Wavelets and filterbanks lab TSIA201

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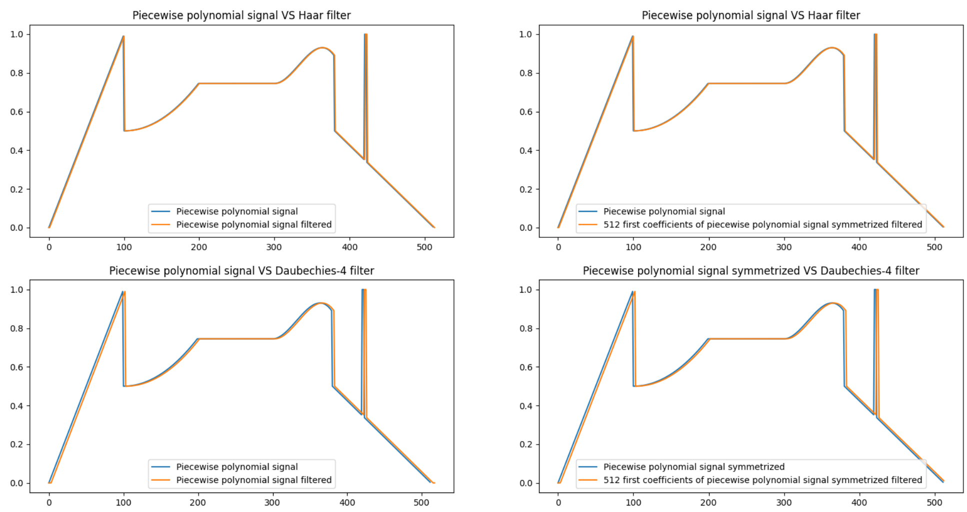
You can also find the high-resolution graphs in the “Graphs” folder of the archive.

1.3



The graphs of this questions are on the left.
Overall, the signal is well reproduced, even though the filtered signal for the Daubechies-4 filter is a bit delayed.
The more vanishing moments we have, the better we can reproduce complex functions. Thus, DB4 having more vanishing moments than Haar, it is supposed to best represent the piecewise polynomial signal.

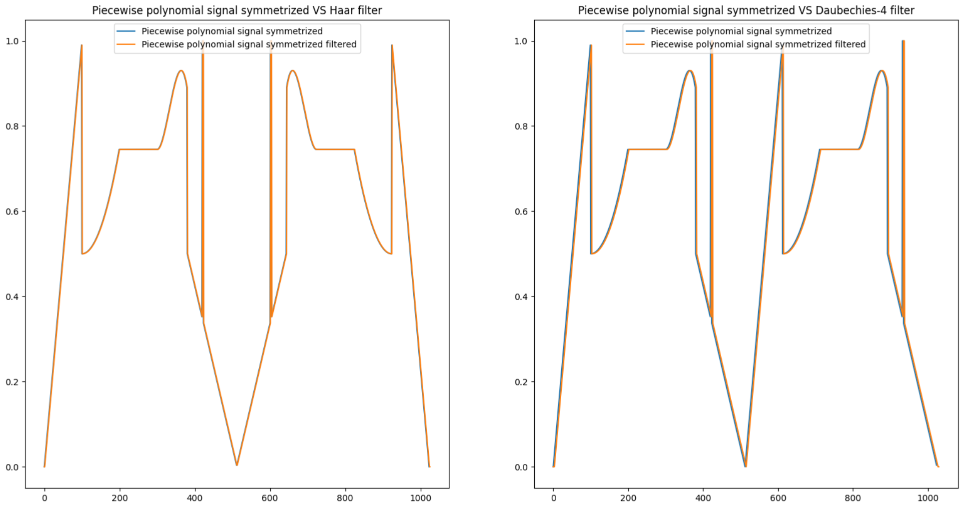
1.4

Yes, we achieve perfect reconstruction (a delay does not invalidate the perfect reconstruction).

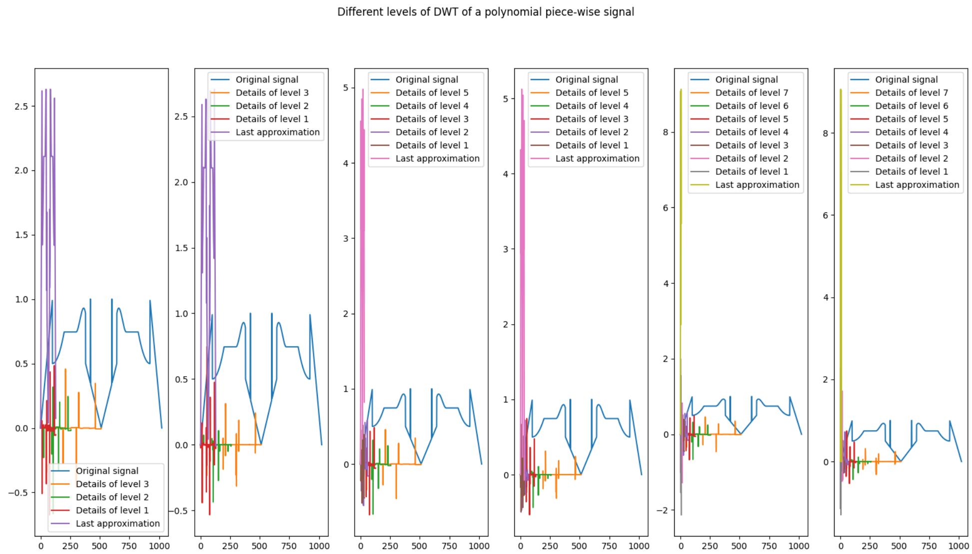
1.5

Yes, we observe coefficient expansion. We have 2 added coefficients in the Haar case, and 6 in the Daubechies-4 case.

1.6

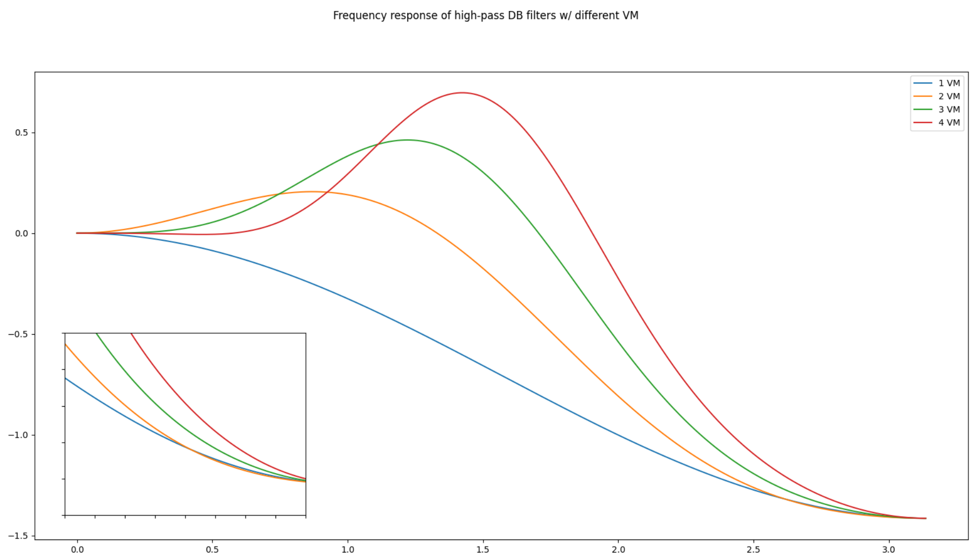


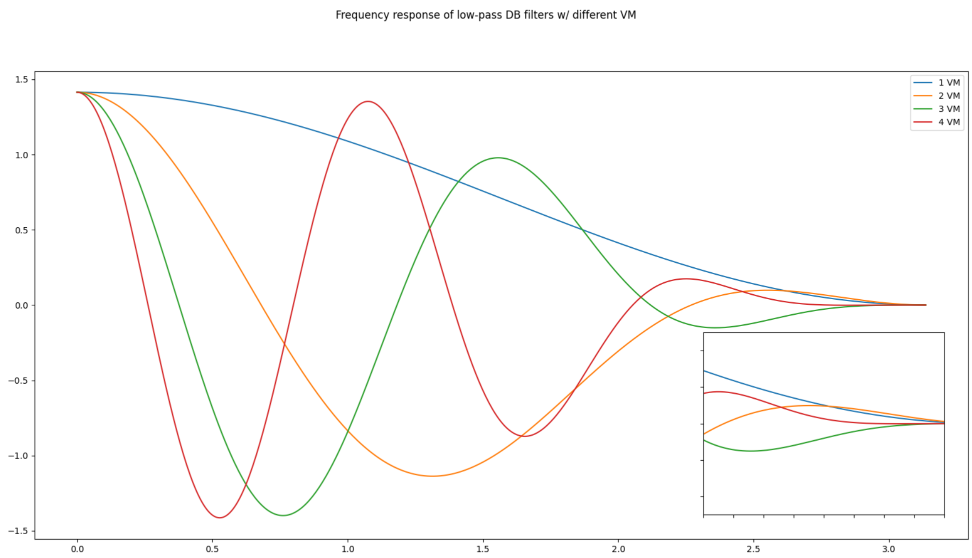
1.8



We indeed have coefficient expansion.
The more levels of DWT, the more spread the discontinuities are.
Only the lower frequency parts produce zero-valued coefficients

1.9 and 1.10



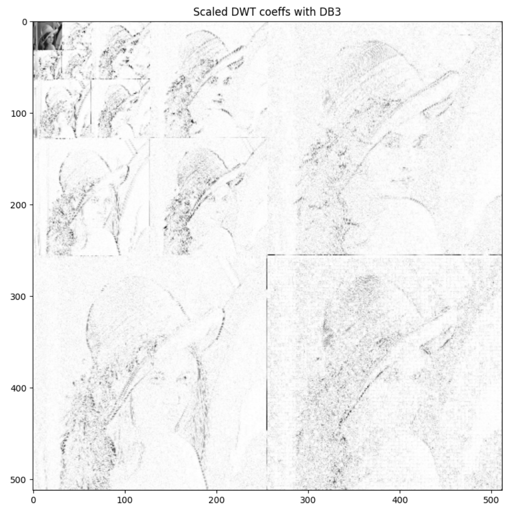


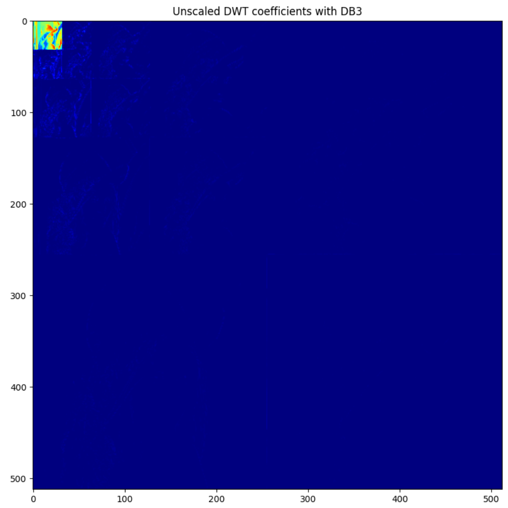
On the zoomed portion, we can observe that the more vanishing moments there is, the less flat the graph becomes approaching $\omega = \pi$.

2.1


0
100
200
300
400
500
0
100
200
300
400
500

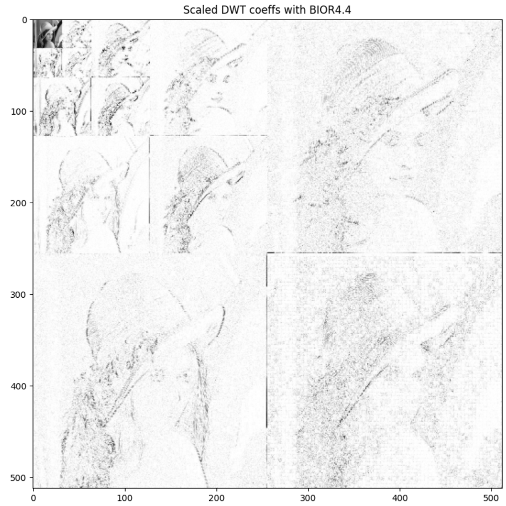
2.2

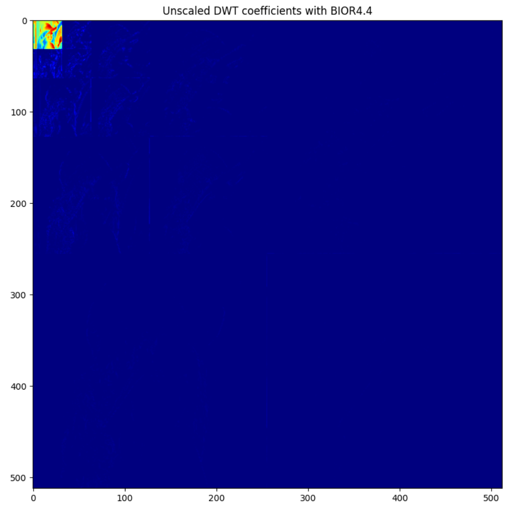

Scaled DWT coeffs with DB3
0
100
200
300
400
500
0
100
200
300
400
500



The more levels of DWT used, the more spread the coefficients are (compared to the little levels where we have a clear separation between high-frequency coefficients (borders) and low-frequency ones (background)). Thus, we can effectively use wavelets in a compression system, given that we can easily reproduce all the details thanks to the less and more spread transforms (the inverse DWT achieving perfect reconstruction confirms that point).

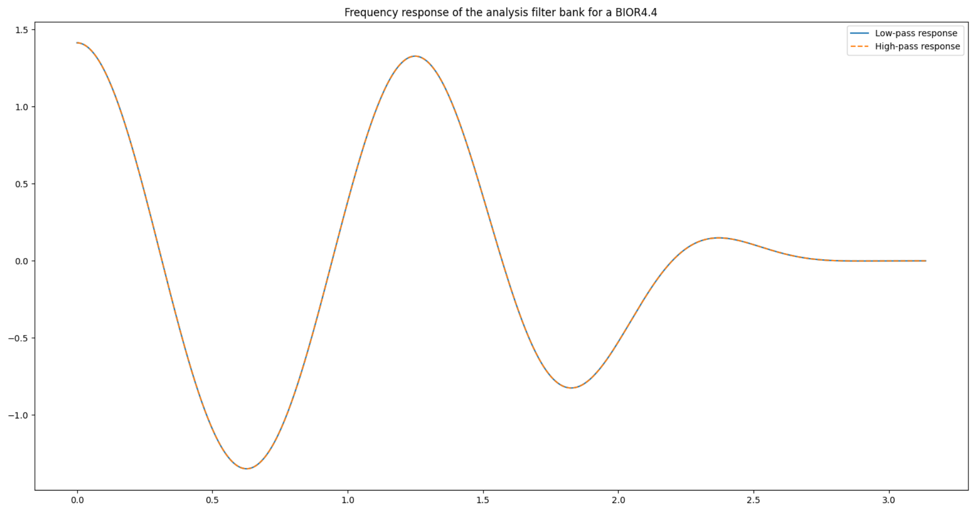
2.3


Scaled DWT coeffs with BIOR4.4

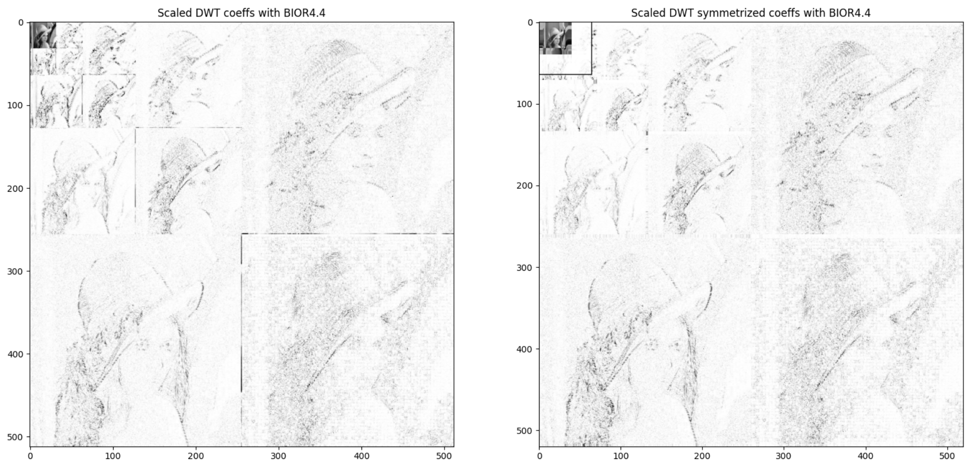

Unscaled DWT coefficients with BIOR4.4


Inverse DWT
0
100
200
300
400
500
0
100
200
300
400
500

2.5


Frequency response of the analysis filter bank for a BIOR4.4
Low-pass response
High-pass response
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

2.6



We observe that using a symetric periodization, we have less border effects (confirmed by the fact that we limit high frequencies by symmetrizing the input signal).