Formes d'arbres possibles

$$\vec{v} = (\underbrace{2^{N-1}, \dots, 1}_{N \text{ vecteurs}}) \qquad x_i - x_j = \begin{cases} 2^{N-i} - 2^{N-j} \\ 0 \end{cases}$$

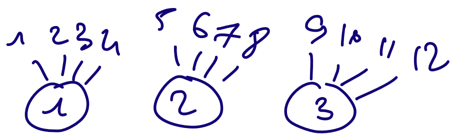
• $n=2$, $N=3$ $\qquad \vec{v} = (4,2,1)$ $\qquad m=3$

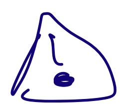
$x_1 - x_2 = 0$ ou 2 $\qquad x_1 - x_3 = 0$ ou 3 $\qquad x_2 - x_3 = 0$ ou 1

$S^-_{2,1} = S_{1,2} = \{0,2\}$ $\qquad S^-_{3,1} = S_{1,3} = \{0 \text{ ou } 3\}$ $\qquad S^-_{3,2} = S_{2,3} = \{0,1\}$

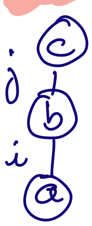
$S^-_{1,2} = S^-_{1,3} = S^-_{2,3} = \{0\}$

132 arbres





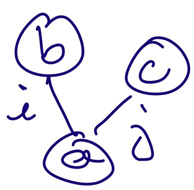
pas deux sur la même étiquette



$i,j \in [\![1;4]\!]$ $\qquad \times 3 \times 2 \times 1 \times 4 \times 4 = 96$

choix de a $\quad$ b $\quad$ c $\qquad$ choix de j

choix de i



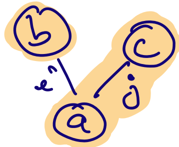
$1 \le i < j \le 4$ $\qquad \times 3! \times 6 = 36$

choix $\{a,b,c\}$ $\qquad$ $i<j$

Definition 4.1. A cadet sequence $(v_1, \dots, v_k)$ of $T \in \mathcal{T}^{(m)}(N)$ is an $\boldsymbol{S}$*-cadet sequence* if for all $1 \le i < j \le k$, $\sum_{p=i+1}^{j} \operatorname{lsib}(v_p) \notin S^-_{v_i,v_j}$. An $\boldsymbol{S}$*-boxed tree* is a boxed tree (T,B) with $T \in \mathcal{T}^{(m)}(N)$, and B containing only **S**-cadet sequences. We denote by $\mathcal{U}_{\mathbf{S}}$ the set of **S**-boxed trees.

"left sibling

↳ les 132 arbres ont des emboîtements directs (a)(b)(c)

Conditions sur  (a,c) S-cadet sequence si $\underset{j-1}{\underset{\parallel}{\mathrm{lsib}(c)}} \notin S^-_{a,c}$

↳ $a=1$ $c=2$ $S^-_{1,2}=\{0\} \to j\neq 1 \rightsquigarrow$ } 12 choix

$c=3$ $S^-_{1,3}=\{0\}$

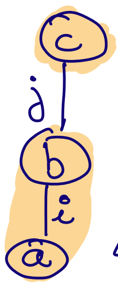
$a=2$ $c=1$ $S^-_{2,1}=\{0,2\} \to j\neq 1,3 \rightsquigarrow$ 4 choix

$c=3$ $S^-_{2,3}=\{0\} \to j\neq 1 \rightsquigarrow$ 6 choix

$a=3$ $c=1$ $S^-_{3,1}=\{0,3\} \to j\neq 1,4 \rightsquigarrow$ 3 choix

$c=2$ $S^-_{3,2}=\{0,1\} \to j\neq 1,2 \rightsquigarrow$ 5 choix

$\underline{30}$



(a,b) S-cadet sequence si $\underset{i-1}{\underset{\parallel}{\mathrm{lsib}(b)}} \notin S_{a,b}$

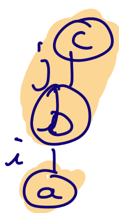
↳ $(a,b)=(1,2),(1,3),(2,3) \to S^-_{a,b}=\{0\} \rightsquigarrow i\neq 1$

9 choix

$(2,1),(3,1),(3,2) \to 2$ valeurs interdites

6 choix

15×4
$=60$

 $\to$ 60 choix

 $\to$ (a,b,c) S-cadet sequence

$i-1 \notin S^-_{a,b}$, $i+j-2 \notin S^-_{a,c}$, $j-1 \notin S^-_{b,c}$

$(a,b,c)=(1,2,3)$ $i\neq 1$ $i+j\neq 2$ $j\neq 1$

↳ 9 choix

$(1,3,2)$ $i\neq 1$, $j\notin\{1,2\}$, $i+j\neq 2$

↳ 6 choix

$(2,1,3)$ $i\notin\{1,3\}$, $j\neq 1$, $i+j\neq 2$

↳ 6 choix

35

$(2,3,1)$ $i \neq 1$, $j \notin \{1,4\}$, $i+j \notin \{2,4\}$
$(2,3)(3,2)(3,3)(4,2)(4,3)$
↳ 5 choix

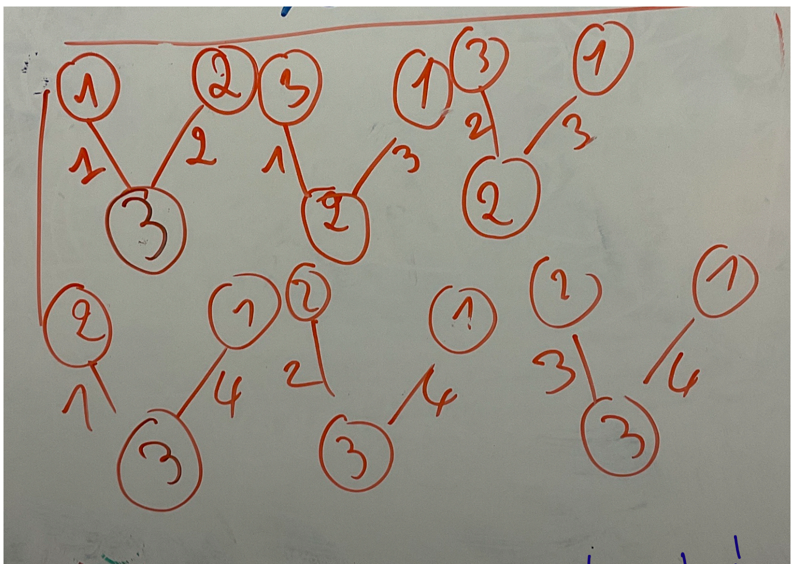
$(3,1,2)$ $i \notin \{1,4\}$, $j \neq 1$, $i+j \notin \{2,3\}$
$(2,2)(2,3)(2,4)(3,2)(3,3)(3,4)$
↳ 6 choix

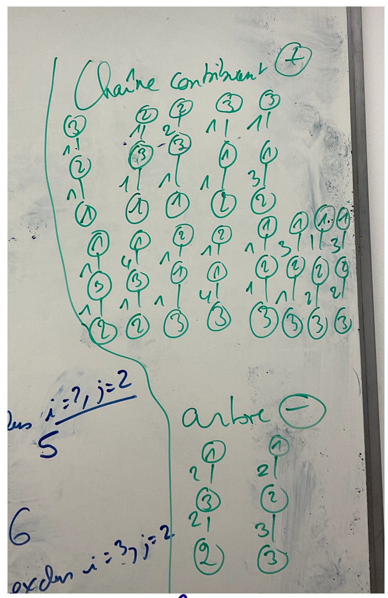
$(3,2,1)$ $i \notin \{1,2\}$, $j \notin \{1,3\}$, $i+j \notin \{2,5\}$
↳ ~~(3,2)~~ $(3,4)(4,2)(4,4)$
3 choix

$132 - 150 + 35$

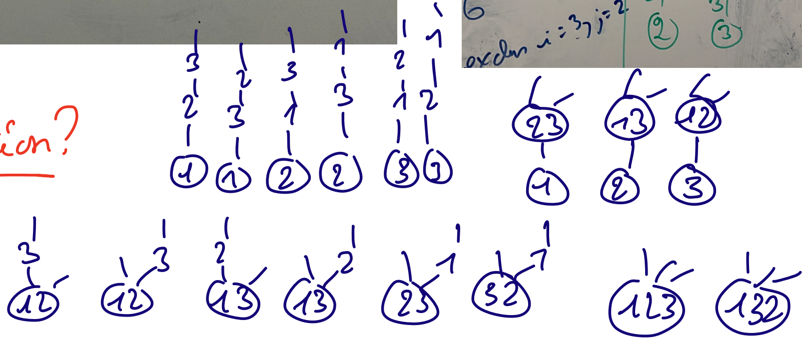
↳ 17

Contribut° : 6 corolles





↳ Bijection ?



Guillaume → Preuve géométrique

Questº : Quelles peuvent être les contributº des arbres ?

- Lien entre compositº d'arbres et contributions (?)

- Symétries séries génératrices (comme Chapoton fait)

- En deux, combien de transitivité ?

Si $i > j > k \to \alpha \neq^{0} u_j - u_i$, $\beta \neq^{0} u_k - u_j$?

$\to \alpha + \beta \neq u_k - u_i$

Contre-ex : $\alpha = u_k - u_j$ $\beta = u_j - u_i$

$j > i > k \to \alpha \neq u_i - u_j$, $\beta \neq u_k - u_j \Rightarrow \alpha + \beta \neq u_k - u_i$

$\hookrightarrow \alpha = u_k - u_j$ $\beta = u_i - u_j$

$i > k > j \to \alpha \neq 0$, $\beta \neq u_j - u_k \Rightarrow \alpha + \beta \neq u_k - u_i$

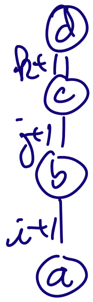
$\hookrightarrow \alpha = \underset{2^{n-j} - 2^{n-k}}{u_j - u_k}$, $\beta = \underset{2^{n+1-k} - 2^{n-j} - 2^{n-i}}{\underbrace{2u_k} - u_j - u_i} \leq 0$!

Pour chaînes plus longues, quelle caractérisatº ?

$\to$ Quelle bijectº ?

Diagonale = morφ de $P_\infty \otimes P_\infty \leftarrow P_\infty$

• Qu'est-ce qu'il se passe pour une chaîne de taille 4?



(a,b,c,d) SCS

ssi $i \notin S^-_{a,b}$, $j \notin S^-_{b,c}$ $k \notin S^-_{c,d}$

$i+j \notin S^-_{a,c}$ $j+k \notin S^-_{b,d}$, $i+j+k \notin S^-_{a,d}$

Si $a>b>c$,
ou $b>c>d$,
ou $b>a>c$,
ou $c>b>d$,
} Cas précédent

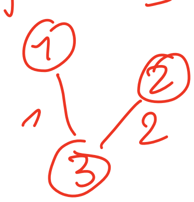
$a<d \longrightarrow S^-_{a,d}=\{0\} \rightarrow$ transitivité vérifiée

$a>d$

- $b>a>d$
  - $c>b>a>d$
  - $b>c>a>d$
  - $b>a>c>a$
  - $b>a>d>c$
- $a>b>d$
  - $c>a>b>d$
  - $a>c>b>d$
  - $a>b>c>d$
  - $a>b>d>c$
- $a>d>b$
  - $c>a>d>b$
  - $a>c>d>b$
  - $a>d>c>b$
  - $a>d>b>c$

pas transitif

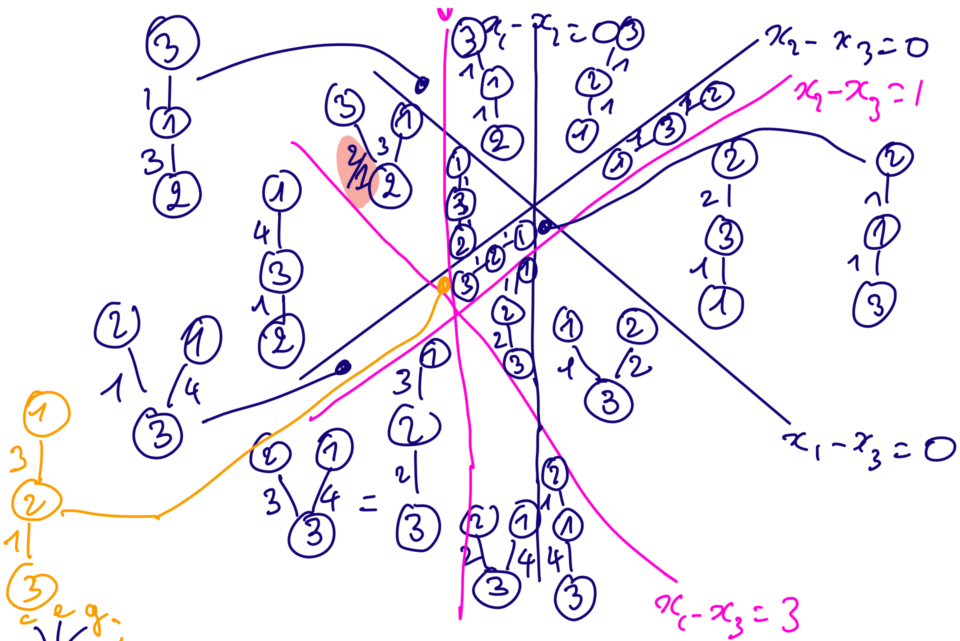
Sujet $\Psi_S \longrightarrow$ p77

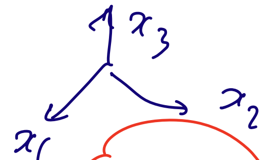


$\Psi_S(T)$

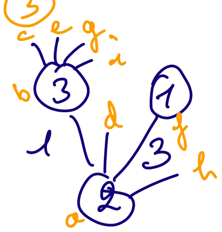
$\longmapsto (x_1,x_2,x_3)$ $\left\{\begin{array}{l} x_1-x_2<0 \\ x_1-x_3>0 \\ x_1-x_3<3 \\ x_1-x_3>1 \end{array}\right.$

$x_1-x_2=2$

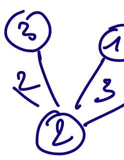




See below for a nicer picture

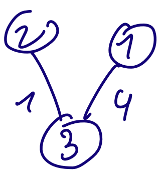


$x_1 - x_2 > 2$
$3 > x_1 - x_3 > 0$
$x_2 - x_3 < 0$

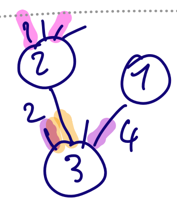


$x_1 - x_2 > 2$
$3 > x_1 - x_3 > 0$
$x_2 - x_3 < 0$

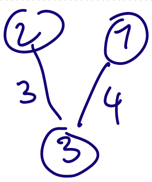
$S_{1,2} = \{0,2\}$
$S_{1,3} = \{0,3\}$
$S_{2,3} = \{0,1\}$



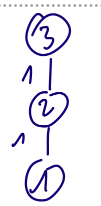
$x_1 - x_2 > 2$
$x_1 - x_3 > 3$
$0 < x_2 - x_3 < 1$



$2 > x_1 - x_2 > 0$
$x_1 - x_3 > 3$
$x_2 - x_3 > 1$



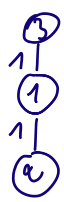
$x_1 - x_2 > 2$
$x_1 - x_3 > 3$
$x_2 - x_3 > 1$



$x_1 - x_2 < 0$
$x_1 - x_3 < 0$
$x_2 - x_3 < 0$

$x_1 - x_2 < 0$
$x_1 - x_3 < 0$
$0 < x_2 - x_3 < 1$

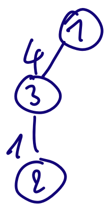
$x_1 - x_2 < 0$
$x_1 - x_3 < 0$
$x_2 - x_3 > 1$



$0 < x_1 - x_2 < 2$
$x_1 - x_3 < 0$
$x_2 - x_3 < 0$

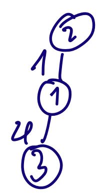
$x_1 - x_2 > 2$
$x_1 - x_3 < 0$
$x_2 - x_3 < 0$

$2 > x_1 - x_2 > 0$
$0 < x_1 - x_3 < 3$
$x_2 - x_3 < 0$

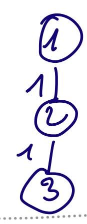


$x_1 - x_2 > 2$
$x_1 - x_3 > 3$
$x_2 - x_3 < 0$

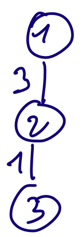
$x_1 - x_2 < 0$
$0 < x_1 - x_3 < 3$
$0 < x_2 - x_3 < 1$



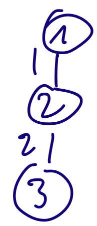
$x_1 - x_2 < 0$
$x_1 - x_3 > 3$
$x_2 - x_3 > 1$



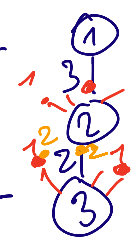
$0 < x_1 - x_2 < 2$
$0 < x_1 - x_3 < 3$
$0 < x_2 - x_3 < 1$



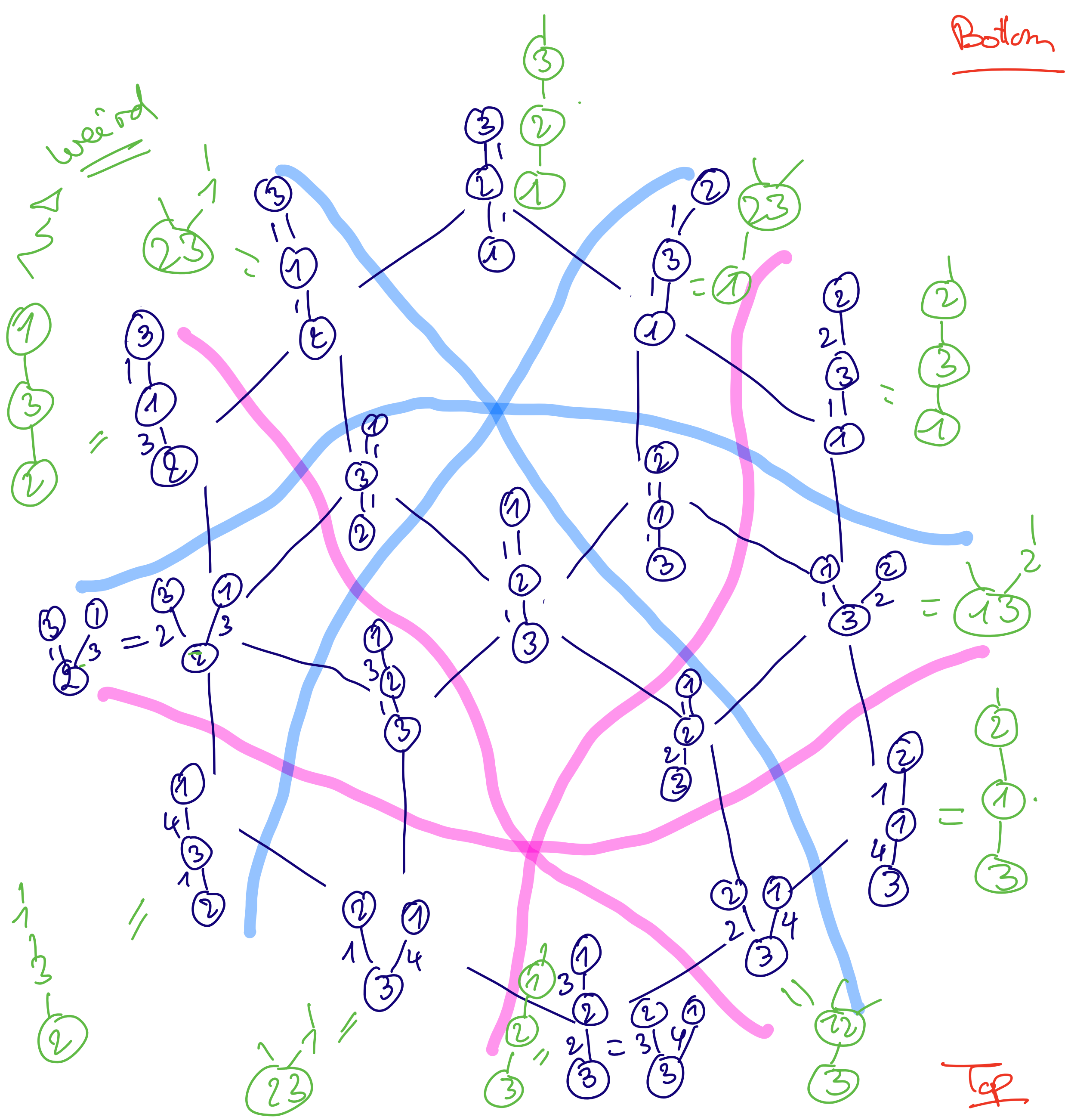
$x_1 - x_2 > 2$
$0 < x_1 - x_3 < 3$
$0 < x_2 - x_3 < 1$



$0 < x_1 - x_2 < 2$
$0 < x_1 - x_3 < 3$
$x_2 - x_3 > 1$



$x_1 - x_2 > 2$
$x_1 - x_3 > 3$
$x_2 - x_3 > 1$



Je ne comprends pas du tt le lien : certains sont fusionnés et d'autres non