Quantitative Portfolio Management

Assignment #7

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Instructions for each assignment . . . I

- Assignment #1 should be done individually.
- The other assignments are to be done in groups of 4 or 5 students.
  - This means that groups of 1, 2, 3, 6, etc. are not allowed.
  - Diversity in groups is strongly encouraged
    (people from different countries, different genders, different finance knowledge, and different coding ability, etc.)

Instructions for each assignment . . . II

- Each assignment should be emailed as a Jupyter file
  - To `Raman.Uppal@edhec.edu`
  - The subject line of the email should be: "QPM: Assignment n," where $n = \{1, 2, \ldots, 8\}$.
  - Assignment n is due before Lecture n, where $n = \{1, 2, \ldots, 8\}$.
  - Assignments submitted late will not be accepted (grade $= 0$), so please do not email me assignments after the deadline.

Instructions for each assignment . . . III

- The Jupyter file should include the following (use Markdown):
  - Section “0” with information about your submission:
    - Line 1: QPM: Assignment n
    - Line 2: Group members: listed alphabetically by last name, where the last name is written in CAPITAL letters
    - Line 3: Any comments/challenges about the assignment
  - Section “k” where $k = \{1, 2, \ldots\}$.
    - First type Question k of Assignment n.
    - Then, below the question, provide your answer.
    - Your code should include any packages that need to be imported.

Data for Assignment 7

- In this question, we apply the parametric portfolio policies developed by Brandt, Santa-Clara, and Valkanov (2009).
- The data file for this assignment has monthly returns for nine firm-specific characteristics: Market, SMB, HML, RMW, CMA, UMD, ROE, IA, BAB.
- Assume that these returns were generated by $N_t = 2000$ stocks and that the number of stocks is constant over time.
- The first five characteristics (Market, SMB, HML, RMW, CMA) are from Fama and French (2015), the sixth (UMD) is from Carhart (1997), the profitability (ROE) and investment (IA) factors are from Hou, Xue, and Zhang (2015), and the betting-against-beta (BAB) factor is from Frazzini and Pedersen (2014).
- All factors are returns in excess of the risk-free rate.
  - In particular, every factor (besides MKT and BAB) is the return of a long-short portfolio of stocks with $1 in the long leg and $1 in the short leg, and thus, their returns equal their excess returns.
  - The MKT and BAB factors are also long-short portfolios because they are returns in excess of the risk-free rate.

Questions for Assignment 7

Q7.1 Explain why one might expect these nine factors to be related to stock returns. Write only a few sentences (2 or 3 sentences) for each factor. (Feel free to use ChatGPT, but reading the original paper would be much more educational.)

Q7.2 Find the optimal θ vector (of dimension 9×1) for a mean-variance investor with risk aversion of $\gamma = 5$ if the investor can invest in only these nine factors. Use the entire dataset to estimate the nine factors' mean and covariance of returns (i.e., we do not need to do out-of-sample analysis).

Q7.3 Find the Sharpe ratio for each of the nine factors and compare it to that of the parametric portfolio you have identified in the previous question.

Q7.4 Having obtained the optimal θ vector, please explain in words how one would obtain the optimal portfolio weights for each of the $N_t = 2000$ assets that are used to form each of the nine factors.

Discussion of Assignment 7: Initial setup and data download

Code to import libraries and download data

```
import numpy as np
from scipy.optimize import minimize
import pandas as pd

# To format numbers in pandas dataframes set up format for entire file
pd.options.display.float_format = '{:,.4f}'.format

# Import data
data = pd.read_excel("QPM-FactorsData.xlsx")

# Transform dates into datetime
data['Dates'] = pd.to_datetime(data['Dates'], format = '%Y%m')

# Set dates as the dataframe index
data.set_index('Dates', inplace = True)
data                                  # should get 647 rows x 9 columns
```

Notation used in the code

- I will solve the problem in more than one way.
- To keep track of which approach is being discussed, it is useful to have good notation.
- The notation I use is listed below.

```
mv_pp_unc = mean-variance parametric-portfolio unconstrained
mv_pp_con = mean-variance parametric-portfolio constrained

w_mv_pp_unc = weights of mean-variance parametric-portfolio unconstrained
w_mv_pp_con = weights of mean-variance parametric-portfolio constrained

ret_mv_pp_unc = return of mean-variance parametric-portfolio unconstrained
ret_mv_pp_con = return of mean-variance parametric-portfolio constrained

sr_mv_pp_unc = Sharpe ratio of mean-variance parametric portfolio unconstrained
sr_mv_pp_con = Sharpe ratio of mean-variance parametric portfolio constrained
```

Discussion of Assignment: Q7.1 . . . I

Q7.1 Explain why one might expect these nine factors to be related to stock returns. Write only a few sentences (2 or 3 sentences) for each factor. (Feel free to use ChatGPT, but reading the original paper would be much more educational.)

1. **Market:** Exposure to the market factor, measured by β, captures the systematic risk of an asset. Investors expect assets with higher betas to pay higher returns, which is consistent with the CAPM.

Discussion of Assignment: Q7.1 ...II

2. **Size** The size effect is a phenomenon where smaller companies, as measured by market capitalization, tend to outperform larger companies. The reasons why smaller firms might exhibit higher returns are:
   - *Risk premium:* Smaller companies are often considered riskier investments because they may be more vulnerable to economic downturns, have less diversified revenue streams, and may face greater challenges in accessing capital.
   - *Market inefficiencies:* Smaller companies may be followed by fewer analysts and institutional investors, leading to less efficient pricing.
   - *Growth potential:* Smaller companies may have more room for growth compared to their larger counterparts.

Discussion of Assignment: Q7.1 . . . III

3. **Value (HML):** High Minus Low (HML) is a *value* premium; it represents the spread in returns between companies with a high book-to-market value ratio and companies with a low book-to-market value ratio.

   "Value" stocks with a high book-to-market ratio may have higher returns because of the following reasons:

   - *Market inefficiencies:* Markets may not always efficiently price stocks, so value stocks are those that are underpriced.
   - *Investor behavior:* During periods of market pessimism or economic downturns, investors may seek out value stocks that are perceived to be undervalued, leading to a potential premium for value stocks.
   - *Market sentiment:* The value factor is often associated with companies that have solid fundamentals but are temporarily undervalued due to investor sentiment.
   - *Dividends and income:* Value stocks often have higher dividend yields, providing investors with a potential income stream, which is valued in times of uncertainty or economic downturns.

Discussion of Assignment: Q7.1 . . . IV

4. **Profitability (RMW):** The robust-minus-weak profitability factor suggests that firms with higher operating profitability will have higher returns in the future. Also, during bad times, investors value firms with high profitability.
5. **Investment (CMA):** The conservative-minus-aggressive investment factor relates to the company's internal investment.
   - According to this factor, companies that invest aggressively in growth projects are likely to underperform in the future because aggressive firms may take on more risk, use higher leverage, and pursue more speculative investments.
   - On the other hand, conservative firms may prioritize financial stability and have lower leverage.

Discussion of Assignment: Q7.1 . . . V

6. **Momentum (UMD):** The momentum factor is grounded in the belief that stocks that have recently performed well will continue to do so, and stocks that have recently performed poorly will continue to underperform. Some reasons why momentum factor may be related to stock returns are:

    - *Investor herding*; i.e., when positive news is observed in a stock, investors may rush to buy, leading to further price appreciation. Similarly, when negative news is released, investors may sell, contributing to further price declines. This herding behavior can create self-reinforcing trends in stock prices.
    - *Information diffusion* to new information may be slow. So, as new positive information is gradually disseminated and incorporated into stock prices, momentum can build.
    - *Underreaction* by investors to positive news can lead to momentum because the stock's price continues to adjust over time.
    - *Trend-following strategies* can lead to momentum.

Discussion of Assignment: Q7.1 . . . VI

7. **ROE** is a factor from Hou, Xue, and Zhang (2015), and is the difference between the return on a portfolio of high profitability (return on equity, ROE) stocks and the return on a portfolio of low profitability stocks.
8. **IA** is a factor from Hou, Xue, and Zhang (2015), and is the difference between the return on a portfolio of low investment stocks and the return on a portfolio of high investment stocks.

Discussion of Assignment: Q7.1 . . . VII

9. **Betting-against-beta: BAB** The BAB factor is based on the empirical observation that low-beta assets tend to earn higher risk-adjusted returns than predicted by traditional asset pricing models like the CAPM.
   - One reason for this may be that low-beta assets, usually stable and less volatile stocks, are more liquid.
   - Thus, investors seeking liquidity or stability may favor low-beta assets, contributing to their outperformance.

Discussion of Assignment: Q7.2 . . . I

Q7.2 Find the optimal θ vector (of dimension 9×1) for a mean-variance investor with risk aversion of $\gamma = 5$ if the investor can invest in only these nine factors. Use the entire dataset to estimate the nine factors' mean and covariance of returns (i.e., we do not need to do out-of-sample analysis).

- There are several ways of thinking about this question,
  - which are outlined on the next page, and
  - then demonstrated on the pages that follow.

Discussion of Assignment: Q7.2 . . . II

1. One approach is to choose the portfolio weights without constraining the weight on the market to be equal to one.
   1.1 This would be appropriate if you were an investor wanting to invest in the nine assets and wanted to find out what weight to assign to each of the nine assets.

2. Another approach is to view the market factor as the benchmark, and then to ask whether the investor can improve her portfolio by investing also in the other assets. In this setting, there are two ways to solve the problem, both of which deliver the same answer:
   2.1 Solve a portfolio choice problem with constraints, constraining the weight on the market, $\theta_{\text{mkt}} = 1$, and optimize over the weights on the other eight factors.
   2.2 A cleverer approach is to choose only the weights on the eight factors other than the market (not shown in these notes).

Code for Q7.2: First approach

- We now choose the weights on the nine factors without constraining the weight on the market to equal one.

Computing the factor weights (no constraints)

```
# Preliminary step to import libraries and data is given earlier

# Function to find the mean-variance parametric-portfolio unconstrained
    weights
def mv_pp_unc(mu, Sigma, gamma):

    # compute the inverse of the var-cov matrix
    Sigmainv = np.linalg.inv(Sigma)

    # portfolio weights:
    w = (1/gamma) * Sigmainv @ mu

    return w
```

Code for Q7.2: First approach (continued)

Code to compute the portfolio weights (unconstrained)

```
# Applying the function to find the mean-variance unconstrained weights

# Mean of factor returns
mu = data.mean()

# Variance-covariance matrix of factor returns
Sigma = data.cov()

# Define gamma
gamma = 5

# Compute the parametric portfolio weights unconstrained
w_mv_pp_unc = mv_pp_unc(mu,Sigma,gamma)

# Put the weights in a dataframe
w_mv_pp_unc = pd.Series(w_mv_pp_unc, index=data.columns)
w_mv_pp_unc_df = pd.DataFrame({"w_mv_pp_unc": w_mv_pp_unc})
w_mv_pp_unc_df
```

Output from Code for Q7.2: First approach

	w_mv_pp_unc
Market	1.1062
SMB	0.9558
HML	-0.1550
RMW	0.2070
CMA	0.1534
UMD	0.2162
ROE	1.7734
IA	2.7261
BAB	0.7783

Now we look at the second approach.

In this approach we constrain the weight on the market factor: $\theta_{\mathsf{mkt}} = 1$.

Code for Q7.2: Second approach with $\theta_{\text{mkt}} = 1$

Code to solve mean-variance portfolio problem with a constraint

```
# In the code below, we constrain the weight on the market factor to
     equal 1, while all the other weights are unconstrained

from scipy.optimize import minimize

def mv_pp_con(n, mu, Sigma, gam):
    A = np.ones((n))
    b = 1
    lb = np.append(1,np.repeat(-9999,n-1))

    ub = np.append(1,np.repeat(9999,n-1))

    H = gam * Sigma

    objective = lambda x: 0.5 * x.T.dot(H).dot(x) - mu.T.dot(x)

    result = minimize(objective, x0=np.ones(n) / n,
                      bounds=list(zip(lb, ub)))
    return result.x
```

Code for Q7.2: Second approach with $\theta_{\mathsf{mkt}} = 1$ (continued)

Code to find the mean-variance weights with a constraint

```
# Use the minimization function to solve the constrained optimization
    problem

# number of factors
n = data.shape[1]

# convert mu into an np object
mu = np.array(mu)

w_mv_pp_con = mv_pp_con(n,mu,Sigma,gamma)

# Put the weights in a dataframe, along with the unconstrained weights
w_mv_pp_con = pd.Series(w_mv_pp_con, index=data.columns)
w_mv_pp_con_df = pd.DataFrame({"w_mv_pp_unc": w_mv_pp_unc,"w_mv_pp_con":
     w_mv_pp_con})
w_mv_pp_con_df
```

Output for Q7.2: Second approach with $\theta_{\text{mkt}} = 1$

	w_mv_pp_unc	w_mv_pp_con
Market	1.1062	1.0000
SMB	0.9558	0.9810
HML	-0.1550	-0.1546
RMW	0.2070	0.1783
CMA	0.1534	0.0351
UMD	0.2162	0.2011
ROE	1.7734	1.7560
IA	2.7261	2.7487
BAB	0.7783	0.7981

- From the above, we see that
  - in the unconstrained solution, the weight on the market factor is close to one (it is 1.1062)
  - thus, when we constrain the weight on the market to be exactly one, the other weights do not change much (except the weight for CMA).

Having found the weights, we now compute the Sharpe ratios

Discussion of Assignment: Q7.3

Q7.3 Find the Sharpe ratio for each of the nine factors and compare it to that of the parametric portfolios.

Code for function to compute the annualized Sharpe ratio

```
# Define a function to compute annualized Sharpe ratio
def SharpeRatio(ret,rf = 0):
    Mu = np.mean(ret-rf) * 12
    std = ret.std() * np.sqrt(12)
    return Mu, std, Mu/std        # note this function returns
     three quantities

# Set up a dataframe for reporting the Sharpe ratios
sr_factors = pd.DataFrame(index=data.columns,
                          columns=['E[R]', 'Volatility', 'SR'])
```

Discussion of Assignment: Q7.3 (continued)

- We now compute the Sharpe ratios for the individual factors.
- We also compute the Sharpe ratios of the constrained and unconstrained parametric portfolios.

Code to compute the annualized Sharpe ratio

```
# Compute the Sharpe ratio for each factor (which is in columns)
for k in data.columns:
    ret_k = data[k]
    Mu, std, sharpe = SharpeRatio(ret_k, rf=0)
    sr_factors.loc[k] = [Mu, std, sharpe]

# Compute the Sharpe ratio of the unconstrained parametric portfolio
ret_mv_pp_unc = data @ w_mv_pp_unc
sr_mv_pp_unc = SharpeRatio(ret_mv_pp_unc)

# Compute the Sharpe ratio of the constrained parametric portfolio
ret_mv_pp_con = data @ w_mv_pp_con
sr_mv_pp_con = SharpeRatio(ret_mv_pp_con)
```

Output for Q7.3: Sharpe ratios

Code to produce dataframe with all the Sharpe ratios

```
# To compare the Sharpe ratios, add to list of sr_factors
sr_factors.loc[len(sr_factors)] = sr_mv_pp_unc
sr_factors.loc[len(sr_factors)] = sr_mv_pp_con
sr_factors = sr_factors.rename(index={9: 'sr_mv_pp_unc'})
sr_factors = sr_factors.rename(index={10: 'sr_mv_pp_con'})
sr_factors
```

Conclusion from analysis of Sharpe ratios

	E[R]	Volatility	SR p.a.
Market	0.0684	0.1579	0.4329
SMB	0.0212	0.1070	0.1979
HML	0.0283	0.1017	0.2779
RMW	0.0320	0.0761	0.4207
CMA	0.0332	0.0694	0.4779
UMD	0.0754	0.1486	0.5073
ROE	0.0608	0.0888	0.6843
IA	0.0409	0.0653	0.6264
BAB	0.1047	0.1164	0.8996
sr_mv_pp_unc	0.4203	0.2899	1.4497
sr_mv_pp_con	0.4095	0.2828	1.4483

- From the table above that reports the Sharpe ratios:
  - We see that the Sharpe ratio for the market factor is 0.4329;
  - Several other factors, have even higher Sharpe ratios;
  - But, the Sharpe ratios of both the unconstrained and constrained parametric portfolios are even higher, about 1.44 per year.

Discussion of Assignment: Q7.4 . . . I

Q7.4 Having obtained the optimal θ vector, please explain in words how one would obtain the optimal portfolio weights for each of the $N_t = 2000$ assets that are used to form each of the nine factors.

- Note that a parametric portfolio consists of investment in K factors;
  - specifically, θ_k is the weight on factor k, with $k = \{1, \ldots, K\}$;
- Each factor k itself consists of investments in N_t assets;
  - specifically, $w_{k,n}$ is the weight in factor k of asset n;
- Thus, the investor's holding of asset n through factor k is $\theta_k \times w_{k,n}$
- Thus, the overall weight on asset n through all the factors is

$$w_n = \sum_{k=1}^{K} \theta_k \times w_{k,n}.$$

Discussion of Assignment: Q7.4 ...II

- We can illustrate this as follows:

$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ \vdots \\ w_N \end{bmatrix}}_{\substack{\text{overall weight} \\ \text{on } N \text{ assets}}} = \theta_1 \underbrace{\begin{bmatrix} w_{11} \\ w_{12} \\ \vdots \\ w_{1n} \\ \vdots \\ w_{1N} \end{bmatrix}}_{\substack{\text{weight on } N \text{ assets} \\ \text{in 1st factor}}} + \theta_2 \underbrace{\begin{bmatrix} w_{21} \\ w_{22} \\ \vdots \\ w_{2n} \\ \vdots \\ w_{2N} \end{bmatrix}}_{\substack{\text{weight on } N \text{ assets} \\ \text{in 2nd factor}}} + \ldots + \theta_K \underbrace{\begin{bmatrix} w_{K1} \\ w_{K2} \\ \vdots \\ w_{Kn} \\ \vdots \\ w_{KN} \end{bmatrix}}_{\substack{\text{weight on } N \text{ assets} \\ \text{in } K\text{th factor}}} .$$

Bibliography

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End of assignment