Optimal Siting, Sizing and Bid Scheduling of a Price-Maker Battery on a Nodal Wholesale Market

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***Abstract*—The market for battery storage is set to boom in the coming years. This trend may be explained by a combination of factors ranging from a falling cost of batteries to a growing need to address uncertain and unflexible renewable energy generation. It is well known that capital costs remain too high for a large-scale lithium-ion battery storage to be profitable solely by arbitrating a wholesale market. However, we show that by carefully selecting a battery's siting and size with respect to its influence on prices and congestion, a battery storage can still be profitable on a nodal wholesale market. To that end, we develop a price-maker mixed-integer optimization framework that maximizes a depreciated battery storage revenue and yields the optimal siting and size of that battery storage. Furthermore, it can be used to optimize the bidding schedule of a battery storage in a nodal transmission-constrained wholesale market. We conducted multiple simulations to illustrate and confirm the need for this approach. Namely, we compared price-maker and price-taker results on available data from the New Zealand nodal wholesale market.**

***Index Terms*—Local Marginal Price, Nodal Network, Wholesale Market, MILP Optimization, Battery Storage optimization**

Nomenclature

λ	Local marginal prices
a	Generator bid prices
B	Normalized cost of 1 MWh of battery energy capacity
c	Battery bid prices vector
d	Nodal demand
g	Generator power injection
H	Shift-factor matrix
h	Line capacity
i_b	Battery storage location
p	Nodal power injection
P^{max}	Generator maximum bid volumes
P^{min}	Generator minimum bid volumes
q	Battery bid volumes vector
T	Horizon of the battery scheduling problem
u	Battery energy injection
z	Storage charge level (SCL)
z_{cap}	Battery capacity

I. Introduction

A. Background & Motivation

Deployment of Grid-Scale batteries is surging in energy markets. In the United States alone, prospective studies suggest that installed capacity will increase by 3.8 GW by 2023 [1] [2]. Different factors explain this trend: a falling cost of this technology, more adapted market rules, new regulations requesting storage, and an increase in penetration of renewable energies in the electricity mix. Batteries are especially well tailored to improve the grid reliability by participating in ancillary and regulation markets [3] [4].

There are three different ways to site a battery on the grid: on the distribution network near the load center, co-located with various renewable energy generators (solar, wind) or on the transmission network [5]. We can broadly classify the literature pertaining to the optimization of operation and sizing of a battery in three groups. The first group takes the Independent System Operator's (ISO) perspective having the duty of optimizing system reliability and reducing cost of transmission [6]. The second one focuses on finding the best joint operation of a battery with other resources such as solar [7] [8], wind [9] or both [10], and increasingly with electric vehicles. Finally, the last group deals with the optimization of a standalone storage system from a merchant operator's perspective and at a transmission level. Our contribution adds to the last group.

A widespread approach to generate an optimal bidding schedule for battery storage is to assume that the storage will be a *price-taker*, and devise an optimization program with exogenous market prices. Some articles have looked into finding an optimal sizing and operation but under exogenous real-time prices [11]. Some have focused on finding the optimal dispatch over multi-markets (day-ahead market, reserve and regulation market) [12]. Additionally, the impact of degradation on the optimal bidding schedule was investigated in [13]. On nodal markets, feasibility studies concerning implementation of battery storage have been conducted but always under the *price-taker* assumption [14] [15]. However, Mohsenian-Rad laid a framework for a price-maker large-scale battery storage program over a nodal-wholesale market [16]. His article only considers large-scale battery storage and does not provide a comparative price-taker approach to illustrate the importance of endogenous prices. It also does not consider the siting and sizing planning problem.

B. Price-taker vs Price-maker approach

Over a wholesale-market, an agent is said to be *price-taker* if its bidding behavior does not influence the market's clearing price. This assumption is usually taken when the

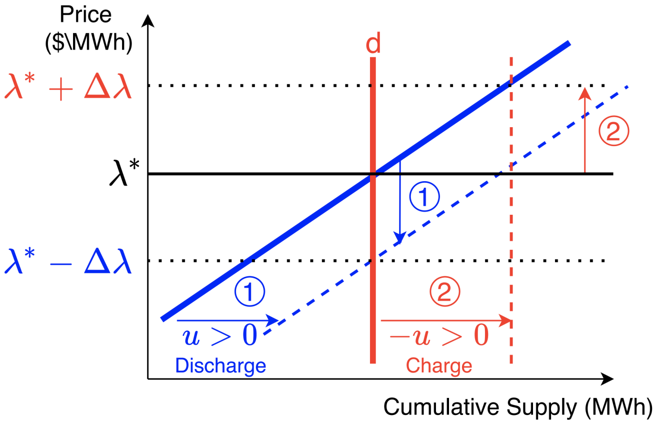

Fig. 1: Discharge (1) and charge (2) effects on prices on single node market

agent bids small volumes with respect to the demand of the market. When an agent bids large volumes, its bid volume and price can both impact market prices to its advantage, it has market-power. When congestion occurs over a node, the nodal demand is disconnected from cheap generation. As a result, the nodal price increases. The battery bids in this case, even if small, are not negligible to the single-node-market demand. As a result, the battery is a *price-maker* on the congested area, i.e, its bids will move the nodal cleared price.

Assuming the battery node is congested, we represented on Fig 1 two main cases of how the battery injection u can impact the prices in both directions. The blue thick supply curve idealizes the accumulated accessible bids and the red thick vertical curve is the demand d. If the battery injects energy into the grid (1), the supply curve shifts the cleared price λ^* to the right, defined as the intersection of supply and demand. Then the cleared price falls by $\Delta\lambda$. In an extreme case, injecting too much energy can remove congestion and the price would fall even further to meet the market marginal price. If it withdraws energy (2), demand d increases, the cleared price will increase. In the following, a *price-taker* (*price-maker* resp.) program is defined as a battery profit maximization optimization program with the local marginal prices (LMPs) taken as exogenous (resp. endogenous).

C. Focus of this article and contribution

In this article, we examine the situation where a battery located at the transmission level and bidding over a wholesale nodal market is a *price-maker* when congestion occurs. Over a congested area, a well-operated battery can both alleviate congestion and maximize its earning by selling energy at a higher price than the system marginal price. To the best of our knowledge, our article is the first to study the optimal sizing and siting for a battery on a nodal wholesale market under endogenous market prices. This allows for taking into account the depth of congestion patterns over the grid. We further illustrated this approach by comparing it to a simpler price-taker strategy on the New Zealand grid.

II. Problem Notations and Formulation

A. Battery storage problem

We consider a standalone battery located at the transmission level and bidding solely on a nodal uniform-pricing (pay-as-clear) wholesale market. The grid's buses are described by a set of nodes $\mathcal{N}$ numbered from 1 to N. The battery is located at node $i_b \in \mathcal{N}$, and has capacity z_{cap}. At each trading period t comprised in $\mathcal{T} = \{1, .., T\}$, the operator submits a bid price c_t and volume q_t to the market and is charged with the cleared price $\lambda_t^{i_b}$. The high level goals are to determine:

1) the optimal siting i_b^* of the battery on the network,
2) the optimal capacity z_{cap}^*,
3) the optimal bidding schedule (c_t^*, q_t^*) for each trading period t.

A key challenge is that the clearing price $\lambda_t^{i_b}$ is a function of the battery's bid price and quantity (c_t, q_t). We will devise an optimization program with variable $z_{cap}, (c, q)$ over a time horizon of T. The optimal siting i_b^* is a parameter that will be found by iterating the optimization program over the nodes of the network. In the following sections, we describe each part of the optimization problem.

1) Storage Charge Level (SCL) Constraints: Denote $u \in \mathbb{R}^T$, the storage energy injection vector. Denote $u_t > 0$ (resp. $u_t < 0$) if the battery is injecting (resp. withdrawing) energy to the grid at time period t. Denote $z \in \mathbb{R}^T$, the Storage Charge Level (SCL) profile of the battery. The SCL profile is constrained by the energy capacity and some minimum level set to zero in this article (1). The power injection vector is bounded by an upward and downward charging rate constraint to limit short and deep cycles (2). An ideal linear model for the charge profile is used with an assumed efficiency of 1 (3). Finally, to prevent a bias in the revenues of the battery, the first SCL is imposed to be equal to the last (4). Note that roundtrip inefficiencies or linear constraints could be added to the framework without loss of generality.

$$\forall t \in \mathcal{T}, \quad 0 \le z_t \le z_{cap} \quad \text{SCL constraint} \tag{1}$$

$$u^{min} \le u_t \le u^{max} \quad \text{Charging rate constraint} \tag{2}$$

$$z_{t+1} = z_t - u_t \quad \text{SCL update equation} \tag{3}$$

$$z_{T+1} = z_1 \quad \text{SCL initial condition} \tag{4}$$

Given the initial state z_1, the injection vector $(u_t)_{t \le T}$ is in bijection with the SCL vector $(z_t)_{t \le T}$. Noting $\mathbb{1}_T$ as the unit vector of size T, and S a sub-triangular matrix filled with -1, the equality constraint 3 is equivalent to $z = z_1 \cdot \mathbb{1}_T + Su$. This allows to rewrite constraints (1, 2, 4) under compact form, as a function of u and drop the dependency on z in the following:

$$\tilde{S}u \le \bar{u} + z_{cap}e \quad \text{Battery Power Injection Feasibility} \tag{5}$$

Where we have :

$$\tilde{S} = \begin{bmatrix} S \\ -S \\ I_T \\ -I_T \end{bmatrix}, \quad \bar{u} = \begin{bmatrix} -z_1 \cdot \mathbb{1}_{T-1} \\ 0 \\ -z_1 \cdot \mathbb{1}_{T-1} \\ 0 \\ u^{max} \cdot \mathbb{1}_T \\ u^{min} \cdot \mathbb{1}_T \end{bmatrix}, \; e = \begin{bmatrix} \mathbb{1}_{T-1} \\ 0 \\ \mathbb{0}_T \\ \mathbb{0}_T \\ \mathbb{0}_T \end{bmatrix}$$

Denote $\mathbf{\Gamma}$ to be a large real scalar that will be used to linearize bi-linear constraints throughout this article. Since the battery cannot sell negative generation at a positive bid price, we introduce a binary variable $r^c \in \{0,1\}^T$ and the three following constraints:

$$\begin{aligned} \forall t \in \mathcal{T}, \qquad & c_t \geq 0 \\ (r_t^c - 1)\mathbf{\Gamma} &\leq q_t \\ c_t &\leq r_t^c \mathbf{\Gamma} \end{aligned} \tag{6}$$

With this formulation if $q_t < 0$ then $c_t = 0$ which removes the battery contribution to the economic dispatch bids. If $q_t \geq 0$, c_t is positively unconstrained.

2) Optimization problem: Let i_b be a selected node to site the battery. Its role as a decision variable will only be introduced when iterating through the nodes of the network. Let $\lambda_t \in \mathbb{R}^n$ be the Local Marginal Prices (LMP) for time period t. They are equivalently defined as the optimal dual variables of the economic dispatch injection constraint (see Prog 10). The LMP are the nodal clearing prices. At each period, the periodic profit $\lambda_t^{i_b} \cdot u_t$ is earned (or paid) where $\lambda_t^{i_b}$ is the LMP of the battery node at time t. Since the battery charging cycles imply dynamic arbitrage, our program will be multi-periodic from $t = 1$ to $t = T$, hence the terminology “bidding schedule”.

Instead of solely optimizing revenues from energy arbitrage, we maximize profit, i.e. revenues minus the cost of the battery. The battery cost will appear as a linear penalty on z_{cap} in the optimization framework and the optimal z_{cap} will appear as a trade-off between the arbitrage revenue it generates and its cost. The unit capacity cost is denoted as b \$/MWh and the battery lifetime, or, investment horizon is y years. Instead of writing the penalty as $b \cdot z_{cap}$, assuming ν trading periods over one year, we normalize the capacity cost to the time horizon T to preserve generality. The normalized cost for z_{cap} is therefore $\frac{T}{\nu y} \cdot b \cdot z_{cap} = B \cdot z_{cap}$. As an extension, a linear penalty on the power capacity could be added in the same way to size storage via both energy and power capacity.

Two programs are now developed: one under the *price-taker* assumption and the other under the *price-maker*’s. Both frameworks will be compared in an offline setup and with perfect foresight in the results’ section (III). The blue color is used hereinafter to highlight optimization variables.

3) Price-taker framework: Under a price-taker framework, the LMP do not depend on the battery bids. They are exogenous. The bidding strategy will be a succession of self-schedule bid volumes sent to the market operator.

$$\begin{aligned} \pi^*(B) = \max_{z_{cap}, u} \quad & \overbrace{\underbrace{\sum_{t=1}^{T} \lambda_t^{i_b} u_t}_{\text{Arbitrage revenue}} - \underbrace{B \cdot z_{cap}}_{\text{Time-normalized capacity cost}}}^{\text{profits}} \\ \text{subject to} \quad & \tilde{S} u \leq \bar{u} + z_{cap} e \end{aligned} \tag{7}$$

However this program has no limit as to how much energy the market can absorb. We thereby show that such a framework cannot yield a finite positive capacity. Let us define π_1^* (resp. u_1^*) as the value (resp. the bidding schedule) of the arbitrage for $z_{cap} = 1$MWh of capacity over the time horizon.

$$\pi_1^* = \max_{u} \sum_{t=1}^{T} \lambda_t^{i_b} u_t, \quad \text{s.t } \tilde{S} u \leq \bar{u} + e$$

Now, Program 7 either yields no capacity or is infeasible depending on the value of B. If $B < \pi_1^*$, then for a given z_{cap}, $u^* = z_{cap} \cdot u_1^*$ is admissible and yields the following profits: $(\pi_1^* - B) \cdot z_{cap}$. To maximize this objective function, we cannot simply increase z_{cap}, therefore there is no upper bound to the objective function and, $\pi^*(B) = +\infty$. Finally, if $B \geq \pi_1^*$, then $\pi_1^* - B < 0$ and $\pi^*(B) = 0$.

Program 7 will be used in the results section for a fixed z_{cap} to estimate the expected arbitrage revenue from a price-taker framework. The unbounded properties of linear program (7) with a price-taker assumption motivates the use of a market-aware program.

4) Price-maker framework: Under the price-maker framework, the LMPs are no longer exogenous. Both the LMPs and the storage power injection are the results of the market clearing operation (or Economic Dispatch) denoted as E.D and introduced in (10). Hence $(\lambda_t, u_t) = \text{E.D}_t(c_t, q_t)$, and the bidding strategy (c, q) is solution of:

$$\begin{aligned} \max_{z_{cap}, c, q, u, \lambda} \quad & \Big(\sum_{t=1}^{T} \lambda_t^{i_b} \cdot u_t\Big) - B \cdot z_{cap} \\ \text{subject to} \quad & \tilde{S} u \leq \bar{u} + z_{cap} e \\ & \forall t \in \mathcal{T}, \; (\lambda_t, u_t) = \text{E.D}_t(c_t, q_t) \end{aligned} \tag{8}$$

Since the E.D is an optimization program, Prog 8 is a bi-level optimization program and, in the following, we will transform it to a single-level optimization program by expressing the KKT conditions of the underlying E.D. This will ultimately lead to program 19.

B. Nodal Wholesale Market

1) Topology of the network and DC Power flow equations: Denote $p_t \in \mathbb{R}^n$ as the nodal power injection vector. The transmission lines $\mathcal{L} = \{1, ..., L\}$, have capacities $\{h_l, l \in \mathcal{L}\}$. To obtain conditions on the feasibility of p_t, we use a linearization of the AC power flow equations known as DC Power Flow equations for computational simplicity. On the one hand, power is balanced over a lossless grid: $1^\top p_t = 0$. On the other, flows over lines are real and linear in the voltage angles. Denoting $H \in \mathbb{R}^{2L,N}$ the shift-factor matrix, line flows can be linked to the nodal power injections p_t to finally obtain the constraint $Hp_t \leq h$. [17]

2) Time Frame of the market: Bids and constraints of market participants are considered in a general multi-periodic setting $\mathcal{T}$. Multiple-time blocks of energy are not considered.

3) Demand: At each time step, $d_t \in \mathbb{R}_+^N$ is assumed given, inelastic, and non-negative.

4) Generators: Let $\mathcal{G} = \{1, ..., G\}$ be the set of generators. Let $g_t \in \mathbb{R}_+^G$ be the generation vector dispatched by the market. More precisely, $g_{t,j}$ is the generation dispatched by the market operator to unit $j \in \mathcal{G}$ and at time step t. A simple Wholesale market with one bidding band per generator and per time-step is considered to simplify notations. At each time period t, each producer sends three values to the operator:

- $P_{t,j}^{min}$, its must-run capacity which is self-scheduled,
- $P_{t,j}^{max}$, the maximum capacity it is willing to offer in the market,
- $a_{t,j}$, the bidding price associated to the volume $P_{t,j}^{max} - P_{t,j}^{min}$

It follows that the cost for the dispatch of g_t is $a_t^\top (g_t - P_t^{min})$. With g_t feasible if and only if : $I_g g_t \leq \bar{g}_t$.

$$I_g = \begin{bmatrix} I_G \\ -I_G \end{bmatrix}; \qquad \bar{g}_t = \begin{bmatrix} P_t^{max} \\ P_t^{min} \end{bmatrix}$$

Since there may be multiple generators at the same node, we denote $M_g \in \mathbb{R}^{N,G}$, the producer-node adjacency matrix. $M_{g_{i,j}} = 1$ if and only if unit j belongs to node i, with $i \in \mathcal{N}$ and $j \in \mathcal{G}$. Otherwise $M_{g_{i,j}} = 0$.

5) Battery : The battery buys on the wholesale market at the clearing price. In that case, the bid volume q_t is negative, therefore u should be negative and act as a supplement of demand. Both buying and selling behaviors are ensured with the following constraint:

$$u_t \leq q_t$$

By construction (see Section II-A1), if $q_t < 0$, $c_t = 0$ and $u_t^* = q_t$ to minimize the objective function. Otherwise, $\epsilon = q_t - u_t^*$ is an extra MWh of demand that has to be met by a generator, resulting in increasing market cost and lost revenue for the battery.

Similarly to the generators, $M_u \in \mathbb{R}^N$ is the battery-node adjacency vector, all its elements are zero except for :

$$M_{u i_b} = 1$$

Note that $M_u^\top$ is a matrix that project a vector onto its value at node i_b. In particular,

$$\lambda_t^{i_b} \cdot u_t = M_u^\top \lambda_t \cdot u_t \tag{9}$$

C. Economic Dispatch

At a time period t, the market operator dispatches the generation g_t and the storage u_t in order to minimize cost for the network while satisfying demand, respecting power balance as well as line and bid constraints. The nodal power injection vector is defined as the nodal difference between what is produced $M_g g_t$ and what is consumed d_t. $M_u u_t$ can either act as a supplement of production or consumption. This leads to $p_t = M_g g_t + M_u u_t - d_t$.

$$\begin{aligned} \min_{p_t, g_t, u_t} \quad & a_t^\top g_t + c_t \cdot u_t \\ \text{subject to} \quad & \gamma : \mathbb{1}_n^\top p_t = 0 \\ & \lambda : p_t = M_g g_t + M_u u_t - d_t \\ & \beta : H p_t \leq h \\ & \sigma_g : I_g g_t \leq \bar{g}_t \\ & \sigma_u : u_t \leq q_t \end{aligned} \tag{10}$$

where the Greek letters to the left of the colons denote corresponding dual variables.

D. Mathematical Program under Equilibrium Constraints

As shown in program (8), dual variable λ and primal variable u are solutions of the Economic Dispatch. In other words, they must satisfy the KKT conditions of the LP program (10). First, the Lagrangian of the Economic Dispatch and its dual function are devised, then a linearization of the KKT conditions and of the objective function are developed.

1) Lagrangian and dual function: The Lagrangian of the problem is denoted by $\mathfrak{L}(p_t, g_t, u_t, \omega)$ where $\omega = (\gamma, \lambda, \beta, \sigma^g, \sigma^u)$ belongs to the set of dual variables $\Omega = \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}_+^{2L} \times \mathbb{R}_+^G \times \mathbb{R}_+$. For clarity, since the time period t is fixed, the dependency of the optimization variables in t is dropped. However, the dependency in t of the exogenous input is kept.

$$\min_{p,g,u} \max_{\omega \in \Omega} \quad \mathfrak{L}(p, g, u, \omega)$$

$$\begin{aligned} \mathfrak{L}(p, g, u, \omega) &= a_t^\top g + c_t \cdot u - \gamma \mathbb{1}_n^\top p + \beta^\top (Hp - h) \\ &+ \lambda^\top (d_t + p - M_g g - M_u u) + \sigma_g^\top (I_g g - \bar{g}_t) + \sigma_u (u - q_t) \\ &= (a_t^\top - \lambda^\top M_g + \sigma_g^\top I_g) g + (c_t - \lambda^\top M_u + \sigma_u) u \\ &+ (-\gamma \mathbb{1}_n^\top + \beta^\top H + \lambda^\top) p - \beta^\top h + \lambda^\top d_t - \sigma_g^\top \bar{g}_t - \sigma_u q_t \end{aligned}$$

We define $\mathcal{A}$ s.t :

$$\mathcal{A} = \left\{ \omega \in \Omega, \quad \begin{aligned} a_t^\top - \lambda^\top M_g + \sigma_g^\top I_g \geq 0 \\ c_t - \lambda^\top M_u + \sigma_u \geq 0 \\ -\gamma \mathbb{1}_n^\top + \beta^\top H + \lambda^\top \geq 0 \end{aligned} \right\}$$

Then, the dual problem can be devised under a closed-form expression:

$$\mathfrak{g}(\omega) = \min_{p,g,u} \mathcal{L}(p, g, u, \omega)$$

$$\mathfrak{g}(\omega) = \begin{cases} \lambda^\top d_t - \beta^\top h - \sigma_g^\top \bar{g}_t - \sigma_u q_t & \text{if } \omega \in \mathcal{A} \\ -\infty \quad \text{otherwise} \end{cases}$$

Since the problem is a linear optimization problem, it is convex. Furthermore, since the economic dispatch has a feasible solution when battery bids are null, there exists a feasible point into the set. Therefore, Slater's conditions are satisfied, and strong duality holds:

$$a_t^\top g + c_t u = d_t^\top \lambda - h^\top \beta - \bar{g}_t^\top \sigma_g - q_t \sigma_u \tag{11}$$

This equality will be used later to linearize the objective function.

2) KKT Conditions for the Economic Dispatch: In this section, a set of constraints that form domain $\mathcal{C}$ is devised such that if there exists $(p, g, u, \omega) \in \mathcal{C}$ then $(\lambda, u) = \text{E.D}_t(c_t, q_t)$. If (p, g, u, ω) satisfy the KKT conditions, then they are optimal and verify:

- **Stationary conditions** for the Lagrangian

$$\nabla_p \mathfrak{L} = -\gamma \mathbb{1}_n + H^\top \beta + \lambda = 0 \tag{12}$$
$$\nabla_g \mathfrak{L} = a_t - M_g^\top \lambda + I_g^\top \sigma_g = 0 \tag{13}$$
$$\nabla_u \mathfrak{L} = c_t - M_u^\top \lambda + \sigma_u = 0 \tag{14}$$

- **Complementary Slackness conditions**

$$\beta^\top (H p_t - h) = 0 \tag{15}$$
$$\sigma_g^\top (I_g g - \bar{g}_t) = 0 \tag{16}$$
$$\sigma_u (u - q_t) = 0 \tag{17}$$

- **Primal Constraints**
  All the other constraints present in program (10) must be satisfied at the optimum.

E. Linearization of the program

The KKT conditions cannot be appended as such into our battery program (8). We explain and resolve this issue next.

a) Slack constraints: The 3 equations of complementary slackness are not affine but can be linearized using integers following Fortuny-Amat and McCarl linearization [18]. The idea is to introduce a discrete variable whose value will either set the dual variable or the inequality constraint to zero. A new set of integer variables is therefore introduced: $r^\beta \in \{0,1\}^{2L}, r^{\sigma_g} \in \{0,1\}^{2G}, r^{\sigma_u} \in \{0,1\}$. $\mathbf{\Gamma}$ is set to be sufficiently large. If there exists a set of variables satisfying the following linear constraints, then complementary slackness conditions are satisfied.

$$\begin{aligned} &\beta \leq (1 - r^\beta)\mathbf{\Gamma} \\ &H p_t - h \leq r^\beta \mathbf{\Gamma} \\ &\sigma_g \leq (1 - r^{\sigma_g})\mathbf{\Gamma} \\ &(I_g g - \bar{g}_t) \leq r^{\sigma_g} \mathbf{\Gamma} \\ &\sigma_u \leq (1 - r^{\sigma_u})\mathbf{\Gamma} \\ &(u - q_t) \leq r^{\sigma_u} \mathbf{\Gamma} \end{aligned} \tag{18}$$

b) Objective function: The objective function of the battery program 8 is not linear because of the cross-product $\lambda_t^{i_b} \cdot u_t$. For t fixed, we transform each bi-linear term following Section 6.4.3.1 of [18].

First, we multiply (14) by u :

$$M_u^\top \lambda u = c_t u + \sigma_u u$$

Then from the complementary slackness condition (17):

$$\sigma_u u = \sigma_u q_t$$

Finally from strong duality (11) :

$$c_t u = \lambda^\top d_t - \sigma_u q_t - \left(a_t^\top g + \beta^\top h + \sigma_g^\top \bar{g}_t\right)$$

The bi-linear terms in the objective function can be rewritten as a linear function in the variables $g, \beta, \lambda, \sigma_g$.

$$\begin{aligned} \lambda^{i_b} \cdot u &= M_u^\top \lambda u && \text{(from (9))} \\ &= c_t u + \sigma_u u && \text{(from (14))} \\ &= c_t u + \sigma_u q_t && \text{(from (17))} \\ &= \lambda^\top d_t - \left(a_t^\top g + \beta^\top h + \sigma_g^\top \bar{g}_t\right) && \text{(from (11))} \\ &= d_t^\top \lambda - \left(a_t^\top g + h^\top \beta + \bar{g}_t^\top \sigma_g\right) \end{aligned}$$

Applying this transformation to every bi-linear term in the sum of the objective function of program 8 will yield the objective function in program 19.

c) Illustration of the objective function linearization for a single-node wholesale market: The above linearization can be understood as a geometrical transformation as highlighted in Fig. (2), where we wish to express the red rectangle as a combination of rectangles in the optimization variables. To illustrate it, a single node wholesale market is considered with no self-scheduled bids ($P_{min} = 0$). The one-time profit made by the battery is:

$$\lambda \cdot u = \underbrace{\lambda \cdot d}_{\text{paid by the loads}} - \underbrace{\sum_{j \in \mathcal{G}} \lambda \cdot g_j}_{\text{paid to the other agents}}$$

The stationary equation (13) multiplied by the generator output g_j yields:

$$\lambda g_j = c_j g_j + \sigma_j g_j, \; j \in \mathcal{G}$$

This corresponds to the splitting of the generator profit rectangle into two sub-rectangles : its surplus (above its bidding price) and its costs (below its bidding price).

Finally, the slack constraints impose that $\sigma_j^{max} g_j = \sigma_j^{max} P_j^{max}$. As a result, the objective function is linear :

$$\lambda \cdot u = \underbrace{\lambda \cdot d}_{\text{paid by loads}} - \sum_{j \in \mathcal{G}} \underbrace{c_j g_j}_{\text{cost of generator } j} + \underbrace{\sigma_j P_{max,j}}_{\text{surplus of generator } j}$$

F. MILP program

Finally, the subscripts relative to the time period are reintroduced. By putting all the constraints together, a multi-period price-maker mixed-integer linear program for battery profit maximization is obtained. This formulation is valid for a given i_b. Iterating the program over $\mathcal{N}$ finally yields

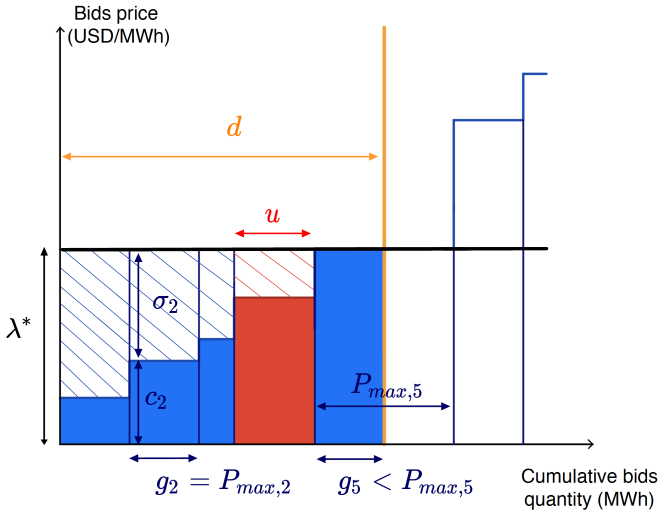

Fig. 2: Linearization of objective function for single-node wholesale market

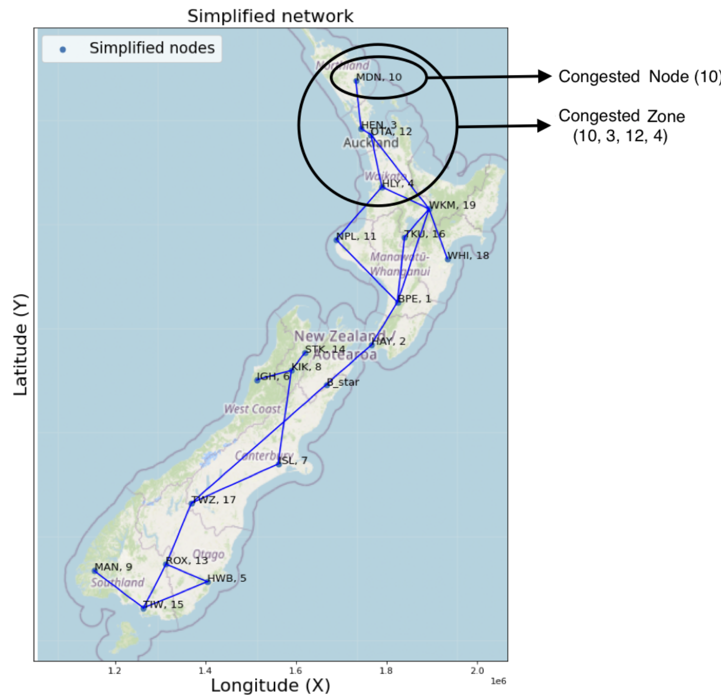

Fig. 3: Simplified New Zealand power grid

the optimal site i_b^*, its associated capacity z^{cap} and bidding schedule (c_t, q_t).

$$
\begin{aligned}
\max \Big(\sum_{t=1}^{T} d_t^\top \lambda_t - (a_t^\top g_t + h^\top \beta_t \; + \bar{g}_t^\top \sigma_t^g) \Big) - B z_{cap} & \\
\text{s.t.} \qquad (5) \qquad\qquad\qquad & \text{Battery Feasibility} \\
\forall t \in \mathcal{T}, (6), (10) \qquad\qquad & \text{E.D. Feasibility} \\
(12), (13), (14), (18) & \text{ KKT conditions}
\end{aligned} \tag{19}
$$

III. Results & Discussion

A. Case Study: New Zealand

New Zealand's Energy Market (NZEM) offers a relevant application to compare the price-maker and price-taker approaches as the clearing economic dispatch algorithm is a nodal wholesale market using DC powerflow equations [19]. The Electrical Authority gives open access to a number of relevant data resources regarding the NZEM, including loads, bids and network characteristics. Note that all data, code and graphs presented in this section can be found at [20].

B. Scenario

The optimization program should be run over at least a year worth of data to yield a battery site consistent with seasonal patterns of the power flow. However, we arbitrarily decided to work on the 2nd day of September 2019 in order to illustrate how the different constraints affect the optimal siting and bidding schedule. On this day, 80 generators offered bids on the market. While we based our experiments on real data, we made some assumptions to simplify the bidding data and topology, to favor congestion over the network while ensuring the plausibility of the LMP and to prevent infeasible situations.

- **Topology:** The NZ network is comprised of 258 nodes. With most of the nodes being relatively minor, we aggregate these nodes in 19 main nodes and we connect them with transmission lines forming the "backbone" of the transmission system as per [21]. A swing node (B^*, 0) is set between the two islands.
- **Loads:** The load curves of the simplified network come from aggregating demand at the interior nodes. We multiplied the load at node 10 by 12, so that for some hours, the demand exceeds the capacity of the single line connecting nodes 10 to 3 (see Fig 3, North end). We applied a factor of 1.3 to the rest of the loads to favor congestion and to prevent infeasible formulation due to high self-schedule volumes.
- **Bids:** For each generator, P_{min} was taken as the volume bid at \$0 and P_{max} as the maximum of the cumulative volume offered on the market. Finally a was set to the weighted average of the bidding prices for bids over \$0. However, with these bids the simulated LMPs are unrealistic and do not match the actual price data. We adjust the bidding prices a so that the simulated marginal cost overfit the realized marginal cost. We also added a swing generator at node 10 with specified bidding prices such that the simulated congestion charge at this node fits the realized values when congestion occurs.
- **Marginal prices and congestion charge:** Figure 4 displays γ, the marginal cost of the system. The congestion charge is reported at a node as the difference between its LMP λ, and γ.

C. Results over the congested zone

The power capacity is set so that the battery can fully discharge in an hour, i.e. we consider a "1-hour battery".

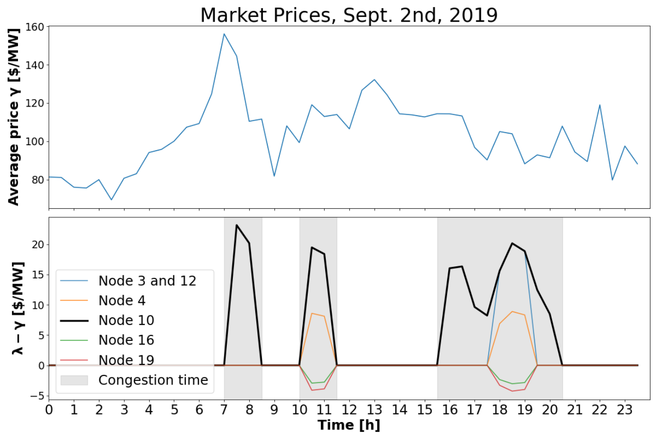

Fig. 4: Gamma prices and congestion charge for the scenario.

Varying this parameter can significantly impact the results as increasing power capacity relaxes the feasible set and thus delivers a better arbitrage for the same installed capacity. Note that we could introduce a linear penalty in the power capacity to obtain the optimal parameter. The normalized unit capacity penalty is set to 180$/MWh, i.e. each MWh has to yield a profit of more than 180$ over the scenario. Considering a current capacity cost of around 600$/kWh for a 1-hour duration battery [22], this penalty corresponds to a roughly 9 year-investment period. Selecting a lower penalty leads to abnormally large capacity because of the existence of a significant arbitrage with perfect foresight. Moreover, it is inconsistent with the current 7-10 year lifetime of Li-ion battery storage [23]. A larger penalty would prevent any investment in the battery.

We implemented the price-maker program (PM) following (19) and report on optimal capacity and profits for different sittings. A price-taker program (PT) following (7), returns an optimal bidding schedule and hence, profits and revenue. A summary of these results are presented on Table I. We can immediately notice that the PT program results in negative profits for nodes 3, 12 and 4, and smaller profits for node 10. We discuss this further in detail at section III-D. The nodes outside the congested area yield a null capacity by design of the penalty and are not reported in the table. That is, the revenue for 1MWh of energy capacity on those nodes is worth \$173 which is smaller its associated cost of \$180. The unit revenues are reported on the third row of the table and would be the measure chosen to install a battery regarding PT. We note that node 10 would be chosen to install a 35MWh battery by PM. Node 10 would also be chosen by PT, only with no information on its size. PT would next select nodes 4 instead of 3 (or 12) because of higher unit revenue. Interestingly, PM informs us that node 4 would yield a smaller revenue as node 4 must be less congested. Finally, we notice a significant offset between actual and expected revenue for PT, illustrating the drawbacks of the price-taker assumption in congested networks.

	Battery Node	**3 or 12**	**4**	**10**
	Unit Profit (z_{cap} = 1MWh) (in $)	17	22	39
PM	Energy capacity $\mathbf{z^*_{cap}}$ (in MWh)	45.5	16.4	35.0
	Profits (Revenue-Cost) (in $)	575.7	63.6	**769.6**

TABLE I: Optimal storage sizes, revenues, and profits on congested nodes and for unit capacity cost of B = 180\$/MWh. PM = price-maker, PT = price-taker.

D. Results over Node 10

Given a 100MWh/100MW battery storage, we plot in Fig 5 the revenues (PT vs PM), the difference between the Baseline LMP (i.e. without battery) λ_{BL} and the LMP with a price-taker battery λ_{PT}, the losses incurred to the PT agent due to this price difference, and finally the injection schedule for both programs. We first note that the revenues for PT are inferior to PM.

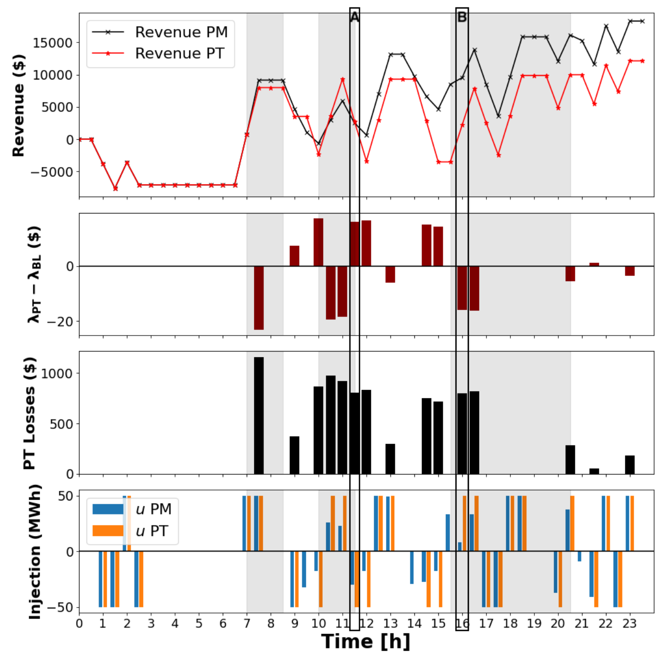

Fig. 5: PM vs PT comparison for accumulated profits, PT-BL difference in price and power injection. The PT assumption can unexpectedly collapse prices or create congestion, which produces less actual revenues than expected.

Looking across the subplots, the PT Losses defined as the benefits for prices fixed to λ_{BL} minus the actual benefits are positive when injecting $u > 0$ (resp. withdrawing $u < 0$) power to the grid when resolving (resp. creating) a congestion event. By providing too much power (e.g. box B at 04:00 p.m., 4th subplot), the battery loses market power as the congestion is solved. The LMP falls to meet the system marginal price (2nd subplot). It sells at a low price, hence incurring large losses (3rd subplot). By withdrawing too much power (e.g. box A at 11:30 a.m.), the battery creates a congestion, the LMP increases (2nd subplot). It buys at a high price, hence incurring large losses (3rd subplot). While this happens for a relatively large capacity (accounting for

20% of the congested node load but only *2%* of the total load) we show that in our scenario, prices react rapidly to even small additions of storage capacity.

In Fig 6, we plot the average of the absolute difference of the PT and PM LMP at node 10 λ_{10}^{PT}, λ_{10}^{PM} and the baseline LMP λ_{10}^{BL}, as a function of installed capacity. We notice that the price-taker assumption is invalidated even for a small capacity. We deduce from this figure that as soon as a congestion event occurs on node 10, a price-taker assumption cannot hold.

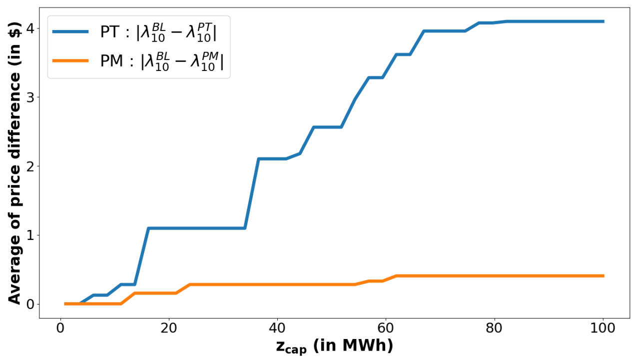

Fig. 6: Average absolute difference between the PM and PT LMPs with the BL LMP, as a function of z_{cap}. A non-zero value indicates the presence of storage bids have altered the clearing price.

IV. Conclusion

In this article, we developed a mixed-integer linear program that sites storage on a transmission network and maximizes profits within a nodal wholesale market where the cleared price and volumes are endogenous to the program. This is unique from previous work that assume cleared prices are exogeneous (the "price-taker" assumption) and/or do not consider optimal siting. To express the market constraints, we formulated and then exploited the KKT conditions of a Nodal Economic Dispatch and linearized the slack constraints. This results in adding a set of auxiliary binary variables. Moreover, the objective function is reformulated using strong duality.

Finally, we tested our program against a simpler price-taker algorithm and on the New Zealand Energy Market (NZEM) and transmission network. On a congested grid, we showed that even a small-scale battery can become a price-maker as soon as it does not take its influence on the market into consideration. On the contrary, the price-maker program generally does not always alleviate congestion since this does not necessarily benefit its price-maker position over the congested area. We believe this market-aware approach to be new and paramount to correctly size a battery over a nodal wholesale market. While these results hold for our simulation and for a specific scenario of congestion, further work should look into characterizing the influence of a battery bidding strategy on the clearing prices and on the consumer and producer surplus of the market. If even a small battery can use its price-maker position over a congested area to benefit from high-price, this anti-competitive behavior when operated by larger-scale batteries could impact social welfare.

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