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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 200
plt.rcParams.update({"text.usetex": True})
import pandas as pd
import random
```

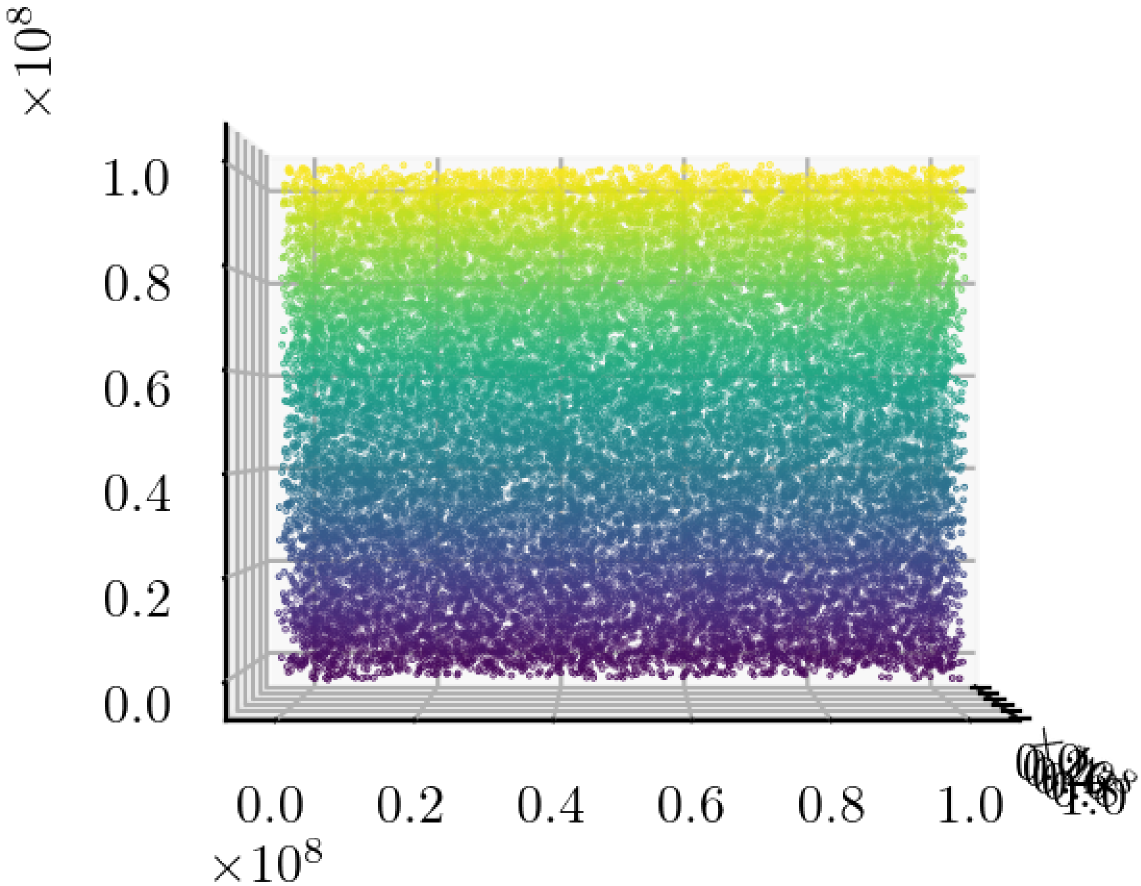
Q1

First loading the "random" points generated with C.

In [2]:
```
f = pd.read_csv("rand_points.txt", sep=" ", header=None)
xdata = np.array(f[0])
ydata = np.array(f[1])
zdata = np.array(f[2])
```

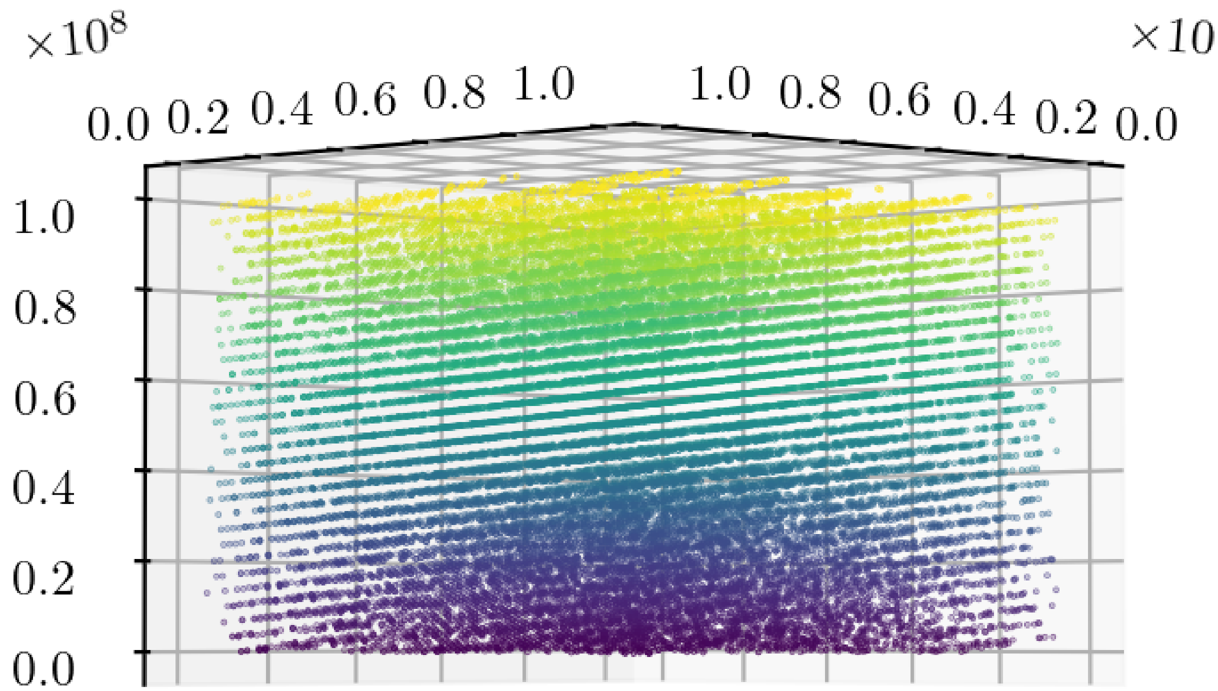
Now I show what it the points look like for some random viewing angle

In [3]:
```
ax = plt.axes(projection='3d')
ax.view_init(0, 0)
ax.scatter3D(xdata, ydata, zdata, c=zdata, s=0.1)
plt.show()
```



Now I show what the points look like when the viewing angle is parallel to the planes on which the points live

In [4]:
```
ax = plt.axes(projection='3d')
ax.view_init(-2, 45)
ax.scatter3D(xdata, ydata, zdata, c=zdata, s=0.1)
plt.show()
```



This way the planes are visible. Now I repeat this procedure using Python's random.randint

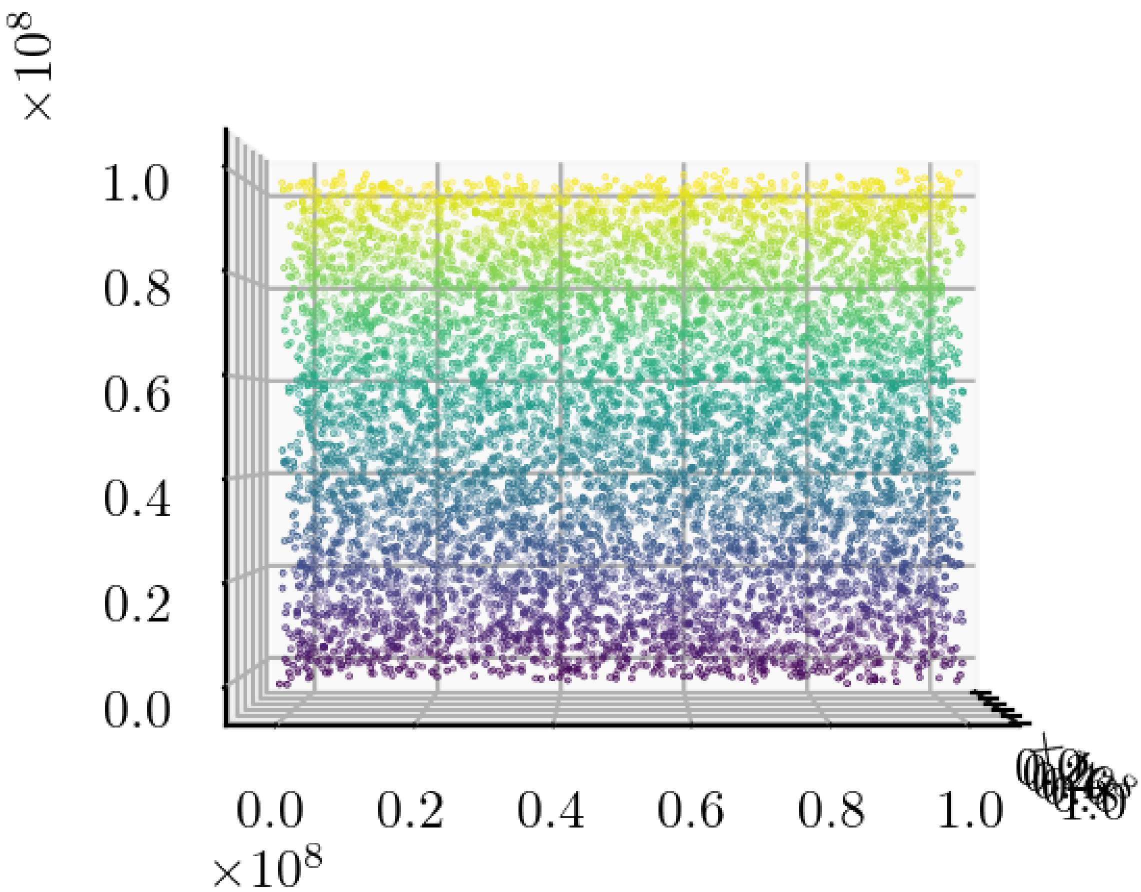
In [5]:
```
# Making vector of random integers between 0 and 2^31
vec=np.empty(300000000,dtype='int32')
for i in range(300000000):
    vec[i] = random.randint(0, 2**31)
# Reshaping the vector into 3d points
vec = np.reshape(vec,(3,100000000))
# Keeping only points with 0 < x,y,z < 1e8
maxval=1e8
vmax=np.max(vec,axis=0)
vv2=vec[:,vmax<maxval]
print(vv2.shape)

xdata = vv2[0]
ydata = vv2[1]
zdata = vv2[2]
```

```
(3, 10070)
```

Plotting the points with a random viewing angle

In [6]:
```
ax = plt.axes(projection='3d')
```

```
ax.view_init(0, 0)
ax.scatter3D(xdata, ydata, zdata, c=zdata, s=0.1)
plt.show()
```



I see no plane here, and searching many viewing angles using an interactive plot, I cannot find any plane, or any other feature suggesting that the numbers are not really random.

I wasn't able to load the C library on my machine.

Q2

The distributions that work as bounding distributions are the Power law and the Lorentzian. The reason is that these ones decays slower than an Exponential as $x \to \infty$. Meanwhile the Gaussian distribution decays faster than an exponential as $x \to \infty$. Therefore, the Gaussian will necessarily be below the Exponential for some values of x, so isn't suitable.

I will generate samples from the exponential distribution with PDF

$$P(x) = e^{-x} \tag{1}$$

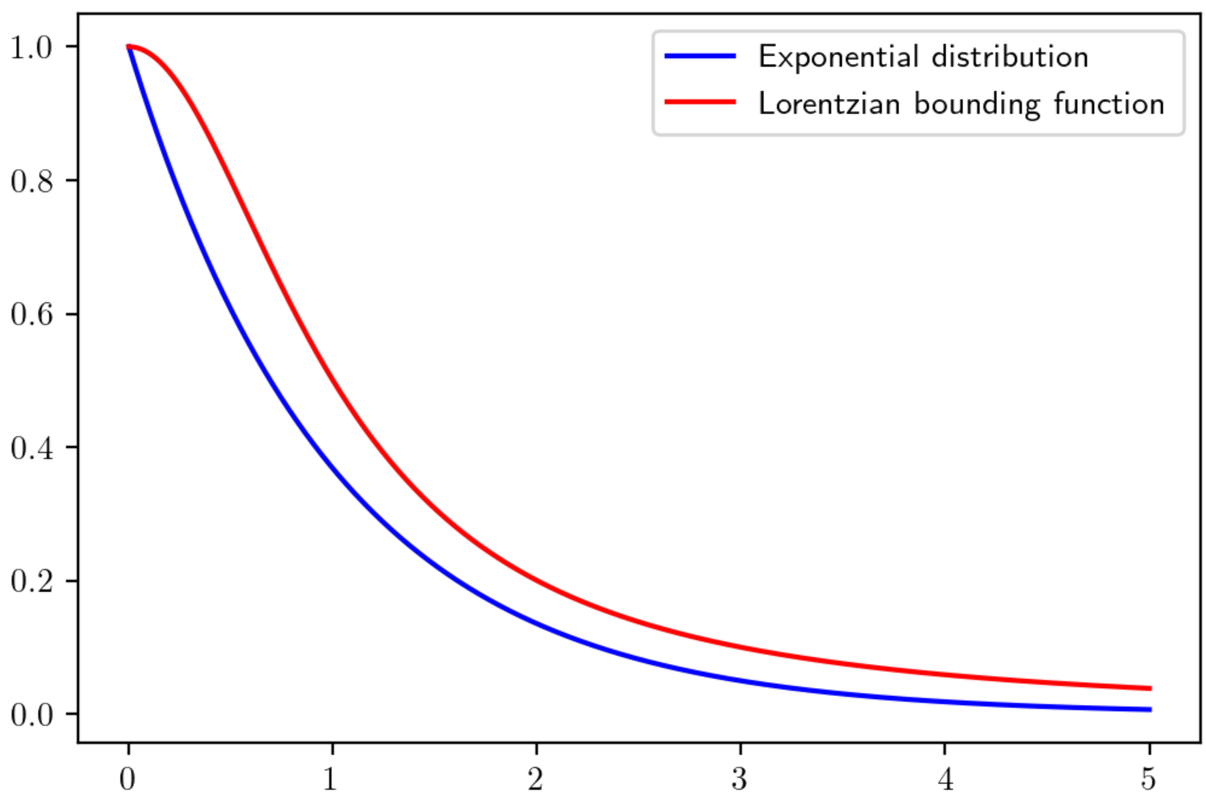
My most efficient bounding function is

$$f(x) = \frac{1}{1+x^2} \tag{2}$$

First I plot the exponential distribution and the bounding function

In [7]:
```
x = np.linspace(0,5,1000)
exponential = np.exp(-x)
lorentzian = 1/(1+x**2)
plt.plot(x,exponential,label='Exponential distribution',color='blue')
plt.plot(x,lorentzian,label='Lorentzian bounding function',color='red')
plt.legend(frameon=True)
```

Out[7]:
```
<matplotlib.legend.Legend at 0x126eb3d1330>
```



Now I generate samples from the Lorenztian and accept/reject them to get samples from the Exponential

In [8]:
```
# Lorentzian distribution
def lorentzian(x):
    return 1/(1+x**2)

# Exponential bounding function
def exponential(x):
    return np.exp(-x)
```

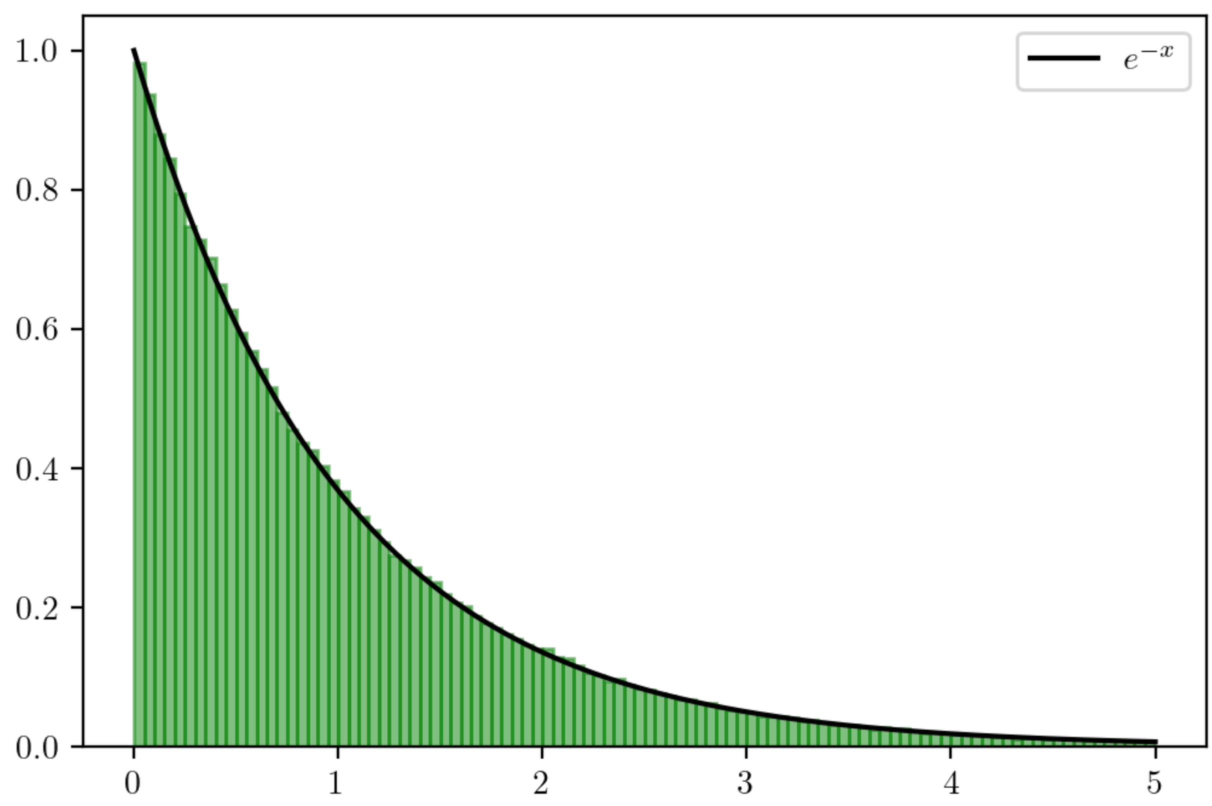
```
# Function to generate samples from exponential distribution
def gen_exp_samples(n):
    # Getting lorentzian samples with uniform distribution over y component
    samples_lorentzian_x = np.random.standard_cauchy(n)
    samples_lorentzian_x = samples_lorentzian_x[samples_lorentzian_x>0]
    samples_lorentzian_y = np.random.rand(samples_lorentzian_x.shape[0])*lorentzian(sa
    # Accepting/Rejecting samples
    samples_exp = samples_lorentzian_x[samples_lorentzian_y<exponential(samples_lorent
    # Printing ratio of accepted samples
    print(samples_exp.shape[0]/n)
    # Removing samples with x>5 just for plotting purposes
    samples_exp = samples_exp[samples_exp<5]
    return samples_exp
```

Now making a histogram with the exponential samples. I print the ratio of accepted samples

In [9]:
```
samples = gen_exp_samples(1000000)
plt.hist(samples,density=True,bins=100,color='green',alpha=0.5,histtype='bar', ec='gre
x = np.linspace(0,5,1000)
plt.plot(x,exponential(x),color='black',label='$e^{-x}$')
plt.legend(frameon=True)
```

```
0.317789
```

Out[9]:
```
<matplotlib.legend.Legend at 0x126e8322ef0>
```



So the efficiency of this model is 32%. Considering that half of the samples are thrown away due to having negative x, it's not so bad.

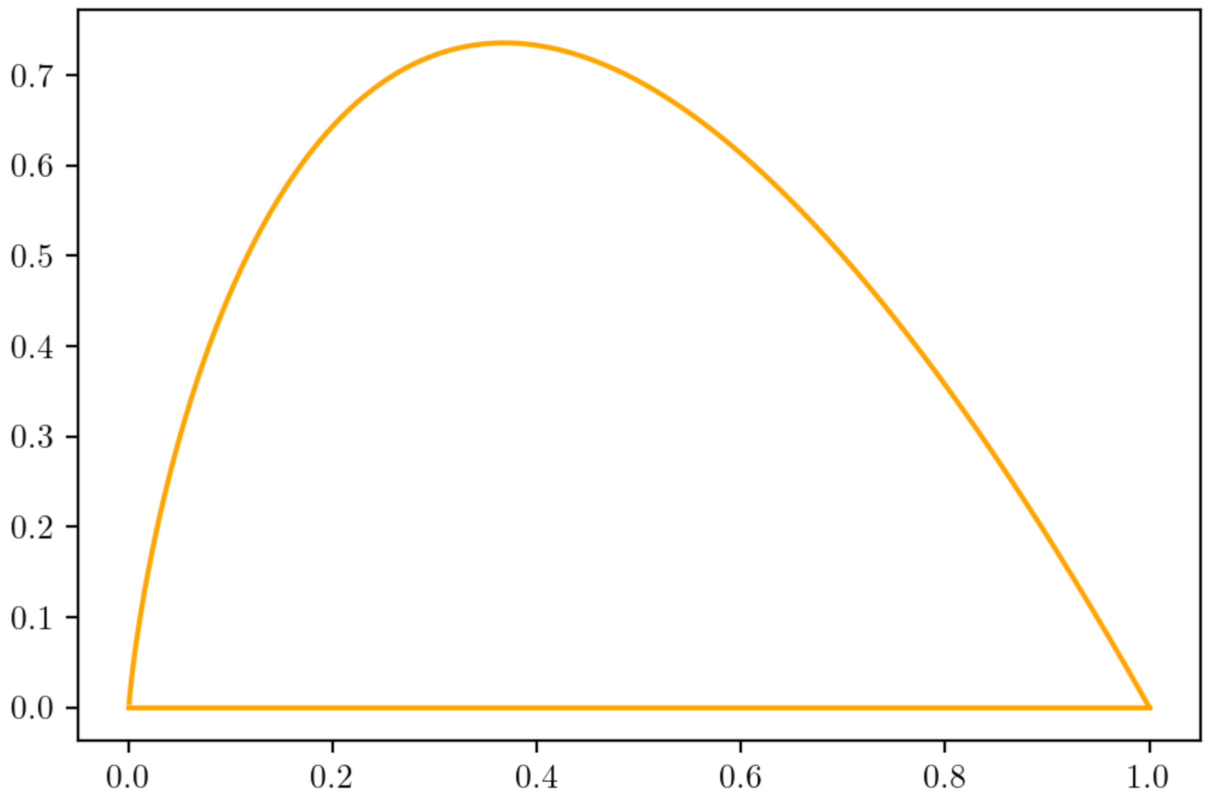
Q3

First I make a plot of the acceptance region

In [10]:
```
x = np.linspace(0,1,3000)
y = np.log(x**2)*(-x)
plt.plot(x,y,color='orange')
plt.plot([0,1],[0,0],color='orange')
print(np.nanmax(y))
```

```
0.7357588602357744
```

```
C:\Users\Guill\AppData\Local\Temp\ipykernel_284\1203540017.py:2: RuntimeWarning: divi
de by zero encountered in log
  y = np.log(x**2)*(-x)
C:\Users\Guill\AppData\Local\Temp\ipykernel_284\1203540017.py:2: RuntimeWarning: inva
lid value encountered in multiply
  y = np.log(x**2)*(-x)
```

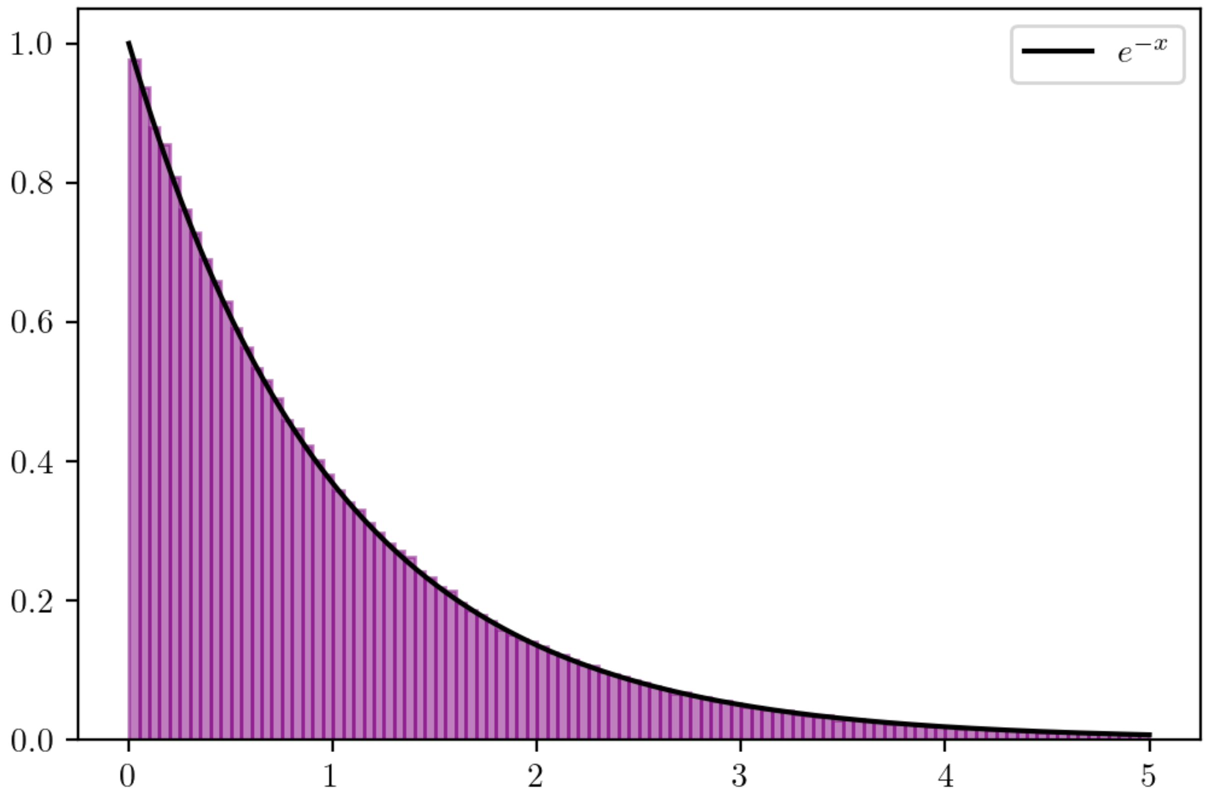


In [11]:
```
# Exponential distribution
def exponential(x):
    return np.exp(-x)

# Drawing samples from the box
def sample_exponential(n):
    samples_box = np.random.rand(2,n)
    samples_box[1,:] = samples_box[1,:]*0.73
    # Accepting/Rejecting the samples depending on whether they are inside the region
    samples_exponential = samples_box[:,samples_box[1,:]<np.log(samples_box[0,:]**2)*(
    samples_exponential = samples_exponential[1,:]/samples_exponential[0,:]
    # Printing ratio of accepted samples
    print(samples_exponential.shape[0]/n)
    # Removing samples with x>5 for plotting purposes
    samples_exponential = samples_exponential[samples_exponential < 5]
    return samples_exponential
```

In [12]:
```
samples = sample_exponential(1000000)
plt.hist(samples,density=True,bins=100,color='purple',alpha=0.5,histtype='bar', ec='pu
x = np.linspace(0,5,1000)
plt.plot(x,exponential(x),color='black',label='$e^{-x}$')
plt.legend(frameon=True)
```

```
0.684199
```

Out[12]:
```
<matplotlib.legend.Legend at 0x126c71e01f0>
```



The histogram matches expectations again, and the efficiency is 68%.