Bridging Direct and Indirect Data-Driven Control Formulations via Regularizations and Relaxations

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***Abstract*—In this article, we discuss connections between sequential system identification and control for linear time-invariant systems, often termed indirect data-driven control, as well as a contemporary direct data-driven control approach seeking an optimal decision compatible with recorded data assembled in a Hankel matrix and robustified through suitable regularizations. We formulate these two problems in the language of behavioral systems theory and parametric mathematical programs, and we bridge them through a multicriteria formulation trading off system identification and control objectives. We illustrate our results with two methods from subspace identification and control: namely, subspace predictive control and low-rank approximation, which constrain trajectories to be consistent with a nonparametric predictor derived from (respectively, the column span of) a data Hankel matrix. In both cases, we conclude that direct and regularized data-driven control can be derived as convex relaxation of the indirect approach, and the regularizations account for an implicit identification step. Our analysis further reveals a novel regularizer and a plausible hypothesis explaining the remarkable empirical performance of direct methods on nonlinear systems.**

***Index Terms*—Optimal control, Pareto optimization, system identification.**

I. Introduction

THE vast realm of data-driven control methods can be classified into *indirect data-driven control* approaches consisting of sequential system identification and model-based control as well as *direct data-driven control* approaches seeking an optimal decision compatible with recorded data. Both approaches have a rich history, and they have received renewed interest cross-fertilized by novel methods and widespread interest in machine learning. Representative recent surveys are in [1]–[8].

The pros and cons of both paradigms have often been elaborated on. The indirect approach is modular with well-understood subtasks, though modeling and identification are cumbersome, their results are often not useful for control (due to, e.g., incompatible uncertainty quantifications), and practitioners often prefer end-to-end methods. Direct approaches promise to resolve these problems by learning control policies directly from data. However, they are often analytically and computationally less tractable and rarely apply to real-time and safety-critical control systems. Selected direct methods that proved themselves in theory and practice are iterative feedback tuning and virtual reference feedback tuning [9]–[11].

Quite a few approaches have bridged the direct and indirect data-driven control paradigms. Of relevance to this article, we note the literature on identification for control [7], [12]–[14] and control-oriented regularized identification [15], which propose that the control objective should bias the identification task. Likewise, dual control dating to Feldbaum [16] addresses the exploration versus exploitation tradeoffs in simultaneous identification and optimal control; see the works in [17]–[19] for recent contributions. Furthermore, Campestrini *et al.* [20] formulate data-driven model reference control as an identification problem, where various degrees of prior information can be incorporated so that the method can range between the direct and the indirect approach.

We take a similar perspective here: the sequential identification and control tasks can be abstracted as a nested bilevel optimization problem: find the best control subject to a model, where the model is the best fit to a dataset within some hypothesis class. This approach is modular and both steps admit tractable formulations, but generally it is also suboptimal: there is no separation principle—aside from special cases, see [7, Sec. 4]—for these two nested optimization problems. An end-to-end direct algorithmic approach may thus outperform indirect methods if a tractable formulation was available. For the latter, we resort to a paradigm square in between behavioral system theory and subspace system identification methods.

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Behavioral system theory [21]–[23] takes an abstract view on dynamical systems as sets of trajectories, and it does not require parametric representations, which makes it appealing from a data-centric perspective. For example, linear time-invariant (LTI) systems are characterized as shift-invariant subspaces within an ambient space of time series. The role of identification is to find such a low-dimensional feature from data. Subspace methods take a similar (albeit more algorithmic) viewpoint [24]–[26] and extract parametric models from the range and null spaces of a low-rank data Hankel matrix.

Both lines of work come together in a result known as the fundamental lemma [27]; see also the works in [6], [28], and [29] for recent extensions. It states that, under some assumptions, the set of all finite-length trajectories (the restricted behavior) of an LTI system equals the range space of a data Hankel matrix. This result serves as the theoretic underpinning for work in subspace identification [29]–[31] and data-driven control, in particular subspace predictive control (SPC) based on nonparametric models [32]–[34], explicit feedback policies parameterized by data matrices [35]–[37], and data-enabled predictive control (DeePC) seeking compatibility of predicted trajectories with the range space of a data Hankel matrix. The latter methods have first been established for deterministic LTI systems in [38] and [39] and have recently been extended by suitably regularizing the optimal control problems. Closed-loop stability was certified in [40]. The regularizations were first mere heuristics [41] but have later been constructively derived by robust control and optimization [42]–[46]. These approaches, albeit recent, have proved themselves in practical nonlinear problems in multiple domains [45]–[49]. We also note the recent maximum-likelihood perspective [50]. We refer to Markovsky and Dörfler [6] surveying results surrounding the fundamental lemma.

In this article, we explore the following questions: how does DeePC relate to a prior system identification? What are principled regularizations? And why does it work so well in the nonlinear case? We start our investigations from indirect data-driven control formulated as a bilevel optimization problem in the general output feedback setting. As a vehicle to transition between indirect and direct approaches, we consider a multicriteria problem trading off identification and control objectives reminiscent of similar approaches [7], [12]–[20] blending the two. We formally show that one tail of its Pareto front corresponds to the bilevel problem, and a convex relaxation results in the regularized DeePC formulations used in [40]–[49].

Most of our results are formulated in the abstract language of behavioral systems theory and parametric mathematical programs, but we also specialize our treatment to two concrete methods: SPC [32]–[34] and low-rank approximation [39]. In both cases, we conclude that the direct regularized data-driven control can be derived as a convex relaxation of the indirect approach, where LTI complexity specifications (selecting the model class) are dropped, and the projection of the data on the set of LTI systems is replaced by regularizations accounting for implicit identification. In particular, starting from indirect data-driven control based on low-rank approximation of a Hankel matrix, we arrive at a DeePC formulation with an ℓ_1-regularizer (see Theorem IV.8). When formulating indirect data-driven control via the SPC framework, our analysis reveals a novel regularizer for DeePC promoting a least-square data fit by projecting on the null space of the Hankel matrix (see Theorem IV.6).

We illustrate our results with numerical studies illustrating the role of regularization, superiority of the new regularizer, and comparisons. Informed by our analysis, we hypothesize and numerically confirm that the indirect approach is superior in case of "variance" error, e.g., for LTI stochastic systems, and the direct approach wins in terms of "bias" error, e.g., for nonlinear systems supporting the empirical observations in [45]–[49]. Similar bias-variance tradeoffs can also be found in [51] discussing suboptimality of direct and indirect methods as function of the data size. These findings also resonate with those of data-driven model reference control [20] concluding that the direct approach is superior in reducing the bias, whereas the indirect one gives better variance—especially if an erroneous model class is selected.

The remainder of this article is organized as follows. Section II reviews representations of LTI systems. Section III formulates the direct and indirect data-driven control problems, and Section IV bridges them. Section V contains our numerical studies. Finally, Section VI concludes this article. Readers familiar with the behavioral approach may skip Section II.

II. LTI Systems and Their Representations

We adopt a behavioral perspective, which allows for system theory independent of parametric representations. We aim at a concise exposition and refer to the works in [6], [21]–[23] for details.

A. Behavioral Perspective on Discrete-Time LTI Systems

Consider the discrete-time axis $\mathbb{Z}$, the signal space $\mathbb{R}^q$, and the associated space of trajectories $\mathbb{R}^{q\mathbb{Z}}$ consisting of all q-variate sequences $(\dots, w(-1), w(0), w(1), \dots)$ with $w(i) \in \mathbb{R}^q$. Consider a permutation matrix P partitioning each $w(i) = P \begin{bmatrix} u(i) \\ y(i) \end{bmatrix}$, where $u(i) \in \mathbb{R}^m$ and $y(i) \in \mathbb{R}^{q-m}$ are free and dependent variables that will later serve as inputs and outputs. The *behavior* $\mathscr{B}$ is defined as a subset of the space of trajectories, $\mathscr{B} \subset \mathbb{R}^{q\mathbb{Z}}$, and a system as the triple $(\mathbb{Z}, \mathbb{R}^q, \mathscr{B})$.

In what follows, we denote a system merely by its behavior $\mathscr{B}$, keeping the signal space $\mathbb{R}^{q\mathbb{Z}}$ fixed throughout. A system is *linear* if $\mathscr{B}$ is a subspace of $\mathbb{R}^{q\mathbb{Z}}$. Let σ denote the shift operator with action $\sigma w(t) = w(t+1)$. A system is *time-invariant* if $\mathscr{B}$ is shift-invariant: $\sigma\mathscr{B} = \mathscr{B}$. Finally, $\mathscr{B}_L$ is the restriction of $\mathscr{B}$ to $\mathbb{R}^{qL}$, i.e., to trajectories of length $L \in \mathbb{Z}_{>0}$.

B. Kernel Representations and Parametric Models

Rather than mere set-theoretic descriptions, one typically works with explicit *parametric representations* (colloquially termed *models*) of LTI systems. For instance, a *kernel representation* with *lag* ℓ specifies an LTI behavior as

$$\mathscr{B} = \operatorname{kernel}(R(\sigma)) = \left\{ w \in \mathbb{R}^{q\mathbb{Z}} \ : R(\sigma)w = 0 \right\}$$

where $R(\sigma) = R_0 + R_1\sigma + \cdots + R_\ell\sigma^\ell$ is a polynomial matrix of degree ℓ, and the matrices $R_0, R_1, \dots, R_\ell$ take values in

$\mathbb{R}^{(q-m)\times q}$. Alternatively, one can unfold the kernel representation by revealing a latent variable: the state $x(t) \in \mathbb{R}^n$. The *input/state/output* (or *state-space*) *representation* is

$$\mathscr{B} = \Big\{ w = P \begin{bmatrix} u \\ y \end{bmatrix} \in \mathbb{R}^{q\mathbb{Z}} \ : \exists x \in \mathbb{R}^{n\mathbb{Z}} \text{ such that } \sigma x = Ax + Bu\,,\ y = Cx + Du \Big\}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{q-m\times n}$, and $D \in \mathbb{R}^{q-m\times m}$. We assume that the lag ℓ (respectively, the state dimension n) is minimal, i.e., there is no other kernel (respectively, state-space) representation with smaller lag (respectively, state dimension).

The dimension n of a minimal state-space representation manifests itself in a minimal kernel representation as $n = \sum_{i=1}^{q-m} \ell_i$, where ℓ_i is the degree of the ith row of $R(\sigma)$.

C. Representation-Free Estimation and Behavior Dimension

Given a state-space representation with m inputs, order n, and lag ℓ, the extended observability and convolution matrices

$$\mathscr{O}_L = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{L-1} \end{bmatrix} \quad \text{and} \quad \mathscr{G}_L = \begin{bmatrix} D & 0 & \cdots & & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ CA^{L-2}B & \cdots & CAB & CB & D \end{bmatrix}$$

parameterize all length-L trajectories in $\mathscr{B}_L$ as

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} I & 0 \\ \mathscr{G}_L & \mathscr{O}_L \end{bmatrix} \begin{bmatrix} u \\ x_{\text{ini}} \end{bmatrix} \tag{1}$$

where $x_{\text{ini}} \in \mathbb{R}^n$ is the initial state. Recall the *observability problem*: given length-L time series of inputs and outputs, can x_{ini} be reconstructed? Equation (1) gives a succinct answer: namely, x_{ini} can be reconstructed if and only if $\mathscr{O}_L$ has full column-rank. The minimum L so that $\mathscr{O}_L$ has full rank n equals the *lag* ℓ of a minimal kernel representation. As readily deducible from (1) and formalized in [38, Lemma 1], in a *representation-free* setting, the initial condition x_{ini} for a trajectory $w \in \mathscr{B}_L$ can be estimated via a prefix trajectory $w_{\text{ini}} = \big(w(-T_{\text{ini}}+1), \ldots, w(-1), w(0)\big)$ of length $T_{\text{ini}} \geq \ell$ so that the concatenation $w_{\text{ini}} \wedge w \in \mathscr{B}_{T_{\text{ini}}+L}$ is a valid trajectory.

Hence, an LTI system is characterized by the complexity parameters (q, m, n, ℓ), and we denote the corresponding class of LTI systems by $\mathscr{L}^{q,n}_{m,\ell}$: namely, LTI systems with m inputs, $q-m$ outputs, minimal state dimension n, and minimal lag ℓ.

The following lemma characterizes the dimension of $\mathscr{B}_L \in \mathscr{L}^{q,n}_{m,\ell}$ in terms of the complexity parameters (q, m, n, ℓ).

Lemma II.1 (Dimension of $\mathscr{B}_L$): Let $\mathscr{B} \in \mathscr{L}^{q,n}_{m,\ell}$. Then, $\mathscr{B}_L$ is a subspace of $\mathbb{R}^{qL}$, and for $L \geq \ell$, its dimension is $mL + n$.

Proof: Due to linearity of $\mathscr{B}$, $\mathscr{B}_L \subset \mathbb{R}^{qL}$ is a subspace. To show that the dimension of $\mathscr{B}_L$ equals $mL + n$ for $L \geq \ell$, we appeal to a minimal state-space representation of $\mathscr{B}$—a state-space-independent proof is in [29, Sec. 3]. We have $w = P \begin{bmatrix} u \\ y \end{bmatrix} \in \mathscr{B}_L$ if and only if (1) holds for some $x_{\text{ini}} \in \mathbb{R}^n$. Since the representation is minimal, $\mathscr{O}_L \in \mathbb{R}^{(q-m)L\times n}$ is of full column-rank for $L \geq \ell$. Therefore, the matrix $\begin{bmatrix} I & 0 \\ \mathscr{G}_L & \mathscr{O}_L \end{bmatrix} \in \mathbb{R}^{qL\times(mL+n)}$ is of full rank $mL + n$ for $L \geq \ell$ and forms a basis for $\mathscr{B}_L$. Thus, $\mathscr{B}_L$ has dimension $mL + n$. ■

Remark II.2 (Complexity bounds): All forthcoming results assume known complexity (q, m, n, ℓ). When only data and no prior information are available, it is reasonable to assume upper bounds on (q, m, n, ℓ). In this case, the anticipated dimension of $\mathscr{B}_L$ is at most $mL + n$, and the forthcoming rank equalities the behavior dimension should be replaced by inequalities.

D. Image Representation of Restricted Behavior

The restricted behavior $\mathscr{B}_L$, the set of all trajectories of length L, can be described by a kernel or state-space representation. As an interesting alternative, we recall the *image representation* of $\mathscr{B}_L$ by a data matrix of a time series.

Consider the sequence $w = \big(w(1), w(2), \ldots, w(T)\big)$ with elements $w(i) \in \mathbb{R}^q$, and define the (block) *Hankel matrix* $\mathscr{H}_L(w) \in \mathbb{R}^{qL\times(T-L+1)}$ of depth L, for some $L \leq T$, as

$$\mathscr{H}_L(w) = \begin{bmatrix} w(1) & w(2) & \ldots & w(T-L+1) \\ w(2) & w(3) & \ldots & w(T-L+2) \\ \vdots & \vdots & \ddots & \vdots \\ w(L) & w(L+1) & \ldots & w(T) \end{bmatrix}.$$

A result due to Willems *et al.* [27] that became known as the *Fundamental Lemma* offers an image representation of the restricted behavior in terms of the column span of a data Hankel matrix. We present a necessary and sufficient version here assuming the following:

(A.1) rank $(\mathscr{H}_L(w)) = mL + n$

Lemma II.3: [29, Corollary 19]): Consider an LTI system $\mathscr{B} \in \mathscr{L}^{q,n}_{m,\ell}$ and an associated trajectory $w = \big(w(1), w(2), \ldots, w(T)\big) \in \mathbb{R}^{qT}$. The followings are equivalent for $L > \ell$:

$$\text{colspan}\,(\mathscr{H}_L(w)) = \mathscr{B}_L \quad \Longleftrightarrow \quad \text{Assumption (A.1).}$$

In words, the Hankel matrix $\mathscr{H}_L(w)$ composed of a single T-length trajectory parameterizes all L-length trajectories if and only if rank$(\mathscr{H}_L(w)) = mL + n$. A plausible reasoning leading up to Lemma II.3 is that every column of $\mathscr{H}_L(w)$ is a trajectory of length L, and the set of all such trajectories has at most dimension $mL + n$ (see Lemma II.1). Lemma II.3 extends the original *Fundamental Lemma* [27, Th. 1], which requires input/output partitioning, controllability, and persistency of excitation of order $L + n$ (i.e., $\mathscr{H}_{L+n}(u)$ must have full row rank) as sufficient conditions. Lemma II.3 also extends to mosaic Hankel, Page, and trajectory matrices [29].

Remark II.4 (Models versus data): It is debatable whether the image representation via the Hankel matrix $\mathscr{H}_L(w)$ should be called a "model," as it is readily available from raw data. Hence, we call colspan$(\mathscr{H}_L(w))$ a *data-driven representation* of $\mathscr{B}_L$ and reserve the term "model" for parametric (kernel or state-space) representations. Models are useful for many reasons: first and foremost the availability of powerful analysis and design

methods. Another readily discernible advantage is that models are vastly compressed compared to the image representation, and the latter holds only on finite horizons unless trajectories are weaved together [31] (see also Remark III.7).

III. Direct and Indirect Data-Driven Control

We present different data-driven control formulations along with assumptions under which the formulations are consistent. These assumptions are used only for consistency statements and not for our main results, but they will prove insightful.

A. Optimal Control Problem

Given a plant with *plant behavior* $\mathscr{B}^P \in \mathscr{L}^{q,n}_{m,\ell}$, a T_{ini}-length prefix trajectory $w_{\text{ini}} = \big(w(-T_{\text{ini}}+1), \dots, w(0)\big) \in \mathscr{B}_{T_{\text{ini}}}$, a L-length reference trajectory $w_r \in \mathbb{R}^{qL}$ in a *reference behavior* $\mathscr{B}^R$, and a set of *admissible trajectories* $\mathcal{W} \subset \mathbb{R}^{qL}$, consider the finite-time *optimal control problem*

$$\boldsymbol{C}: \underset{w \in \mathcal{W}}{\text{minimize}} \; c_{\text{ctrl}}(w - w_r) \quad \text{subject to } w_{\text{ini}} \wedge w \in \mathscr{B}^P_{T_{\text{ini}}+L}. \tag{2}$$

For $T_{\text{ini}} \geq \ell$, the prefix trajectory w_{ini} implicitly sets the initial condition for the optimal control problem (2) (see Section II-C). In case of uncertain initial condition, the prefix w_{ini} can be made a decision variable and included via a penalty term in the cost; c.f., [39]–[43]. We refrain from such extensions here.

Typically, the cost $c_{\text{ctrl}} : \mathbb{R}^{qL} \to \mathbb{R}_{\geq 0}$ includes a running and a terminal cost. The set $\mathcal{W} \subset \mathbb{R}^{qL}$ captures constraints on admissible trajectories (e.g., capturing input saturation). We denote a minimizer (if it exists) of the optimization problem $\boldsymbol{C}$ in (2) by $w^\star_C$. We make the following regularity assumptions.

(A.2) $c_{\text{ctrl}} : \mathbb{R}^{qL} \to \mathbb{R}_{\geq 0}$ is a convex function that achieves its minimum when $w = w_r$; $\mathcal{W} \subset \mathbb{R}^{qL}$ is closed, convex, and nonempty; and $(\mathbb{R}^{qT_{\text{ini}}} \oplus \mathcal{W}) \cap \mathscr{B}^P_{T_{\text{ini}}+L}$ is nonempty.

The last assumption ensures that $\mathcal{W}$ is *viable*, i.e., a trajectory of $\mathscr{B}^P$ originating anywhere can be contained within $\mathcal{W}$ for L steps. Problem (2) is thus convex with closed, convex, and nonempty feasible set due to Assumption (A.2) and because $\mathscr{B}^P_{T_{\text{ini}}+L}$ is a subspace (see Lemma II.1). Under further standard assumptions existence and uniqueness of a (global) minimum can be assured, but we do not impose further structure.

For problem (2), we do not necessarily assume $\mathscr{B}^P = \mathscr{B}^R$, since we often ask systems to track nonplant behavior (e.g., steps). Likewise, we generally do not assume feasibility: $w_r \in \mathcal{W}$. However, such assumptions connect to model reference control and allow to state consistency results as presented next.

(A.3) $w_{\text{ini}} \wedge w_r \in (\mathbb{R}^{qT_{\text{ini}}} \oplus \mathcal{W}) \cap \mathscr{B}^P_{T_{\text{ini}}+L}$, i.e., the reference $w_r \in \mathscr{B}^R_L$ is compatible with the prefix trajectory w_{ini}, the plant $\mathscr{B}^P$, and the constraints $\mathcal{W}$.

Fact III.1: Under Assumptions (A.2) and (A.3), the minimum of the control problem $\boldsymbol{C}$ in (2) is achieved for $w^\star_C = w_r$.

Fact III.1 (and similar consistency results later) follows since $w^\star_C = w_r$ is feasible and achieves the minimum of the cost. Fact III.1 [and consistency Assumption (A.3)] serve to establish ground-truth for comparing different problem formulations.

Problem (2) becomes a "classical" control problem if a parametric model for the plant $\mathscr{B}^P$ is available. The latter is usually obtained from data through system identification.

B. Indirect Data-Driven Control via System Identification

Given a T-length trajectory $w_d \in \mathbb{R}^{qT}$ as *identification data*, conventional system identification and control consists of three steps. The first step, *model class selection*, amounts to choosing the set of candidate models, e.g., $\mathscr{L}^{q,n}_{m,\ell}$, specified by the complexity (q, n, m, ℓ). The second step, *model fitting*, chooses an element from the model class that fits the data best in some specified sense, e.g., distance between data w_d and model $\mathscr{B}$. This step is often synonymous to learning a parametric model (e.g., PEM), though some classic (e.g., ETFE) and modern (e.g., kernel-based) methods are nonparametric and by-pass the model order selection; see Pillonetto *et al.* [2] for a review (and the acronyms). However, for control design, the nonparametric models again have to be projected on a behavior in $\mathscr{L}^{q,n}_{m,\ell}$. Both approaches can be abstracted as

$$\boldsymbol{ID}: \underset{\hat{w}_d, \widehat{\mathscr{B}}}{\text{minimize}} \; c_{\text{id}}(\hat{w}_d - w_d) \quad \text{subject to } \hat{w}_d \in \widehat{\mathscr{B}}_T \,, \widehat{\mathscr{B}} \in \mathscr{L}^{q,n}_{m,\ell}. \tag{3}$$

It is useful to think of the identification loss $c_{\text{id}} : \mathbb{R}^{qT} \to \mathbb{R}_{\geq 0}$ as a distance. Given the data w_d, problem (3) seeks the closest LTI behavior within the class $\mathscr{L}^{q,n}_{m,\ell}$, i.e., the closest subspace with dimensions as in Lemma II.1. We denote a minimizer of (3) by $\big(\hat{w}^\star_{d,ID}, \widehat{\mathscr{B}}^\star_{ID}\big)$ and assume the following about $c_{\text{id}}(\cdot)$:

(A.4) $c_{\text{id}}(\cdot)$ achieves its minimum when $\hat{w}_d = w_d$.

Note that existence and uniqueness of minimizers of (3) does not only hinge upon the regularity of cost and constraint functions, but also on the data. In general, identification problems are nonconvex. For now, we keep problem (3) abstract and general and resort to more specific formulations in Section IV.

Exact identification of the true system requires exact data $w_d \in \mathscr{B}^P_T$ and an identifiability assumption [29, Th. 15], which assures that $\mathscr{B}^P$ can be recovered from w_d:

(A.5) $w_d \in \mathscr{B}^P_T$, i.e., w_d is a valid trajectory of $\mathscr{B}^P_T$; and
(A.6) $\text{rank}(\mathscr{H}_{\ell+1}(w_d)) = m(\ell+1) + n$.

Fact III.2: Under Assumptions (A.4)–(A.6), the minimum value of the system identification problem $\boldsymbol{ID}$ in (3) is achieved for $\hat{w}^\star_{d,ID} = w_d$ and $\widehat{\mathscr{B}}^\star_{ID} = \mathscr{B}^P$.

We again note that the (arguably strong) Assumptions (A.3), (A.5), and (A.6) are used only for consistency statements (such as Fact III.2) and not for our later main results and simulations.

Finally, equipped with an identified behavior $\widehat{\mathscr{B}}^\star \in \mathscr{L}^{q,n}_{m,\ell}$, the third step is *certainty-equivalence control*: solve the optimal control problem (2) subject to the identified model

$$\underset{w \in \mathcal{W}}{\text{minimize}} \; c_{\text{ctrl}}(w - w_r) \quad \text{subject to} \quad w_{\text{ini}} \wedge w \in \widehat{\mathscr{B}}^\star_{T_{\text{ini}}+L}. \tag{4}$$

In (4), $c_{\text{ctrl}}(w - w_r)$ is merely a *surrogate* (predicted) control error since $w \in \widehat{\mathscr{B}}^\star$, the identified model, rather than $w \in \mathscr{B}^P$.

Putting both the system identification (3) and certainty-equivalence control (4) together, we arrive at indirect data-driven control formulated as the *bilevel problem*

$$\begin{aligned} \boldsymbol{BL}: \underset{w\in\mathscr{W}}{\text{minimize}}\ & c_{\text{ctrl}}(w - w_r) \\ \text{subject to}\ \ & w_{\text{ini}} \wedge w \in \widehat{\mathscr{B}}^\star_{T_{\text{ini}}+L} \\ \text{where}\ \ & \widehat{\mathscr{B}}^\star \in \underset{\hat{w}_d,\ \widehat{\mathscr{B}}}{\arg\min}\, c_{\text{id}}(\hat{w}_d - w_d) \\ \text{subject to}\ \ & \hat{w}_d \in \widehat{\mathscr{B}}_T, \widehat{\mathscr{B}} \in \mathscr{L}^{q,n}_{m,\ell}. \end{aligned} \tag{5}$$

The bilevel problem structure in (5) reflects the sequential system identification and control tasks, that is, first a model is fitted to the data in the inner identification problem before the model is used for control in the outer problem. We denote a minimizer for the inner problem of (5) by $\big(\hat{w}^\star_{d,BL}, \widehat{\mathscr{B}}^\star_{BL}\big)$ and a minimizer for the outer problem of (5) by $w^\star_{BL}$.

Remark III.3 (Further problem levels and the value of models): The bilevel formulation (5) is only the tip of the iceberg, and the overall design may feature further nested levels, e.g., optimization of the model selection hyperparameters (n, ℓ), uncertainty quantification, etc. We deliberately neglect these levels here and focus on identification and control.

Since our ultimate interest is control, we treat models in a disregarding manner, i.e., they serve merely an auxiliary purpose. Of course, models are desired for other reasons: system design, analysis, the reasons in Remark II.4, etc.

Under suitable consistency assumptions, the sequential system identification and control approach in (5) is optimal.

Fact III.4: Consider the optimal control problem $\boldsymbol{C}$ in (2) and the bilevel problem $\boldsymbol{BL}$ in (5). Then, the following holds.

1) Under Assumptions (A.4)–(A.6), the bilevel problem $\boldsymbol{BL}$ reduces to the optimal control $\boldsymbol{C}$,
2) Under the additional Assumptions (A.2) and (A.3), the minimum value of the bilevel problem $\boldsymbol{BL}$ is achieved for $\hat{w}^\star_{d,\text{BL}} = w_d$, and $\widehat{\mathscr{B}}^\star_{\text{BL}} = \mathscr{B}^P$, $w^\star_{\text{BL}} = w_r$.

The first statement echoes the "model as well as possible" paradigm and a separation of control and identification, albeit in a simple setting; see [7, Sec. 4.2] for further reading.

C. Direct Data-Driven Control via the Image Representation

The direct data-driven control approach pursued here hinges upon the Fundamental Lemma II.3. A direct corollary of the latter is that the prediction and estimation trajectories have to be within the column span of the data Hankel matrix.

Corollary III.5 (Direct data-driven control): Assume that Assumptions (A.1) and (A.5) hold with L replaced by $T_{\text{ini}} + L$, then the optimal control problem $\boldsymbol{C}$ in (2) is equivalent to

$$\begin{aligned} \boldsymbol{D}: \underset{w\in\mathscr{W}}{\text{minimize}}\ & c_{\text{ctrl}}(w - w_r) \\ \text{subject to}\ \ & \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} \in \text{colspan}\left(\mathscr{H}_{T_{\text{ini}}+L}(w_d)\right), \end{aligned} \tag{6}$$

i.e., the minimizers and minima of (2) and (6) coincide.

Fact III.6: Under Assumptions (A.1) with L replaced by $T_{\text{ini}} + L$, (A.2), (A.3), and (A.5), the minimum value of (6) is achieved for $w^\star_D = w_r$.

Remark III.7 (Data lengths): It is instructive to compare the sample complexity of direct and indirect approaches (6) and (5). Due to Assumption (A.1), (6) requires more data than the identification Assumption (A.6). This discrepancy is due to (6) seeking a multistep predictor, whereas identification (3) seeks a single-step predictor to be applied recursively. By weaving multiple trajectories of length $\ell + 1$, Assumption (A.1) can be eased so that the data lengths coincide; see [31, Lemma 3].

In comparison, with system identification, the model order selection is implicit in Assumption (A.1) and encoded in the rank of the Hankel matrix $\mathscr{H}_{T_{\text{ini}}+L}(w_d)$—at least, for exact data $w_d \in \mathscr{B}^P_T$. If the data w_d is noisy, then $\mathscr{H}_{T_{\text{ini}}+L}(w_d)$ likely has full rank, and the constraint of (6) is vacuous. Thus, $w = w_r$ uniquely minimizes the surrogate control error, but the realized control error may be arbitrarily different. In short, certainty equivalence can fail arbitrarily poorly in direct data-driven control, and the direct approach has to be robustified. This is a major difference with the indirect (first identify, then control) approach (5): one purpose of identification is to filter noisy data by projecting on a deterministic behavior.

To go beyond certainty equivalence, the DeePC approaches [40]–[46] reformulate the constraint in (6) as $\text{col}(w_{\text{ini}}, w) = \mathscr{H}_{T_{\text{ini}}+L}(w_d)g$ for some g and add a robustifying regularizer

$$\begin{aligned} \boldsymbol{D}_\lambda: \underset{w\in\mathscr{W},g}{\text{minimize}}\ & c_{\text{ctrl}}(w - w_r) + \lambda \cdot h(g) \\ \text{subject to}\ \ & \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} = \mathscr{H}_{T_{\text{ini}}+L}(w_d)g. \end{aligned} \tag{7}$$

To provide an intuition, every column of $\mathscr{H}_{T_{\text{ini}}+L}(w_d)$ is a trajectory of $\mathscr{B}^P_{T_{\text{ini}}+L}$, and the decision variable g linearly combines these columns for the optimal trajectory w—consistent with the prefix trajectory w_{ini} and regularized by $h(g)$. The regularization function $h(\cdot)$ and parameter λ are nonnegative. Choices for $h(\cdot)$ are one-norms [41], two-norms [44], squared two-norms [40], [45], or arbitrary p-norms [42], [43], [46].

The regularizers can be related to robust optimization formulations in deterministic [44]–[46] or stochastic settings [42], [43], where λ is a design parameter specifying the size of the assumed uncertainty set. The regularized formulation (7) has proved itself in practical (nonlinear) control systems [45]–[49].

IV. Bridging Direct and Indirect Approaches

A. Multiobjective Data-Driven Control

From an optimization perspective, it is natural to lift the bilevel problem (5) to a *multicriteria problem* simultaneously optimizing for identification and control objectives. Using weighted sum scalarization, the multicriteria problem is

$$\boldsymbol{MC}_\gamma : \underset{w\in\mathcal{W},\hat{w}_d,\widehat{\mathscr{B}}}{\text{minimize}}\ \gamma\cdot c_{\text{id}}(\hat{w}_d - w_d)\ +\ c_{\text{ctrl}}(w - w_r)$$
$$\text{subject to}\ \ w_{\text{ini}}\wedge w \in \widehat{\mathscr{B}}_{T_{\text{ini}}+L}\,,\ \hat{w}_d\in\widehat{\mathscr{B}}_T,\ \widehat{\mathscr{B}}\in\mathscr{L}^{q,n}_{m,\ell} \tag{8}$$

where the tradeoff parameter $\gamma\geq 0$ traces the Pareto front between the identification and optimal control objectives.

The multicriteria problem (8) can be interpreted as fitting a model $\widehat{\mathscr{B}}$ simultaneously to two datasets: the identification data w_d and the reference w_r. From a control perspective, the identification criterion biases the solution $w\in\widehat{\mathscr{B}}$ to adhere to the observed data w_d rather than merely matching the to be tracked reference w_r. Likewise, from the other side, the identification criterion is biased by the control objective. In short, control and identification *regularize* each other, in the spirit of identification for control [7], [12]–[14]. A similar formulation has been proposed in [15] interpolating between PEM identification and a model-reference control objective. Likewise, the data-driven model reference control formulation in [20] interpolates between a direct and an indirect approach. Finally, dual control approaches consider similar multicriteria formulations balancing exploration (for identification) and exploitation (i.e., optimal control) [16]–[19].

We denote a minimizer of (8) by $\big(w^\star_{MC},\hat{w}^\star_{d,MC},\widehat{\mathscr{B}}^\star_{MC}\big)$.

Fact IV.1: Under Assumptions (A.2)–(A.6), for any $\gamma\geq 0$, the minimum of the parametric multicriteria problem $\boldsymbol{MC}_\gamma$ is achieved for $\hat{w}^\star_{d,MC}=w_d$, $\widehat{\mathscr{B}}^\star_{MC}=\mathscr{B}^P$, and $w^\star_{MC}=w_r$.

Different points on the Pareto front of (8) have different emphasis regarding the control and identification objectives. Below, we formalize that for γ sufficiently large, the multicriteria problem (8) recovers the bilevel problem (5) corresponding to sequential system identification and control.

We follow standard penalty arguments from bilevel optimization [52], [53], which are particularly tractable here since (5) is only weakly coupled: the inner problem does not depend on the decision variable w of the outer problem. Assume there is a minimum (termed value function) of the inner problem

$$\varphi = \underset{\hat{w}_d,\ \widehat{\mathscr{B}}}{\text{minimize}}\ c_{\text{id}}(\hat{w}_d - w_d)$$
$$\text{subject to}\ \ \hat{w}_d\in\ \widehat{\mathscr{B}}_T\,,\ \widehat{\mathscr{B}}\in\mathscr{L}^{q,n}_{m,\ell}. \tag{9}$$

The bilevel problem (5) reads then equivalently as

$$\underset{w\in\mathcal{W},\hat{w}_d,\widehat{\mathscr{B}}}{\text{minimize}}\ \ c_{\text{ctrl}}(w - w_r)$$
$$\text{subject to}\ \ w_{\text{ini}}\wedge w\in\widehat{\mathscr{B}}_{T_{\text{ini}}+L},\hat{w}_d\in\widehat{\mathscr{B}}_T,$$
$$\widehat{\mathscr{B}}\in\mathscr{L}^{q,n}_{m,\ell}, c_{\text{id}}(\hat{w}_d - w_d)-\varphi = 0. \tag{10}$$

At this point, the reader is encouraged to review the definition and salient properties of a constraint qualification termed partial calmness [52], [53] (see the Appendix). If problem (10) is partially calm at a local minimizer and $c_{\text{ctrl}}(\cdot)$ is continuous, then there is $\gamma^\star>0$ so that, for all $\gamma>\gamma^\star$ (10) equals

$$\underset{w\in\mathcal{W},\hat{w}_d,\widehat{\mathscr{B}}}{\text{minimize}}\ \gamma\cdot\big|c_{\text{id}}(\hat{w}_d - w_d)-\varphi\big|\ +\ c_{\text{ctrl}}(w - w_r)$$
$$\text{subject to}\ \ w_{\text{ini}}\wedge w\in\widehat{\mathscr{B}}_{T_{\text{ini}}+L},\hat{w}_d\in\widehat{\mathscr{B}}_T,\ \widehat{\mathscr{B}}\in\mathscr{L}^{q,n}_{m,\ell} \tag{11}$$

that is, the local minimizers of (10) and (11) coincide (see Proposition A.2). We now drop the absolute value (since $c_{\text{id}}(\hat{w}_d - w_d)-\varphi\geq 0$) and the constant φ (which in our case does not depend on the variable w of the outer problem) from the objective of (11) to recover problem (8). We have thus established a chain of equivalences relating the bilevel and multicriteria problems. We summarize our discussion below.

Proposition IV.2 (Upper tail of the Pareto front of $\boldsymbol{MC}_\gamma$): Consider the parametric multicriteria problem $\boldsymbol{MC}_\gamma$ in (8) and the bilevel problem $\boldsymbol{BL}$ in (5). Assume that the inner identification problem admits a minimum as in (9), (10) is partially calm at any local minimizer, and $c_{\text{ctrl}}(\cdot)$ is continuous. Then, there is $\gamma^\star>0$ so that for $\gamma>\gamma^\star$, the problem $\boldsymbol{MC}_\gamma$ is equivalent to $\boldsymbol{BL}$, i.e., $w^\star_{MC}=w^\star_{BL}$, $\hat{w}^\star_{d,MC}=\hat{w}^\star_{d,BL}$, and $\widehat{\mathscr{B}}^\star_{MC}=\widehat{\mathscr{B}}^\star_{BL}$. Moreover, the optimal values of $\boldsymbol{MC}_\gamma$ and $\boldsymbol{BL}$ coincide up to the constant $\gamma\cdot\varphi$ with φ defined in (9).

The following comments are in order regarding partial calmness. As discussed in Proposition A.2, partial calmness is equivalent to the constraint $c_{\text{id}}(\hat{w}_d - w_d)-\varphi\geq 0$ serving as an exact penalty. Partial calmness is satisfied, for instance, appealing to Proposition A.3, if the identification cost $c_{\text{id}}(\cdot)$ can be phrased as a distance [see the discussion following the identification problem (3)] and $c_{\text{ctrl}}(\cdot)$ is Lipschitz continuous over the feasible set, e.g., the feasible set is either compact (due to constraints) or the control performance is measured by a norm or Huber loss. The Lipschitz constant then serves as a lower estimate for $\gamma^\star$. A non-Lipschitz cost requires $\gamma\to\infty$ as a sufficient condition. Note that for $\gamma\to\infty$, Proposition IV.2 holds without assumptions, since (11) is merely an indicator function reformulation of (10). Our relaxations in the next sections will, among others, drop the requirement on γ sufficiently large as well as the LTI complexity specification $\widehat{\mathscr{B}}\in\mathscr{L}^{q,n}_{m,\ell}$.

Even if the identification (3) is convex the multicriteria problem (8) is not, since it simultaneously optimizes over the to-be-identified model $\mathscr{B}$ and the to-be-designed trajectory w. This can be spotted in a kernel representation: the constraint $w\in\widehat{\mathscr{B}}_L$ takes the form $\widehat{R}(\sigma)w=0$, where both $\widehat{R}$ and w are variables. Other representations lead to the same conclusions.

Proposition IV.3: Consider the multicriteria problem (8) and a kernel representation of the to-be-identified behavior: $\widehat{\mathscr{B}}=\text{kernel}(\widehat{R}(\sigma))$. Then, the feasible set of (8) is not convex.

We believe that the multicriteria problem is interesting in its own right: studying its Pareto front and choosing an optimal tradeoff parameter may possibly yield superior performance.

Our problem setup thus far was conceptual rather than practically useful. Below, we consider concrete problem formulations and turn our conceptual insights into concise results.

B. Bridging Toward SPC

We explain SPC from the perspective of the Fundamental Lemma II.3 stating that any trajectory $w_{\text{ini}}\wedge w\in\mathscr{B}^P_{T_{\text{ini}}+L}$ lies in colspan$(\mathscr{H}_{T_{\text{ini}}+L}(w_d))$. Recall that w_{ini} is a prefix trajectory

of length $T_{\text{ini}} \geq \ell$ setting the initial condition, and w is a future trajectory of length $L > 1$ to be designed via optimal control. Accordingly, permute and partition w and the Hankel matrix

$$\begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} \sim \left[\begin{array}{c} u_{\text{ini}} \\ u \\ \hline y_{\text{ini}} \\ y \end{array}\right], \; \mathscr{H}_{T_{\text{ini}}+L}(w_d) \sim \left[\begin{array}{c} U_{\text{p}} \\ U_{\text{f}} \\ \hline Y_{\text{p}} \\ Y_{\text{f}} \end{array}\right] = \left[\begin{array}{c} \mathscr{H}_{T_{\text{ini}}+L}(u_d) \\ \hline \mathscr{H}_{T_{\text{ini}}+L}(y_d) \end{array}\right]$$

where $u_{\text{ini}} \in \mathbb{R}^{mT_{\text{ini}}}$, $y_{\text{ini}} \in \mathbb{R}^{(q-m)T_{\text{ini}}}$, and $\sim$ denotes similarity under a coordinate permutation. The subscripts "p" and "f" are synonymous to "past" and "future." We seek a linear model, i.e., a matrix K, relating past and future as

$$y = \underbrace{\begin{bmatrix} K_p & | & K_f \end{bmatrix}}_{=K} \cdot \left[\begin{array}{c} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \end{array}\right]. \tag{12}$$

The multistep predictor K is found from Hankel matrix data by means of the least-square criterion [34, Sec. 3.4]

$$\underset{K}{\text{minimize}} \left\| Y_{\text{f}} - K \cdot \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} \right\|_F^2 \tag{13}$$

where $\|\cdot\|_F$ is the Frobenius norm. Via the Moore–Penrose inverse, the solution of (13) is the classic SPC predictor [32]

$$K = Y_{\text{f}} \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix}^{\dagger}. \tag{14}$$

It is insightful to compare (12) and the matrices K_p, K_f to (1) and the extended observability and impulse response matrices $\mathscr{O}_L$ and $\mathscr{G}_L$, respectively. One realizes that for exact data, (12) is an ARX model with $\text{rank}(K_p) = n$ assuring LTI behavior of desired complexity and a lower block-triangular zero pattern of K_f assuring causality. For inexact data, LTI behavior of desired complexity is promoted by low-rank approximation (typically via singular-value thresholding of K_p) [32]; and one aims to gain causality by heuristically thresholding K_f toward a desired zero pattern [34, Remark 10.1], [33, Sec. 3]. The causality requirement can also be omitted for offline or receding horizon control, but it is useful to condition the data on the set of causal models. These steps bring the linear relation (12) half-way toward an LTI model. Though a model has further structure, e.g., K_f is Toeplitz, and the entries of K_p and K_f are coupled; see (1).

Hence, in this case, the identification problem (3) is relaxed to the single, monolithic, and nonconvex program

$$\begin{aligned} \underset{K}{\text{minimize}} \; & \left\| Y_{\text{f}} - K \cdot \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} \right\|_F^2 \\ \text{subject to} \; & K = \begin{bmatrix} K_p & | & K_f \end{bmatrix} \\ & K_f \text{ lower-block triangular,} \\ & \text{rank}(K_p) = n, \end{aligned}$$

where the lower-block triangular specification means that all entries above the diagonal $(q-m) \times m$ blocks equal zero.

We obtain a parametric version of the indirect data-driven approach (5), where $w_{\text{ini}} \wedge w \in \widehat{\mathscr{B}}^{\star}_{T_{\text{ini}}+L}$ and $w \in \mathcal{W} = \mathcal{U} \times \mathcal{Y}$ are replaced by (12) and $(u, y) \in \mathcal{U} \times \mathcal{Y}$, respectively

$$\begin{aligned} \underset{u \in \mathcal{U}, y \in \mathcal{Y}}{\text{minimize}} \; & c_{\text{ctrl}}\left(\begin{bmatrix} y - y_r \\ u - u_r \end{bmatrix} \right) \\ \text{subject to} \; & y = K^{\star} \cdot \left[\begin{array}{c} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \end{array}\right] \\ \text{where} \; & K^{\star} \in \underset{K}{\arg\min} \left\| Y_{\text{f}} - K \cdot \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} \right\|_F^2 \\ \text{subject to} \; & K = \begin{bmatrix} K_p & | & K_f \end{bmatrix} \\ & K_f \text{ lower-block triangular} \\ & \text{rank}(K_p) = n. \end{aligned} \tag{15}$$

We stress that (15) is generally not an equivalent reformulation of (5) since the inner identification does not necessarily lead to an LTI model; see the comments following (14).

For comparison, consider also an instance of the direct regularized problem (7) with regularizer $h(g) = \|(I - \Pi)g\|_p$

$$\begin{aligned} \underset{u \in \mathcal{U}, y \in \mathcal{Y}, g}{\text{minimize}} \; & c_{\text{ctrl}}\left(\begin{bmatrix} y - y_r \\ u - u_r \end{bmatrix} \right) + \lambda \cdot \|(I - \Pi)g\|_p \\ \text{subject to} \; & \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \\ Y_{\text{f}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \\ y \end{bmatrix}. \end{aligned} \tag{16}$$

here, $\|\cdot\|_p$ is any p-norm, $\Pi = \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix}^{\dagger} \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix}$, and $(I - \Pi)$ is an orthogonal projector on the kernel of the first three block-constraint equations. The proof of Theorem IV.6 will later show that this regularizer is in fact *induced* by the least-square identification (13), i.e., $\|(I - \Pi)g\|_p = 0$ if and only if the least-square criterion is minimized. Hence, it robustifies the problem akin to least squares. We state the following consistency result.

Fact IV.4: Under Assumptions (A.1) with L replaced by $T_{\text{ini}} + L$, (A.2), (A.3), and (A.5), for any $\lambda \geq 0$, the minimum of the regularized problem (16) is achieved for $y^{\star} = Y_f g^{\star} = y_r$ and $u^{\star} = U_f g^{\star} = u_r$, where $\|(I - \Pi)g^{\star}\|_p = 0$.

Remark IV.5 (Consistency of regularizers): Fact IV.4 may not appear insightful at first glance, but it highlights an important fact. The projection-based regularizer $h(g) = \|(I - \Pi)g\|_p$ is consistent since it penalizes only the homogenous solution to the constraint equations (16) and does not affect the variables (u, y). In comparison, the conventional-norm-based regularizer $h(g) = \|g\|_p$ is not consistent: it penalizes the heterogeneous solution of the constraint equations in (16) and, thus, also (u, y).

Hence, even with ideal consistency, Assumptions (A.1), (A.2), (A.3), and (A.5) in place, the norm-based regularizer $h(g) = \|g\|_p$ with $\lambda \neq 0$ does not lead to the ground-truth solution $y^\star = Y_f g^\star = y_r$ and $u^\star = U_f g^\star = u_r$ (see also Remark IV.7).

The following is the main result of this section.

Theorem IV.6 (SPC relaxation): Consider the indirect data-driven control problem (15) and the direct data-driven control problem (16) parameterized by $\lambda \geq 0$. Let Assumption (A.2) hold and assume that $c_{\text{ctrl}}(\cdot)$ is Lipschitz continuous. For λ sufficiently small, (16) is a convex relaxation of (15), that is, the following hold:

i) (16) is convex;
ii) any feasible (u, y) in (15) is feasible for (16); and
iii) the optimal value of (16) lower bounds that of (15).

Proof: First, we perform a convex relaxation by dropping the rank and block-triangularity constraints in (15). Second, observe that the explicit solution of the inner problem, the predictor (14), is equivalently derived as least-norm solution

$$y = Y_{\text{f}} g^\star \text{ where } g^\star = \arg\min_g \|g\|_2 \quad \text{subject to} \quad \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \end{bmatrix}.$$

We now insert this reformulation in the relaxation of (15)

$$\begin{aligned} \underset{u\in\mathcal{U}, y\in\mathcal{Y}}{\text{minimize}} \quad & c_{\text{ctrl}}\left(\begin{bmatrix} y - y_r \\ u - u_r \end{bmatrix}\right) \\ & \text{subject to } y = Y_{\text{f}} g^\star \\ & \text{where } g^\star \in \arg\min_g \|g\|_2 \\ & \text{subject to } \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \end{bmatrix}. \end{aligned} \tag{17}$$

We now follow the arguments from Section IV-A to reduce the bilevel problem (17) to a single-level multicriteria problem.

As in (10), the inner problem can be replaced by a constraint assuring that it achieves its minimum. Here, we add an orthogonality constraint to the constraints of the inner problem

$$\begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \end{bmatrix} \quad \text{and} \quad 0 = \|(I - \Pi)g\|_p.$$

The orthogonality constraint $0 = \|(I - \Pi)g\|_p$ poses the inner optimality constraint as the distance to the subspace containing the minimizers of the inner problem. Retaining all constraints, (17) can then be formulated as the single-level problem

$$\begin{aligned} \underset{u\in\mathcal{U}, y\in\mathcal{Y}, g}{\text{minimize}} \quad & c_{\text{ctrl}}\left(\begin{bmatrix} y - y_r \\ u - u_r \end{bmatrix}\right) \\ \text{subject to} \quad & \begin{bmatrix} U_{\text{p}} \\ Y_{\text{p}} \\ U_{\text{f}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \end{bmatrix} \\ & \|(I - \Pi)g\|_p = 0. \end{aligned} \tag{18}$$

We now apply Proposition A.3, lift the distance constraint $\|(I - \Pi)g\|_p = 0$ to the objective, and recover problem (16) with λ larger than the Lipschitz constant of $c_{\text{ctrl}}(\cdot)$.

Hence, (16) is equivalent to (18) for λ sufficiently large. Our final convex relaxation is to choose λ small rather than large. Namely, from the view-point of the objective: it lowers the cost; or from the bilevel viewpoint: it turns the inner optimality constraint into a weaker suboptimality constraint, i.e., we allow for solutions satisfying $\|(I - \Pi)g\|_p \geq 0$.

Conclusion (i) now follows since (16) is convex; (ii) follows since we have only enlarged the feasible set when passing from (15) to (16); and (iii) follows due to the enlarged feasible set, since the costs of (15) and (18) coincide, and since (16) is a relaxation of (18) if λ is not sufficiently large. ■

Remark IV.7 (Comments on Theorem IV.6): First, we summarize the salient arguments to pass from indirect to direct data-driven control: we relaxed problem (15) by dropping causality (block-triangularity) and LTI complexity (rank) specifications, replaced the least-square criterion (13) by the equivalent least-norm formulation (17), and lifted the problem from bilevel to multicriteria, where the least-square objective induces the regularization $\|(I - \Pi)g\|_p$. For equivalence to the least-square objective, the proof requires λ larger than the (global) Lipschitz constant of $c_{\text{ctrl}}(\cdot)$, similar to robustification-induced regularizations [42], [43]. If $c_{\text{ctrl}}(\cdot)$ is only locally Lipschitz, e.g., in case of a quadratic cost, then choosing a finite (small) λ is a relaxation that allows the predicted trajectory to not adhere to the least-square fit of the data. Though as we will see in Section V-B, its effect is minor for λ not overly small.

Second, continuing on the magnitude of λ: For exact data and under consistency assumptions, (16) achieves the exact minimizer for any $\lambda \geq 0$ (see Fact IV.4). When departing from these ideal assumptions, the least-square fit of the data is enforced only for λ sufficiently large. Generally, λ should be regarded as a tune-able hyperparameter chosen by the designer to control how much the predicted trajectory should adhere to the data (versus the control objective) and to ultimately improve the realized performance. The proof of Theorem IV.6 suggests a sufficiently large value, which is also confirmed by our later empirical findings (see, e.g., Fig. 2).

Third, the regularization based on the projector $\|(I - \Pi)g\|_p$ differs from the standard p-norm regularizers $h(g) = \|g\|_p$ [42]–[44] (or squared two-norms $\|g\|_2^2$ [40], [45]). Actually, it is this projection that recovers the least-square criterion (13). In contrast, norm-based regularizers $\|g\|_p$ are not consistent and bias the optimal solution $(u^\star, y^\star)$ (see Remark IV.5). This is undesirable from an identification perspective: the regularizer should induce a least-square fit of the data. While for small values of λ, both regularizers have a similar effect, for sufficiently large λ, the

identification-induced regularizer $\|(I-\Pi)g\|_p$ demonstrates a superior performance (see Fig. 2 later).

Fourth, our proof strategy reveals an entire class of regularizers. In fact, we can choose any p-norm $\|(I-\Pi)g\|_p$, use more general penalty functions, such as the (squared) merit functions in [53], or attack problem (18) with other penalty or augmented Lagrangian methods. These degrees of freedom reflect the intuition that the Pareto-front of (16) is invariant under certain (e.g., monotone) transformations of objectives, such as taking squares; see [54, Appendix A] for a formal reasoning. For our later simulations in Section V-B, we choose the computationally attractive regularization $\|(I-\Pi)g\|_2^2$.

Fifth and finally, our proof arguments are obviously "qualitative" crossing out rank and causality constraints similar to most SPC implementations and using nonquantifiable "sufficiently large" reasoning. Hence, the convex relaxation (16) of (15) should not be expected to be tight. Nevertheless, the formulation (16) (without projector) has proved itself in many case studies and often outperforms (15), as testified in [45]–[49]. Section V-B will compare the different formulations.

C. Bridging Toward Structured Low-Rank Approximation

We now present an entirely nonparametric problem formulation, namely a version of subspace identification based on structured low-rank approximation [39], and we relate the resulting bilevel problem to direct data-driven control (7).

Given the model class $\mathscr{L}_{m,\ell}^{q,n}$, we project the identification data $w_d \in \mathbb{R}^{qT}$ on $\widehat{\mathscr{B}}_{T_{\text{ini}}+L} \in \mathscr{L}_{m,\ell}^{q,n}$. By Lemma II.3, the latter set is characterized by all trajectories $\hat{w} \in \mathbb{R}^{q(T_{\text{ini}}+L)}$ so that the associated Hankel matrix satisfies $\text{rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w})) \leq m(T_{\text{ini}}+L)+n$ for $(T_{\text{ini}}+L) > \ell$. An implicit assumption is, of course, $T \gg T_{\text{ini}}+L$: the identification data are much longer than the estimation plus control prediction horizons.

In presence of noise, $\mathscr{H}_{T_{\text{ini}}+L}(w_d)$ will not have low rank and has to be approximated by a low-rank matrix in an identification step. Thus, the identification problem (3) reads as

$$\begin{aligned} &\underset{\hat{w}_d}{\text{minimize}}\; c_{\text{id}}(\hat{w}_d - w_d) \\ &\text{subject to rank } (\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)) \leq m(T_{\text{ini}}+L)+n. \end{aligned} \tag{19}$$

Problem (19) is to be read as low-rank approximation problem: given the identification data assorted in a Hankel matrix $\mathscr{H}_{T_{\text{ini}}+L}(w_d)$, we seek the closest sequence $\hat{w}_d$ so that the Hankel matrix $\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)$ has rank no more than $m(T_{\text{ini}}+L)+n$.

Since $\hat{w}_d \in \widehat{\mathscr{B}}_T$, we have $\text{rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)) \leq m(T_{\text{ini}}+L)+n$. Since also $w_{\text{ini}} \in \widehat{\mathscr{B}}_{T_{\text{ini}}}$ and $w \in \widehat{\mathscr{B}}_L$, we conclude

$$\text{rank}\left([\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)\ \text{col}(w_{\text{ini}}, w)]\right) \leq m(T_{\text{ini}}+L)+n\,.$$

Assuming that $\text{rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)) = m(T_{\text{ini}}+L)+n$, which is generically the case, $\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)g = \text{col}(w_{\text{ini}}, w)$ for some vector g. Hence, the bilevel problem (5) takes the form

$$\begin{aligned} &\underset{w\in\mathcal{W},g}{\text{minimize}}\quad c_{\text{ctrl}}(w - w_r) \\ &\text{subject to}\quad \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} = \mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d^\star)g \\ &\qquad\qquad \hat{w}_d^\star \in \underset{\hat{w}_d}{\arg\min}\, c_{\text{id}}(\hat{w}_d - w_d) \\ &\qquad\qquad \text{subject to } \text{rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)) = m(T_{\text{ini}}+L)+n. \end{aligned} \tag{20}$$

Theorem IV.8 (ℓ_1-norm relaxation): Consider the indirect data-driven control problem (20) and the direct data-driven control problem (7) for $h(g) = \|g\|_1$ and parameterized by $\lambda \geq 0$. Let Assumptions (A.2) and (A.4) hold. For λ sufficiently small, (7) is a convex relaxation of (20), that is, the following hold:

i) (7) is convex;
ii) any feasible (w, g) in (20) is also feasible for (7); and
iii) the optimal value of (7) lower bounds that of (20).

Proof: To prove the claim, one can resort to a proof strategy via the multicriteria problem (8), as in the previous section. Instead, we present a more direct approach here.

We start by massaging the rank constraint in (20). First, since $\text{rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d)) = m(T_{\text{ini}}+L)+n$, we may without loss of generality add the constraint $\|g\|_0 \leq n + m(T_{\text{ini}}+L)$ to the outer problem, where $\|g\|_0$ denotes the cardinality (number of nonzero entries) of g. Second, we perform a convex relaxation and drop the rank constraint. Third, another convex relaxation (popular in LASSO problems [55]) is to replace $\|g\|_0 \leq n + m(T_{\text{ini}}+L)$ by $\|g\|_1 \leq \alpha$ for $\alpha > 0$ sufficiently large. As a result of these three steps, (20) is relaxed to

$$\begin{aligned} &\underset{w\in\mathcal{W},g}{\text{minimize}}\quad c_{\text{ctrl}}(w - w_r) \\ &\text{subject to}\quad \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} = \mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d^\star)g\,,\ \|g\|_1 \leq \alpha \\ &\text{where}\quad \hat{w}_d^\star \in \underset{\hat{w}_d}{\arg\min}\, c_{\text{id}}(\hat{w}_d - w_d). \end{aligned} \tag{21}$$

Observe that under Assumption (A.4), the inner problem admits a trivial solution: $\hat{w}_d^\star = w_d$. Thus, (21) reduces to

$$\begin{aligned} &\underset{w\in\mathcal{W},g}{\text{minimize}}\quad c_{\text{ctrl}}(w - w_r) \\ &\text{subject to}\quad \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} = \mathscr{H}_{T_{\text{ini}}+L}(w_d)g\,,\ \|g\|_1 \leq \alpha. \end{aligned} \tag{22}$$

Next, we lift the one-norm constraint to the objective

$$\begin{aligned} &\underset{w\in\mathcal{W},g}{\text{minimize}}\quad c_{\text{ctrl}}(w - w_r) + \lambda \cdot \|g\|_1 \\ &\text{subject to}\quad \begin{bmatrix} w_{\text{ini}} \\ w \end{bmatrix} = \mathscr{H}_{T_{\text{ini}}+L}(w_d)g \end{aligned} \tag{23}$$

where $\lambda \geq 0$ is a scalar weight. In particular, for each value of α in (22), there is $\lambda \geq 0$ so that the solution of (23) coincides with (22), and vice versa. These equivalences are standard in ℓ_1-regularized problems and follow from strong duality (applicable since $c_{\text{ctrl}}(\cdot)$ is convex and Slater's condition holds) [55]. The precise value of λ depends on the Lagrange multiplier of the constraint $\|g\|_1 \leq \alpha$ and, thus, on the data. In either case, there is a selection of parameters so that both problems are equivalent, and choosing λ sufficiently small is a relaxation.

Thus, we arrived at the direct data-driven control (7) for λ sufficiently small and $h(g) = \|g\|_1$. Conclusion (i) follows due

to convexity (7); (ii) follows since we have enlarged the feasible set passing from (20) to (7); and (iii) follows due to the enlarged feasible set, since the costs of (20) and (22) coincide, and since (7) is a relaxation of (22) for λ small. ■

In summary, to pass from indirect data-driven control (20) to direct data-driven control (7), we performed a sequence of convex relaxations effectively replacing the rank constraint of the system identification by a ℓ_1-norm regularizer. Hence, the one-norm regularizer accounts for selecting the model complexity. Similar remarks as those following Theorem IV.6 on tightness of the relaxation apply to Theorem IV.8, too (see Remark IV.7).

D. Hybrid Relaxations

Theorems IV.6 and IV.8 reveal the roles of the two regularizers: $\|g\|_1$ controls the model complexity, whereas $\|(I-\Pi)g\|_2$ accounts for least-square fitting the data. To blend the two, consider a hybrid formulation of (16) and (20)

$$\begin{aligned}
&\underset{w\in\mathcal{W},g}{\text{minimize}}\; c_{\text{ctrl}}(w-w_r)+\lambda_1\cdot\|(I-\Pi)g\|_2^2\\
&\text{subject to}\begin{bmatrix}w_{\text{ini}}\\ w\end{bmatrix}=\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d^\star)g\\
&\text{where }\hat{w}_d^\star\in\underset{\hat{w}_d}{\arg\min}\, c_{\text{id}}(\hat{w}_d-w_d)\\
&\text{subject to rank}(\mathscr{H}_{T_{\text{ini}}+L}(\hat{w}_d))=m(T_{\text{ini}}+L)+n
\end{aligned}\tag{24}$$

where $\lambda_1\geq 0$. Observe that this formulation is consistent.

Fact IV.9: Under Assumptions (A.1) with L replaced by $T_{\text{ini}}+L$, (A.2), (A.3), and (A.5), for any $\lambda_1\geq 0$, the minimum of (24) and achieved for $w^\star=w_r$ and $\|(I-\Pi)g^\star\|_2^2=0$.

The arguments in the previous section then lead us to

$$\begin{aligned}
&\underset{w\in\mathcal{W},g}{\text{minimize}}\quad c_{\text{ctrl}}(w-w_r)\;+\;\lambda_1\cdot\|(I-\Pi)g\|_2^2\;+\;\lambda_2\cdot\|g\|_1\\
&\text{subject to}\quad\begin{bmatrix}w_{\text{ini}}\\ w\end{bmatrix}=\mathscr{H}_{T_{\text{ini}}+L}(w_d)g
\end{aligned}\tag{25}$$

where $\lambda_2\geq 0$. We will validate the performance of the hybrid regularizer in Section V-B below (see specifically Fig. 3).

E. Possible Pitfalls of Relaxations

Note that the two convex relaxation results in Theorems IV.6 and IV.8 are *trivially* true in the limit when $\lambda=0$. In fact, even the abstract multicriteria formulation (8) can be related to a relaxation of the abstract bilevel problem (5) in the limit $\gamma=0$. Namely, for $\gamma=0$, (8) reduces to

$$\begin{aligned}
&\underset{w,\hat{w}_d,\widehat{\mathscr{B}}}{\text{minimize}}\quad c_{\text{ctrl}}(w-w_r)\\
&\text{subject to}\quad w_{\text{ini}}\wedge w\in\widehat{\mathscr{B}}_{T_{\text{ini}}+L},\hat{w}_d\in\widehat{\mathscr{B}}_T,\widehat{\mathscr{B}}\in\mathscr{L}_{m,\ell}^{q,n}.
\end{aligned}\tag{26}$$

The variable $\hat{w}_d$ and the constraint $\hat{w}_d\in\widehat{\mathscr{B}}_T$ can be removed, and (26) amounts to matching the model $\widehat{\mathscr{B}}$ to the reference w_r. The next result is followed by a discussion on regularizers.

Corollary IV.10: Consider the indirect data-driven control (5) and multicriteria problem (26) in the limit $\gamma=0$, and let Assumption (A.2) hold. Then, problem (26) is a relaxation of problem (5), that is, the following hold:

i) any feasible $(w,\hat{w}_d,\widehat{\mathscr{B}})$ in (5) is also feasible for (26); and
ii) the optimal value of (26) lower bounds that of (5).

Proof: Consider the equivalent formulation (10) of (5), and note that (26) equals (10) when the inner optimality constraint $c_{\text{id}}(\hat{w}_d-w_d)-\varphi=0$ is dropped. The conclusions now follow analogously as in Theorems IV.6 and IV.8. ■

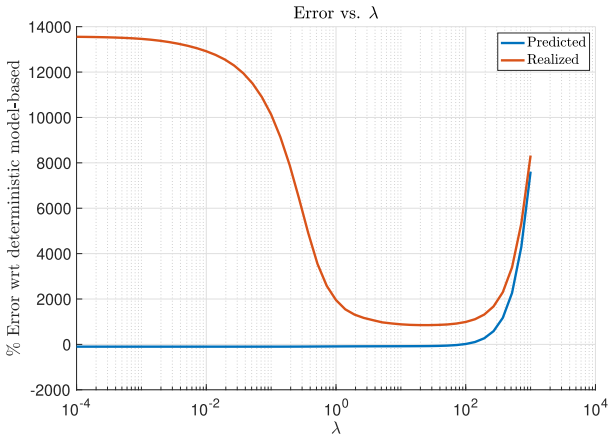

Fig. 1. Predicted and realized errors (relative to the ground-truth optimal performance and averaged over 100 datasets) with one-norm regularizer $\lambda\|g\|_1$.

Analogous corollaries can be stated for Theorems IV.6 and IV.8 for $\lambda=0$. Given such results, one may wonder whether Theorems IV.6 and IV.8 are vacuous since they are trivially true for $\lambda=0$. We offer several answers. First, the limit $\lambda=0$ clearly leads to a better solution $w^\star$ (i.e., a lower surrogate tracking error) for the *open-loop* optimal control problem. However, this solution merely matches the reference w_r and does not adhere to the identification data w_d in the sense of meeting any fitting criterion. Hence, the optimal solution $w^\star$ may not be a trajectory of the true system behavior, and the actual *realized* control performance can be arbitrarily poor. Obviously, such a situation is not desirable, and one may want to regularize with a small but nonzero λ—an observation consistent with the works in [41]–[46] albeit derived from a different perspective. Second, Theorems IV.6 and IV.8 require λ to be sufficiently small, but not zero. According to the proofs, depending on Lipschitz constants and multipliers of the respective problems, there is a smallest value for λ so that the behavior $\widehat{\mathscr{B}}$ matches (in the $c_{\text{id}}(\cdot)$ fitting criterion) the plant behavior $\mathscr{B}^P$. In [41]–[46], the coefficient λ relates to a desired robustness level. In either case, λ can hardly be quantified *a priori* and without cross-validation (see also Remark IV.7).

We follow up on this set of questions in Section V.

V. Numerical Analysis and Comparisons

We now numerically investigate the effect of the hyperparameter λ, confirm the superiority of the regularizer $h(g)=\|(I-\Pi)g\|_2^2$, and compare direct and indirect approaches.

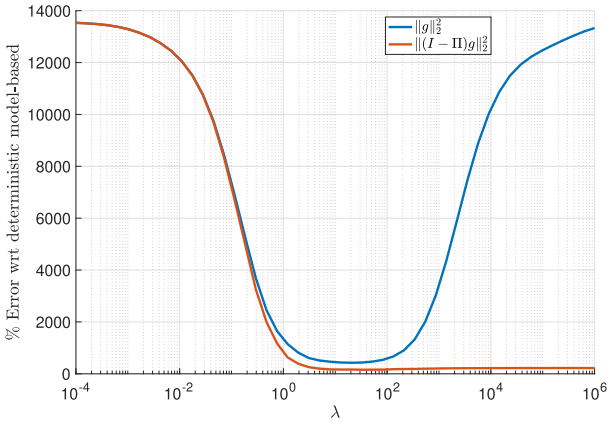

Fig. 2. Comparison of the realized performance (relative to the ground-truth optimal performance and averaged over 100 datasets) for the two-norm $\|g\|_2^2$ and identification-induced regularization $\|(I-\Pi)g\|_2^2$ as function of λ.

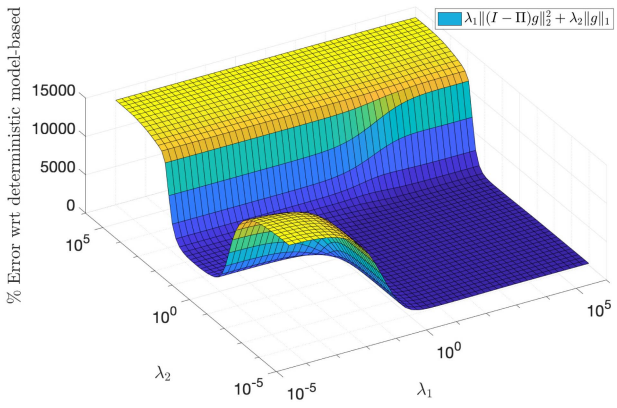

Fig. 3. Realized error (relative to the ground-truth optimal performance and averaged over 100 datasets) for a hybrid regularizer $\lambda_1\|(I-\Pi)g\|_2^2+\lambda_2\|g\|_1$.

A. Choice of Regularization Parameter

We first study the parameter λ regularizing direct data-driven control (7). Consider the benchmark single-input, single-output, fourth order, LTI system [56]. Denoting the tth element of the concatenated input and output by $w(t)=(u(t),y(t))$, the control cost was chosen as $c_{\text{ctrl}}(w-w_r)=(w-w_r)^\top W(w-w_r)$ with reference $w_r(t)=(u_r(t),y_r(t))=(0,\sin(2\pi t/(L-1)))$ for $t\in\{0,1,\dots,L-1\}$, prediction horizon $L=20$, $W=I_L\otimes\text{diag}(0.01,2000)$, where I_L is the $L\times L$ identity, and $\otimes$ denotes the Kronecker product. In this entire section, we disregard constraints, i.e., $\mathcal{W}\equiv\mathbb{R}^{qL}$. We used a one-norm regularizer $h(g)=\|g\|_1$ in (7) and a prefix-trajectory of length $T_{\text{ini}}=5$ (see Section II-C).

We collected one noise-free input/output time series of length $T=250$ by applying a random Gaussian input. From this noise-free dataset, 100 independent noisy datasets were constructed by adding Gaussian noise with a noise-to-signal ratio of 5%. For each dataset and each value of $\lambda\in(0,10^3)$, optimal control inputs were computed from (7). We define the *predicted* error as $c_{\text{ctrl}}(w^\star-w_r)$, where $w^\star$ is an optimizer of (7). We define the *realized* error as $c_{\text{ctrl}}(w_{\text{true}}-w_r)$, where w_{true} is the realized trajectory of the system after applying the computed optimal inputs. The predicted and realized errors were converted to a percentage increase in error with respect to the ground-truth optimal performance (i.e., if the deterministic system was exactly known), and were averaged over the 100 independent datasets. The results are plotted in Fig. 1.

It is apparent that choosing λ too small leads to an optimistic predicted error but very poor realized performance. Furthermore, the performance is poor for large values of λ indicating that the regularization parameter should be chosen carefully (though a wide range delivers equally good results). These observations are consistent with those in [41]–[46] and the hypotheses discussed at the end of Section IV-E.

B. Role of Projection in Two-Norm Regularization

Theorem IV.6 suggests that the identification-induced regularizer $h(g)=\|(I-\Pi)g\|_2^2$ is superior to a two-norm regularizer $h(g)=\|g\|_2^2$ if one is interested in consistency and the predicted trajectory adhering to a least-square fit of the data. To test this hypothesis, we consider the same case study from Section V-A and report the averaged cost in Fig. 2.

Both regularizers perform similarly for small λ, but the identification-induced regularizer shows a superior and surprisingly constant performance for sufficiently large λ. By the proof of Theorem IV.6, for λ sufficiently large, the direct and indirect problems (16) and (15) are equivalent—up to causality and complexity constraints. Thus, a sufficiently large λ forces the least-square fit (13) and results in excellent performance independent of the specific value of λ. While there is a small window where the two-norm excels, the identification-induced regularizer shows overall much more robust performance.

Next, we study the merits of hybrid regularization (25). For the same case study, Fig. 3 shows the averaged realized performance plotted over the regularization parameters. The $\{\lambda_1=0\}$ and $\{\lambda_2=0\}$ slices recover Figs. 1 and 2. As before, the regularizer $\|(I-\Pi)g\|_2^2$ is more robust though a hybrid regularizer yields a minor albeit robust improvement. A closer examination of the data underlying Fig. 3 reveals that a hybrid regularization can improve up to 15% over the best results achievable with the regularizer $\|(I-\Pi)g\|_2^2$ only.

C. Effect of Data Length

We continue with the same case study and discuss the effect of data length on direct and indirect methods. For the direct method, we used the identification-induced regularizer $h(g)=\|(I-\Pi)g\|_2^2$ with sufficiently large weight $\lambda=10000$, as indicated in Fig. 2. For the indirect method, the inner system identification problem (3) is solved using the subspace approach N4SID [26] with prefix horizon $T_{\text{ini}}=5$, prediction horizon $L=20$, and (correct) model-order selection $n=4$.

For our case study, Lemma II.3 demands at least $T=53$ data points. Fig. 4 shows the beneficial effects of including more data on the realized *median* performance of the direct and indirect methods. The main findings are as follows: First, both methods are asymptotically consistent. Second, the indirect method is superior in the low data regime echoing that models are compressed and denoised representations (see Remark II.4).

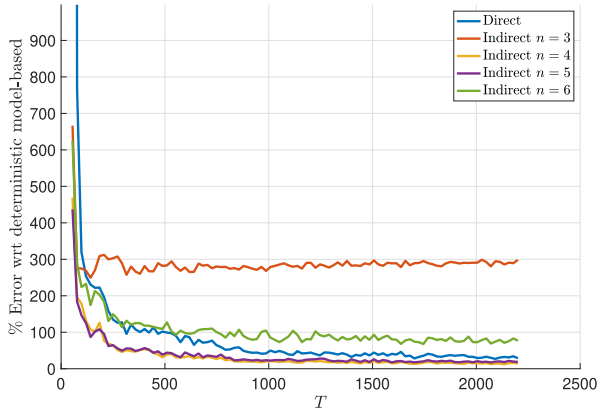

Fig. 4. Realized median error (over 100 datasets) for the direct and indirect (with different model order selections) methods for varying amount of data.

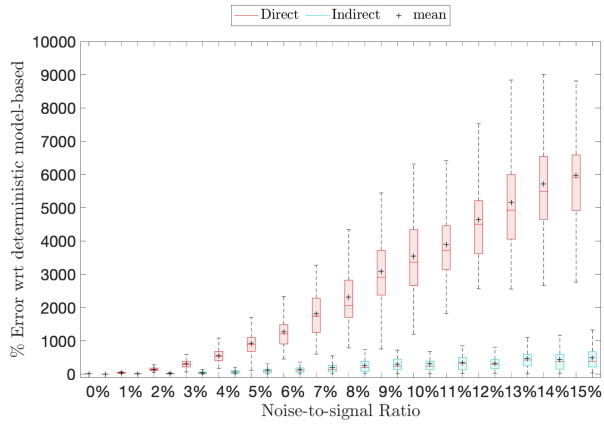

Fig. 5. Comparison of direct and indirect methods for varying noise.

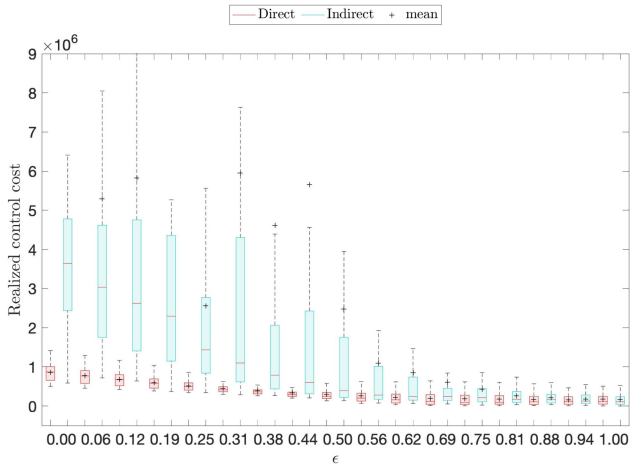

Fig. 6. Comparison of direct and indirect methods for varying nonlinearity.

Third and finally, when an incorrect model-order $n \in \{3, 5, 6\}$ is selected for the indirect method (resulting in a bias), then consistency could be lost. We see that the direct method is superior for $n \in \{3, 6\}$, whereas the indirect method still performs well for $n = 5$. This bias effect is even more pronounced when studying the average (as opposed to the median) error due to several outliers of the indirect method.

This third point hints at a bias-variance tradeoff between the direct and indirect methods, which will be studied below.

D. Comparison and Bias-Variance Hypotheses

We now compare the direct and indirect approaches through two case studies. The first study evaluates the performance of both methods on the basis of "variance" error, i.e., on a linear system with noisy measurements. The second study evaluates the performance on the basis of "bias" error, i.e., on a nonlinear system with noise-free measurements.

We expect the direct method to perform better on the nonlinear system since the indirect method erroneously selects a linear model class thus leading to a larger "bias" error. On the other hand, we expect the indirect method to perform better on the linear system with noisy outputs since the identification step filters noise thus leading to a lower "variance" error.

1) Comparison: Stochastic Linear System: Consider the same case study as in Section V-A, i.e., same LTI system, cost, and reference. We collected data for varying levels of noise-to-signal ratio, i.e., we considered measurements that were affected by Gaussian noise with noise-to-signal ratio in the set $\{0\%, 1\%, \ldots, 15\%\}$. For each noise-to-signal ratio, $T = 250$ input/output data samples were collected by applying a random Gaussian input. These data were then used for both the direct and indirect methods.

For the indirect method, the inner system identification problem (3) is again solved using N4SID [26] with prefix horizon $T_{\text{ini}} = 5$ and prediction horizon $L = 20$. Equipped with a (correct) fourth-order identified model, optimal control inputs are computed by solving (4). The indirect method was compared to the direct method (7), with $h(g) = \|g\|_1$, $T_{\text{ini}} = 5$, and $\lambda = 27$. The hyperparameters of both methods were kept constant for all simulations below and chosen to give good realized control performance for all noise-to-signal ratios.

For both methods, we recorded the realized performance after applying the open-loop inputs and converted it to a percentage error with respect to the best possible performance (i.e., if the deterministic model was exactly known). For each noise-to-signal ratio, 100 simulations were conducted with different random datasets. The results are displayed in the box plot in Fig. 5 and show that both methods perform well for low levels of noise (up to approximately 2% noise-to-signal ratio). As the data becomes noisier, the performance of the direct method degrades significantly, whereas the performance of the indirect method remains relatively constant. We remark that a slightly better albeit qualitatively similar result is obtained with the regularizer $\|(I - \Pi)g\|_2^2$.

We attribute these observations to the fact that identification denoises the data. These results confirm our hypothesis that the indirect method is superior in terms of "variance" error.

2) Comparison: Deterministic Nonlinear System: We now consider the scenario where the direct and indirect methods are subject to a "bias" error, but not a "variance" error. Consider the discrete-time nonlinear Lotka–Volterra dynamics considered for

direct data-driven control in [57]

$$x(t_{k+1}) = f_{\text{nonlinear}}(x(t_k), u(t_k)) = \begin{bmatrix} x_1(t_k) + \Delta t(a x_1(t_k) - b x_1(t_k) x_2(t_k)) \\ x_2(t_k) + \Delta t(d x_1(t_k) x_2(t_k) - c x_2(t_k) + u(t_k)) \end{bmatrix}$$

where $t_{k+1} - t_k = \Delta t = 0.01$, $a = c = 0.5$, $b = 0.025$, $d = 0.005$, and $x(t_k) = \begin{bmatrix} x_1(t_k) & x_2(t_k) \end{bmatrix}^\top$. Here, $(x_1(t_k), x_2(t_k))$ denote prey and predator populations, and $u(t_k)$ is the input. A linearization about the equilibrium $(\bar{u}, \bar{x}_1, \bar{x}_2) = (0, c/d, a/b)$ yields the affine linear system

$$x(t_{k+1}) = f_{\text{linear}}(x(t_k), u(t_k), \bar{x}_1, \bar{x}_2) = \begin{bmatrix} x_1(t_k) + \Delta t\left((a - b\bar{x}_2)(x_1(t_k) - \bar{x}_1) \right. \\ \left. - b\bar{x}_1(x_2(t_k) - \bar{x}_2)\right) \\ x_2(t_k) + \Delta t\left(d\bar{x}_2(x_1(t_k) - \bar{x}_1) \right. \\ \left. + (d\bar{x}_1 - c)(x_2(t_k) - \bar{x}_2) + u(t_k)\right) \end{bmatrix}.$$

We expect direct data-driven control (7) to perform well on such a nonlinear system for two reasons: First, nonlinear systems can be well approximated by LTI systems of sufficiently high complexity; and second, the direct method (7) does not specify the LTI system complexity (e.g., by enforcing rank constraints).

We compare the direct and indirect methods for varying degree of nonlinearity by interpolating between $f_{\text{nonlinear}}$ and f_{linear}, i.e., we study the interpolated system

$$x(t_{k+1}) = \epsilon \cdot f_{\text{linear}}(x(t_k), u(t_k), \bar{x}_1, \bar{x}_2) + (1 - \epsilon) \cdot f_{\text{nonlinear}}(x(t_k), u(t_k)) \tag{27}$$

for $\epsilon \in [0, 1]$. For $\epsilon = 1$ (respectively, $\epsilon = 0$), the dynamics are purely affine (respectively, nonlinear). For each $\epsilon \in \{0, 0.1, \ldots, 1\}$, $T = 2415$ data points were collected by applying a noisy sinusoidal input $u(t_k) = 2(\sin(t_k) + \sin(0.1t_k)))^2 + v(t_k)$ with $v(t_k)$ sampled from a Gaussian random variable. Full state measurement was assumed. The data collection was repeated for 100 different initial conditions. For each degree of nonlinearity $\epsilon \in \{0, 0.1, \ldots, 1\}$ and each initial condition, the data were used to compute optimal open-loop control inputs using direct and indirect methods. The control cost was chosen as $c_{\text{ctrl}}(w - w_r) = \|w - w_r\|_2^2$ with equilibrium reference $w_r = (0, 100, 20, \ldots, 0, 100, 20)$, $L = 600$, and $w = (u, x)$.

For the indirect method, the inner system identification optimization problem given by (3) is solved using the subspace approach N4SID [26] with initial condition horizon $T_{\text{ini}} = 4$, and prediction horizon $L = 600$. A model order of 4 was chosen, as it produced the best performance as measured by the realized control cost. Optimal control inputs were then computed by solving (4). For comparison, we chose the direct method (7) with $h(g) = \|g\|_1$, $T_{\text{ini}} = 4$, and $\lambda = 8000$. The performance was measured with the realized control cost after applying the open-loop inputs to system (27). As before, the hyperparameters of both direct/indirect methods were judicially chosen and kept constant for all simulations.

The results displayed in Fig. 6 show that both methods perform well for low levels of nonlinearity: $\epsilon \in [0.7, 1]$. As the system becomes increasingly nonlinear, the performance of the indirect method degrades significantly, whereas the performance of the direct method remains relatively constant. We attribute this observation to the fact that the indirect method incurs a "bias" error from selecting a linear model class and applying certainty-equivalence control, whereas the direct method uses data from the nonlinear system without bias. These findings confirm our earlier bias-variance observations from Fig. 4.

VI. Conclusion

We studied the relationship between indirect and direct data-driven control formulated as bilevel (first-identify, then control) and single-level regularized (based on the Fundamental Lemma) optimization problems, respectively. An intermediate multicriteria problem allowed us to efficiently transition between both formulations. We concluded that the regularized direct approach can be viewed as a convex relaxation of the indirect approach, where the choice regularizer depended on the problem formulation and accounted for an implicit identification step. We also discovered a novel regularizer that is consistent and accounts for least-square identification.

Our results suggested the use of the indirect method in case of "variance" errors and the use of the direct method in presence of "bias" errors (e.g., a nonlinear system or when selecting a wrong model order). These insights echo the bias-variance tradeoffs previously encountered for direct and indirect methods in [20] and [51], and they shed some partial light on the remarkable empirical performance of (direct) DeePC applied to nonlinear systems.

As a limitation, our results concern only the open-loop predictive control problem, though we ultimately care about the realized performance, especially in a receding horizon closed-loop implementation. Some preliminary results on the realized performance of regularized control formulations were obtained in [46] through the lens of robust optimization, but the topic remains largely open. Moreover, we believe that the proposed multicriteria data-driven control formulation is important in its own right and may deliver excellent performance if one were to find a convex formulation and appropriate tradeoff parameter. Both of these are formidable tasks for future work.

Finally, we believe that our approach is also applicable to other identification and control formulations and may deliver interesting and novel direct data-driven control formulations.

Appendix

Consider the mathematical program (MP)

$$\boldsymbol{MP}: \underset{x \in C}{\text{minimize}}\ f(x) \quad \text{subject to} \quad g(x) \leq 0, h(x) = 0, \tag{28}$$

where $C \subset \mathbb{R}^n$ is closed, and f, g, and h are lower semicontinuous maps from $\mathbb{R}^n$ to $\mathbb{R}$, $\mathbb{R}^m$, and $\mathbb{R}$. Consider the perturbation

$$\boldsymbol{MP}_\epsilon: \underset{x \in C}{\text{minimize}}\ f(x) \quad \text{subject to} \quad g(x) \leq 0, h(x) = \epsilon \tag{29}$$

for $\epsilon \in \mathbb{R}$. We recall the definition of partial calmness [53].

Definition A.1: Let $x^\star$ solve $\boldsymbol{MP}$, and let $\mathbb{B}_n$ denote the open unit ball in $\mathbb{R}^n$. Then, $\boldsymbol{MP}$ is said to be *partially calm* at $x^\star$ provided that there are $\mu > 0$ and $\delta > 0$ such that, for all $\epsilon \in \delta\mathbb{B}_1$ and all $x \in x^\star + \delta\mathbb{B}_n$ feasible for $\boldsymbol{MP}_\epsilon$, one has

$$f(x) + \mu|h(x)| \geq f(x^\star)\,. \tag{30}$$

Partial calmness is equivalent to *exact penalization*. In particular, consider for $\mu \geq 0$, the penalized mathematical program

$$\begin{aligned} \boldsymbol{PMP}_\mu : \underset{x\in C}{\text{minimize}}\; & f(x) \,+\, \mu \cdot |h(x)| \\ \text{subject to}\quad & g(x) \leq 0. \end{aligned} \tag{31}$$

We summarize [52, Prop. 3.3], [53, Prop. 2.2].

Proposition A.2: Assume that f is continuous, and let $x^\star$ be a local minimizer of $\boldsymbol{MP}$. Then, $\boldsymbol{MP}$ is partially calm at $x^\star$ if and only if there is $\mu^\star > 0$ so that $x^\star$ is a local minimizer of $\boldsymbol{PMP}_\mu$ for all $\mu \geq \mu^\star$. Moreover, any local minima of $\boldsymbol{PMP}_\mu$ with $\mu > \mu^\star$ are also local minima of $\boldsymbol{MP}$.

Partial calmness has been studied for a range of problems, particularly bilevel problems [52], [53], [58]. Of specific importance to us is a related result due to Clarke [59, Prop. 2.4.3], which allows for exact penalization (or equivalently partial calmness) and reads in our notation as follows.

Proposition A.3: Consider the mathematical program $\boldsymbol{MP}$ and its penalized version $\boldsymbol{PMP}_\mu$. Assume that f is Lipschitz continuous with Lipschitz constant L, the equality constraint takes the form of a distance to a closed set $S \subset C$

$$0 = h(x) = \text{distance}(x, S) = \inf_{y\in S}\|x - y\|$$

and $\boldsymbol{MP}$ attains a minimum. Then, for $\mu > \mu^\star = L$, any local minimum of $\boldsymbol{PMP}_\mu$ is also a local minimum of $\boldsymbol{MP}$.

We note that $\|\cdot\|$ in Proposition A.3 can be an arbitrary norm. For a more general problem setup with a parametric set S depicting the value function of an (inner) optimization problem, the reader is referred to [53, Th. 2.5]. Additionally, the setup can be extended to (squared) *merit functions* as penalty functions [53, Ths. 2.6 and 2.9]. These generalize the notion of distance but are easier to formulate and compute.

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