Question 1: Simulating a Time Series

Take the stochastic process that follows

$$y_{t+1} = \gamma + \theta y_t + \sigma w_{t+1} \tag{1}$$

where $w_{t+1} \sim \mathrm{N}(0,1)$.

1.1 Simulation

Let $\gamma = 1, \sigma = 1, \theta = 0.8$ and $y_0 = 0$.

(a) Setup the problem as a linear state space model (i.e. LSS in QuantEcon)

(b) Simulate the data generating process $\{y_t\}_{t=0}^{T}$ for $T = 1000$

(c) Plot a rolling mean of the process, i.e. for each $1 \leq \tau \leq T$, plot $\frac{1}{\tau}\sum_{t=1}^{\tau} y_t$

1.2 Stationary Distribution

(a) For some large T (maybe 30 is high enough? You can play around with it) simulate y_T for a large I (maybe 200, but feel free to play around with it). Plot a histogram of the stationary distribution for y_T (which we can call y_∞ if we assume that T is large enough).

(b) Numerically find the mean and variance of this as an ensemble average, i.e. $\mathbb{E}[y_\infty] \approx \sum_{i=1}^{I} \frac{y_T^i}{I}$ and $\mathbb{V}[y_\infty] \approx \sum_{i=1}^{I} \frac{(y_T^i)^2}{I} - \mathbb{E}[y_\infty]^2$

(c) Find the stationary distribution for this plot an overlay of the pdf of a normal distribution with parameters $\mathbb{E}[y_\infty]$ and $\mathbb{V}[y_\infty]$ vs. the histogram.[1]

Question 2: Linear Asset Pricing

Assume that the aggregate component of payoffs follows

$$Z_{t+1} = \alpha + \rho_2 Z_t + \rho_3 Z_{t-1} + \sigma_1 w_{1,t+1} \tag{2}$$

And the idiosyncratic component of payoffs follows

$$z_{t+1} = \rho_1 z_t + \sigma_2 w_{2,t+1} + \theta w_{2,t} + c w_{1,t+1} \tag{3}$$

While the payoff for a given $\{Z_t, z_t\}$ state is

$$y_t = \lambda Z_t + (1-\lambda) z_t \tag{4}$$

where $\begin{bmatrix} w_{1,t+1} \\ w_{2,t+1} \end{bmatrix} \sim N(0, I)$.

Define the expected PDV of payoffs as

$$p_t \equiv \mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j y_{t+j}\right] \tag{5}$$

[1] Hint: when plotting a histogram, you are getting a count rather than a density. To normalize it so it sums to 1, like a density, you can use `histogram(yourdata, normed=true)`

2.1 Setting up the Model

For parameter values, use $\alpha = 0.1, \rho_1 = 0.8, \rho_2 = 0.75, \rho_3 = 0.1, \theta = 0.1, c = 0.05, \lambda = 0.5, \beta = 0.95, \sigma_1 = 0.1, \sigma_2 = 0.05$

(a) Pick an appropriate state, and setup this problem in our canonical linear state space model.

(b) From theory, find an expression for p_t from the LSS model in terms of the underlying state, write a function which implements the price based on the state and parameters

(c) Calculate the stationary distribution of the $\{Z_t\}$ process with these parameters(note that you will not need to deal with a joint distribution).[2]

2.2 Simulating the Model

Let Z_0 be the mean of the stationary distribution of Z_t you calculated previously and $z_0 = z_1 = w_{2,0} = 0$. Let $T = 20$

(a) Simulate $I = 5$ paths of $\{Z_t, z_t, y_t, p_t\}_{t=1}^T$

(b) Plot the I paths for the y_t and p_t series in a way you think is easy to interpret

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[2] Hint on stationary distributions: as you saw in the lecture notes, QuantEcon has a `stationary_distributions` function for the LSS model. However, you will need to be a little careful when using calling that function if there is a constant term in your state, because it uses an iterative method. To do this, make sure to pass in the `mu_0` argument with something valid. For example, this is an AR(1) process in the LSS model:

```
using QuantEcon
A1 = 0.6;
a = 1.0
A = [0.6 2.0; 0 1] #Note that the 2nd value in the state is a 1
C = [0.1;0] #Note that the 2nd value in the state is a 1
G = [1 0]
x_0 = [0; 1.0] #Important that the initial condition has the 1 in the correct place!
lss = LSS(A, C, G; mu_0 = x_0) #Using the initial condition with the 1 in the right place
mu_x, mu_y, Sigma_x, Sigma_y = stationary_distributions(lss)
```