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Practical work on spectral estimation

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In this practical work, we propose to compare nonparametric spectral estimators (periodogram, smoothed periodogram) with parametric estimators (linear prediction, Capon, Durbin methods), in the case of three types of processes commonly encountered : autoregressive processes, sum of noisy sinusoids, and autoregressive-moving-average processes.

This practical work will be carried out either with Matlab or Python. The different estimators will be applied to synthetic signals, as well as speech signals.

1 Autoregressive process

1.1 Synthesis

1. Code a function that synthesizes an autoregressive (AR) process of length N, defined by its AR coefficients and the innovation variance σ^2.

1.2 Non-parametric estimation : periodogram

2. Code a function that computes and displays the periodogram of a given input signal, allowing to choose between the standard method and the Bartlett and Welsh methods.
3. Display the periodograms obtained with the different methods (in dB), in the case of an AR process of order 1 whose parameters can be chosen by hand. We will superimpose in each case the periodogram with the known power spectral density (PSD) of the AR process. We will pay attention to the correct normalization of the different estimators.
4. Experimentally compare these different estimators in terms of bias and variance.

1.3 Parametric estimation : linear prediction

5. Estimate the AR model using either the code you wrote in the previous practical work of TSIA202a, or the function `lpc` (linear prediction method) from Matlab or from the librosa Python package.
6. Superimpose the spectral envelope of the estimated AR model with that of the exact AR model and with the periodogram. Comment.

1.4 Application to speech signals

7. Apply the above methods to a vowel sound (such as 'a' for example, that we can extract from file `aeiou.wav`, downloadable from the eCampus website of TSIA202b), and superimpose the estimated spectra. Comment.

2 Sum of noisy sinusoids

2.1 Synthesis

1. Code a function that synthesizes a signal of length N, consisting of the sum of K real sinusoids defined by their frequencies, amplitudes and origin phases, and a Gaussian white noise of variance σ^2.

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2.2 Non parametric estimation : periodogram

2. Display the periodograms obtained with the standard method and the Bartlett and Welsh methods, in the case of $K = 2$ sinusoidal frequencies ν_1 and ν_2, zero origin phase and same amplitude equal to 1. We will superimpose in each case the periodogram with two vertical lines centered at frequencies ν_1 and ν_2, and a horizontal line centered at the variance σ^2.
3. Compare the resolution of the various periodograms, by varying the difference $\Delta\nu = |\nu_1 - \nu_2|$ between both frequencies, as well as the signal-to-noise ratio (SNR) in dB.

2.3 Parametric estimation : linear prediction

4. Estimate a fourth order AR model using the linear prediction method.
5. Superimpose the estimated AR model with the previous figures, always by varying the gap $\Delta\nu$, as well as the SNR in dB. Comment on the results in terms of resolution.

2.4 Parametric estimation : Capon estimator

6. Code a function that implements the Capon estimator.
7. Compare the resolution of this method with the two previous methods, always by varying $\Delta\nu$, as well as the SNR in dB.

3 Autoregressive-moving-average process

3.1 Synthesis

1. Code a function that synthesizes an autoregressive-moving-average (ARMA) process of length N, defined by its AR and MA coefficients and by the innovation variance σ^2.

3.2 Non-parametric estimation : periodogram

2. Display the periodograms obtained with the different methods, in the case of an ARMA process of order $(1, 1)$ whose parameters can be chosen by hand. We will superimpose in each case the periodogram with the known PSD of the ARMA process.
3. Compare these different estimators in terms of bias and variance.

3.3 Parametric estimation : Durbin method

4. Code a function that implements the Durbin method (estimation of the AR part by shifting the autocorrelation, estimation of the MA part via a long AR).
5. Superimpose the spectral envelope of the estimated ARMA model with that of the exact ARMA model and with the periodogram. Comment.

3.4 Application to speech signals

6. Apply the above methods to a nasal vowel sound (such as 'in' for example, that we will be able to extract from file `an_in_on.wav`, downloadable on the eCampus website of TSIA202b), and superimpose the estimated spectra.

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