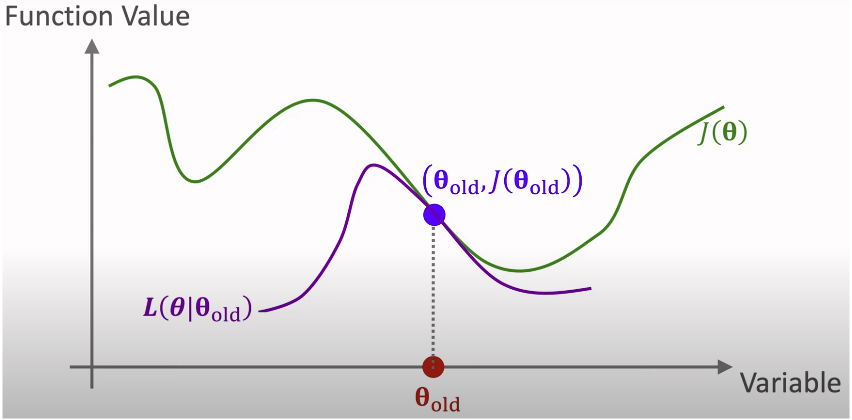
Line search
(like gradient ascent)

Trust region

Source.[1]

[1] https://jonathan-hui.medium.com/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9

Trust Region Policy Optimization (TRPO)



Source.[1]

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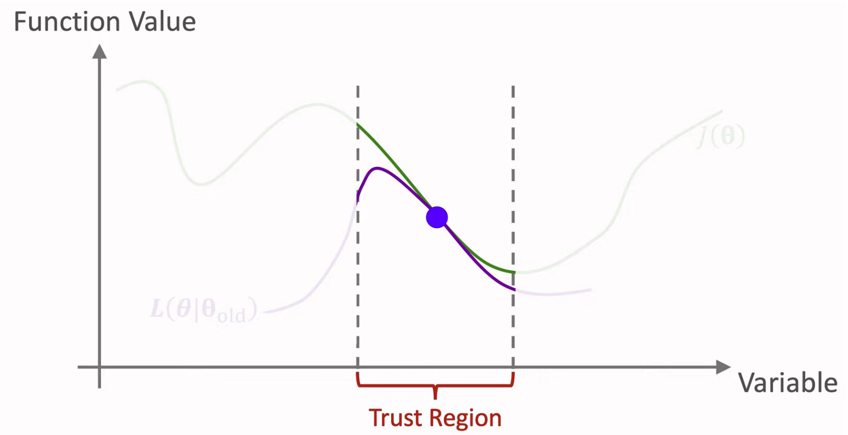
[1] https://www.youtube.com/watch?v=fcSYiyvPjm4&ab_channel=ShusenWang

Trust Region Policy Optimization (TRPO)



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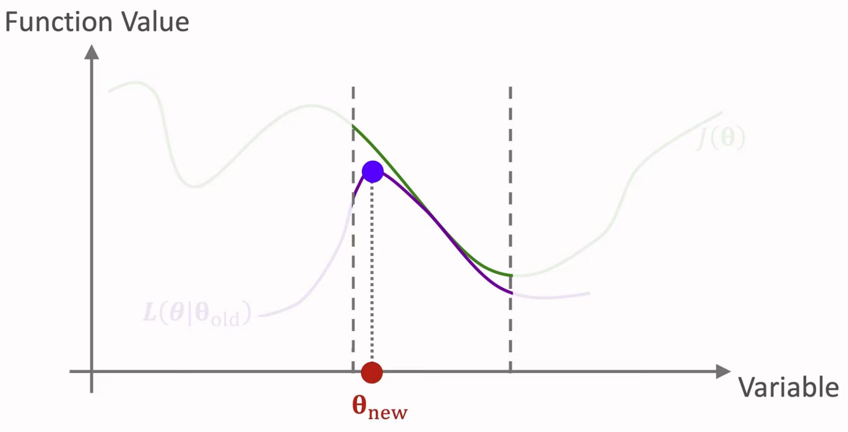
Trust Region Policy Optimization (TRPO)



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Trust Region Policy Optimization (TRPO)

Using the expression of the KL divergence, we have

$$
\begin{aligned}
d_{\mathrm{KL}}(P(\tau;\theta)\|P(\tau;\theta+\delta\theta)) &= \sum_{\tau} P(\tau;\theta)\log\frac{P(\tau;\theta)}{P(\tau;\theta+\delta\theta)} \\
&= \sum_{\tau} P(\tau;\theta)\log\frac{P(s_0)\prod_{t=0}^{T-1}\pi_\theta(a_t\mid s_t)P(s_{t+1}\mid s_t,a_t)}{P(s_0)\prod_{t=0}^{T-1}\pi_{\theta+\delta\theta}(a_t\mid s_t)P(s_{t+1}\mid s_t,a_t)} \\
&= \sum_{\tau} P(\tau;\theta)\log\frac{\prod_{t=0}^{T-1}\pi_\theta(a_t\mid s_t)}{\prod_{t=0}^{T-1}\pi_{\theta+\delta\theta}(a_t\mid s_t)}.
\end{aligned}
$$

Trust Region Policy Optimization (TRPO)

With M samples, this term can be approximated by the sample average and we may rewrite the maximization problem to be

$$\begin{aligned} \max_{\delta\theta} \quad & g^\top \delta\theta \\ \text{subject to} \quad & \frac{1}{M} \sum_{(s,a)} \log \frac{\pi_\theta(a \mid s)}{\pi_{\theta+\delta\theta}(a \mid s)} \leq \varepsilon \,. \end{aligned}$$

This maximization problem with the constraint can be hard to enforce given complicated policies like neural networks.

Trust Region Policy Optimization (TRPO)

We would need to approximate the KL divergence further for a feasible objective. This is done through second-order approximation with fisher matrix F_θ.

$$
\begin{aligned}
d_{\mathrm{KL}}\left(\pi_\theta(a \mid s) \| \pi_{\theta+\delta\theta}(a \mid s)\right) &\approx \delta\theta^\top \left(\sum_{(s,a)\sim\theta} \nabla_\theta \log \pi_\theta(a \mid s) \nabla_\theta \log \pi_\theta(a \mid s)^\top \right) \delta\theta \\
&= \delta\theta^\top F_\theta \delta\theta \,.
\end{aligned}
$$

Our problem is simplified to linear objective quadratic constrained optimization:

$$
\begin{aligned}
\max_{\delta\theta} \quad & g^\top \delta\theta \\
\text{subject to} \quad & \delta\theta^\top F_\theta \delta\theta \le \varepsilon \,,
\end{aligned}
$$

Trust Region Policy Optimization (TRPO)

The above linear objective quadratic constrained optimization problem could be solved analytically using the Karush-Kuhn-Tucker (KKT) conditions. Thus the final TRPO objective is given as

$$\textit{Surrogateloss}: \max_{\pi} L(\pi) = \mathbb{E}_{\pi_{\text{old}}}\left[\frac{\pi(a \mid s)}{\pi_{\text{old}}(a \mid s)} A^{\pi_{\text{old}}}(s, a)\right]$$

$$\text{Constraint: } \mathbb{E}_{\pi_{\text{old}}}\left[d_{\mathrm{KL}}\left(\pi \| \pi_{\text{old}}\right)\right] \leq \varepsilon,$$

where A denotes the advantage function. This corresponds to a general policy gradient form we have mentioned earlier.

Proximal Policy Optimization

The PPO method enforces a **"soft"** constraint by adding a proximal value to the objective function. The objective is the following

$$L(\pi) = \mathbb{E}_{\pi_{\text{old}}}\left[\frac{\pi_\theta\left(a_t \mid s_t\right)}{\pi_{\theta_{\text{old}}}\left(a_t \mid s_t\right)} A^{\pi_{\text{old}}}(s, a) - \beta\, d_{\text{KL}}\left(\pi_{\theta_{\text{old}}}, \pi_\theta\right)\right].$$

The β can be fixed or adaptively chosen (or simply set to 0). One reason why one may wish to adaptively choose β is because it can be hard to find one β that performs well across different problems.

Proximal Policy Optimization

The policy's performance can fluctuate greatly when $\rho_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}$ changes too quickly. Thus PPO limits ρ to a range of $[1-\varepsilon, 1+\varepsilon]$ such that no abrupt updates to the policy will be made. The surrogate objective is then written as

$$L^{CLIP}(\pi) = \mathbb{E}\left[\min\{\rho_t(\theta)A(s,a), \text{clip}(\rho_t(\theta), 1-\varepsilon, 1+\varepsilon)A(s,a)\}\right].$$

We take the minimum of the constrained and unconstrained objectives such that our final objective is a lower bound of the unclipped objective. With this scheme, we only ignore the change in probability ratio when it would make the objective improve, and we include it when it makes the objective worse.

Proximal Policy Optimization

Algorithm 1 PPO-Clip

```
Input: initial policy parameters θ_0, initial value function parameters φ_0
for k = 0, 1, 2, ... do
    Collect set of trajectories D_k = {τ_i} by running policy π_k = π(θ_k) in the environment.
    Compute rewards-to-go R̂_t.
    Compute advantage estimates, Â_t (using any method of advantage estimation) based
    on the current value function V_{φ_k}.
    Update the policy by maximizing the PPO-Clip objective:
```

$$\theta_{k+1} = \arg\max_{\theta} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \min\left(\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)} A^{\pi_{\theta_k}}(s_t, a_t),\ \ g(\epsilon, A^{\pi_{\theta_k}}(s_t, a_t)) \right),$$

```
    typically via stochastic gradient ascent with Adam.
    Fit value function by regression on mean-squared error:
```

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(V_\phi(s_t) - \hat{R}_t \right)^2,$$

```
    typically via some gradient descent algorithm.
end for
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Question and Answering (Q&A)