Lecture 1 - Markov Decision Process

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DDA4230: Reinforcement Learning
Course Page: [Click]

DDA 4230 Resources

Please join our Slack group.

https://join.slack.com/t/slack-us51977/shared_invite/zt-22g8b40v8-0qSs9o0G3~8hXHwWydlCpw

Please check our course page.

https://guiliang.github.io/courses/cuhk-dda-4230/dda_4230.html

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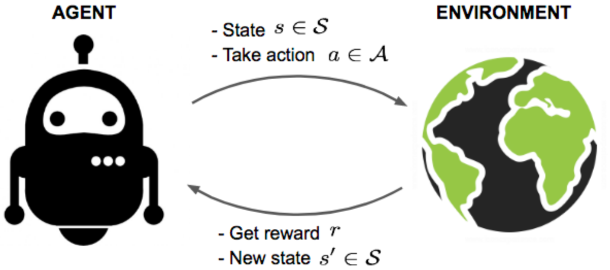
Reinforcement Learning

A reinforcement learning **agent** interacts with its **world** and from that learns how to **maximize cumulative reward** over time.



Reference: https://lilianweng.github.io/posts/2018-02-19-rl-overview/

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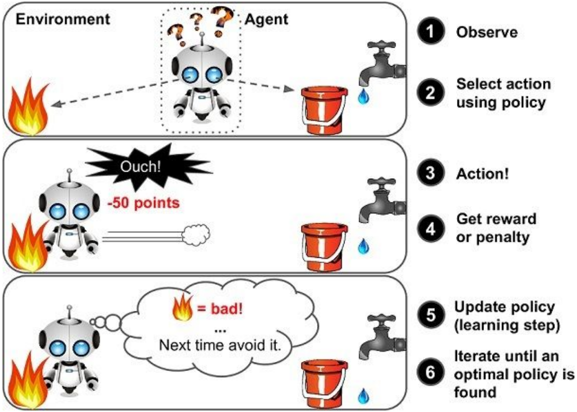
Reinforcement Learning



Reference: https://www.odinschool.com/

No **teacher** or **knowledge** of the world model. Learn to act through **trial and error**.

- **E1**: Agent + Fire -> -50.
- **E2**: Agent + Bucket + Fire -> -50.
- **E3**: Agent + Bucket + Water + fire -> +50.
- **E4**: What will the agent do?

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Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).

Supervised Learning

Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).
- **Interact** with the environment.

Supervised Learning

- Given a dataset, which consists of examples and labels (knowledge).

Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).
- **Interact** with the environment.
- Making **sequential decisions**.

Supervised Learning

- Given a dataset, which consists of examples and labels (knowledge).
- No interaction. Only an offline dataset.

Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).
- **Interact** with the environment.
- Making **sequential decisions**.
- Learn a policy to maximize **cumulative** rewards.

Supervised Learning

- Given a dataset, which consists of examples and labels (knowledge).
- No interaction. Only an offline dataset.
- Making one-step predictions.

Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).
- **Interact** with the environment.
- Making **sequential decisions**.
- Learn a policy to maximize **cumulative** rewards.

Supervised Learning

- Given a dataset, which consists of examples and labels (knowledge).
- No interaction. Only an offline dataset.
- Making one-step predictions.
- Learn a predictor to maximizing point-wise prediction accuracy.

Connection with other learning problems

Reinforcement Learning

- No **teacher** or **knowledge** of the world model (e.g., environment).
- **Interact** with the environment.
- Making **sequential decisions**.
- Learn a policy to maximize **cumulative** rewards.

Unsupervised (Contrastive) Learning

- Given a dataset, which consists of only examples (No labels).
- No interaction. Only an offline dataset.
- Learning latent structure of dataset.
- Learn the latent features for classification or identification.

Challenges in Reinforcement Learning

- How to balance exploration and exploitation?
- How to generalize its experience?
- How to model the delayed consequences of actions (look-ahead).

Markov Decision Process (MDP)

Discrete-time Markov decision process (MDP), denoted as the tuple $(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \rho_0, \gamma)$.

- $\mathcal{S}$ the state space;
- $\mathcal{A}$ the action space. $\mathcal{A}$ can depend on the state s for $s \in \mathcal{S}$;
- $\mathcal{T} : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$ the environment transition probability function;
- $\mathcal{R} : \mathcal{S} \times \mathcal{A} \to \Delta(\mathbb{R})$ the reward function;
- $\rho_0 \in \Delta(\mathcal{S})$ the initial state distribution;
- $\gamma \in [0, 1]$ the discount factor.

Note that $\Delta(\mathcal{X})$ denotes the set of all distributions over set $\mathcal{X}$.

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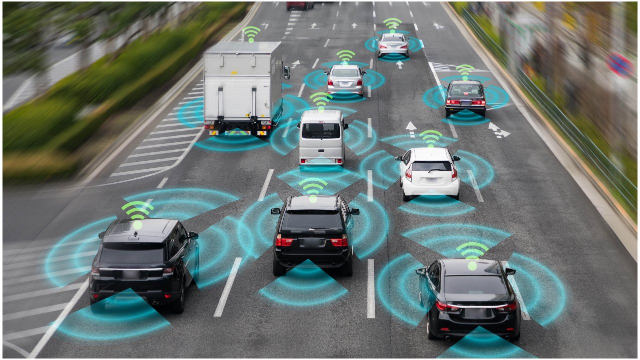
Markov Decision Process (MDP)

Map the Reinforcement Learning (RL) environment to an MDP.

Application 1: Autonomous Driving.



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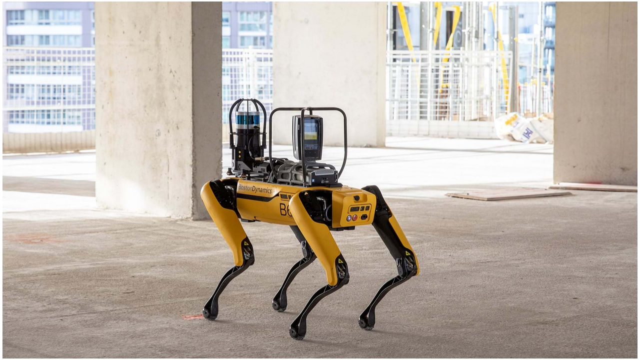
Reference: https://www.topgear.com/

Markov Decision Process (MDP)

Map the Reinforcement Learning (RL) environment to an MDP.

Application 2: Robot Control.



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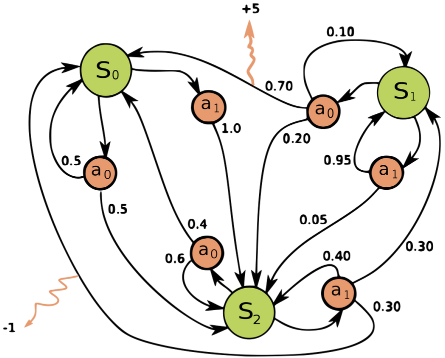
Reference: https://bostondynamics.com/

Markov Decision Process (MDP)

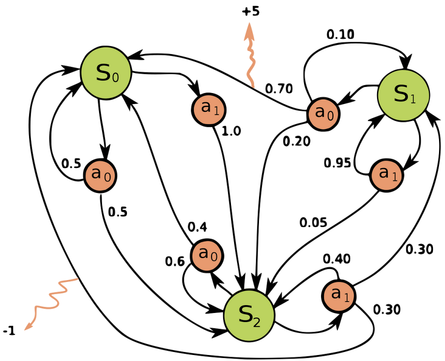


For $t = 0, 1, \ldots,$ start with $s_0 \sim \rho_0$.

- The agent observes the state s_t;
- The agent chooses an action a_t;
- The agent receives the reward $r_t \sim \mathcal{R}(s_t, a_t)$;
- The environment transitions to a subsequent state: $s_{t+1} \sim \mathcal{T}(s_t, a_t)$.

Reference: https://en.wikipedia.org/wiki/Markov_decision_process

Markov Decision Process (MDP)



Trajectory generation.

- This process generates the sequence $s_0, a_0, r_0, s_1, \ldots,$ until when s_T is a terminal state, or indefinitely.
- The sequence up to time t is defined as the trajectory indexed by t, as $\tau_t = (s_0, a_0, r_0, s_1, \ldots, r_t)$.

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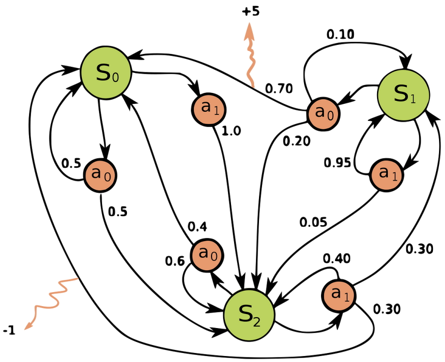
Markov Decision Process (MDP)



In an MDP, the optimal choice of action a_t depends only on the state s_t. A policy defines the mapping from $\mathcal{S}$ to $\mathcal{A}$.

- Stochastic policy: $\pi : \mathcal{S} \to \Delta(\mathcal{A})$
- Deterministic policy: $a_t = \pi(s_t)$

Reference: https://en.wikipedia.org/wiki/Markov_decision_process

Important Variables in MDPs

- The return is defined as the discounted cumulative reward as a random variable.

$$R_t = \sum_{t' \geq t}^{\infty} \gamma^{t'} r_{t'} \, .$$

- The expectation of the return is the objective to be maximized by the agent

$$J = \mathbb{E}_{s_t, a_t, r_t, t \geq 0}\left[R_0\right] = \mathbb{E}_{s_t, a_t, r_t, t \geq 0}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right] \text{ and } \pi = \arg\max_{\pi} J$$

  The expectation is subject to random variables $(s_0, a_0, r_0, s_1, \ldots, r_\infty)$ with a complicate state space $(\mathcal{S} \times \mathcal{A} \times \mathcal{R})^T$. The optimization problem is not characterisable (non-linear, non-convex, non-quadratic..) in general.

Important Variables in MDPs

The stochasticity of a Markov chain given the MDP and the policy can come from three components:

- Stochastic Markovian dynamics: $P_{\mathcal{T}}(s_{t+1}|s_t, a_t)$;
- Stochastic rewards: $P_{\mathcal{R}}(r_{t+1}|s_t, a_t)$;
- Stochastic policies: $\pi(a_t|s_t)$;

Important Variables in MDPs

- The action value Q-function of a given policy π

$$Q^{\pi}(s,a) = \mathbb{E}_{s_t,a_t,r_t,t\geq 0}\Big[\sum_{t=0}^{\infty} \gamma^t r(s_t,a_t) \mid s_0 = s, a_0 = a\Big]$$

which is the expected return of policy π at state s after taking action a.

- The state value function of a given policy π

$$V^{\pi}(s) = \mathbb{E}_{a\sim\pi(s)}\left[Q^{\pi}(s,a)\right]$$

which is the expected return given the initial state only.

Important Variables in MDPs

Example: Game Go

- Rewards: $r_0 = 0, r_2 = 0, ..., r_{T-1} = 0$. If win $r_T = 1$ otherwise $r_T = 0$.
- Discount: $\gamma \to 1$.
- $Q^\pi(s, a)$: winning probability of making a move a under state s by following policy π.
- $V^\pi(s)$: winning probability of at the state s by following policy π.

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Important Variables in MDPs

- The advantage function of a given policy π can be defined as:

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

- Based on the value functions, define the temporal-difference error

$$\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t).$$

Remark:

1. r_t: rewards at t.
2. $V(s_{t+1})$: expected cumulative rewards at $t+1, t+2, \ldots$.
3. $V(s_t)$: expected rewards at $t, t+1, t+2, \ldots$.
4. if π is optimal, $\mathbb{E}[\delta_t] = 0$.

The Bellman Equation

- State-value Bellman equation (named after Richard E. Bellman):

$$V(s_t) = \mathbb{E}[r_t + \gamma V(s_{t+1})] \quad \text{and} \quad V(s_T) = \mathbb{E}[r_T] \,.$$

  for non-terminal and terminal states, respectively.

- Action-value Bellman equation:

$$Q(s_t, a_t) = \mathbb{E}[r_t + \gamma Q(s_{t+1}, a) \mid a \sim \pi(a \mid s_{t+1})] \quad \text{and} \quad Q(s_T, a_T) = \mathbb{E}[r_T]$$

  for non-terminal and terminal states, respectively.

Stationarity of MDPs and Agents

Stationarity MDP

- Markovian dynamics of s_{t+1} depends only on s_t and a_t as $s_{t+1} \sim \mathcal{T}(s_t, a_t)$.
- The reward r_t depends only on s_t and a_t as $r_t \sim \mathcal{R}(s_t, a_t)$.
- A policy is stationary if the action depends only on the state: $a_t \sim \pi(s_t)$.

Remark: If there is an optimal policy, there is an optimal stationary policy if the process has a non-fixed horizon.

Non-Stationarity MDP

- The transition dynamics depend on the time t as $s_{t+1} \sim \mathcal{T}_t(s_t, a_t)$.
- The reward depend on the time t as $r_t \sim \mathcal{R}_t(s_t, a_t)$.
- A policy is non-stationary if the action also depends on t: $a_t \sim \pi_t(s_t)$.

State and Action Spaces

Two common settings of the state space and the action space are:

- $\mathcal{S} \in \mathbb{R}^n$ the n dimensional state space, $\mathcal{A} \in \mathbb{R}^m$ the m dimensional action space;
- $\mathcal{S} \in [n]$ the size-n discrete state space, $\mathcal{A} \in [m]$ the size-m discrete action space.

Discount of Rewards

The discount factor $\gamma \in [0,1]$ balances the short-term and long-term rewards. When the objective is discounted $(\gamma < 1)$

$$R_0 = r_0 + \gamma r_1 + \gamma^2 r_2 + \dots,$$

Two extreme cases are $\gamma = 0$ and $\gamma = 1$, where the former corresponds to $R_0 = r_0$ as a one-step MDP and the latter corresponds to $R_0 = r_0 + r_1 + r_2 + \dots$.

Agents of RL

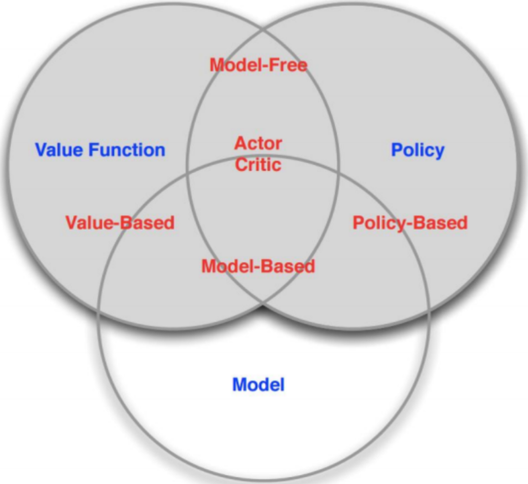
We can classify our agents in a number of ways:

Agent type	Policy	Value Function	Model
Value-based	Implicit	✓	?
Policy-based	✓	✗	?
Actor-critic	✓	✓	?
Model-based	?	?	✓
Model-free	?	?	✗

- ✓ indicates that the agent has the component.
- x indicates that it must not have the component.
- ? indicates that the agent may have that component.

Agents of RL

Classification of different reinforcement learning agents.



Classification of Markov structures

<table>
<tr><td colspan="2" rowspan="2">Markov structure</td><td colspan="2">Do actions have influence over the state transitions?</td></tr>
<tr><td>NO</td><td>YES</td></tr>
<tr><td rowspan="2">Are the states fully observable?</td><td>YES</td><td>Markov process (Markov chain)</td><td>Markov decision process</td></tr>
<tr><td>NO</td><td>Hidden Markov model</td><td>Partially observable Markov decision process</td></tr>
</table>

Classification of Markov structures

Fully observable Markov decision process

- All the players are observable.

Classification of Markov structures

Partially observable Markov decision process

- Only a partial number of the players are observable.

Question and Answering (Q&A)