Aero 3 — Ma 322 (2023-2024)
TP2 — Numerical Solution of Differential Equations

Question 1

Write a function that implements the explicit Euler method to solve a differential equation. This function `EulerExplicit(F,a,b,y0,N)` takes the following arguments:

- F the function describing a Cauchy problem $y' = F(t, y)$.
- a and b the bounds of t.
- y_0 the value set for $y(a)$ in the initial condition (provided as either a single number or a one-dimensional vector).
- N the number of subdivisions, determining the step size $h = \dfrac{b-a}{N}$.

The function returns:

- t containing (as a one-dimensional array) the values t_0 to t_N of the subdivision of the interval $[a, b]$.
- Y containing the calculated values y_n. Y will be a two-dimensional array, containing $N+1$ columns and p rows. So the first column corresponds to y_0, the next one to y_1, and so on.

Question 2

Experiment with the written function to solve the following differential equations. These problems are simple and the explicit solution is known and provided. Each time, graphically represent the calculated numerical solution and the provided exact solution. Choose $N = 100$.

1. $y' = y$ with $y(0) = 1$.
2. $y' = -y + t$ with $y(0) = 1$. The solution is $y(t) = t - 1 + 2e^{-t}$. Solve it on the interval $[0, 2]$.
3. $y'' + y = 2t$ with initial conditions $y(0) = 0$ and $y'(0) = 1$. The solution is $y = 2t - \sin t$. Solve it on $[0, 5]$.

Question 3

Program the second and fourth order Runge-Kutta methods (write functions `RK2` and `RK4` with the same inputs and outputs as `EulerExplicit`).

Test them on the problems from the previous question.

Question 4

Studying the motion of a simple pendulum without friction leads to the study of solving the following nonlinear differential equation.

$$L\theta''(t) = -g \sin \theta(t)$$

Here, L denotes the length of the pendulum, g the acceleration due to gravity, and $\theta(t)$ the angle formed by the pendulum and the vertical.

Take $g = 9.81$ m/s^2 and $L = 1$ m.

The initial conditions considered are $\theta(0) = \dfrac{\pi}{2}$ and $\theta'(0) = 0$. We will limit the calculations to $t \in [0, 8]$.

Solve the pendulum problem with $h = 0.04$ using the three coded methods. Graphically represent $\theta(t)$ as a function of t over the considered interval. Do the obtained results seem relevant? Try with other discretization steps and comment.