

Tentative schedule of/Déroulement prévu pour MACS201

- Part I

Topic: Projections and Conditional expectation.

Time slot/TH-1 **Lecture/Cours :** Hilbert spaces: reminders, Hilbert bases, projection, examples: $L^2(\Omega, \mathcal{F}, \mu)$, case $\mu = \text{Lebesgue}$.

Time slot/TH-2 **Classwork, homework/TD, travail perso :** Exercises 4.1 and 4.2 (both about Gaussian vectors, taken from Chapter 4), Exercises 1.1 and 1.2.

Time slot/TH-3 **Lecture/Cours :** Remainders in probability: existence and uniqueness of measures. Conditional expectation: L^2 case, L^1 case, immediate properties.

Time slot/TH-4 **Classwork, homework/TD, travail perso :** Exercises 2.1 to 2.3 (up to (i)).

Topic: Conditional distributions, conditional density.

Time slot/TH-5 **Lecture/Cours :** Kernels, regular version of conditional distribution (notation $\mathbb{P}^{\mathcal{G}}, \mathbb{P}^{Y|\mathcal{G}}, \mathbb{P}^{Y|X}$), link with conditional expectation, disintegration of measures on product spaces. Conditional density.

Time slot/TH-6 **Classwork, homework/TD, travail perso :** Finish Exercise 2.3, start Exercises 2.4 to 2.8,

Time slot/TH-7 **Lecture/Cours :** Riesz representation theorem, Radon-Nikodym theorem.

Time slot/TH-8 **Classwork, homework/TD, travail perso :** Finish Exercises 2.5 to 2.7.

Topic: Statistical applications

Time slot/TH-9 **Lecture/Cours :** Statistical model. Dominated models. Sufficient statistics. Fisher factorization theorem.

Time slot/TH-10 **Classwork, homework/TD, travail perso :** Finish Exercise 2.8. Exercises 2.9 and 2.10.

Time slot/TH-11 **Lecture/Cours :** Kullback-Leibler divergence, Likelihood, statistical testing,

Time slot/TH-12 **Classwork, homework/TD, travail perso :** Exercises 2.14 and 3.1 to 3.5.

Time slot/TH-13 **Exam/CC :** (2 hours). Exam on projections, conditional expectation and probabilities. Documents such as lecture notes (printed and hand written) are allowed during the exam.

Time slot/TH-14 **Classwork, homework/TD, travail perso :** Finish Exercises 3.2 to 3.5.

- Part II

Topic: Stochastic Processes, introduction.

Time slot/TH-15 **Lecture/Cours :** Proc, def, paths, fidi distribution, filtration, examples (iid, gaussian processes), Stopping times.

- Time slot/TH-16 **Lecture/Cours** :Strict/weak stationnarity, (Herglotz), autocovariance, (exples: iid, gaussiens, harmonic processes, martingale increments)
- Time slot/TH-17 **Classwork, homework/TD, travail perso** :Revision : finish all previous exercises. Start Exercises 4.3 to 4.6.
- Time slot/TH-18 **Lecture/Cours** :Finish Exercises 4.3 to 4.6.
- Time slot/TH-19 **Lecture/Cours** :Extension of unitary operators for Hilbert spaces, Random measures with orthogonal increments, spectral representation of weakly stationary processes.
- Time slot/TH-20 **Classwork, homework/TD, travail perso** :Exercises 5.1 and 5.2.
- Time slot/TH-21 **Lecture/Cours** :Examples: randomly weighted sum of dirac point masses and harmonic processes. Innovation process.
- Time slot/TH-22 **Classwork, homework/TD, travail perso** :Exercises 5.3 and 5.5 to 5.9.
- Topic:** Markov chains on general state spaces, an introduction.
- Time slot/TH-23 **Lecture/Cours** :Definition, conditioning the future given the past, Homogeneous Markov chains, Markov kernels
- Time slot/TH-24 **Classwork, homework/TD, travail perso** :Exercises 7.4 and 7.5
- Time slot/TH-25 **Lecture/Cours** :Composition, tensor product of kernels, finite distribution of Homogeneous Markov chains, canonical chain, notation \mathbb{P}_ξ , Invariant measure.
- Time slot/TH-26 **Classwork, homework/TD, travail perso** :Exercises 7.2 and 7.3. Look at the examples in Chapter 9.
- Time slot/TH-27 **Lecture/Cours** :Stationary Markov chains, Strong Markov property.
- Time slot/TH-28 **Classwork, homework/TD, travail perso** :Exercises 8.1 to 8.3.
- Time slot/TH-29 **Lecture/Cours** :Finite state-space irreducible Markov chains : construction of invariant measure.
- Time slot/TH-30 **Classwork, homework/TD, travail perso** :Exercises 8.4 to 8.7.
- Time slot/TH-31 **Exam/CC** :(3 hours). Exam on the whole program. Documents such as lecture notes (printed and hand written) are allowed during the exam.