bp神经网反向传播

符号定义：

X: 输入矩阵，维度为$n \times t$ 。n 为数据个数，t 为特征向量维数。

y^m: 第m层神经元的输出矩阵。维度为$1 \times size(m)$

y_i^m: 第m层第i个神经元的输出值。

ω^m: 第$m-1$ 层神经元到第m层神经元的权重。维度为 $size(m-1) \times size(m)$

ω_{ji}^m: 第$m-1$ 层第j个神经元到第m层第i个神经元的权重

ϵ_i^m: 第m层第i个神经元的线性组合。即$\epsilon_i^m = \sum_{j=1}^{size(m-1)} \omega_{ji}^m y_j^{m-1} + b_i^m$

ϵ^m: 第m层神经元的线性组合。$1 \times size(m)$

b^m: 第m层神经元的偏置。维度为$1 \times size(m)$

D: 神经元层数。标量

l_i: 输出层第i个神经元所造成的。维度为$1 \times size(D)$

L: 总损失。标量

Act_m : 第m层神经元的激活函数

$Loss$: 损失函数

运算定义：

$y^m = Act_m(y^{m-1} \cdot \omega^m + b^m)$

$L = \sum_{i=1}^{size(D)} l_i = \sum_{i=1}^{size(D)} Loss(y_i^D)$

反向传播推理：

ω_{kj}^m: 第$m-1$ 层第k个神经元到第m层第j个神经元的权重

$$\frac{\partial L}{\partial w_{kj}^m} = \frac{\partial L}{\partial \epsilon_j^m} \frac{\partial \epsilon}{\partial w_{kj}^m} = \frac{\partial L}{\partial \epsilon_j^m} y_k^{m-1}$$

考虑第$m-1$ 层所有神经元到第m层第j个神经元的权重

$$\frac{\partial L}{\partial w_{\cdot j}^m} = \frac{\partial L}{\partial \epsilon_j^m} \frac{\partial \epsilon_j^m}{\partial w_{\cdot j}^m} = \frac{\partial L}{\partial \epsilon_j^m} (y^{m-1})^T$$

考虑矩阵

$$d\omega^m = \frac{\partial L}{\partial w^m} = \frac{\partial L}{\partial \epsilon^m} \frac{\partial \epsilon^m}{\partial w^m} = (y^{m-1})^T \frac{\partial L}{\partial \epsilon^m}$$

再来考虑 $\frac{\partial L}{\partial \epsilon^m}$，先考虑其中任意一个值

$$\frac{\partial L}{\partial \epsilon_j^m}=\sum_{i=1}^{size(m+1)}\frac{\partial L}{\partial \epsilon_i^{m+1}}\frac{\partial \epsilon_i^{m+1}}{\partial y_j^m}\frac{\partial y_j^m}{\partial \epsilon_j^m}=\frac{\partial y_j^m}{\partial \epsilon_j^m}\sum_{i=1}^{size(m+1)}\frac{\partial L}{\partial \epsilon_i^{m+1}}\omega_{ji}^{m+1}=Act_m^{'}(\epsilon_j^m)(\frac{\partial L}{\partial \epsilon^{m+1}}\cdot(\omega_{j\cdot}^{m+1})^T)$$

考虑矩阵

$$\frac{\partial L}{\partial \epsilon^m}=Act_m^{'}(\epsilon^m)\odot[\frac{\partial L}{\partial \epsilon^{m+1}}\cdot(\omega^{m+1})^T]$$

当$m=D$时

$$\frac{\partial L}{\partial \epsilon_i^D}=\frac{\partial L}{\partial y_i^D}\frac{\partial y_i^D}{\partial \epsilon_i^D}=Loss^{'}(y_i^D)\cdot Act_D^{'}(\epsilon_i^D)$$

考虑矩阵

$$\frac{\partial L}{\partial \epsilon_i^D}=Loss^{'}(y^D)\odot Act_D^{'}(\epsilon^D)$$

考虑b^m

$$db^m=\frac{\partial L}{\partial \epsilon^m}$$

如果综合，即b^m为标量

$$db^m=\sum_{i=1}^{size(m)}\frac{\partial L}{\partial \epsilon_i^m}$$

正向传播

$$y_0=X$$

$$y^m=Act_m(\epsilon^m)=Act_m(y^{m-1}\cdot\omega^m+b^m)$$

记录所有 ϵ^m 和 y^m

反向传播

step1：计算

$$\frac{\partial L}{\partial \epsilon_i^D}=Loss^{'}(y^D)\odot Act_D^{'}(\epsilon^D)$$

step2：自最深层向前遍历依次计算 `for m from D to 1`

$$d\omega^m=\frac{\partial L}{\partial w^m}=\frac{\partial L}{\partial \epsilon^m}\frac{\partial \epsilon^m}{\partial w^m}=(y^{m-1})^T\frac{\partial L}{\partial \epsilon^m}$$

$$db^m=\frac{\partial L}{\partial \epsilon^m}\text{或}db^m=\sum_{i=1}^{size(m)}\frac{\partial L}{\partial \epsilon_i^m}$$

$$\omega^m=\omega^m-\alpha d\omega^m//\text{不考虑}L1\text{、}L2\text{回归}$$

$$b^m=b^m-\alpha db^m$$

$$\frac{\partial L}{\partial \epsilon^{m-1}} = Act_m^{'}(\epsilon^{m-1}) \odot [\frac{\partial L}{\partial \epsilon^m} \cdot (\omega^m)^T]$$