

Lecture 12 - Discrete Q-learning

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DDA4230: Reinforcement Learning

Course Page: [\[Click\]](#)

Model-based v.s. Model-free Algorithms

The model indicates the **transition** function and the **reward** function. This estimation could be in form of point estimation or distribution estimation like posterior sampling.

- **Model-based Algorithm**: maintains an estimate of the model and uses the model when interacting with the environment.
- **Model-free Algorithm**: does not estimate the world model.

When we do not have a reasonable estimation of the model (under large state and action spaces and continuous settings), an error will be induced by a wrongly estimated model as the **model bias** (maybe accumulate during learning).



Q-Learning

We start with the value iteration algorithm and discuss how the model could be lifted.

Algorithm 1: Value iteration

Input: ϵ

For all states $s \in S$, $V'(s) \leftarrow 0$, $V(s) \leftarrow \infty$

while $\|V - V'\|_\infty > \epsilon$ **do**

$V \leftarrow V'$

 For all states $s \in S$, $V'(s) = \max_{a \in A} [R(s, a) + \gamma \sum_{s' \in S} P(s' | s, a) V(s')]$

$V^* \leftarrow V$ for all $s \in S$

$\pi^* \leftarrow \arg \max_{a \in A} [R(s, a) + \gamma \sum_{s' \in S} P(s' | s, a) V^*(s')] \quad , \quad \forall s \in S$

return $V^*(s)$, $\pi^*(s)$ for all $s \in S$



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Q-Learning

- The terms $\sum_{s' \in \mathcal{S}} P(s' | s, a) V(s')$ and $\sum_{s' \in \mathcal{S}} P(s' | s, a) V^*(s')$ could remove the dependency on P by representing the **action values**.
- Introducing the **step size** so that the update only takes at α portion of the action value while the $1 - \alpha$ portion of the action value remains the same.

Algorithm 2: Q-learning

Input: ϵ, α

For all $(s, a) \in \mathcal{S} \times \mathcal{A}$, $Q'(s, a) \leftarrow 0$, $Q(s, a) \leftarrow \infty$

while $\|Q - Q'\|_\infty > \epsilon$ **do**

$Q \leftarrow Q'$

 Sample a trajectory τ from the policy $\pi(a | s) = \arg \max_{a \in \mathcal{A}} Q(s, a)$

 For all state-action-reward-state tuple $(s, a, r, s') \in \tau$,

$Q'(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha \max_{a' \in \mathcal{A}} [r + \gamma Q(s', a')]$

$Q^* \leftarrow Q$ for all $(s, a) \in \mathcal{S} \times \mathcal{A}$

$\pi^* \leftarrow \arg \max_{a \in \mathcal{A}} Q(s, a)$

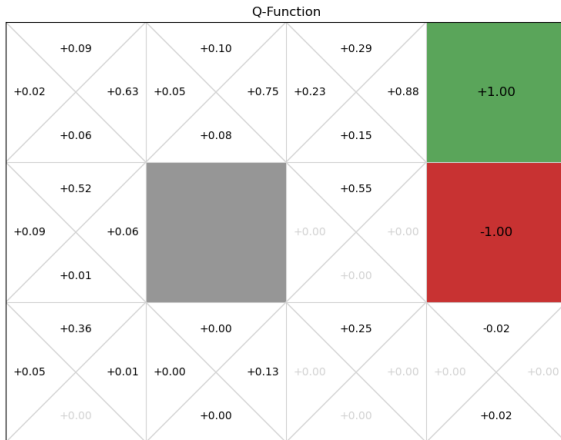
return $Q^*(s, a), \pi^*(s)$ for all s, a

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Q-Learning

Q Learning in Grid World.



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Exploration and ϵ -greedy Q-learning

In Q-learning, the trajectory sampled is subject to the current policy and thereof the current value estimation. However,

- It is possible that the algorithm is **stuck at a suboptimal action value estimate** and does not update itself.
- It is possible that **some states are never explored** with some initialization of the policy and value functions.

A simple way of involving exploration is to force the algorithm to **select a random action with probability ϵ** . This ϵ could decay over the iterations, as is in the ϵ -greedy algorithm for multi-armed bandits.



Exploration and ϵ -greedy Q-learning

Algorithm 3: Q-learning with ϵ -greedy exploration

Input: ϵ, α

For all $(s, a) \in \mathcal{S} \times \mathcal{A}$, $Q'(s, a) \leftarrow 0$, $Q(s, a) \leftarrow \infty$

while $\|Q - Q'\|_\infty > \epsilon$ **do**

$Q \leftarrow Q'$

 Sample a trajectory τ from the policy

$$\pi(a | s) = \begin{cases} \arg \max_{a \in \mathcal{A}} Q(s, a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

 For all state-action-reward-state tuple $(s, a, r, s') \in \tau$,

$$Q'(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha \max_{a' \in \mathcal{A}} [r + \gamma Q(s', a')]$$

$Q^* \leftarrow Q$ for all $(s, a) \in \mathcal{S} \times \mathcal{A}$

$\pi^* \leftarrow \arg \max_{a \in \mathcal{A}} Q(s, a)$

return $Q^*(s, a)$, $\pi^*(s)$ for all s, a

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Q-learning with UCB

In spite of simplicity, ε -greedy Q-learning does not have a rigorous regret guarantee.

- We present another variant of Q-learning with UCB exploration. This algorithm is the first Q-learning variant that has a rigorous regret guarantee of \sqrt{K} .
- We again use $Q_h(s, a)$ as the time-dependent action-value function, which is necessary when the horizon of each episode is constant.



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Q-learning with UCB

Algorithm 4: Q-learning with UCB exploration

Input: α : adaptive step size; p : probability tolerance

Initialize $Q_h(s, a) \leftarrow H$, $N_h(s, a) \leftarrow 0$

while $k \leq K - 1$ **do**

 Start an episode with s_0

for $h \leq H - 1$ **do**

 Take action $a_h^k = \arg \max_a Q_h(s_h^k, a)$ and observe s_{h+1}^k

$N_h(s_h^k, a_h^k) \leftarrow N_h(s_h^k, a_h^k) + 1$

 Update the action value as

$$Q_h(s_h^k, a_h^k) \leftarrow (1 - \alpha)Q_h(s_h^k, a_h^k) + \alpha \left[r_h(s_h^k, a_h^k) + V_{h+1}(s_{h+1}^k) + c \sqrt{\frac{H^3 \log(nmHK/\delta)}{N_h(s_h^k, a_h^k)}} \right]$$

 Update the state value as

$$V_h(s_h^k) = \min \left\{ \max_a Q_h(s_h^k, a), H \right\}$$

$Q_h^* \leftarrow Q_h$

$\pi_h^* \leftarrow \arg \max_a Q_h(s, a)$

return Q_h^* , π_h^* for all $h \in [H]$

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Q-learning with UCB

Theorem

By choosing $\alpha = \frac{H+1}{H+N}$ with the visitation count $N = N_h(s_h^k, a_h^k)$, there exists an absolute constant c such that with probability at least $1 - \delta$ the regret of Q-learning with UCB exploration is at most $O(\sqrt{nmH^5 K \log(nmHK/\delta)})$.

The proof relies on the cast of the variables into a filtration and therefore the use of the Azuma-Hoeffding inequality (introduced in LN3). For those students that are interested in the proof we could host you with a presentation of it.



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Question and Answering (Q&A)



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