



香港中文大學(深圳)  
The Chinese University of Hong Kong, Shenzhen

# Introduction to Computer Science: Programming Methodology

## Lecture 10 Linked List

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# List in Python is a referential structure

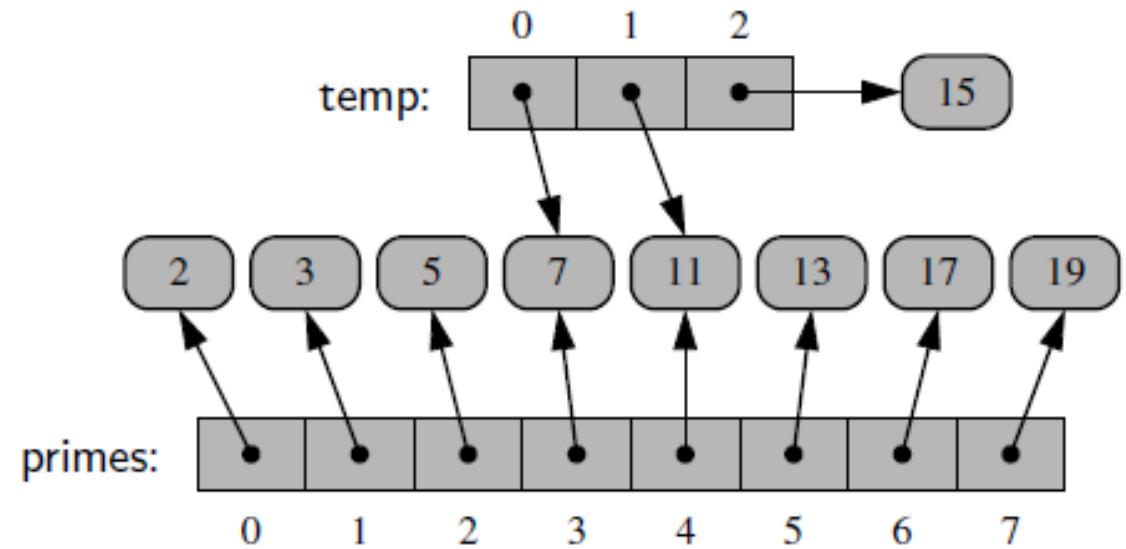
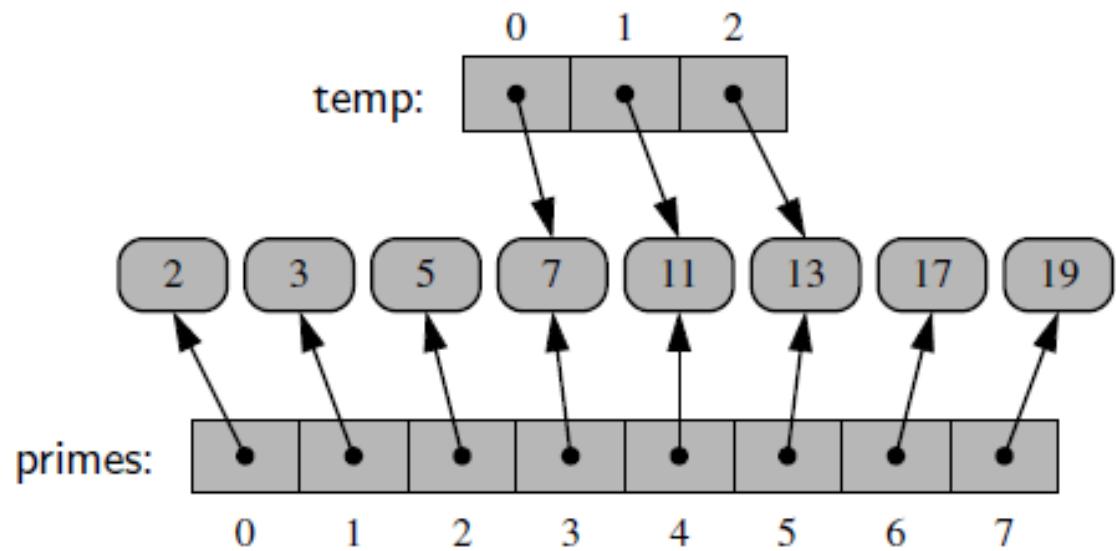
```
>>> a=[1, 2, 3, 4, 5]
>>> for i in range(0, 5):
    print(id(a[i]))
```

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1546964784  
1546964816  
1546964848

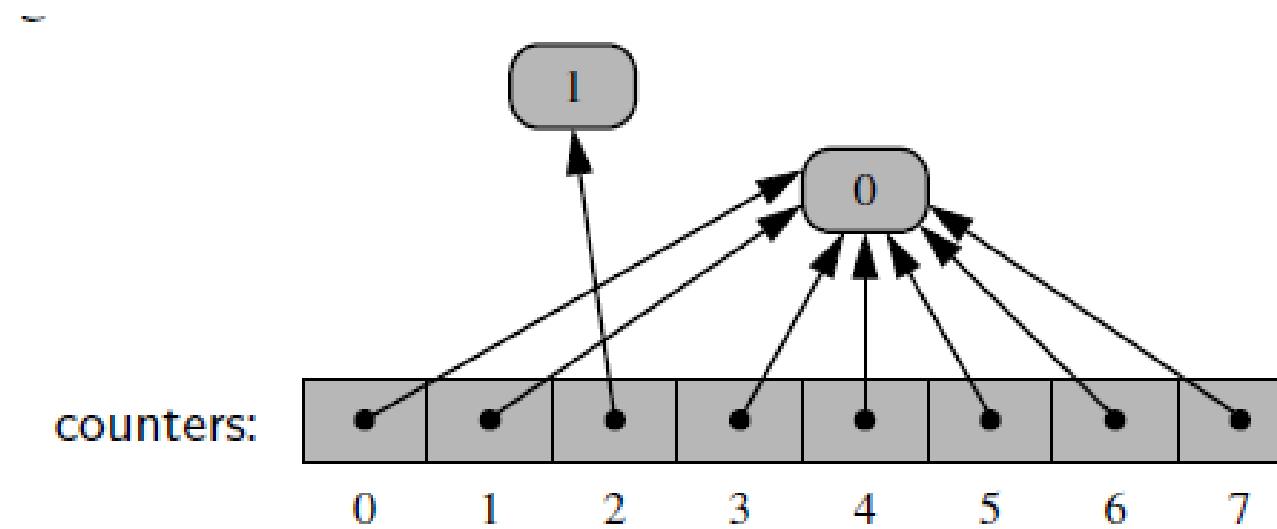
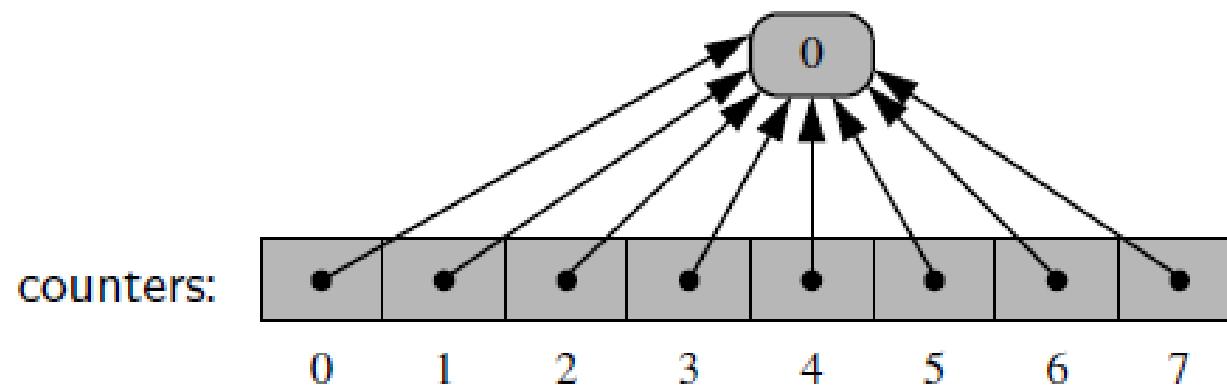
```
>>> a.insert(2, 10)
>>> a
[1, 2, 10, 3, 4, 5]
>>> for i in range(0, 6):
    print(id(a[i]))
```

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# List in Python is a referential structure



# List in Python is a referential structure



# Compact array

- A collection of numbers are usually stored as a **compact array** in languages such as C/C++ and Java
- A compact array is storing the bits that represent the primary data (**not reference**)
- The overall memory usage will be much lower for a compact structure because there is no overhead devoted to the explicit storage of the sequence of memory references (in addition to the primary data)

# Why we need another list data type

- Python's list class is **highly optimized**, and often a great choice for storage
- However, many programming languages **do not** support this kind of optimized list data type

# Common Questions

Why do we need to build linked lists in Python?

## When are linked lists preferred over lists?

Asked 2 years, 11 months ago

Modified 2 years, 11 months ago Viewed 1k times

I'm learning data structures and algorithms using Python. I've learnt that the advantage of linked list is that it does not have a maximum number of nodes, unlike arrays in other languages.

1. Since Python automatically resizes our lists for us, the advantage has been abstracted away for us.

2. Therefore, I've always thought that the only advantage linked lists have was that adding a node at the front or back of the linked list was  $O(1)$ , whereas adding an element to a list could end up being  $O(n)$  from Python having to resize the array and copying every element over.

## 为什么要用python实现链表？

最近学校在学“python及其数据结构”，学完用python实现链表后感觉完全没有实用性啊，实属困惑，链表的功能用python自己的list不是都能实现吗？

收起 ▲

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尾翼

你们学校安排这门课，主要是想让你们学习数据结构，学习链表之类的数据结构的关键不在于用什么语言实现，而是让你们理解，你所谓那些list等已...

3 赞同

2018-10-15

夏韬

python作为 数据结构 这部分知识的载体 传授给你。简化了你的阅读。如果你想复杂点，有意义一点，请出门右转谭浩强C。

1 赞同

2018-10-16

# Common Questions

- Linked list is a **classic** data structure as a good example.
- Rarely used in **high** level programming languages like Python. As a practitioner, you may never use it in Python.
- We use Python to demonstrate the **concept**, so you know what it is when using non-Python languages.

# More about linked lists

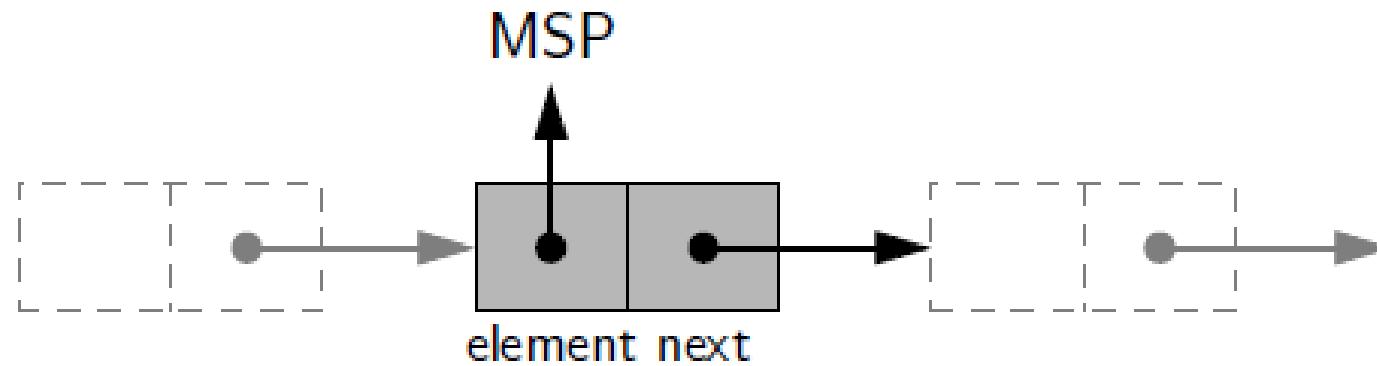


# More about linked lists



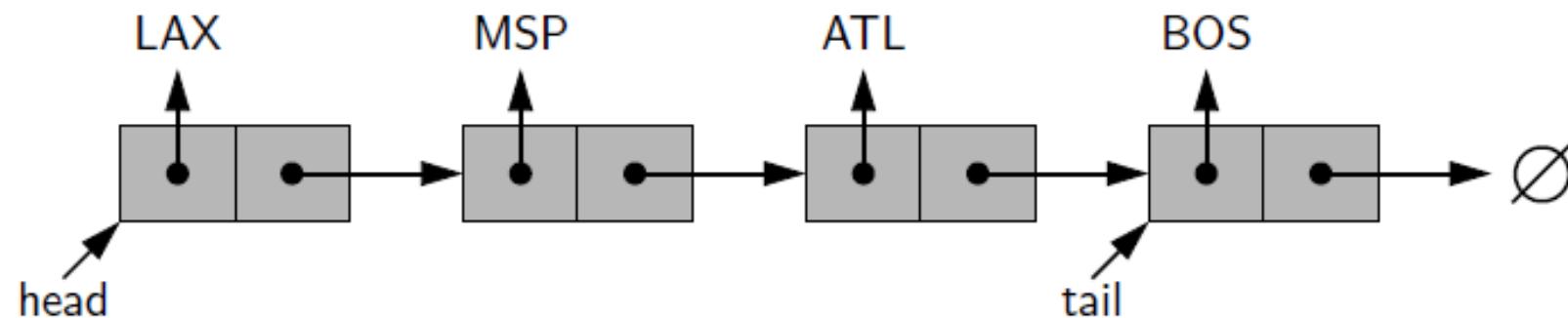
# Linked List

- A singly linked list, in its simplest form, is a collection of nodes that collectively form a linear sequence
- Each node stores a reference to an object that is an element of the sequence, as well as a reference to the next node of the list

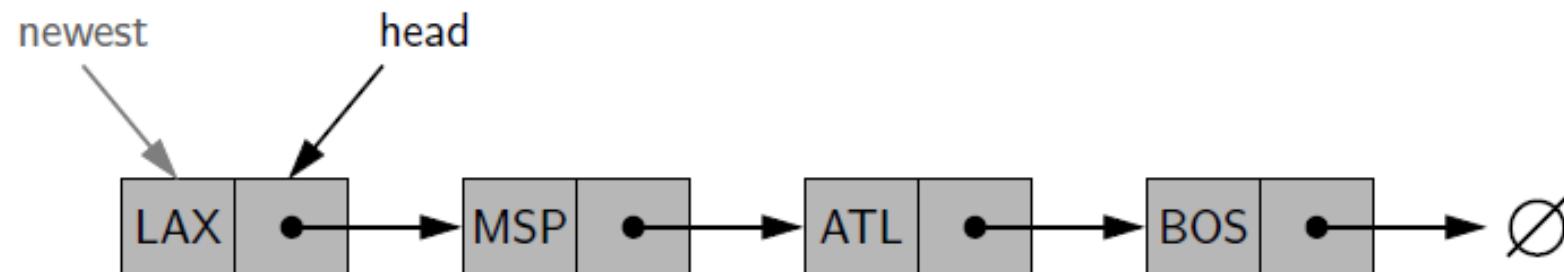
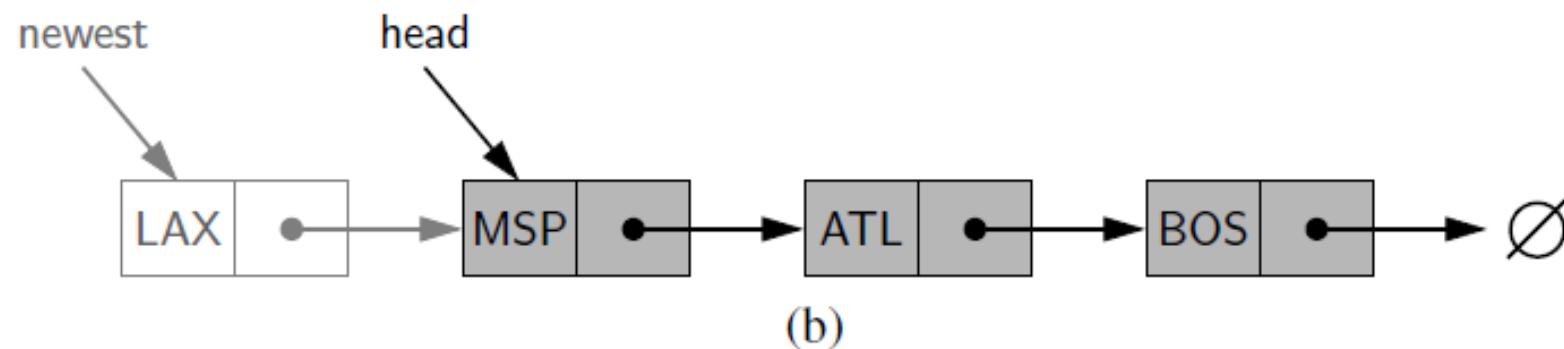
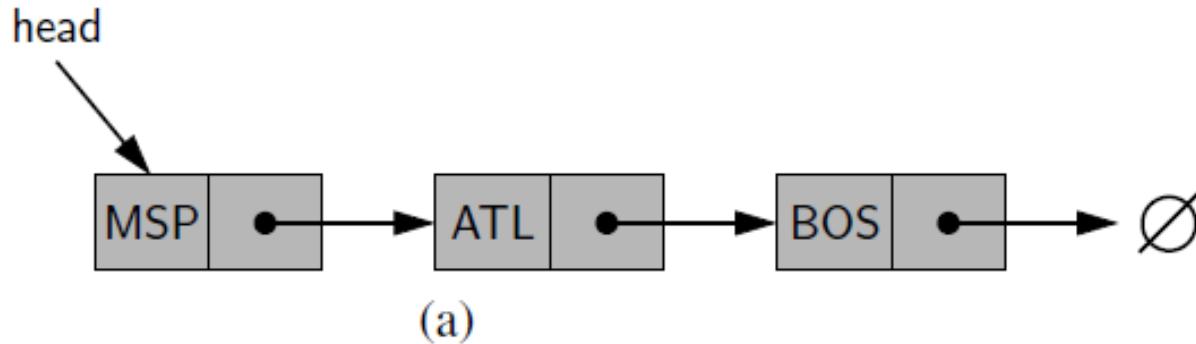


# Linked List

- The first and last nodes of a linked list are known as the **head** and **tail** of the list, respectively
- By starting at the head, and moving from one node to another by following each node's next reference, we can reach the tail of the list
- We can identify the tail as the node having **None** as its next reference. This process is commonly known as **traversing the linked list**.
- Because the next reference of a node can be viewed as a link or pointer to another node, the process of traversing a list is also known as **link hopping** or **pointer hopping**



# Inserting an Element at the Head of a Singly Linked List

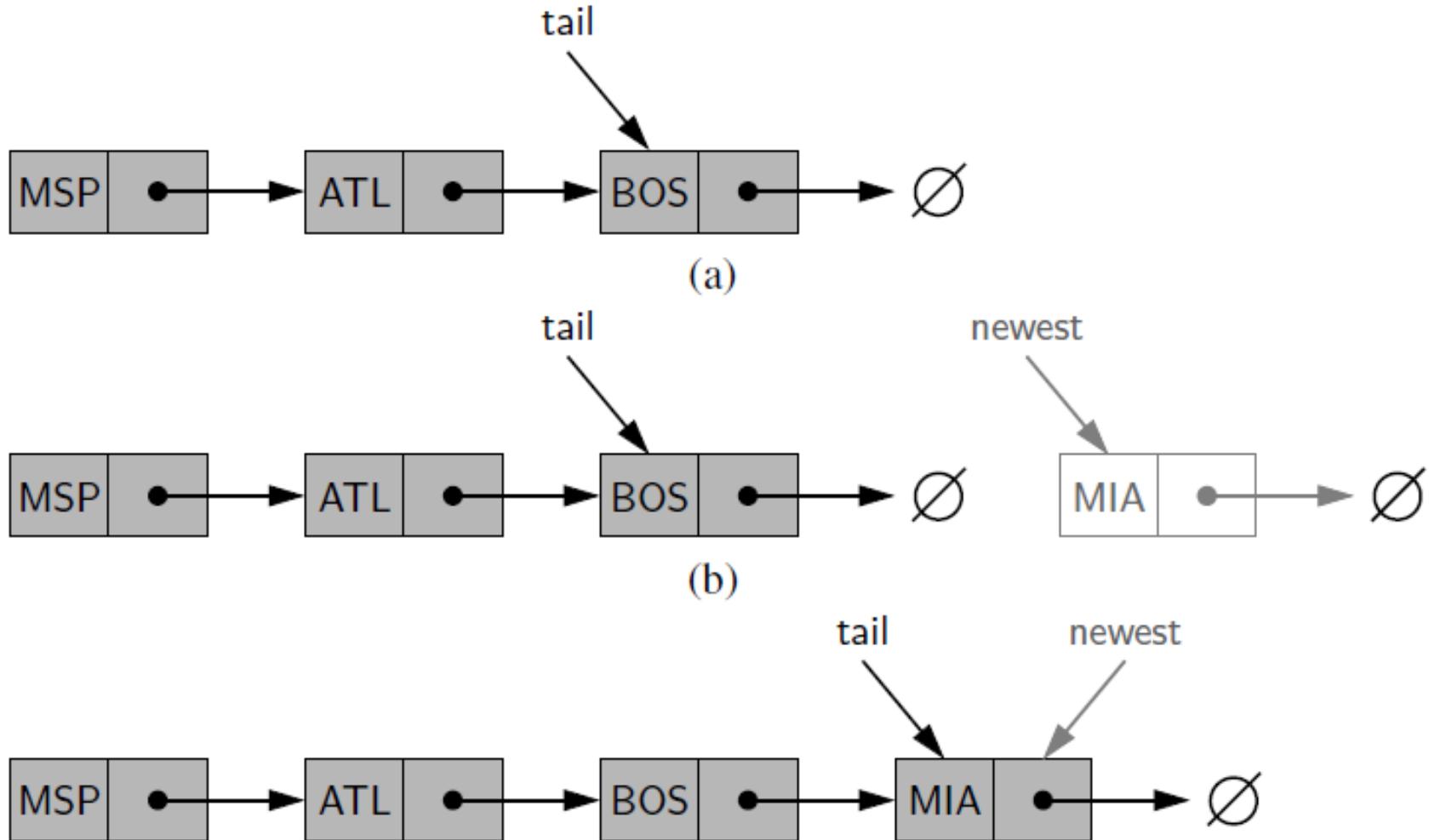


# Pseudo code for inserting a node at the head

**Algorithm** add\_first(L, e):

```
newest = Node(e) {create new node instance storing reference to element e}  
newest.next = L.head {set new node's next to reference the old head node}  
L.head = newest {set variable head to reference the new node}  
L.size = L.size + 1 {increment the node count}
```

# Inserting an Element at the Tail of a Singly Linked List

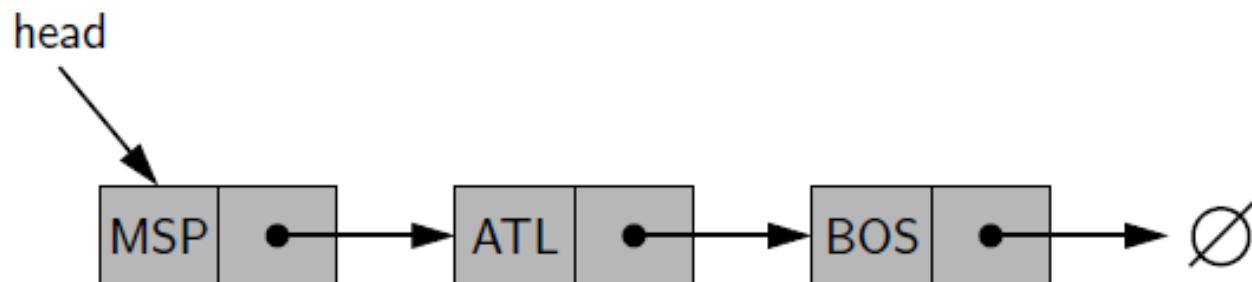
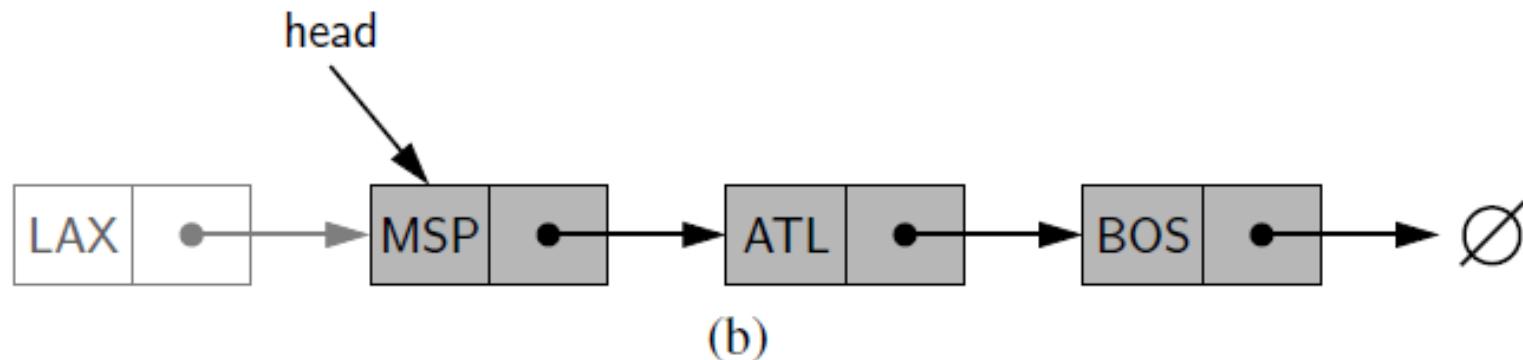
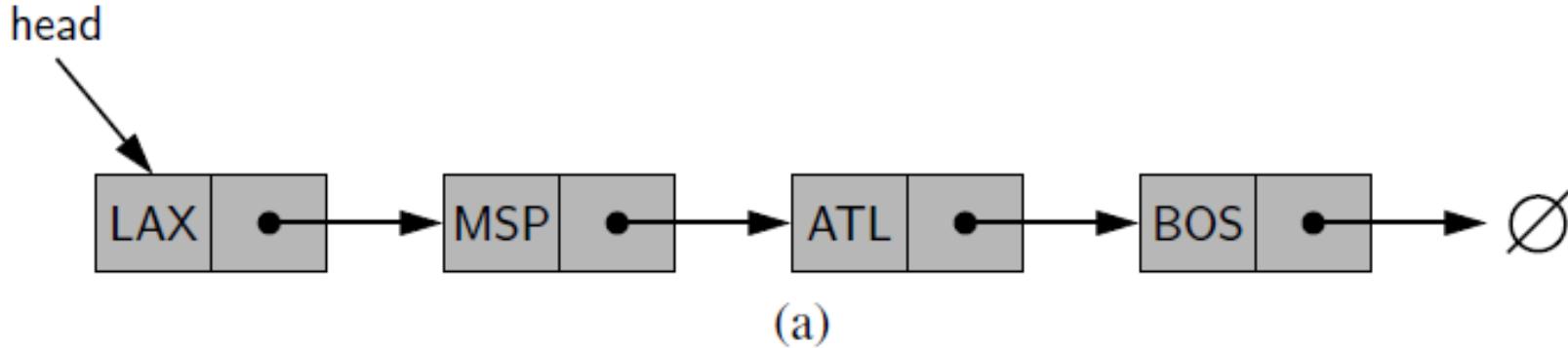


# Pseudo code for inserting at the tail

**Algorithm** add\_last(L, e):

```
newest = Node(e) {create new node instance storing reference to element e}
newest.next = None {set new node's next to reference the None object}
L.tail.next = newest {make old tail node point to new node}
L.tail = newest {set variable tail to reference the new node}
L.size = L.size + 1 {increment the node count}
```

# Removing an Element from the head of a Singly Linked List



# Pseudo code for removing a node from the head

**Algorithm** remove\_first(L):

**if L.head is None then**

Indicate an error: the list is empty.

L.head = L.head.next

{make head point to next node (or None)}

`L.size = L.size - 1`

{decrement the node count}

# Summary: Creating a linked list



```
class Node:  
    def __init__(self, element, pointer=None):  
        self.element = element  
        self.pointer = pointer
```

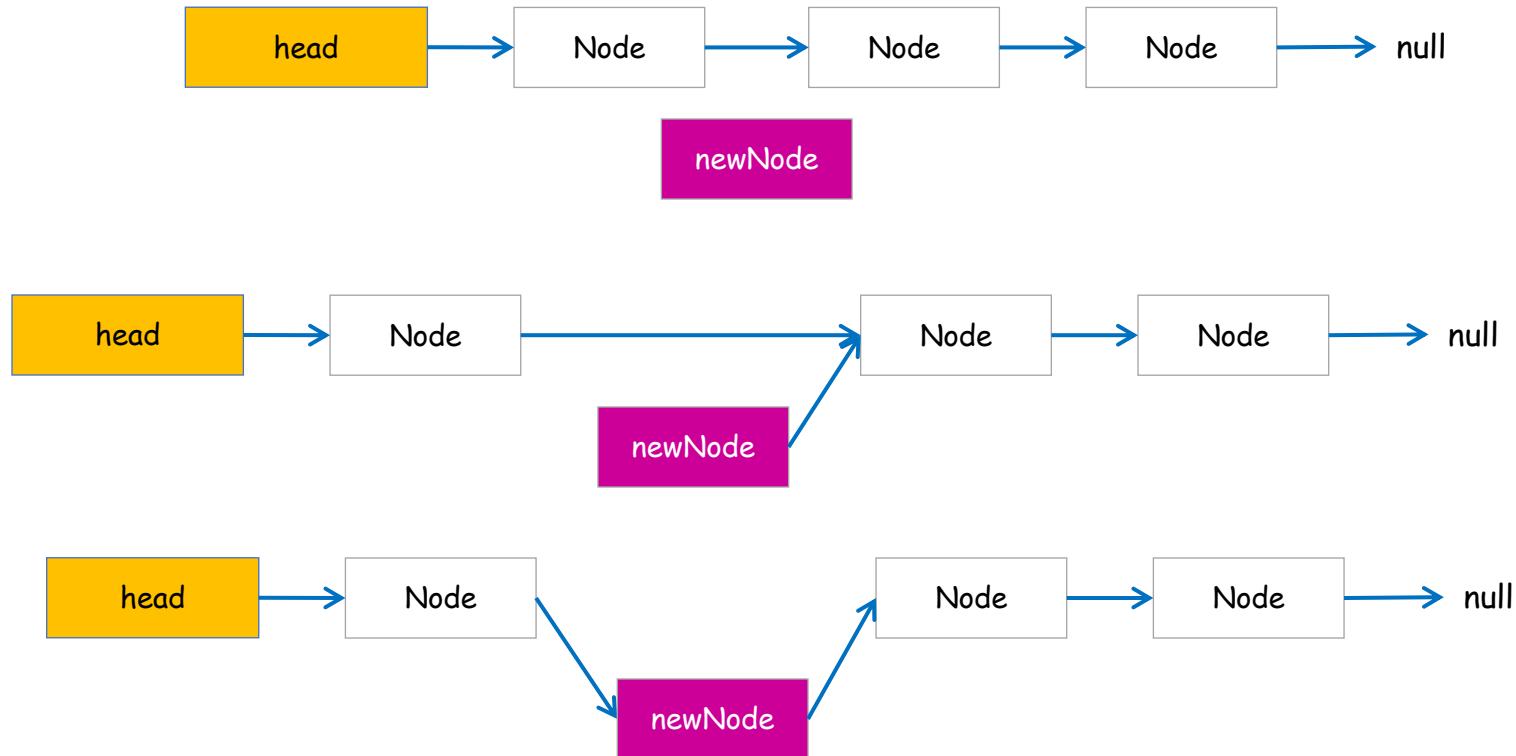
```
head = Node(0)  
node1 = Node(1)  
node2 = Node(2)  
tail = Node(3)
```

```
head.pointer = node1  
node1.pointer = node2  
node2.pointer = tail
```

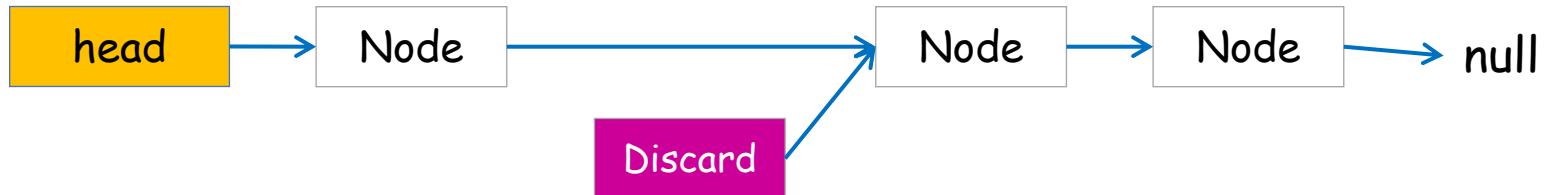
```
p = head  
while(p!=None):  
    print(p.element)  
    p = p.pointer
```

0  
1  
2  
3

# Summary: Inserting an element



# Summary: Removing an element



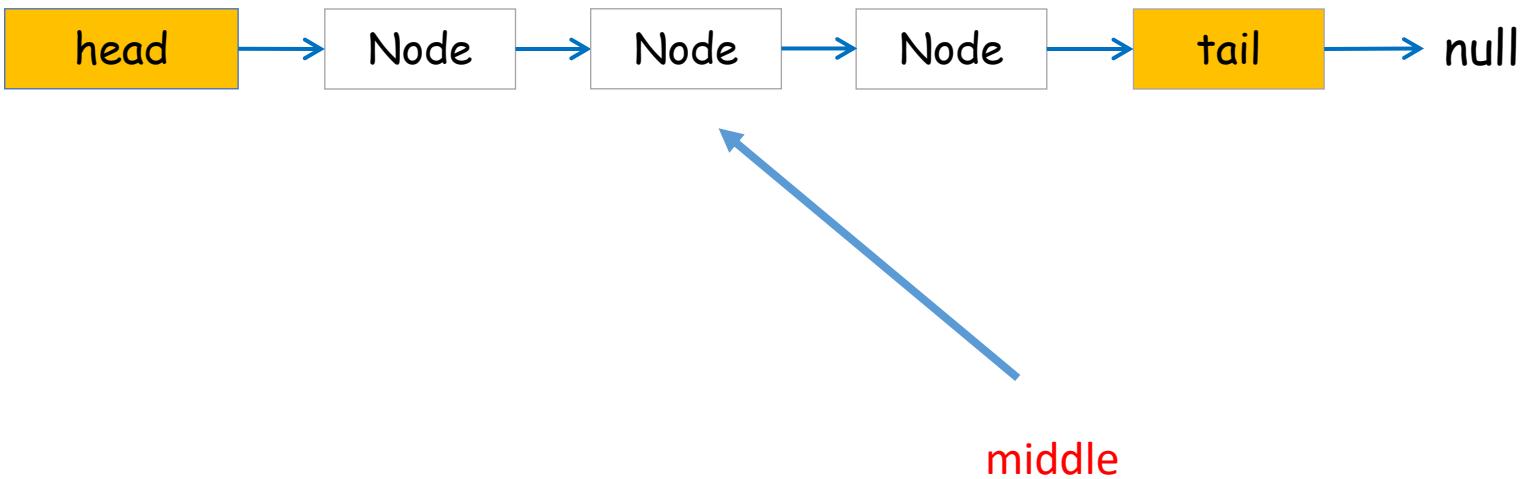
# Practice: Implement stack with a singly linked list

```
class Node:  
    def __init__(self, element, pointer):  
        self.element = element  
        self.pointer = pointer  
  
class LinkedStack:  
  
    def __init__(self):  
        self.head = None  
        self.size = 0  
  
    def __len__(self):  
        return self.size  
  
    def is_empty(self):  
        return self.size == 0  
  
    def push(self, e):  
        self.head = Node(e, self.head)  
        self.size += 1  
  
    def top(self):  
        if self.is_empty():  
            print('Stack is empty. ')  
        else:  
            return self.head.element  
  
    def pop(self):  
        if self.is_empty():  
            print('Stack is empty. ')  
        else:  
            answer = self.head.element  
            self.head = self.head.pointer  
            self.size -= 1  
            return answer
```

# Practice: Implement queue with a singly linked list

```
class LinkedQueue:  
  
    def __init__(self):  
        self.head = None  
        self.tail = None  
        self.size = 0  
  
    def __len__(self):  
        return self.size  
  
    def is_empty(self):  
        return self.size == 0  
  
    def first(self):  
        if self.is_empty():  
            print('Queue is empty.')  
        else:  
            return self.head.element  
  
    def dequeue(self):  
        if self.is_empty():  
            print('Queue is empty.')  
        else:  
            answer = self.head.element  
            self.head = self.head.pointer  
            self.size -= 1  
            if self.is_empty():  
                self.tail = None  
            return answer  
  
    def enqueue(self, e):  
        newest = Node(e, None)  
  
        if self.is_empty():  
            self.head = newest  
        else:  
            self.tail.pointer = newest  
        self.tail = newest  
        self.size += 1
```

# Practice: how to find the middle node



# Practice: how to find the middle node

```
def findMiddleByLength(head):
    # First loop to calculate the length of the linked list
    length = 0
    current = head
    while current:
        length += 1
        current = current.next

    # Calculate the middle index for the first middle node
    middle_index = (length - 1) // 2

    # Second loop to traverse to the middle node
    current = head
    index = 0
    while index < middle_index:
        current = current.next
        index += 1

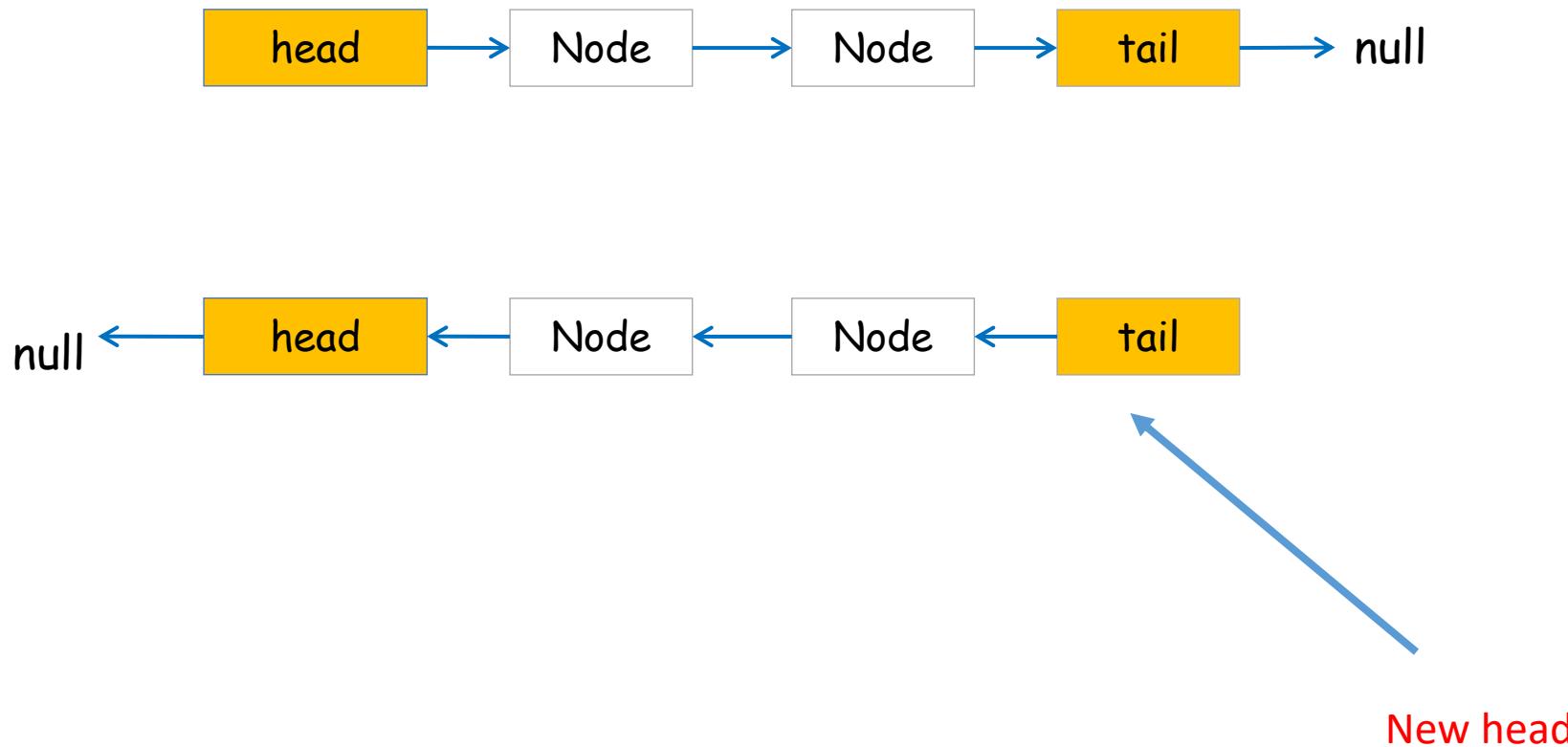
    print("The middle node's data is:", current.data)
    return current
```

# Practice: how to find the middle node

```
# Define the Node class
class Node:
    def __init__(self, data):
        self.data = data # Store data
        self.next = None # Pointer to the next node

# Function to find the middle node of the linked list
def findMiddle(head):
    slow = head # Slow pointer moves one step at a time
    fast = head # Fast pointer moves two steps at a time
    while fast and fast.next:
        slow = slow.next # Move slow pointer by one
        fast = fast.next.next # Move fast pointer by two
    print("The middle node's data is:", slow.data)
    return slow
```

# Reverse a linked list



# Reverse a linked list

```
class Node:
    def __init__(self, data):
        self.data = data # Store data
        self.next = None # Pointer to the next node

# Function to reverse the linked list
def reverseLinkedList(head):
    prev = None
    current = head
    while current:
        next_node = current.next # Save the next node
        current.next = prev # Reverse the link
        prev = current # Move prev to current
        current = next_node # Move to next node
    return prev # New head of the reversed list
```

# Reverse a linked list via recursion

```
# Define the Node class
class Node:
    def __init__(self, data):
        self.data = data # Store data
        self.next = None # Pointer to the next node

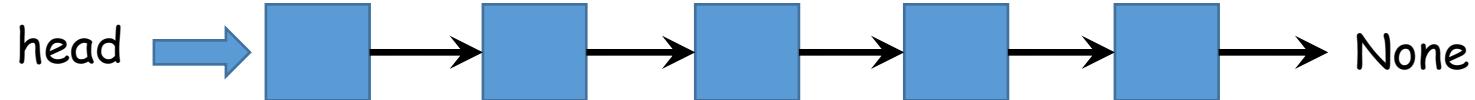
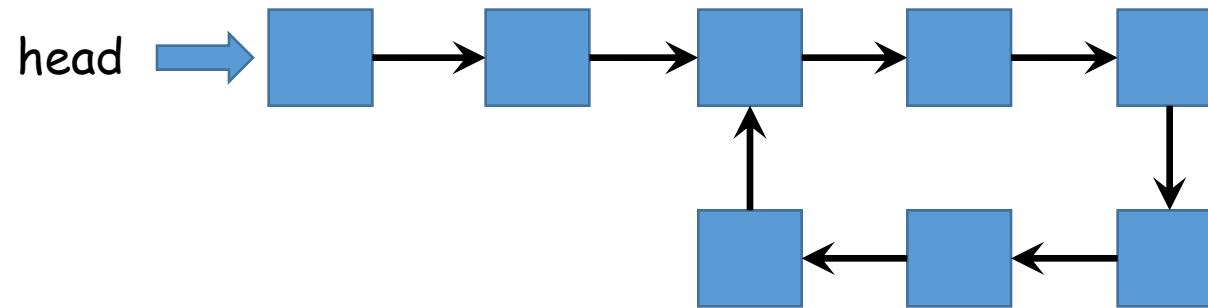
# Function to reverse the linked list recursively
def reverseLinkedListRec(head):
    # Base case: if head is empty or only one node, it's already reversed
    if head is None or head.next is None:
        return head

    # Recursively reverse the rest of the list
    rest = reverseLinkedListRec(head.next)

    # Adjust the pointers
    head.next.next = head
    head.next = None

    # Return the new head of the reversed list
    return rest
```

# Practice: check if there exists a cycle in a linked list



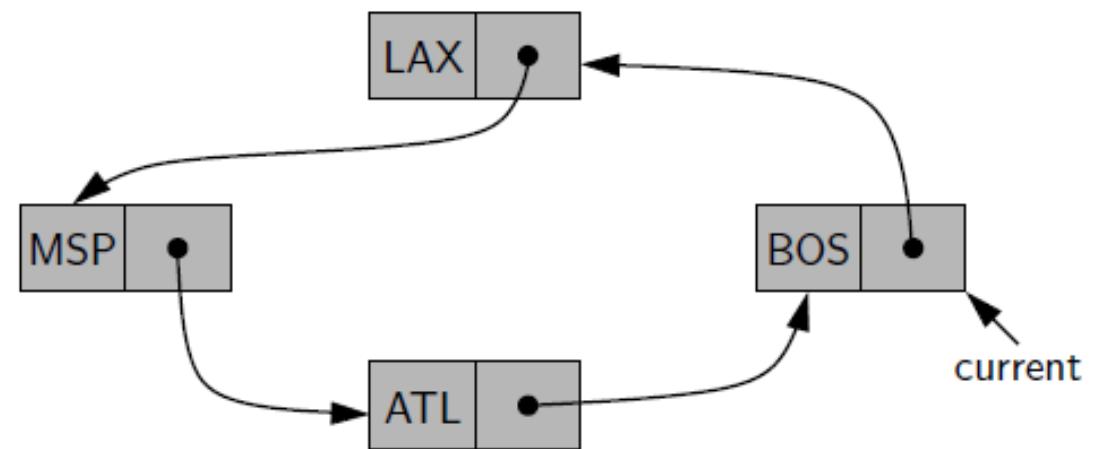
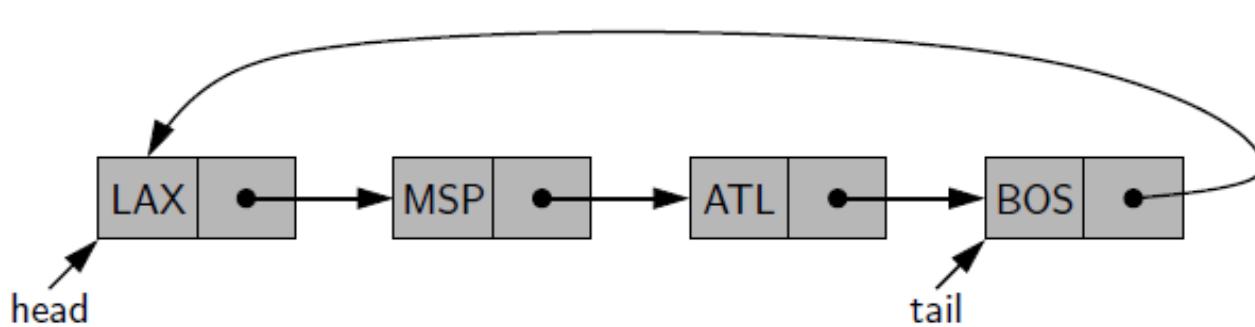
# Practice: check if there exists a cycle in a linked list

```
# Define the Node class
class Node:
    def __init__(self, data):
        self.data = data # Store data
        self.next = None # Pointer to the next node

# Function to check if the linked list has a cycle
def hasCycle(head):
    slow = head # Tortoise moves one step at a time
    fast = head # Hare moves two steps at a time
    while fast and fast.next:
        slow = slow.next # Move slow pointer by one
        fast = fast.next.next # Move fast pointer by two
        if slow == fast:
            print("Cycle detected at node with data:", slow.data)
            return True
    print("No cycle detected.")
    return False
```

# Circularly Linked List

- The **tail** of a linked list can use its next reference to point back to the **head** of the list
- Such a structure is usually called a **circularly linked list**



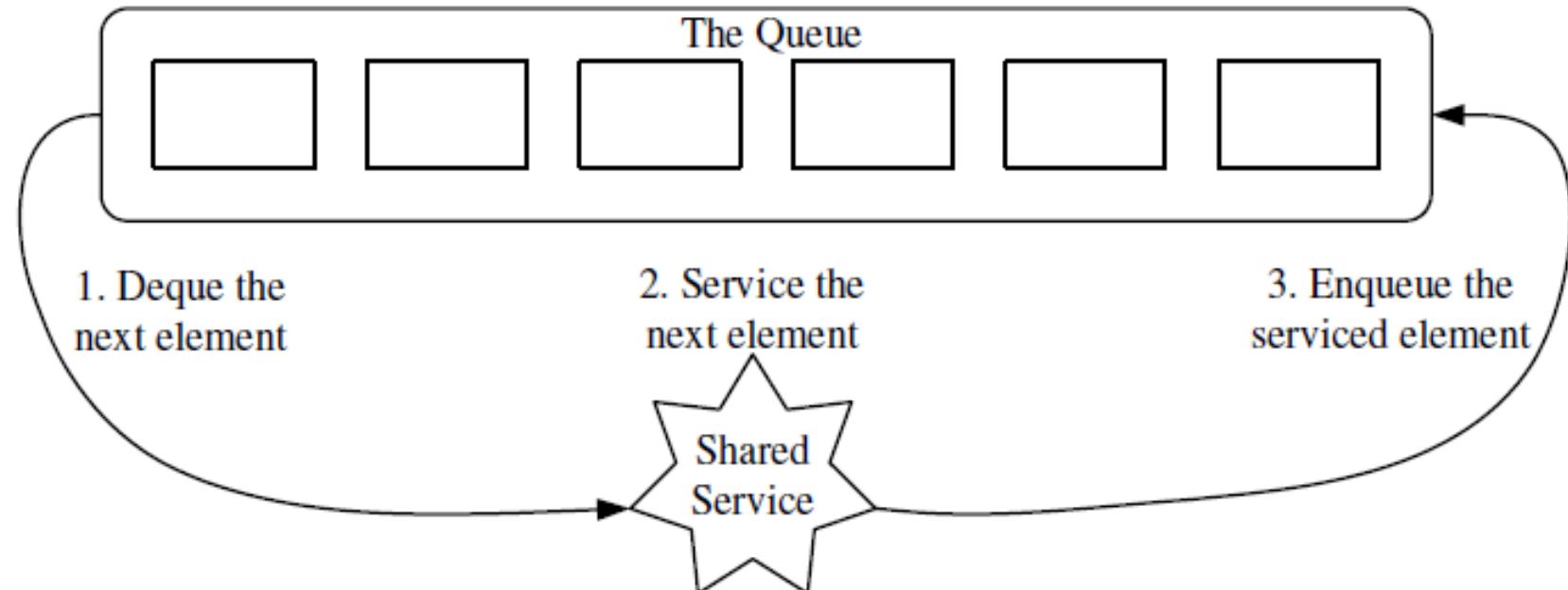
## Example: Round-robin scheduler

- A **round-robin scheduler** iterates through a collection of elements in a circular fashion and “serves” each element by performing a given action on it
- Such a scheduler is used, for example, to **fairly allocate** a resource that must be shared by a collection of clients
- For instance, round-robin scheduling is often used to allocate slices of CPU time to various applications running concurrently on a computer

# Implementing round-robin scheduler using standard queue

- A round-robin scheduler could be implemented with the standard queue, by repeatedly performing the following steps on queue Q:

- 1)  $e = Q.dequeue()$
- 2) Service element e
- 3)  $Q.enqueue(e)$



# Implement a Queue with a Circularly Linked List

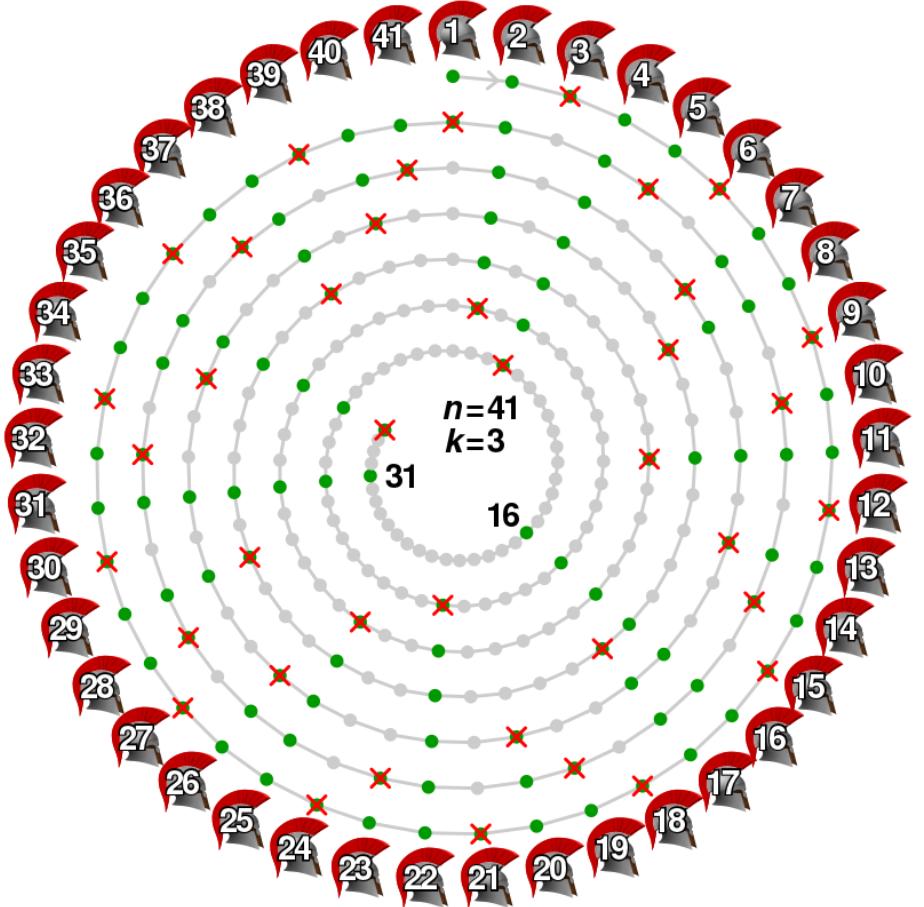
```
class Node:  
    def __init__(self, element, pointer):  
        self.element = element  
        self.pointer = pointer  
  
class CQueue:  
  
    def __init__(self):  
        self.__tail = None  
        self.__size = 0  
  
    def __len__(self):  
        return self.__size  
  
    def is_empty(self):  
        return self.__size == 0  
  
    def first(self):  
        if self.is_empty():  
            print('Queue is empty. ')  
        else:  
            head = self.__tail.pointer  
            return head.element
```

```
def dequeue(self):
    if self.is_empty():
        print('Queue is empty.')
    else:
        oldhead = self.__tail.pointer
        if self.__size == 1:
            self.__tail = None
        else:
            self.__tail.pointer = oldhead.pointer    Skip the old head
        self.__size -= 1
    return oldhead.element

def enqueue(self, e):
    newest = Node(e, None)
    if self.is_empty():
        newest.pointer = newest                      A single self-pointed node
    else:
        newest.pointer = self.__tail.pointer
        self.__tail.pointer = newest                  Insert after the tail!
    self.__tail = newest
    self.__size += 1
```

# Exercise: The Josephus Problem

- The Josephus Problem
  - There are  $n$  people standing in a circle waiting to be executed. After the first man is executed,  $k - 1$  people are skipped and the  $k$ -th man is executed. Then again,  $k-1$  people are skipped and the  $k$ -th man is executed. The elimination proceeds around the circle (which is becoming smaller and smaller as the executed people are removed), until only the last man remains, who is given freedom.
  - The task is to choose the place in the initial circle so that you survive, given  $n$  and  $k$ .

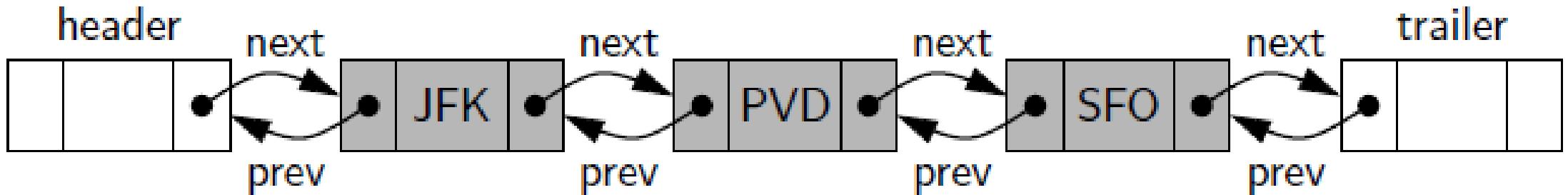


# Doubly linked list

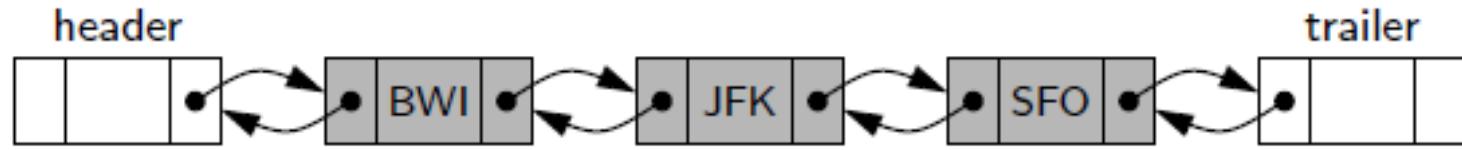
- For a singly linked list, we can efficiently **insert** a node at either end of a singly linked list, and can **delete** a node at the **head** of a list
- But we **cannot** efficiently **delete** a node at the **tail** of the list
- We can define a linked list in which each node keeps an explicit reference to the node before it and a reference to the node after it
- This kind of data structure is called **doubly linked list**

# Head and tail sentinels

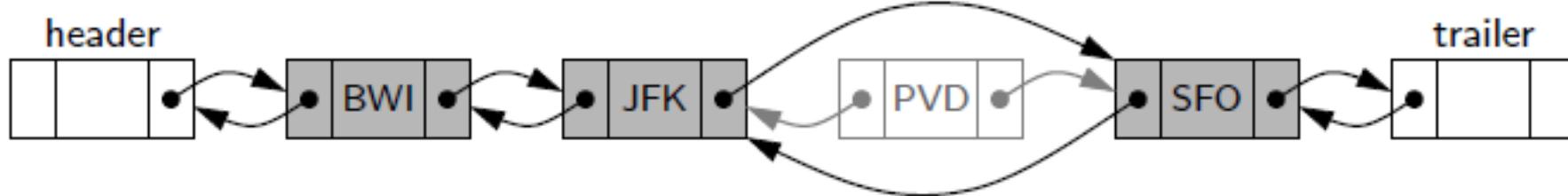
- In order to avoid some special cases when operating near the boundaries of a doubly linked list, it helps to add special nodes at both ends of the list: a **header node** at the beginning of the list, and a **trailer node** at the end of the list
- These “**dummy**” nodes are known as **sentinels** (or guards), and they **do not store** elements of the primary sequence



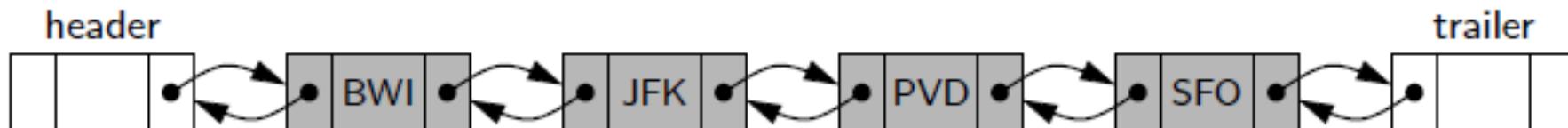
# Inserting in the middle of a doubly linked list



(a)

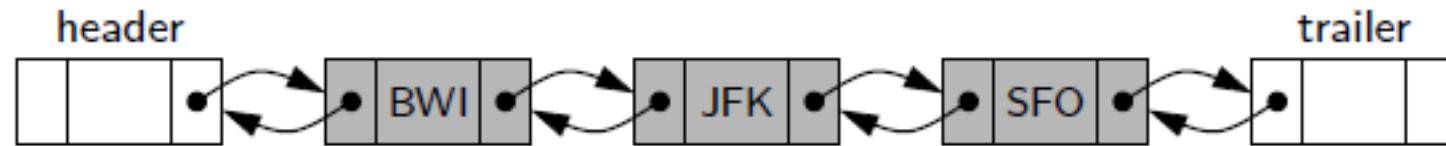


(b)

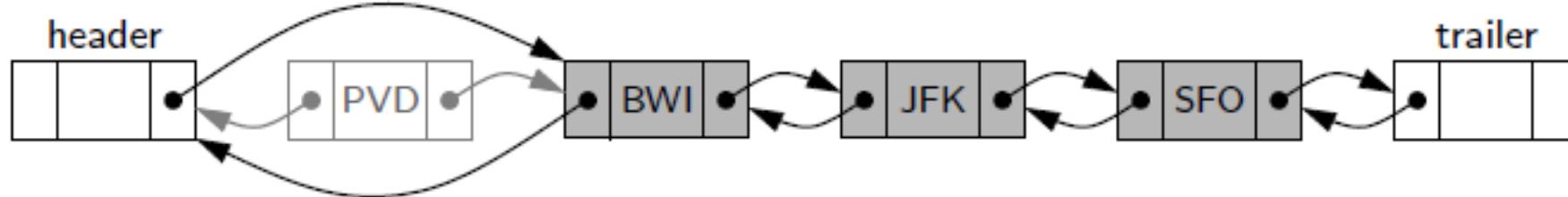


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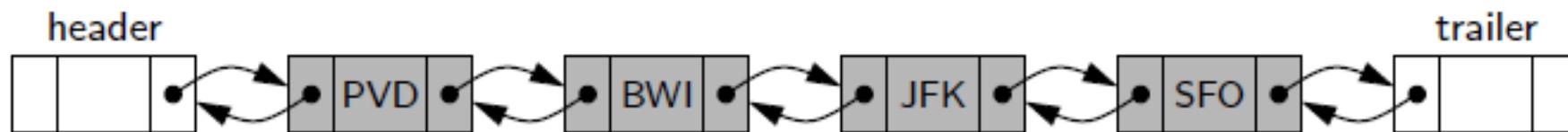
# Inserting at the head of the doubly linked list



(a)

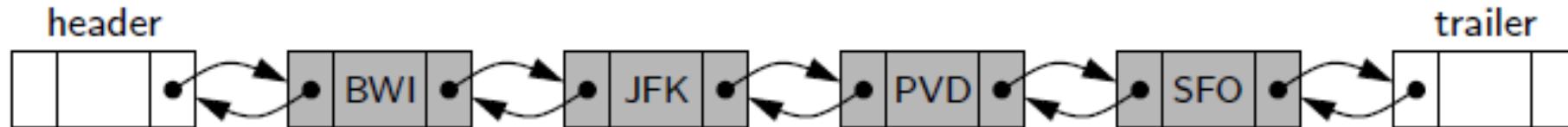


(b)

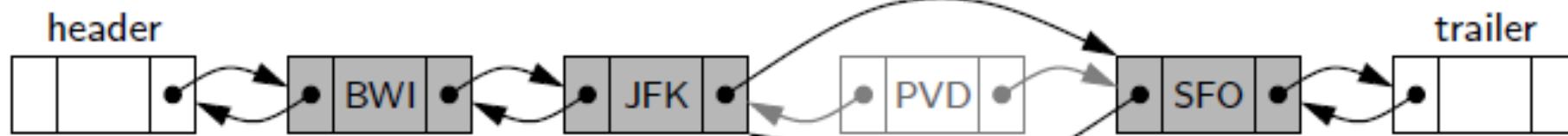


(c)

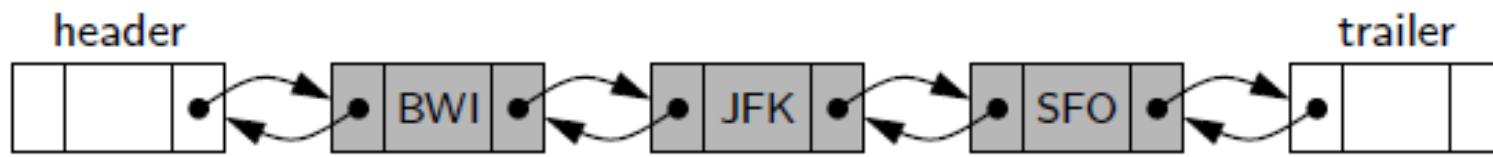
# Deleting from the doubly linked list



(a)



(b)



(c)

# Code for the doubly linked list

```
class Node:  
    def __init__(self, element, prev, nxt):  
        self.element = element  
        self.prev = prev  
        self.nxt = nxt  
  
class DLLList:  
  
    def __init__(self):  
        self.header = Node(None, None, None)  
        self.trailer = Node(None, None, None)  
        self.header.nxt = self.trailer  
        self.trailer.prev = self.header  
        self.size = 0  
  
    def __len__(self):  
        return self.size  
  
    def is_empty(self):  
        return self.size == 0
```

```
def insert_between(self, e, predecessor, successor):  
    newest = Node(e, predecessor, successor)  
    predecessor.nxt = newest  
    successor.prev = newest  
    self.size+=1  
    return newest  
  
def delete_node(self, node):  
    predecessor = node.prev  
    successor = node.nxt  
    predecessor.nxt = successor  
    successor.prev = predecessor  
    self.size -=1  
    element = node.element  
    node.prev = node.nxt = node.element = None  
    return element  
  
def iterate(self):  
    pointer = self.header.nxt  
    print('The elements in the list: ')  
    while pointer != self.trailer:  
        print(pointer.element)  
        pointer = pointer.nxt
```

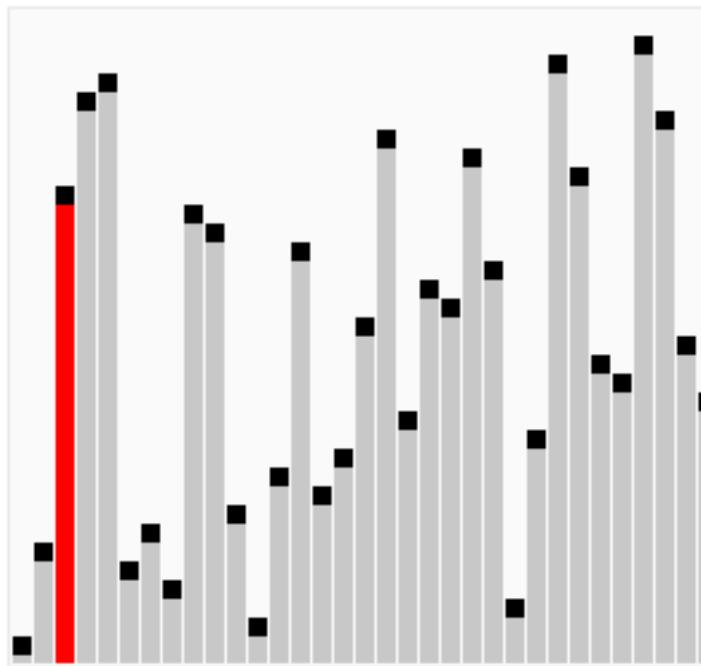
```
def main():  
    d=DLLList()  
    d.__len__()  
  
    newNode = d.insert_between(10, d.header, d.trailer)  
    newNode = d.insert_between(20, newNode, d.trailer)  
    newNode = d.insert_between(30, newNode, d.trailer)  
    d.iterate()  
    d.delete_node(d.header.nxt.nxt)  
    d.iterate()
```

# Bubble sort (Optional)

- **Bubble sort** is a simple sorting algorithm
- Its general procedure is:
  - 1) Iterate over a list of numbers, compare every element  $i$  with the following element  $i+1$ , and swap them if  $i$  is larger
  - 2) Iterate over the list again and repeat the procedure in step 1, but ignore the last element in the list
  - 3) Continuously iterate over the list, but each time ignore one more element at the tail of the list, until there is only one element left

- 
- 3 4 2 1 9 8 7 6 5

# A longer example



# Practice: Bubble sort over a standard list

```
def bubble(bubbleList):
    listLength = len(bubbleList)
    while listLength > 0:
        for i in range(listLength - 1):
            if bubbleList[i] > bubbleList[i+1]:
                buf = bubbleList[i]
                bubbleList[i] = bubbleList[i+1]
                bubbleList[i+1] = buf
        listLength -= 1
    return bubbleList
```

```
def main():
    bubbleList = [3, 4, 1, 2, 5, 8, 0, 100, 17]
    print(bubble(bubbleList))
```

# Practice: Bubble sort over a singly linked list

?

# Solution:

```
from LinkedQueue import LinkedQueue

def LinkedBubble(q):
    listLength = q.size

    while listLength > 0:
        index = 0
        pointer = q.head
        while index < listLength-1:
            if pointer.element > pointer.pointer.element:
                buf = pointer.element
                pointer.element = pointer.pointer.element
                pointer.pointer.element = buf
                Swap values
                index += 1
                pointer = pointer.pointer
            listLength -= 1
    return q
```

```
def outputQ(q):
    pointer = q.head

    while pointer:
        print(pointer.element)
        pointer = pointer.pointer

def main():
    oldList = [9, 8, 6, 10, 45, 67, 21, 1]
    q = LinkedQueue()

    for i in oldList:
        q.enqueue(i)

    print('Before the sorting... ')
    outputQ(q)

    q = LinkedBubble(q)
    print()
    print('After the sorting... ')
    outputQ(q)
```

# Quick sort (Optional)

- Quick sort is a widely used algorithm, which is more efficient than bubble sort
- The main procedure of quick sort algorithm is:
  - 1) Pick an element, called a **pivot**, from the array
  - 2) **Partitioning**: reorder the array so that all elements with values less than the pivot come before the pivot, while all elements with values greater than the pivot come after it (equal values can go either way). After this partitioning, the pivot is in its final position. This is called the **partition operation**
  - 3) Recursively apply the above steps to the sub-array of elements with smaller values and separately to the sub-array of elements with greater values

**Unsorted Array**



3 7 8 5 2 1 9 5 4

3 7 8 5 2 1 9 5 4

3 5 8 5 2 1 9 4 7

3 9 8 5 2 1 4 5 7

3 1 8 5 2 4 9 5 7

3 1 2 5 4 8 9 5 7

3 1 2 4 5 8 9 5 7

3 1 2

1 2 3

5 8 9 5 7

5 5 9 7 8

5 5 7 9 8

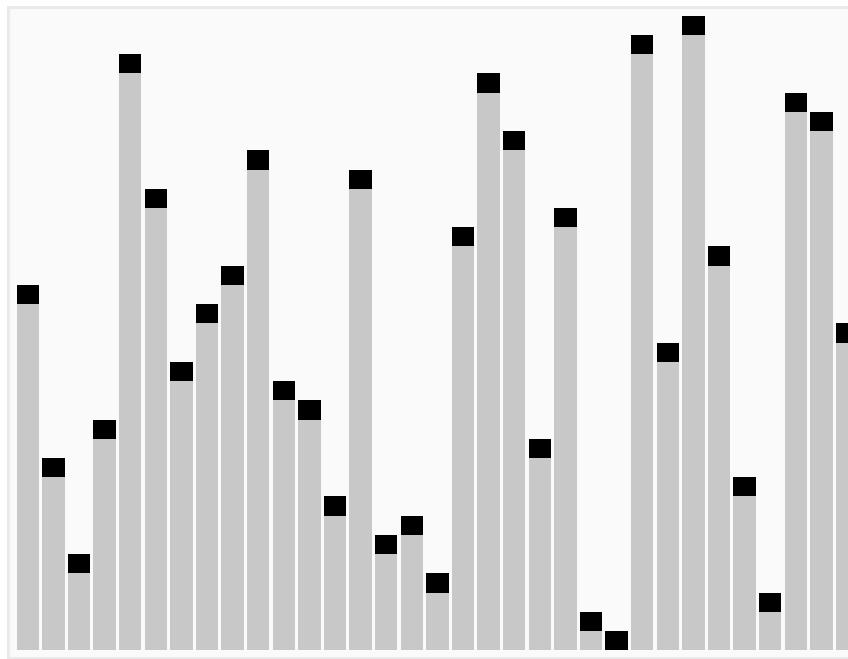
5 5

8 9

8 9

1 2 3 4 5 5 7 8 9

# A longer example



# Practice: Quick sort over a standard list

```
def quickSort(L, low, high):
    i = low
    j = high
    if i >= j:
        return L
    key = L[i]
    while i < j:
        while i < j and L[j] >= key:
            j = j-1
        L[i] = L[j]
        while i < j and L[i] <= key:
            i = i+1
        L[j] = L[i]
    L[i] = key
    quickSort(L, low, i-1)
    quickSort(L, j+1, high)
return L
```

# Practice: Quick sort over a singly linked list

?