

(Fast) Experimental evidence of the Collatz conjecture

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Vocabulary

Syracuse serie

The syracuse recurrence is defined this way:

$$\begin{aligned} f(n) &= n/2 \text{ if } n \text{ is a multiple of } 2 \\ f(n) &= 3n + 1 \text{ if } n \text{ is not a multiple of } 2 \end{aligned}$$

The k^{th} syracuse serie is defined the following way:

$$\begin{aligned} u_0 &= k \\ u_{n+1} &= f(u_n) \end{aligned}$$

Collatz conjecture

The Collatz conjecture (Still an open problem in mathematics) says that: For all integer k the t^{th} syracuse serie eventually reach 1.

Flight time of a Syracuse serie

The flight time of a syracuse serie is the smallest n such that $u_n = 1$

Project proposition

The aim of this project is to check for as much possible integers that the Collatz conjecture is true and possibly compute it's flight time.

Why is this project interesting

At first sight it might seem that having a parallel implementation is trivial. But we can consider a very simple optimisation. We can maintain a set of integers that we know they satisfy the Collatz conjecture. If at some point we find that u_n is in this set then there is no need to continue the computation we know that k satisfies the conjecture too. The sequential implementation can be quite fast with this simple heuristic. It is hard to have a parallel implementation with a good absolute speedup.