Question 1: Unemployment Markov Chain

A worker can be in 3 states: Unemployed(U), Employed(E), and Studying (S).[1] The probabilities of transitioning between these states are:

1. An unemployed worker finds a job with probability α
2. An unemployed worker gets on the job training and begins studying with probability γ
3. An employed worker loses a job with probability β, and never transitions to studying directly
4. A studying worker only spends 1 period studying (i.e., never transitions to studying) and then immediately has a chance to δ to find a job. Otherwise, they become unemployed again.

Assume the parameter values are: $\alpha = 0.15$, $\beta = 0.1$, $\gamma = 0.05$, and $\delta = 0.4$

1.1 Markov Chain and Probabilities

(a) Define a transition matrix (P) for the Markov chain of the worker status. Be explicit on the ordering of states.

(b) Assume that a worker is employed (E). What is the probability they are studying (S) 4 periods from now?

(c) Assume that a worker is unemployed (U) at period 0. Make a graph which shows the probability they will be unemployed in period t for $t = 1, \ldots 30$.

(d) What is the stationary distribution of Employment, Unemployment, and Studying in the economy?

1.2 Simulation of a Worker

Take a worker who is currently unemployed. Define the fraction of time up to period T spent in the employment state as

$$\bar{X}_t := \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{1}\left\{X_\tau = E\right\}$$

where $\mathbb{1}\left\{X_t = E\right\} = \begin{cases} 1 & X_t = E \\ 0 & \text{otherwise} \end{cases}$

(a) Use the transition matrix from the previous part to simulate a path of $T = 1000$ states of employment states for the individual.

(b) Plot the proportion of time spent employed for the worker for each $t = 1, \ldots T$, i.e $\left\{\bar{X}_t\right\}_{t=1}^{T}$

(c) Explain the connection between $\left\{\bar{X}_t\right\}_{t=1}^{T}$ and the stationary distribution?

[1] This is a variation of `https://lectures.quantecon.org/jl/finite_markov.html#exercise-1`. Hint: you can see the code for the solution to that problem.

1.3 Transition Dynamics of Policy Change

Assume that the economy is at the stationary distribution of E, U, and S. Now the government funds a program where they will provide more funding for on the job training for unemployed, which increases the probability $\gamma = 0.08$. However, in order to finance the program they have to tax operating firms, which increases the probability $\beta = 0.12$

(a) Calculate the new transition matrix (P) for the Markov chain after the policy change.

(b) Calculate the new stationary distribution of E, U, and S

(c) Plot the E, U, and S transition dynamics from the old stationary distribution after the policy is implemented for $t = 1, \ldots, 50$.[2]

[2]Hint: if you start at ψ_0 as the old stationary distribution, you can find $\psi_1 = \psi_0 P$ using the new transition matrix P