1 Quantities

- $J \in \mathbb{Z}^+$ = number of scale decomposition
- $T = 2^J$ = number of time periods
- $N \in \mathbb{Z}^+$ = number of cross-section elements

2 Multivariate Locally Stationary Wavelet process (Park et al. (2014))

The vector $(N \times 1)$ of stochastic processes $\boldsymbol{X_{t;T}}$ follows the given decomposition : $\boldsymbol{X_{t;T}} = \sum_{j=-J}^{-1} \sum_{k=0}^{T} \boldsymbol{W_j\left(k/T\right)} \boldsymbol{\xi_{j,k}} \psi_{j,k}(t)$ where

- $\boldsymbol{W_j(z)}$ is a $(N \times N)$ matrix.
  For each (m,n)-element,
  - $W_j^{(m,n)}(z)$ is a Lipschitz continuous function on $z \in (0,1)$.
  - $\sum_{j=-\infty}^{-1} \left| W_j^{(m,n)}(z) \right|^2 < \infty$ (finite energy).
  - $\sum_{j=-\infty}^{-1} 2^{-j} L_j < \infty$ (uniformly bounded Lipschitz constants L_j).
- $\boldsymbol{\xi_{j,k}}$ is the vector $(N \times 1)$ of random orthonormal increments.
  - $Cov(\xi_{j,k}^{(u)}, \xi_{j',k'}^{(u')}) = \delta_{j,j'} \delta_{k,k'} \delta_{u,u'}$ ($\delta_{j,j'}$ is the Kronecker delta).
- $\psi_{j,k}(t)$ is a scalar representing a non-decimated wavelet.

We can define the *cross-Evoluationary Wavelet Spectrum* $(N \times N)$ matrix : $\boldsymbol{S_j(z)} = \boldsymbol{W_j(z)} \boldsymbol{W_j(z)'}$. This gives us the ability to express the *local autocovariance* : $c^{(u,u')}(z,\tau) = \sum_{j=-\infty}^{-1} S_j^{(u,u')}(z) \boldsymbol{\Psi}_j(\tau)$ where $\boldsymbol{\Psi}_j(\tau) =$, the *autocorrelation wavelet*.

3 Notes

- $\boldsymbol{W_j(z)}$ is assumed to have a lower-triangular form in Park et al. (2014) to ease interpretation. However proofs don't depend on this form. Here full matrix.
- The dependence structure is entirely in $\boldsymbol{W_j(z)}$, not in $\boldsymbol{\xi_{j,k}}$.