DEMOGRAPHIC & ENVIROMENTAL NOISE IN PREDATOR-PREY MODELS

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APPROACHES:

- DETERMINISTIC
- DEMOGRAPHIC NOISE
- ENVIROMENTAL NOISE

MODELS:

- 1 PREY - 1 PREDATOR
- 1 PREY - 2 PREDATOR
- 2 PREY - 1 PREDATOR

(PHENOTYPES!)

+ ECOEVOLUTION

Intraspecific variation stabilizes classic predator-prey dynamics
Stefano Allesina, Zachary R. Miller, Carlos A. Serván

DETERMINISTIC

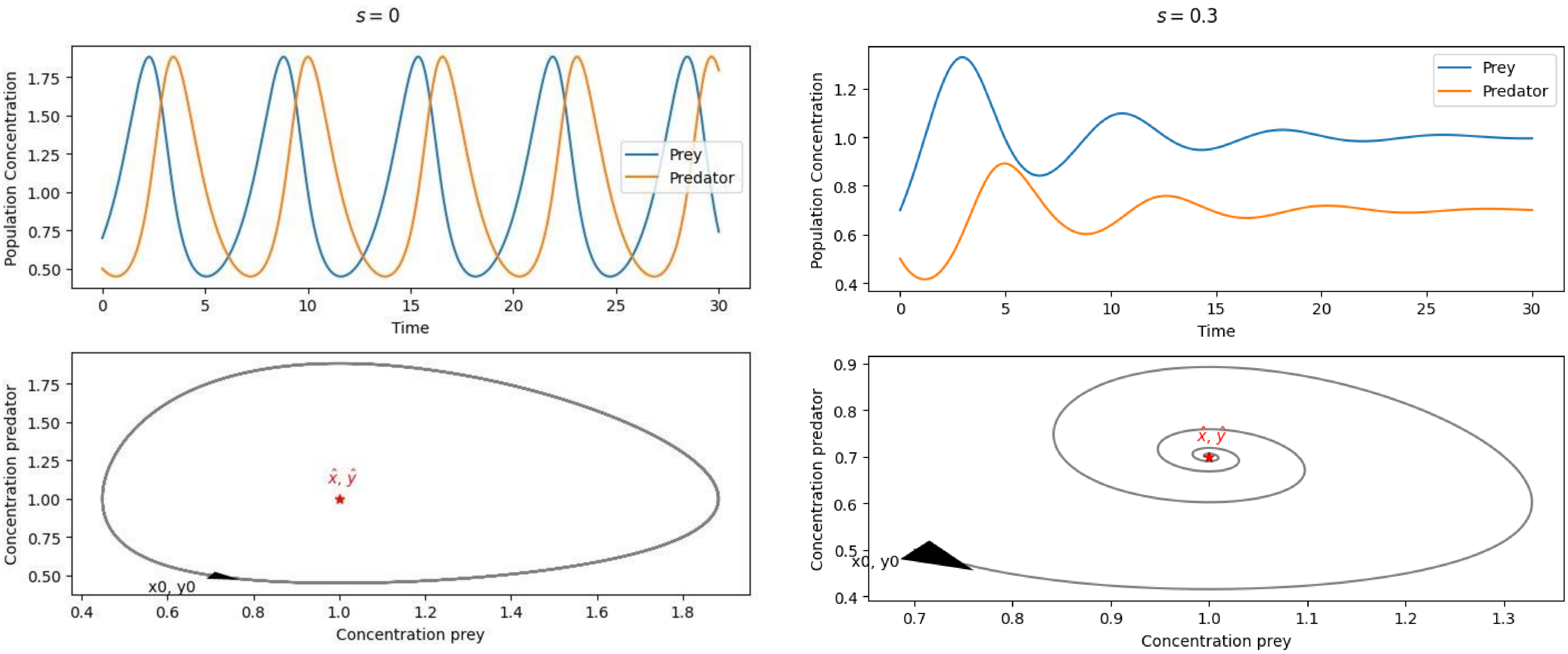
- DIFFERENTIAL EQUATIONS
- EULER METHOD

CLASSIC LOKTA-VOLTERRA

$$\begin{cases} \dot{x} = x(r - ay - sx) \rightarrow Prey \\ \dot{y} = y(bx - d) \rightarrow Predator \end{cases}$$

$$\begin{cases} \hat{x} = d/b \\ \hat{y} = r/a - sd/ab \end{cases}$$

CLASSIC LOKTA-VOLTERRA



TWO PREYS

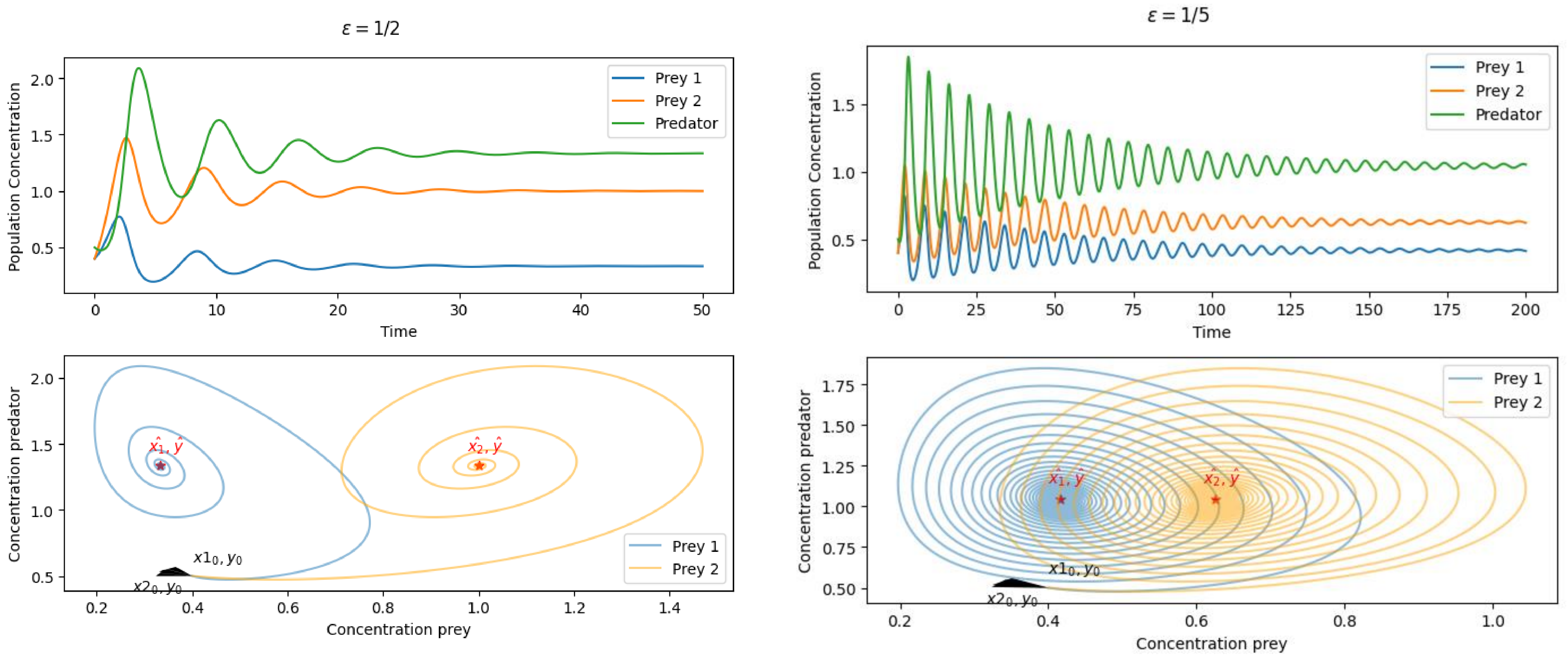
ϵ !

$$\begin{cases} \dot{x_1} = r\frac{x_1+x_2}{2} - a(1+\epsilon)x_1 y) \rightarrow Prey1 \\ \dot{x_2} = r\frac{x_1+x_2}{2} - a(1-\epsilon)x_2 y) \rightarrow Prey2 \\ \dot{y} = y(b[(1+\epsilon)x_1 + (1-\epsilon)x_2] - d) \rightarrow Predator \end{cases}$$

- One prey dies more easily
- Predator takes different benefit from preys
- Same inheritance probability

$$\begin{cases} \hat{x_1} = \frac{d}{2b(1+\epsilon)} \\ \hat{x_2} = \frac{d}{2b(1-\epsilon)} \\ \hat{y} = \frac{r}{a}\frac{1}{1-\epsilon^2} \end{cases}$$

TWO PREYS



$\varepsilon = 1/2$
$\varepsilon = 1/5$
Population Concentration
Time
Prey 1
Prey 2
Predator
Concentration predator
Concentration prey
$\hat{x_1}, \hat{y}$
$\hat{x_2}, \hat{y}$
$x1_0, y_0$
$x2_0, y_0$

TWO PREDATORS

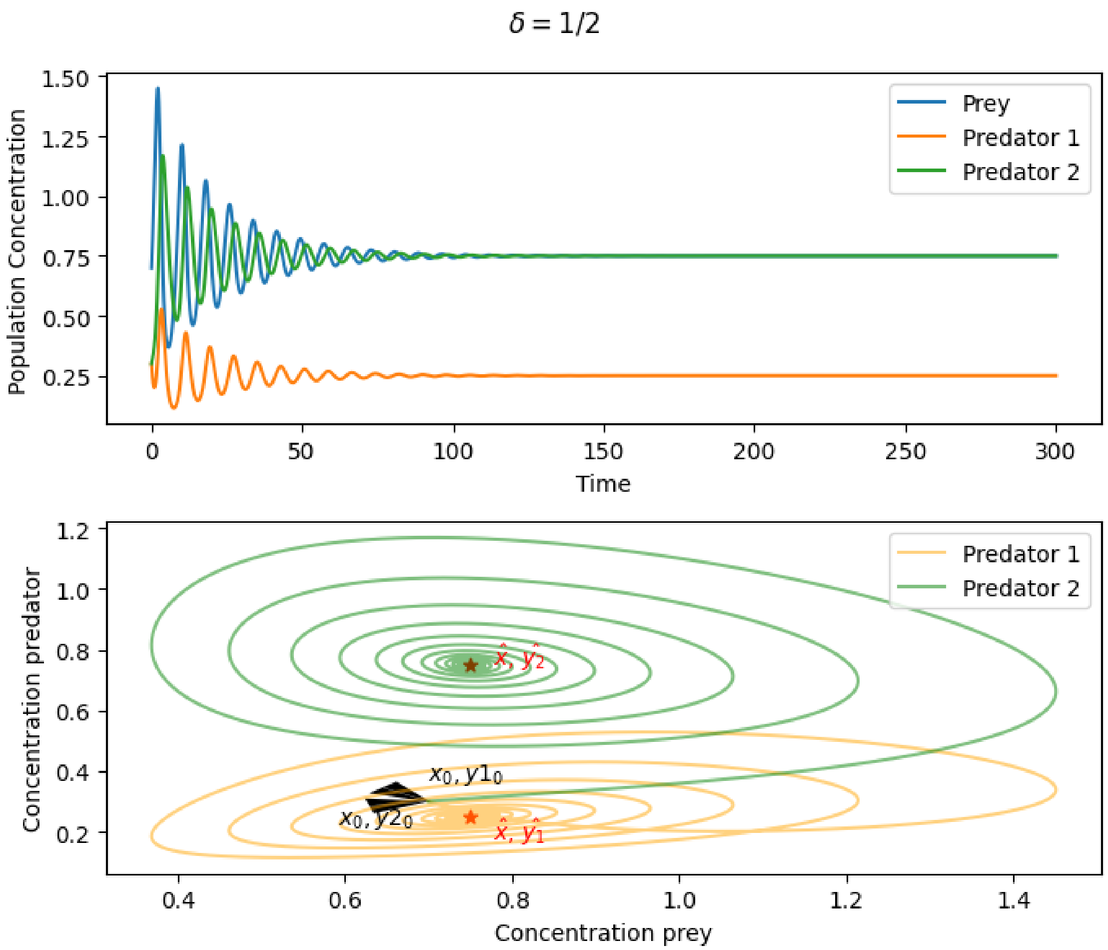
$$\delta\,!$$

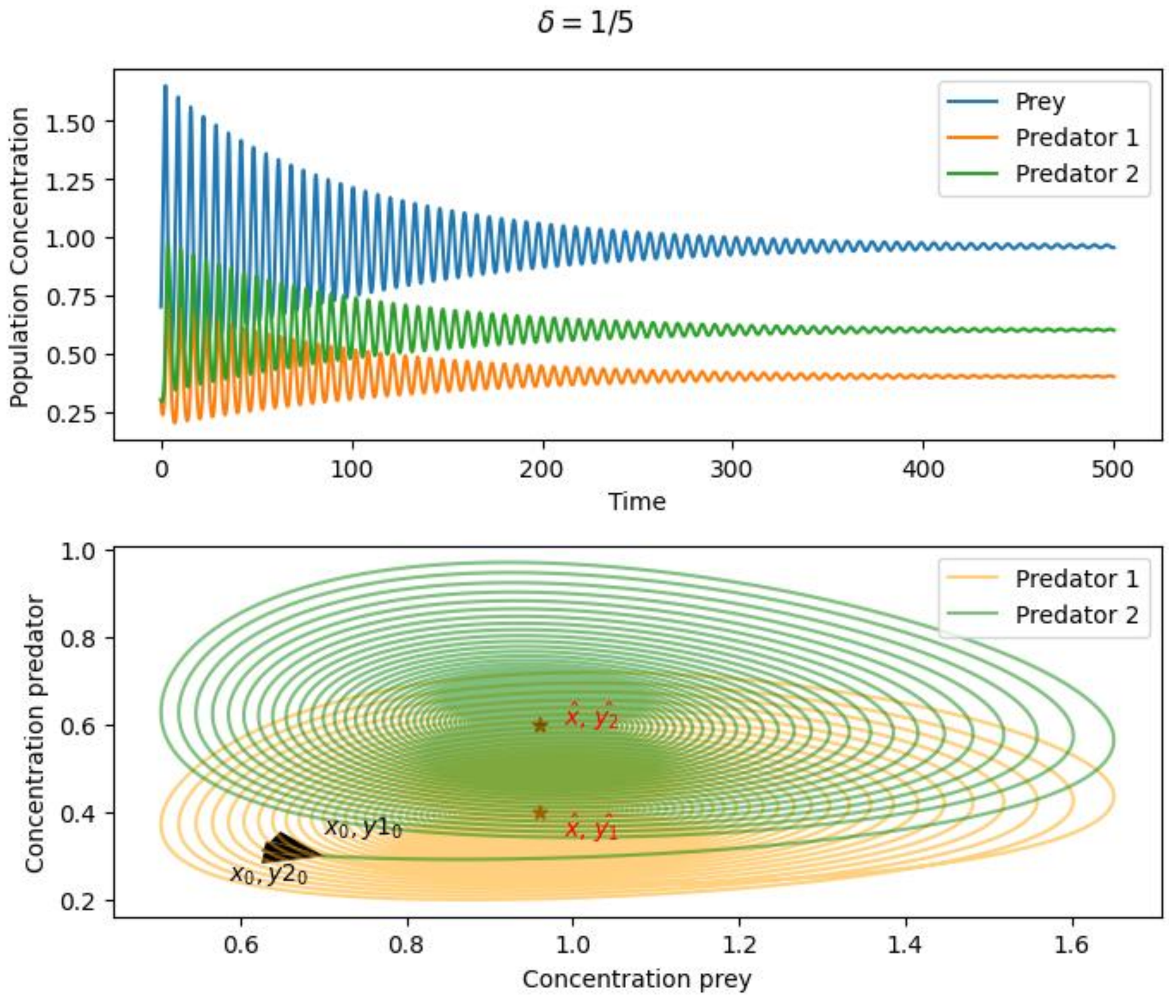
$$\begin{cases} \dot{x} = x(r - a[y_1 + y_2]) \rightarrow Prey \\ \dot{y_1} = -d(1+\delta)y_1 + b\frac{y_1+y_2}{2}x \rightarrow Predator1 \\ \dot{y_2} = -d(1-\delta)y_2 + b\frac{y_1+y_2}{2}x \rightarrow Predator2 \end{cases}$$

- One predator has a higher death rate
- The prey is attacked evenly by predators
- Same inheritance probability

$$\begin{cases} \hat{x} = \frac{d}{b}(1-\delta^2) \\ \hat{y_1} = \frac{r}{a}\frac{1-\delta}{2} \\ \hat{y_2} = \frac{r}{a}\frac{1+\delta}{2} \end{cases}$$

TWO PREDATORS





DEMOGRAPHIC NOISE

- SIMULATION *(size = N)* :

$$prob = rate \cdot \Delta t$$

For each Δt:

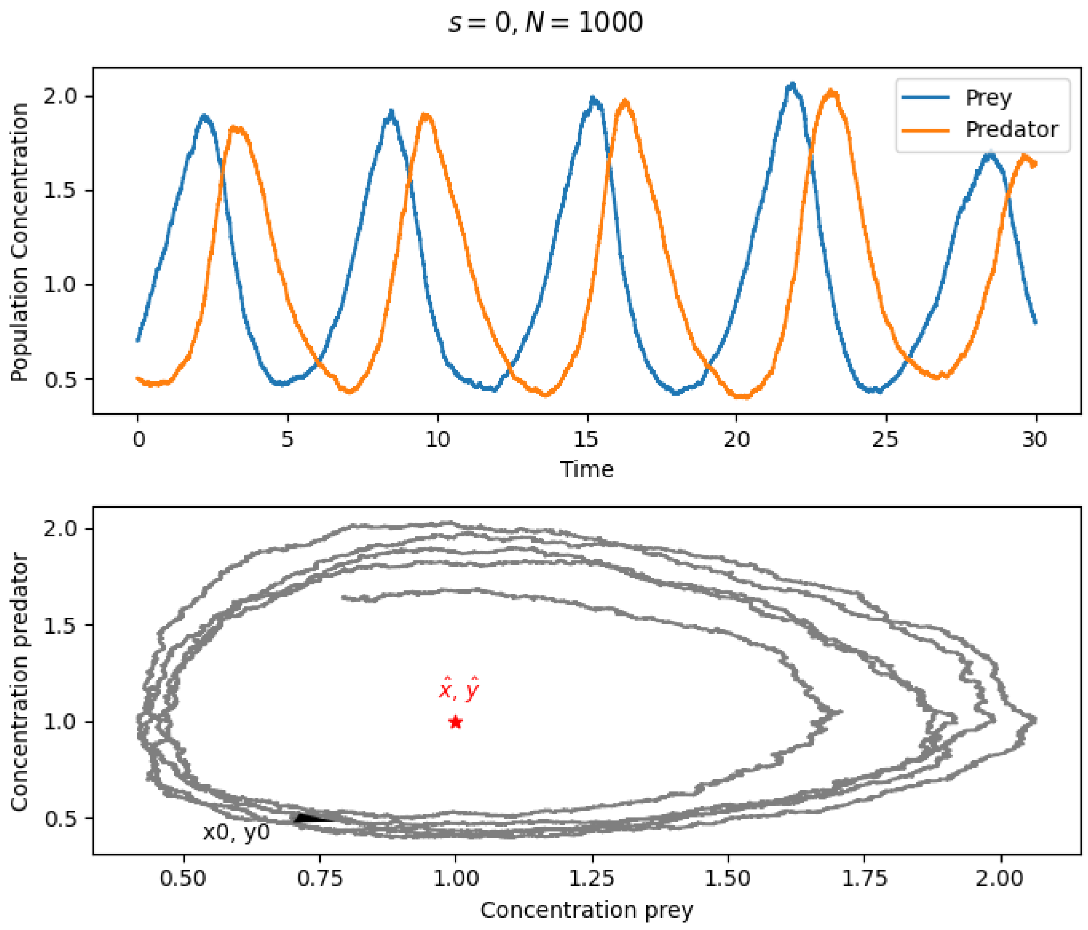
generate $x \cdot N$ random numbers ($\vec{\mu}$)

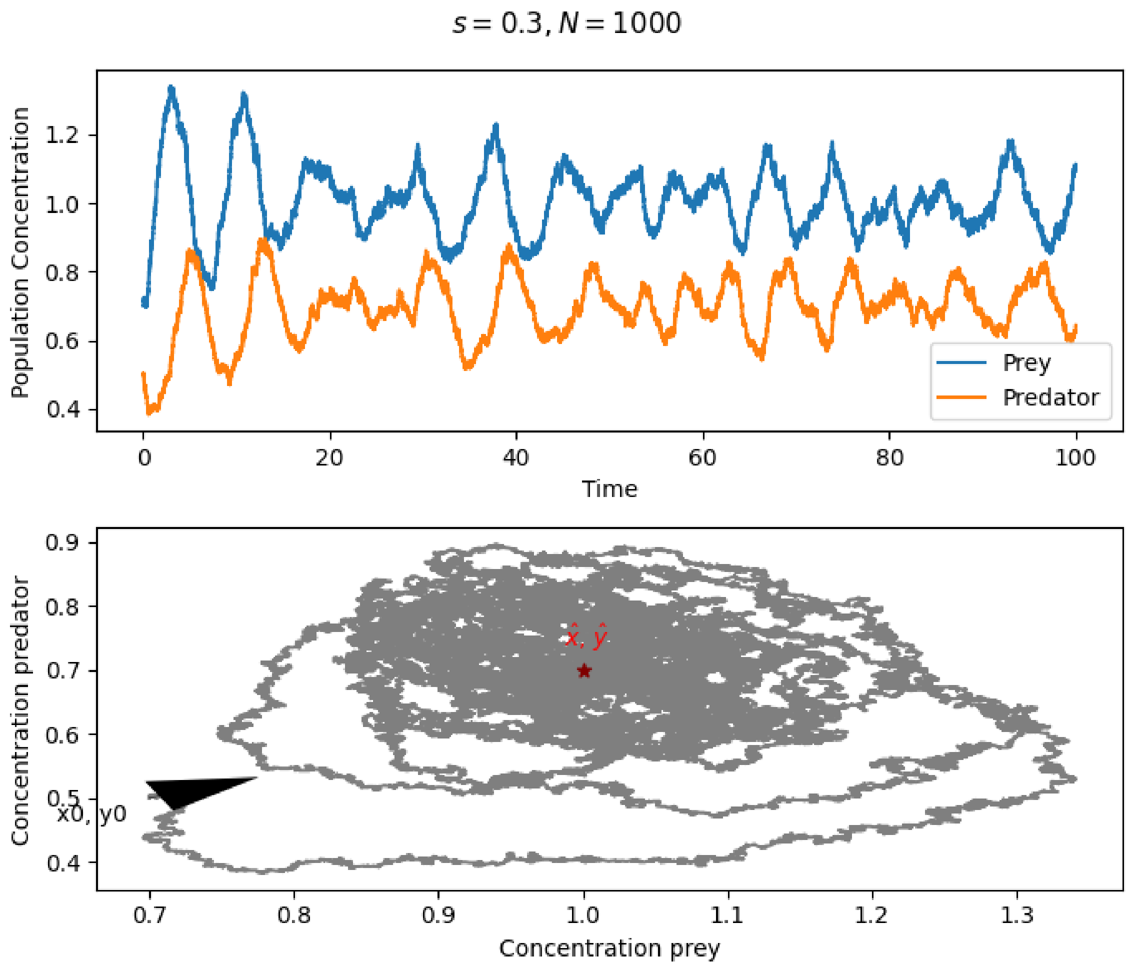
if $\mu_i < prob$ the reaction i is done

CLASSIC LOKTA-VOLTERRA

$$\begin{cases} X \to X + X : r \\ X + Y \to \emptyset + Y : a \\ X + Y \to Y + Y : b \\ Y \to \emptyset : d \\ X + X \to X + \emptyset : s \end{cases}$$

CLASSIC LOKTA-VOLTERRA

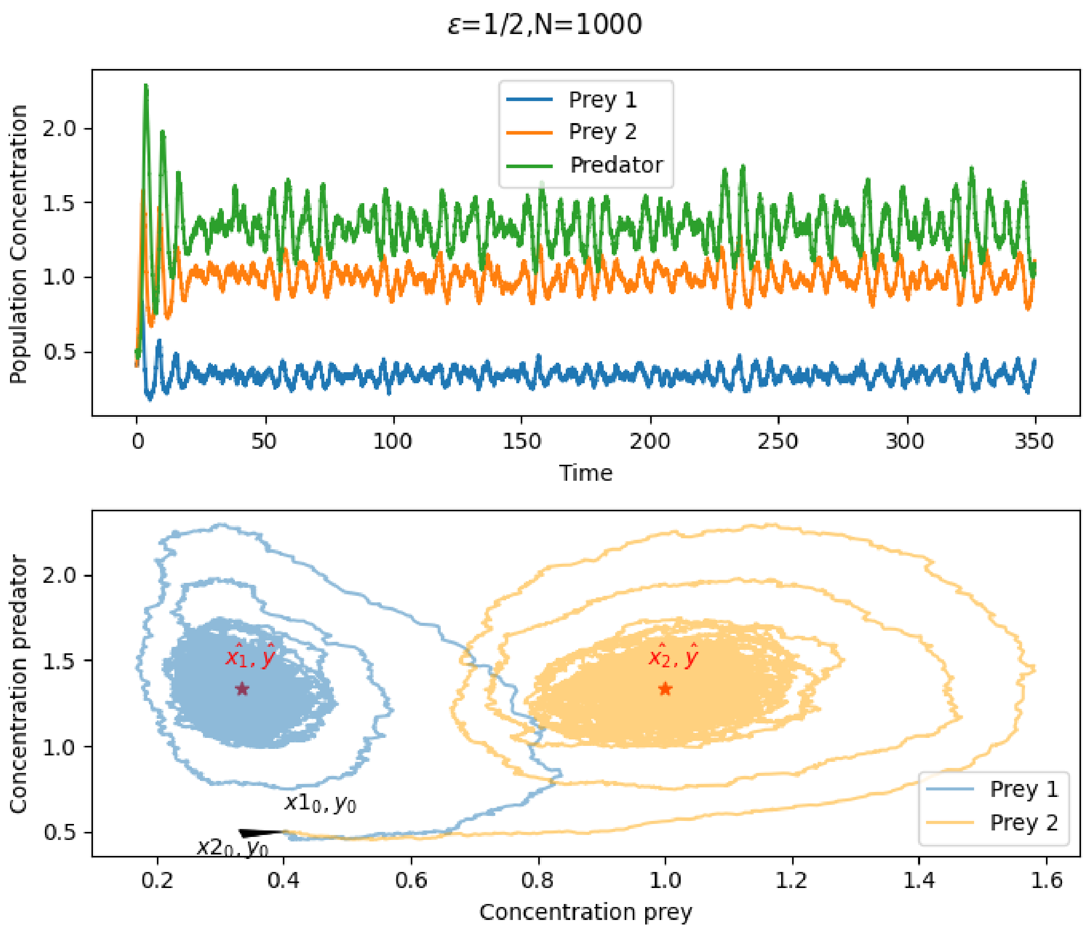


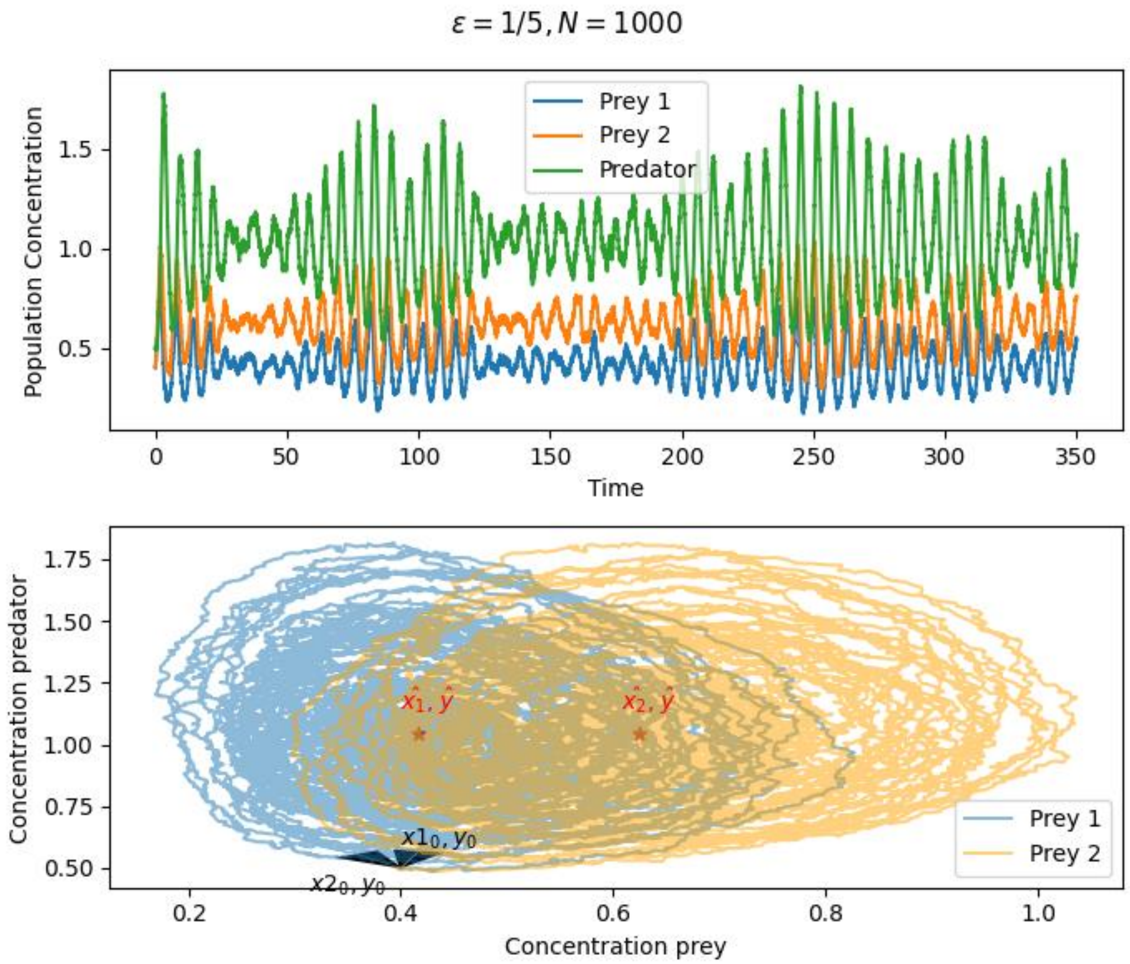


TWO PREY

$$
\begin{cases}
X_i \to X_i + X_j : r/2 \\
X_1 + Y \to \emptyset + Y : a(1+\epsilon) \\
X_2 + Y \to \emptyset + Y : a(1-\epsilon) \\
X_1 + Y \to Y + Y : b(1+\epsilon) \\
X_2 + Y \to Y + Y : b(1-\epsilon) \\
Y \to \emptyset : d
\end{cases}
$$

TWO PREY

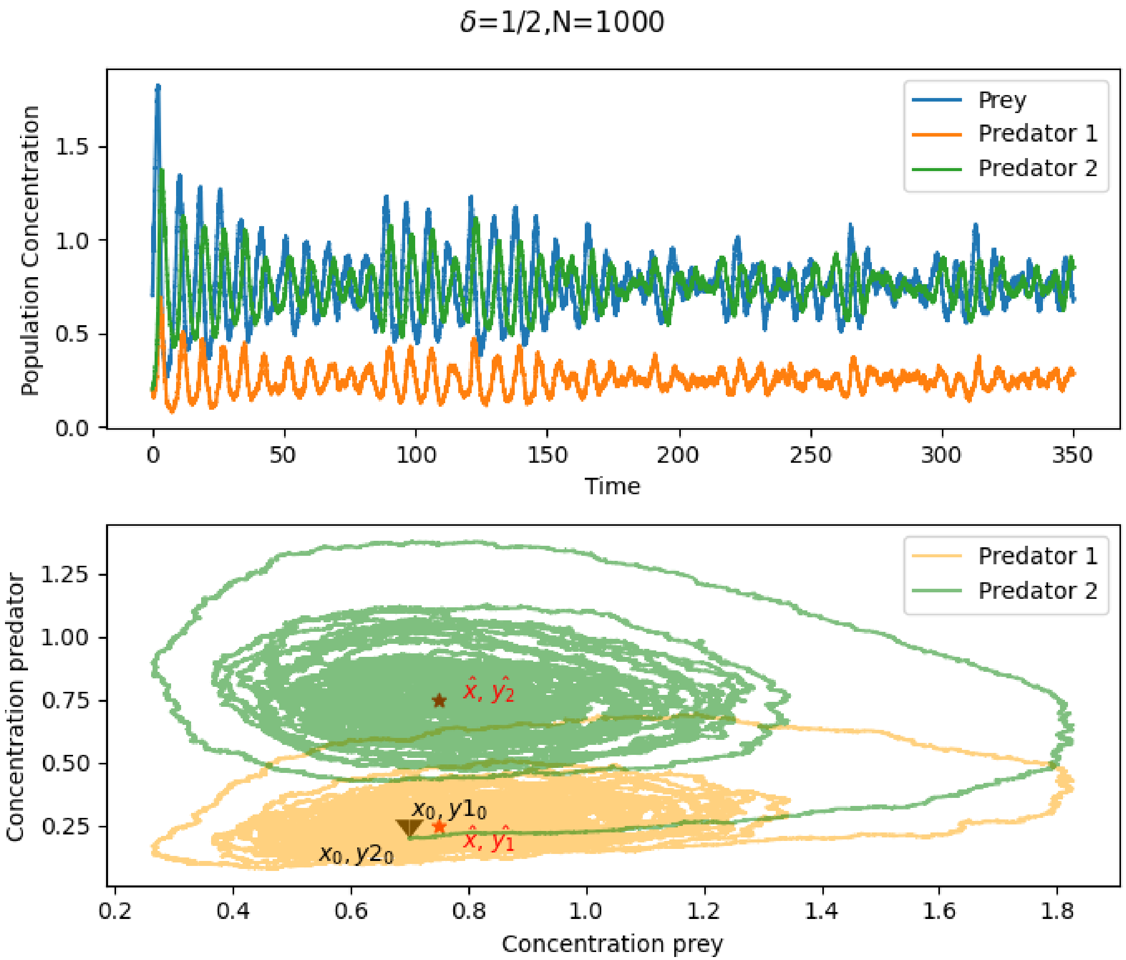

ε=1/2,N=1000
Prey 1
Prey 2
Predator
Population Concentration
Time
Concentration predator
Concentration prey
$\hat{x_1}, \hat{y}$
$\hat{x_2}, \hat{y}$
$x1_0, y_0$
$x2_0, y_0$

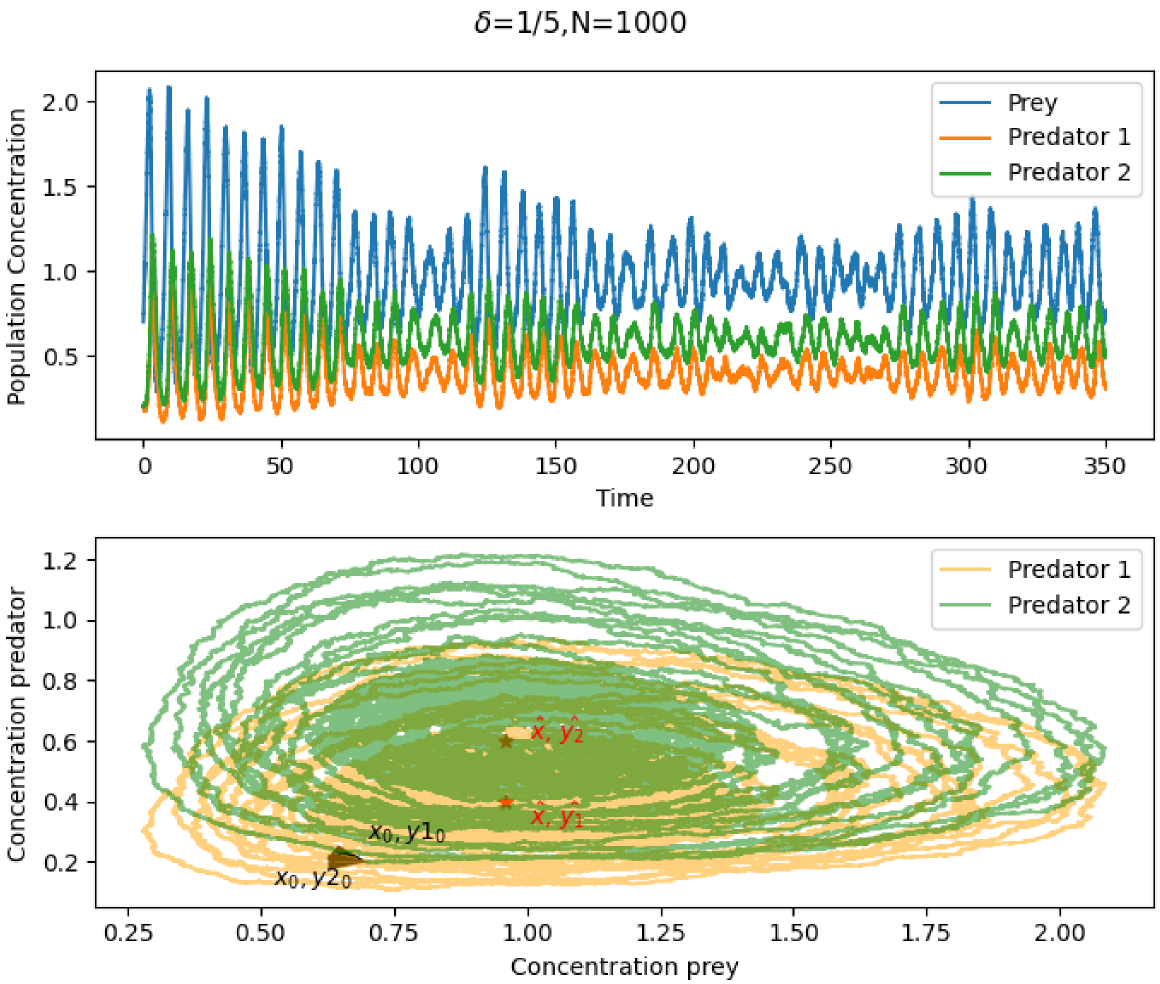

$\varepsilon = 1/5, N = 1000$
Prey 1
Prey 2
Predator
Population Concentration
Time
Concentration predator
Concentration prey
$\hat{x_1}, \hat{y}$
$\hat{x_2}, \hat{y}$
$x1_0, y_0$
$x2_0, y_0$

TWO PREDATOR

$$\begin{cases} X \to X + X : r \\ X + Y_i \to \emptyset + Y_i : a \\ X + Y_i \to Y_j + Y_i : b/2 \\ Y_1 \to \emptyset : d(1+\delta) \\ Y_2 \to \emptyset : d(1-\delta) \end{cases}$$

TWO PREDATOR





ECOEVOLUTION

- Model evolution of species
- Different inheriting ratios for the different phenotypes

TWO PREY ρ !

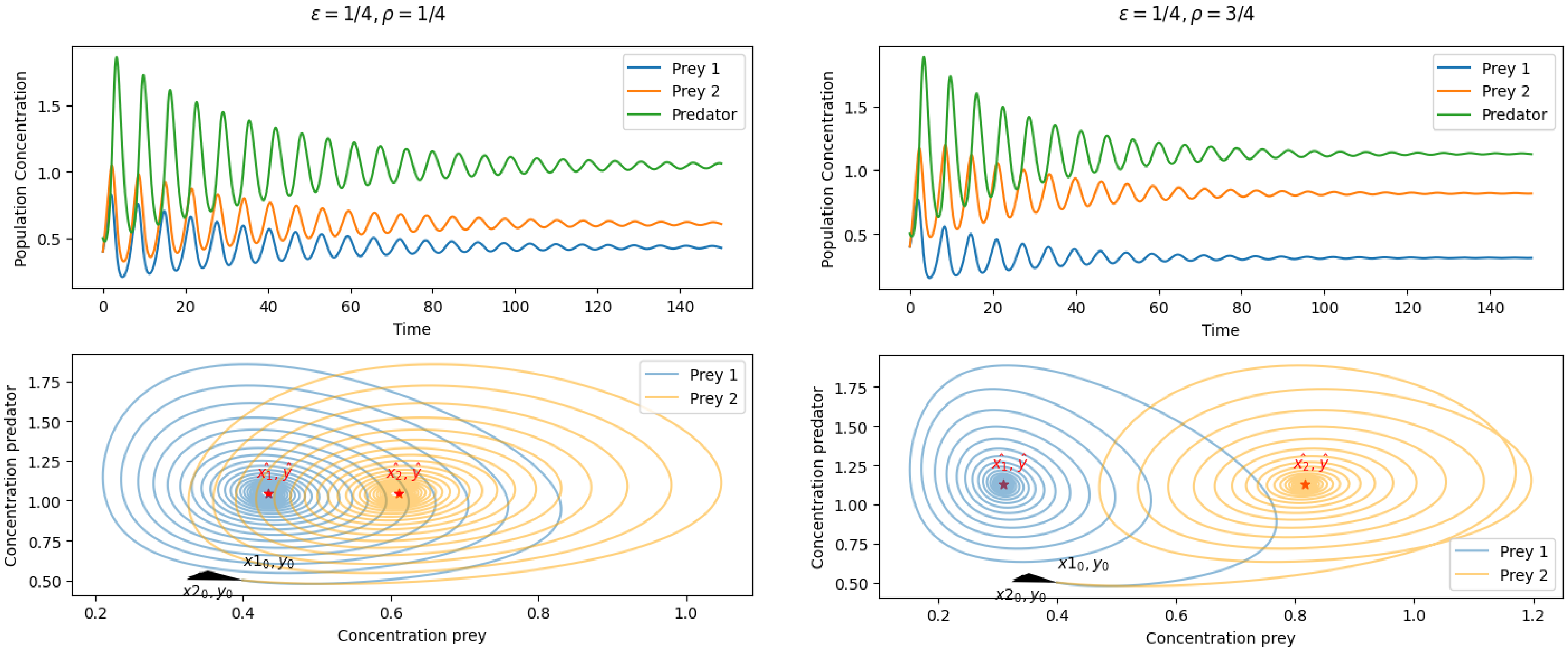
DETERMINISTIC

$$\begin{cases} \dot{x}_1 = r([\rho x_1 + (1-\rho)x_2] - a(1+\epsilon)x_1 y) \\ \dot{x}_2 = r([\rho x_2 + (1-\rho)x_1] - a(1-\epsilon)x_2 y) \\ \dot{y} = y(b[(1+\epsilon)x_1 + (1-\epsilon)x_2] - d) \end{cases}$$

STOCHASTIC

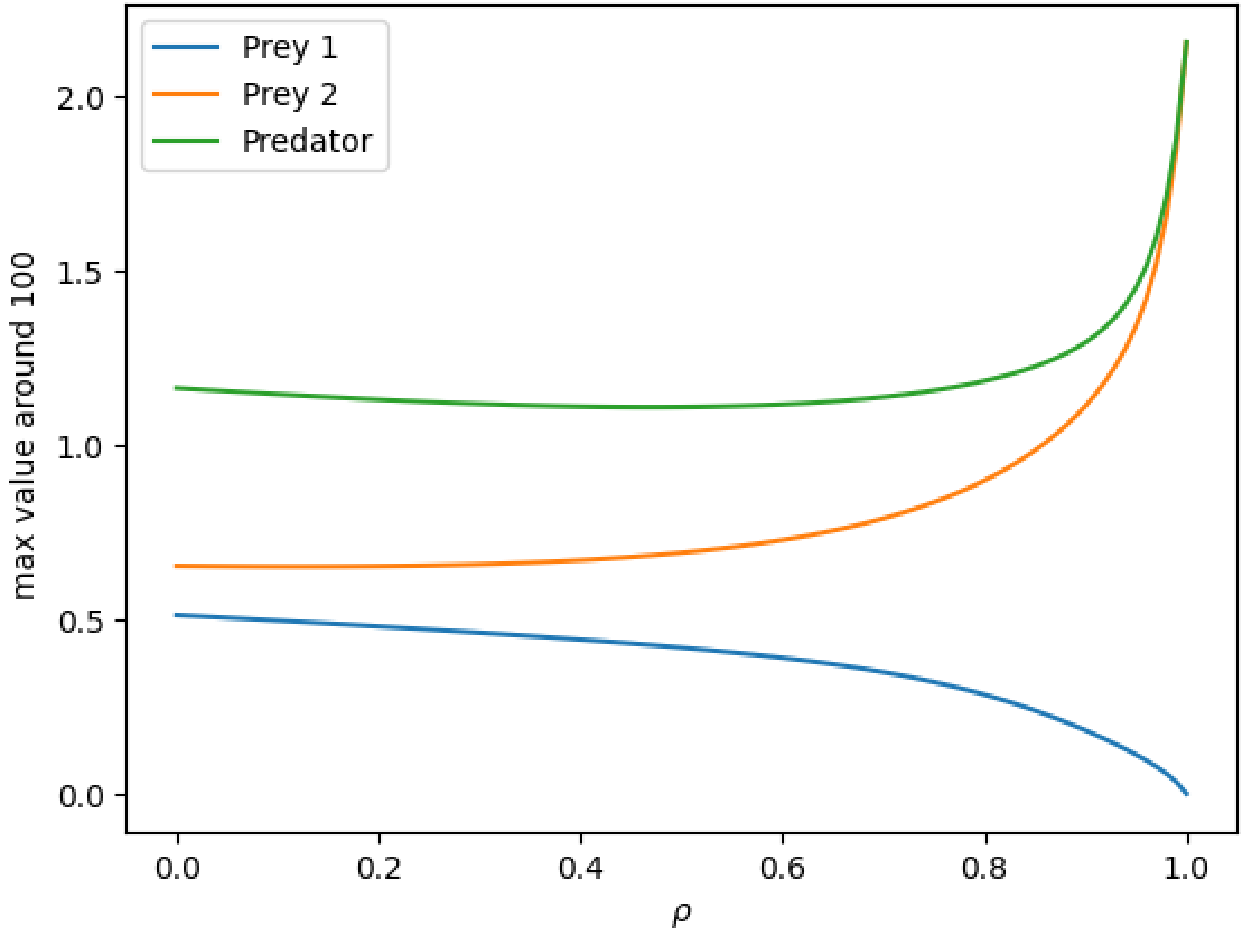
$$\begin{cases} X_i \to X_i + X_i : r\rho \\ X_i \to X_i + X_j : r(1-\rho) \\ X_1 + Y \to \emptyset + Y : a(1+\epsilon) \\ X_2 + Y \to \emptyset + Y : a(1-\epsilon) \\ X_1 + Y \to Y + Y : b(1+\epsilon) \\ X_2 + Y \to Y + Y : b(1-\epsilon) \\ Y \to \emptyset : d \end{cases}$$

TWO PREY (deterministic)

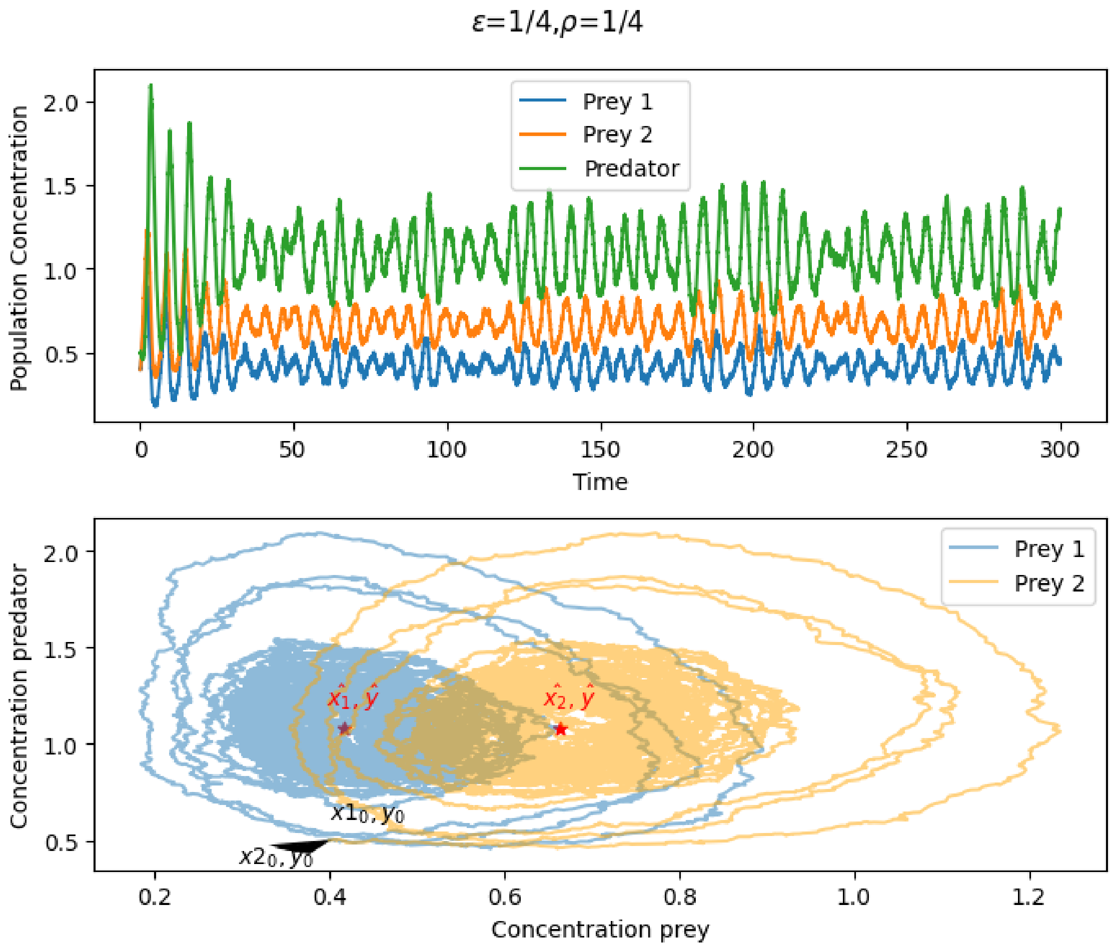


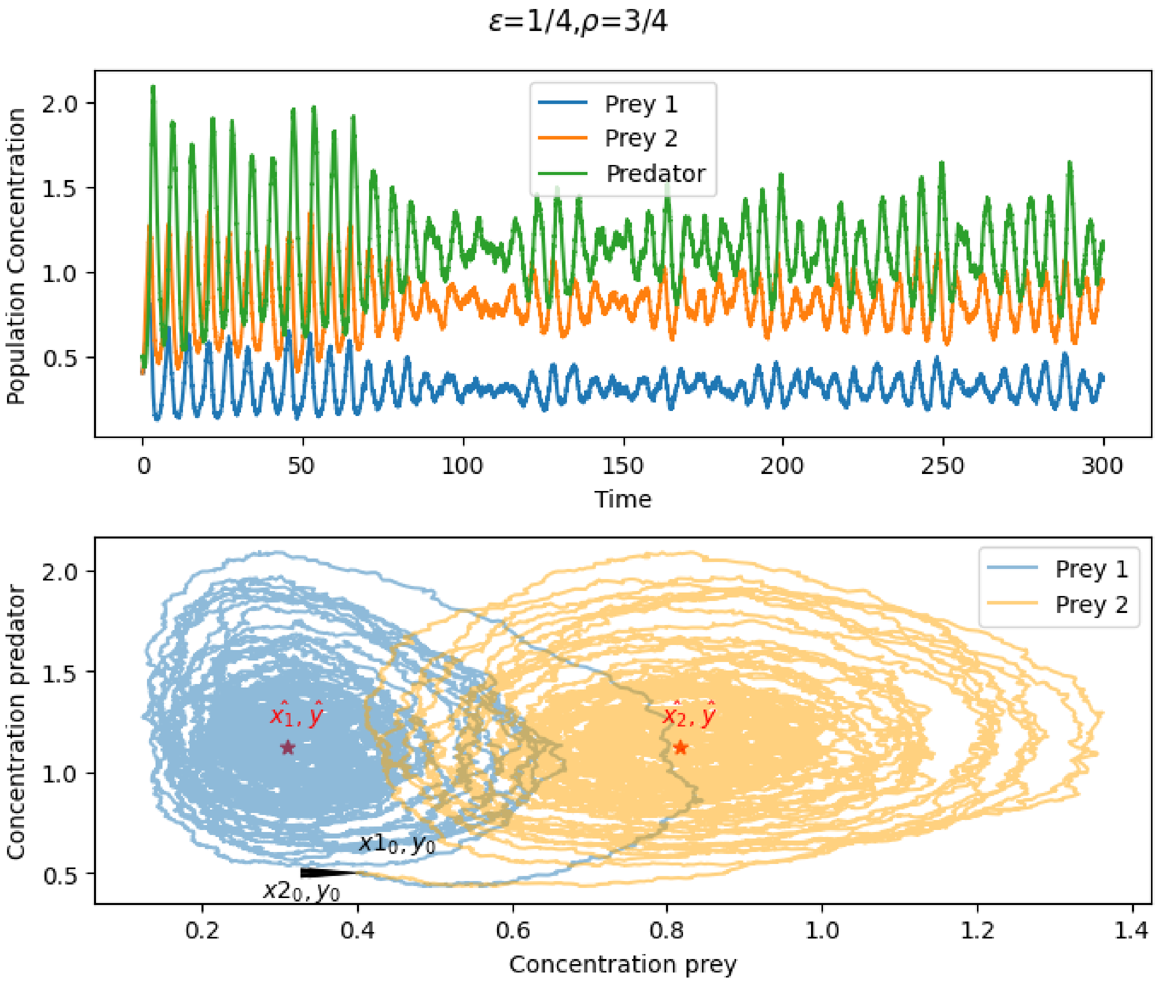
TWO PREY (deterministic)

How fast is the damping???



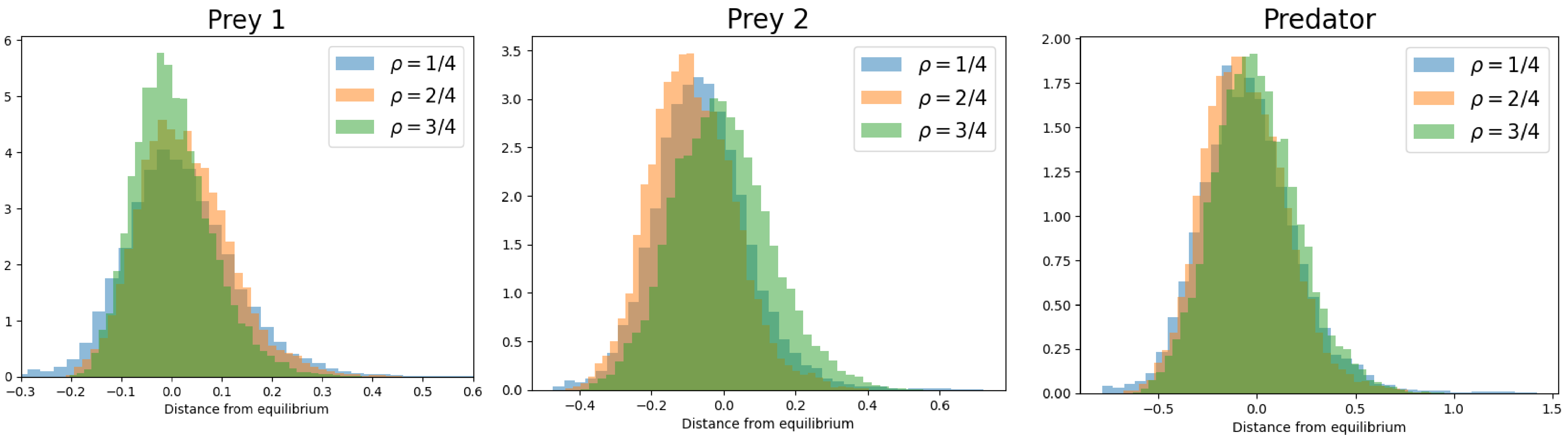
TWO PREY (stochastic)





TWO PREY (stochastic)

How big are the sustained oscillations?



TWO PREDATOR ρ !

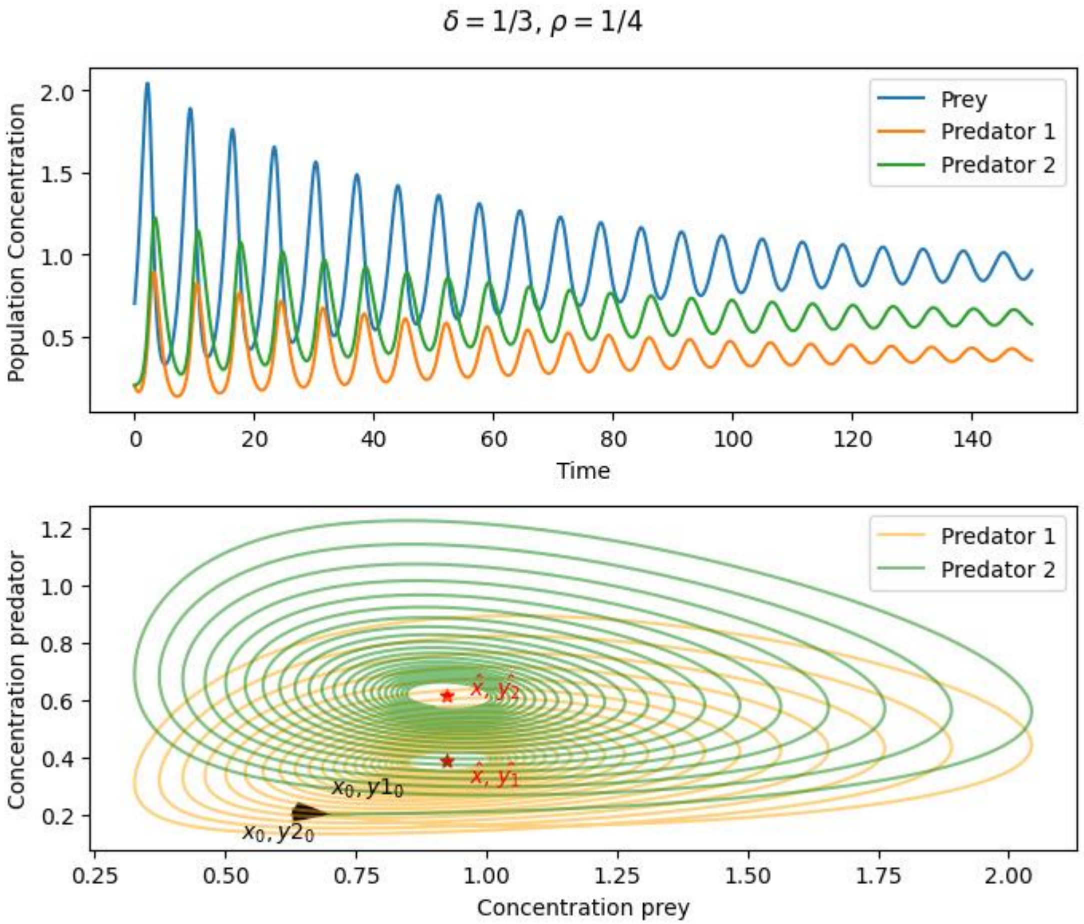
DETERMINISTIC

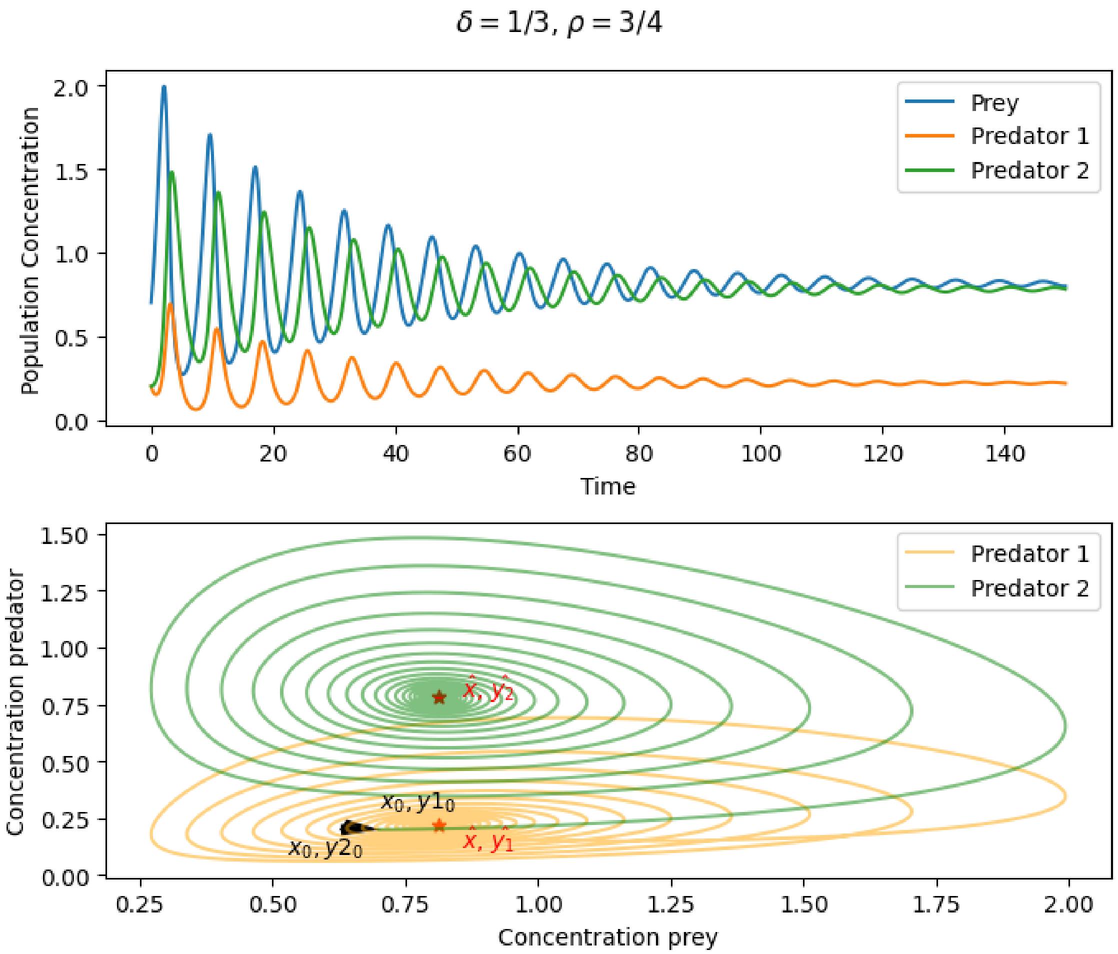
$$\begin{cases} \dot{x} = x(r - a[y_1 + y_2]) \\ \dot{y_1} = -d(1+\delta)y_1 + b[\rho y_1 + (1-\rho)y_2]x \\ \dot{y_2} = -d(1-\delta)y_2 + b[\rho y_2 + (1-\rho)y_1]x \end{cases}$$

STOCHASTIC

$$\begin{cases} X \to X + X : r \\ X + Y_i \to \emptyset + Y_i : a \\ X + Y_i \to Y_i + Y_i : \rho b \\ X + Y_i \to Y_j + Y_i : (1-\rho)b \\ Y_1 \to \emptyset : d(1+\delta) \\ Y_2 \to \emptyset : d(1-\delta) \end{cases}$$

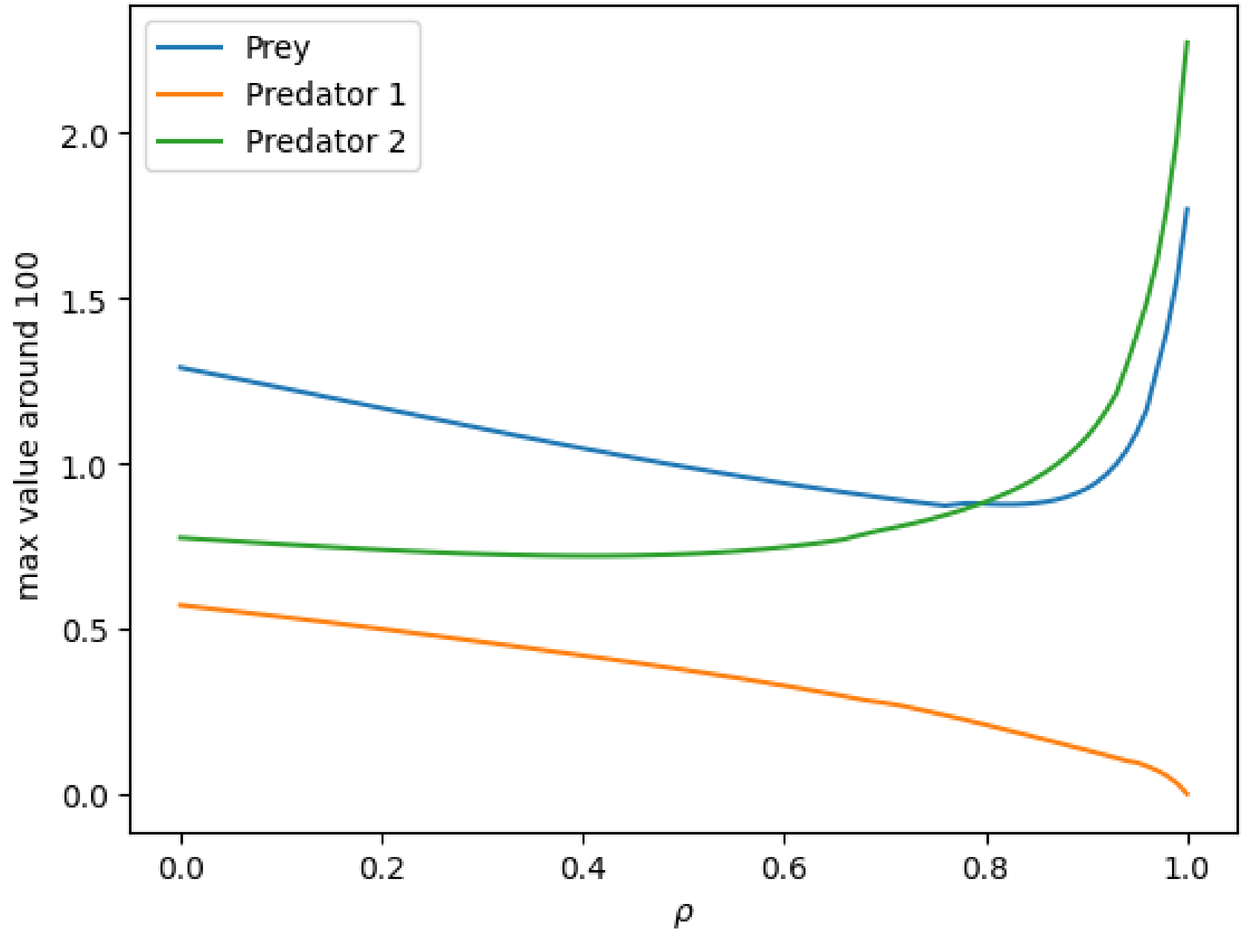
TWO PREDATOR (deterministic)



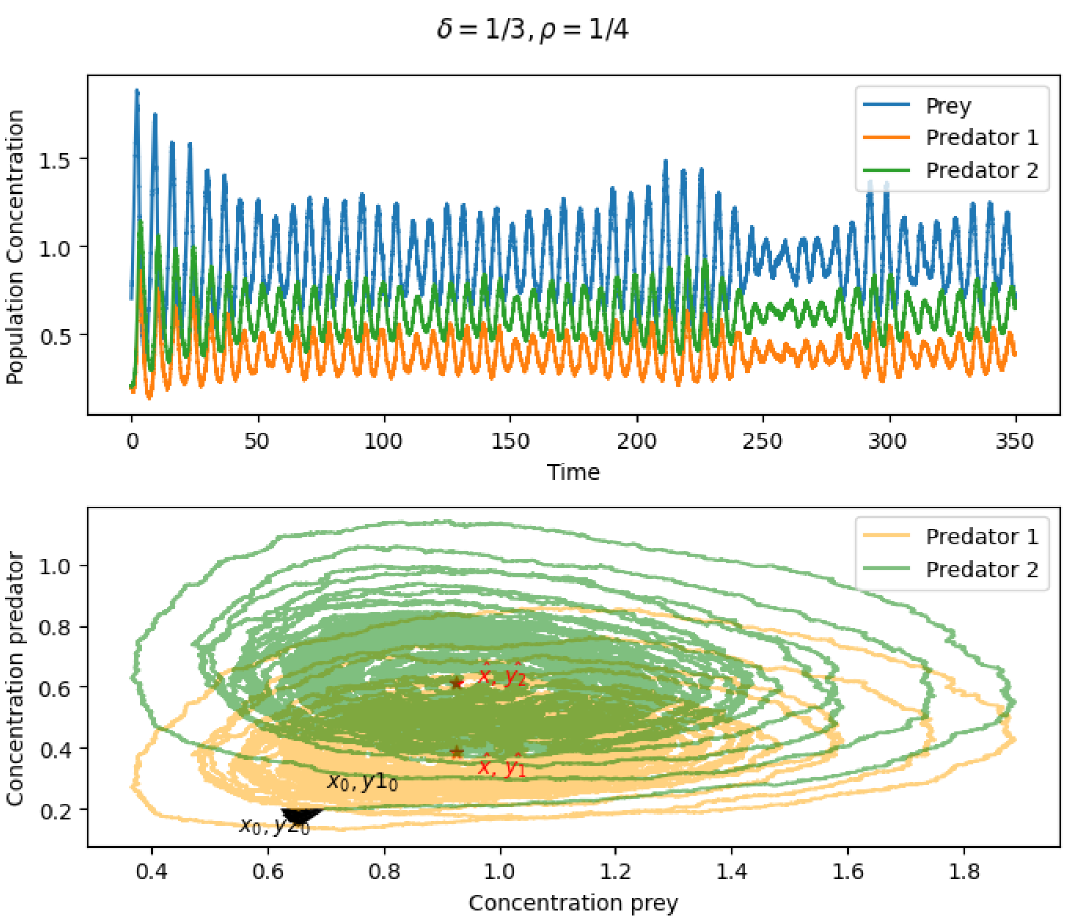


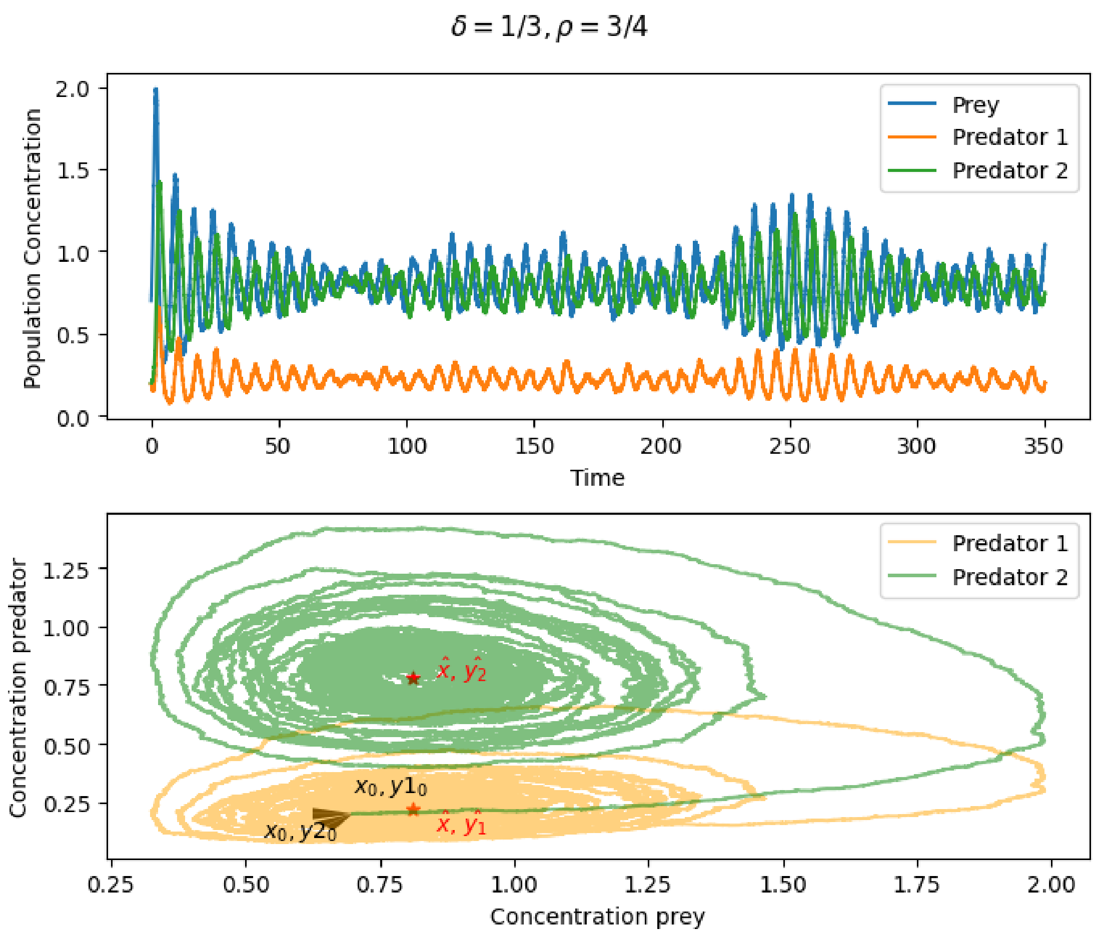
TWO PREDATOR (deterministic)

How fast is the damping???



TWO PREDATOR (stochastic)



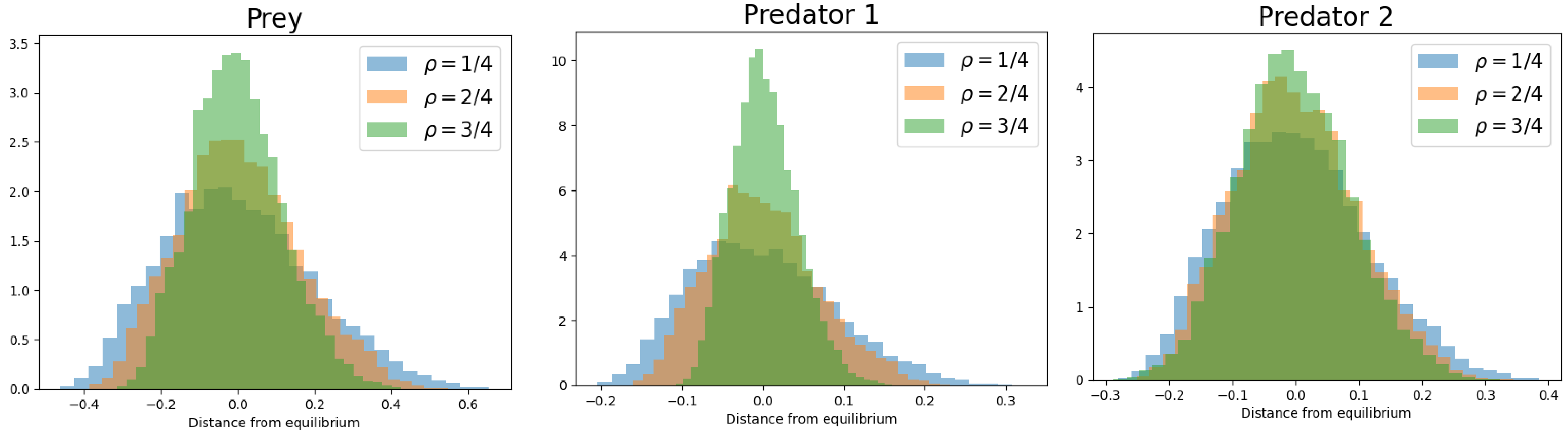


TWO PREDATOR (stochastic)

How big are the sustained oscillations?

$$Wider \leftarrow$$

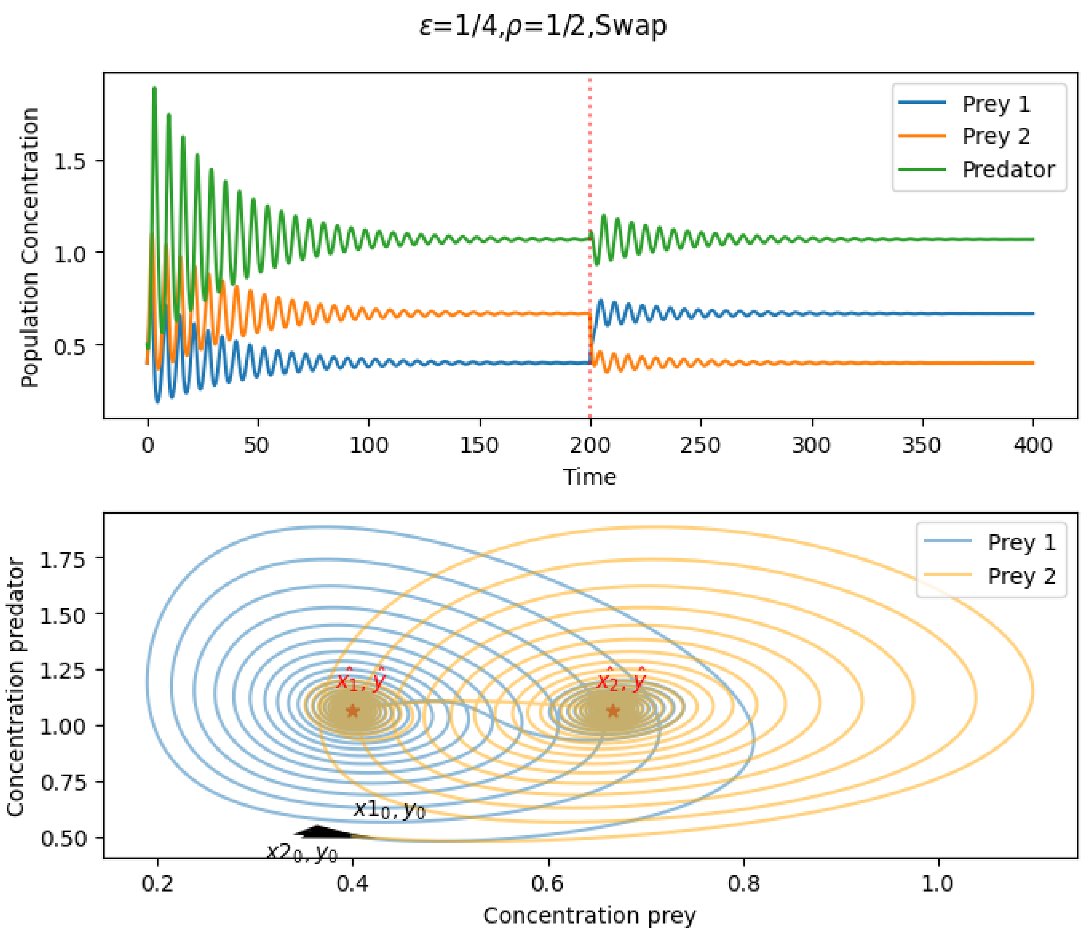
$$\rho = 1/4 > \rho = 1/2 > \rho = 3/4$$

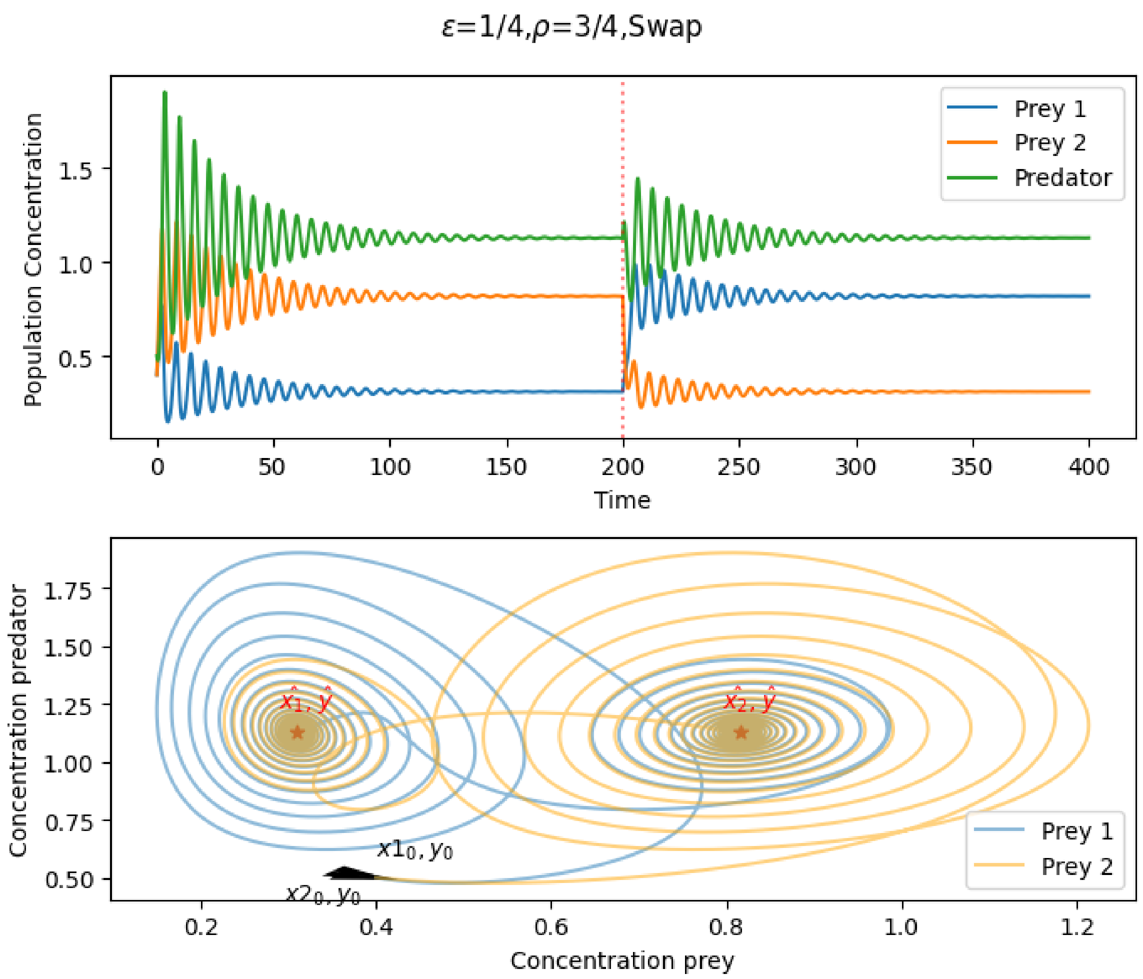


ENVIROMENTAL NOSIE

- Add enviromental noise to our ecoevolutionary **deterministic** model
- Two different types:
  - Swaping sign of $\epsilon \; and \; \delta$
  - Gaussian noise for $\epsilon \; and \; \delta$

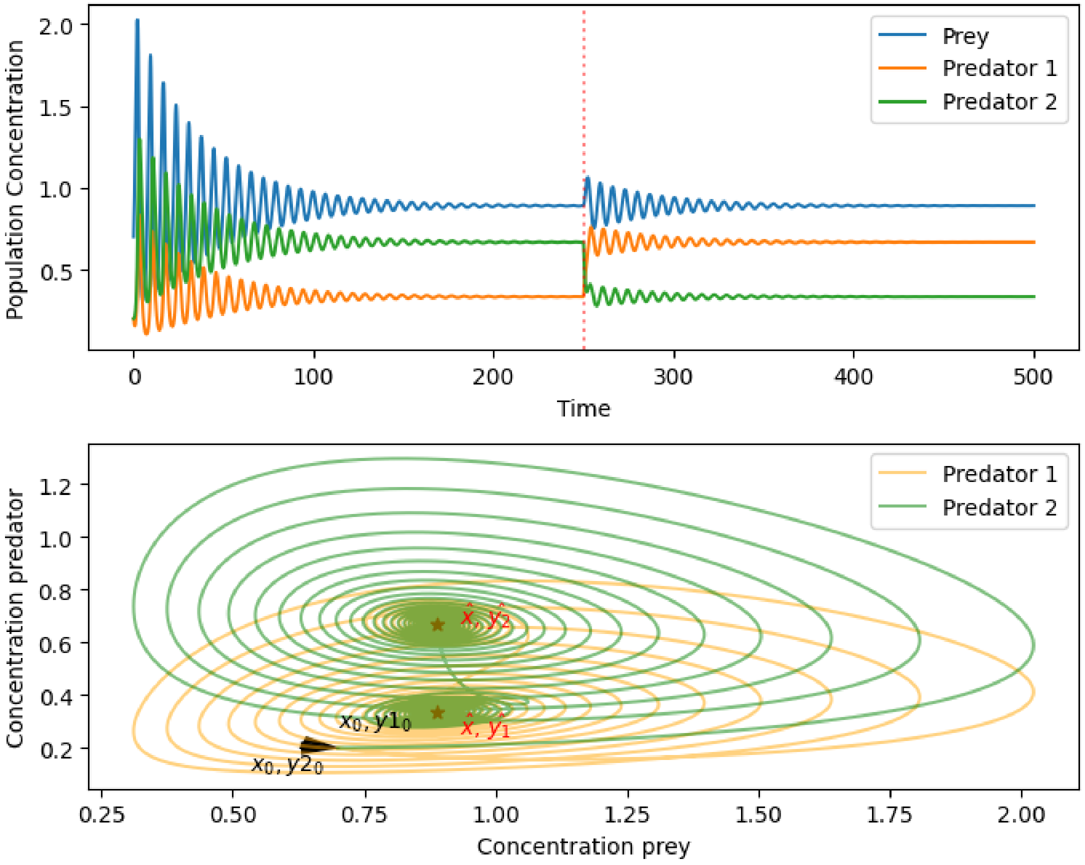
TWO PREY (swap)



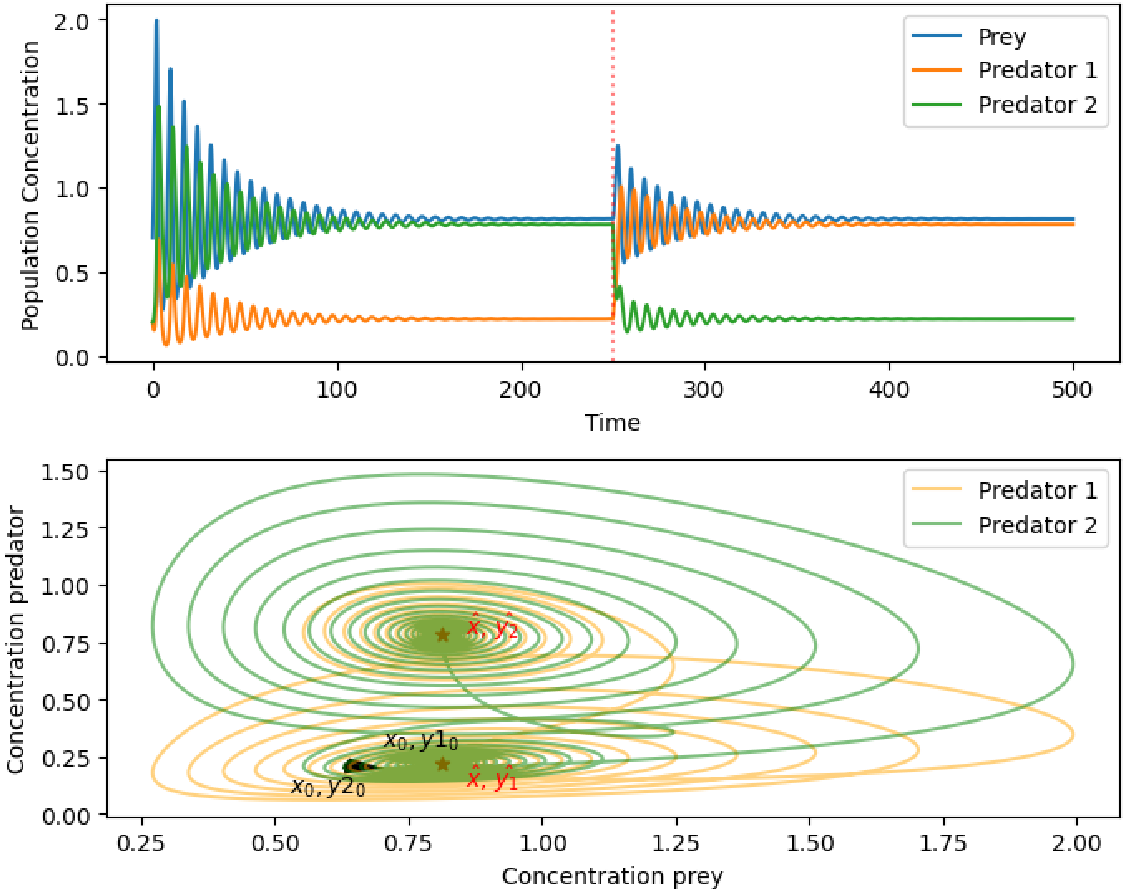


TWO PREDATOR (swap)

$\delta = 1/3, \rho = 1/2, Swap$



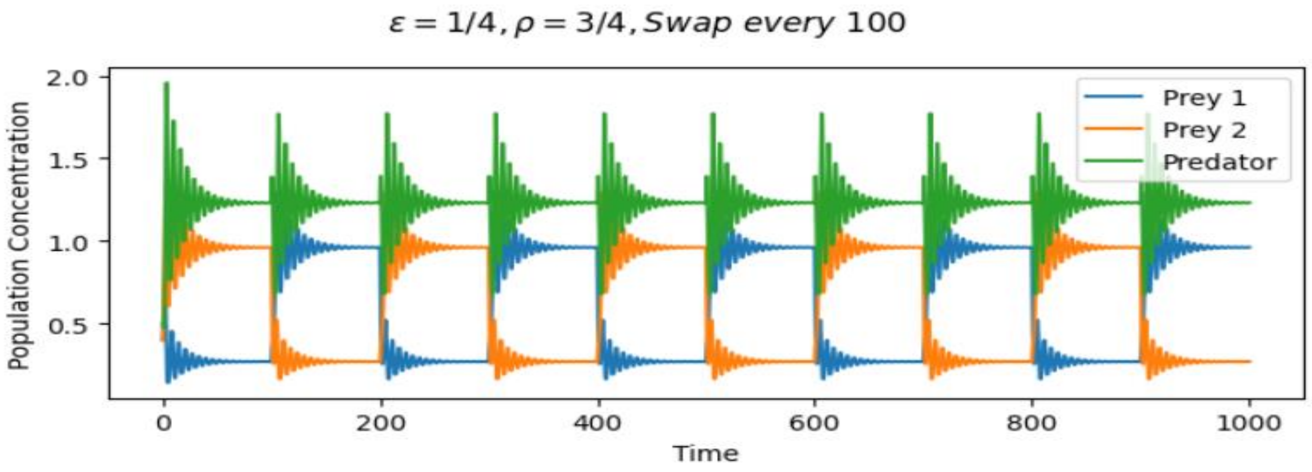
$\delta = 1/3, \rho = 3/4, Swap$

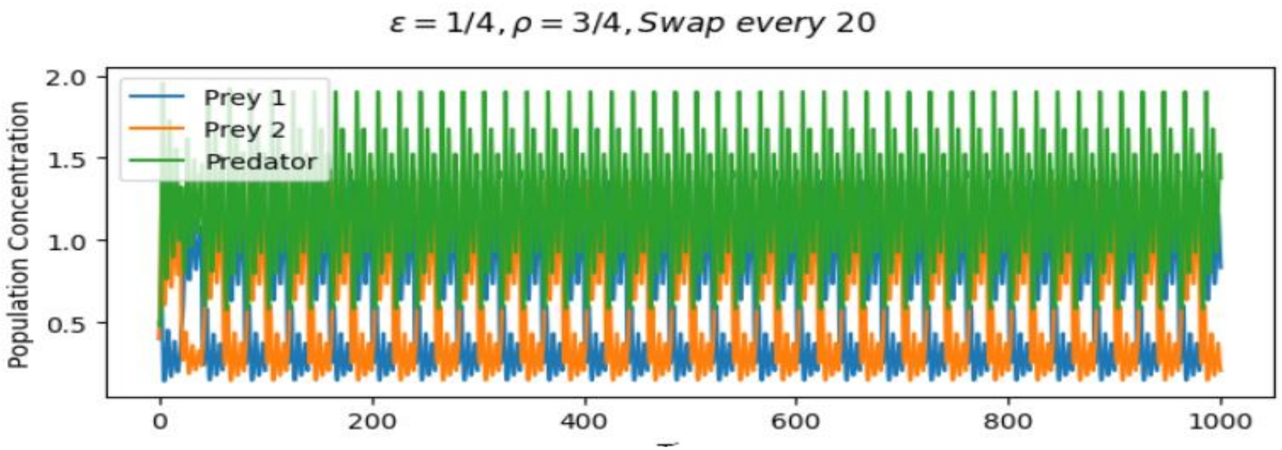


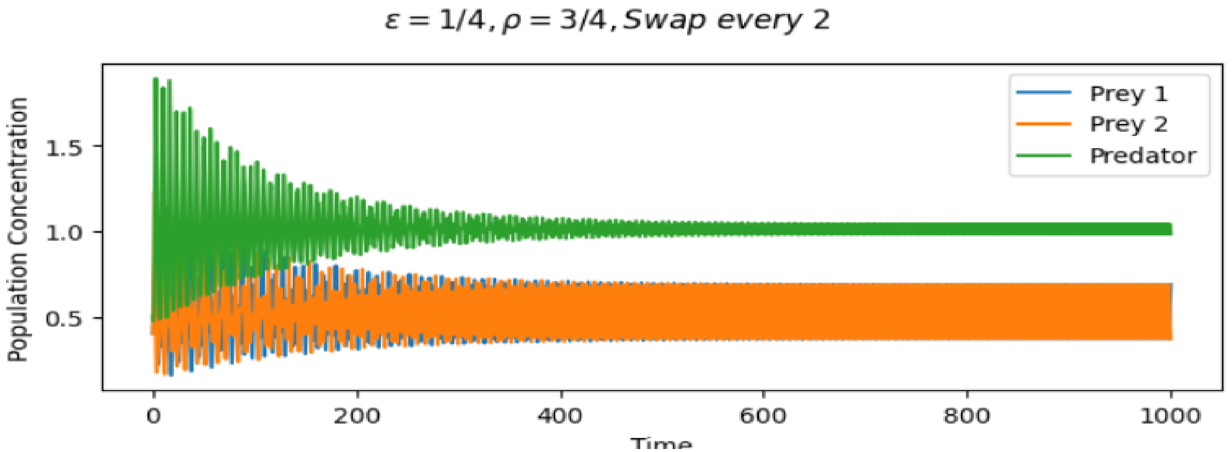
Can we extend this idea to get sustained oscillations???

Swap every appropiate number of time steps

Less swaps

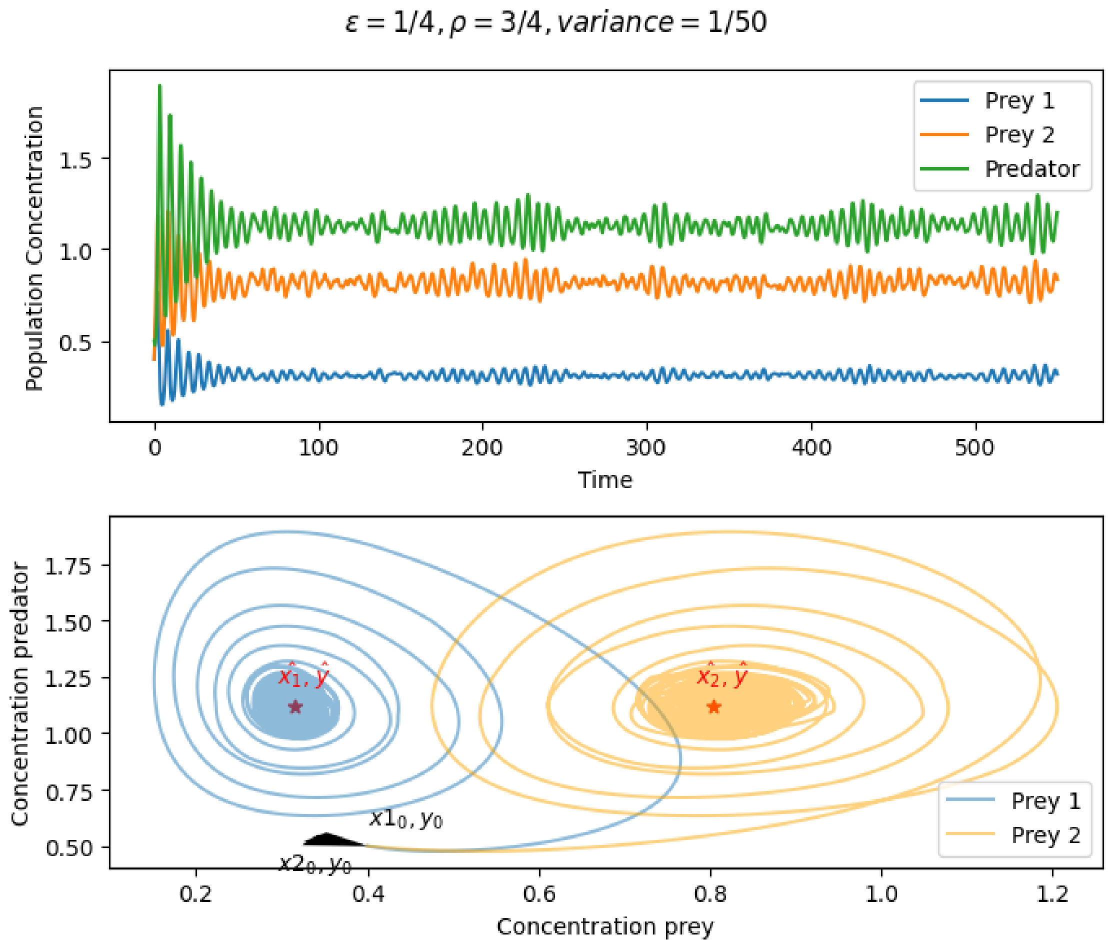


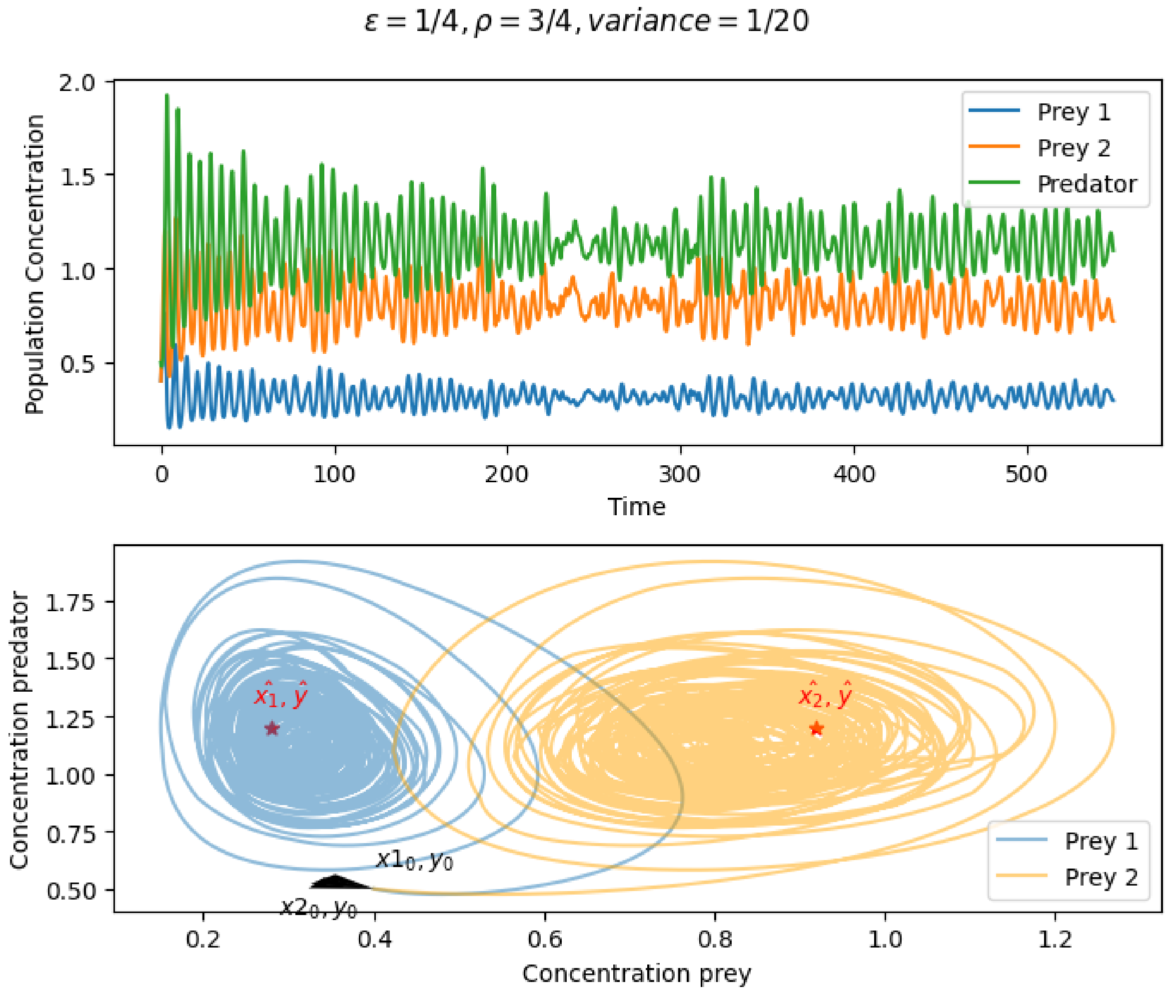




TWO PREDATOR (Gaussian)

Changes every time step (not every Δt!!)



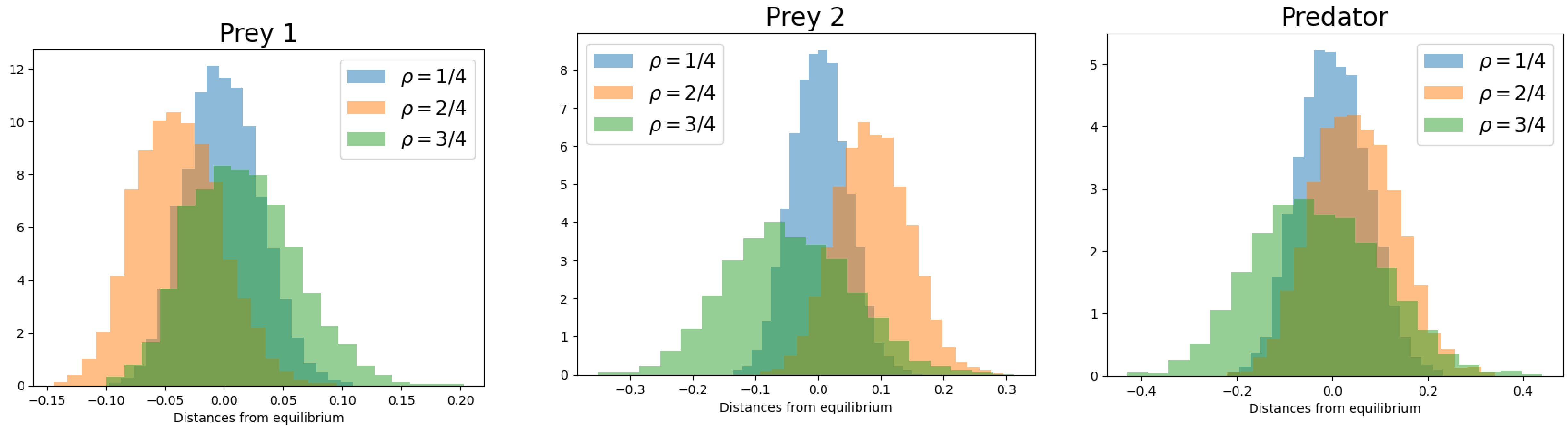


What happens for different ρ ?? *(variance=1/20)*

$$Wider \leftarrow$$

$$\rho = 3/4 > \rho = 1/2 > \rho = 1/4$$

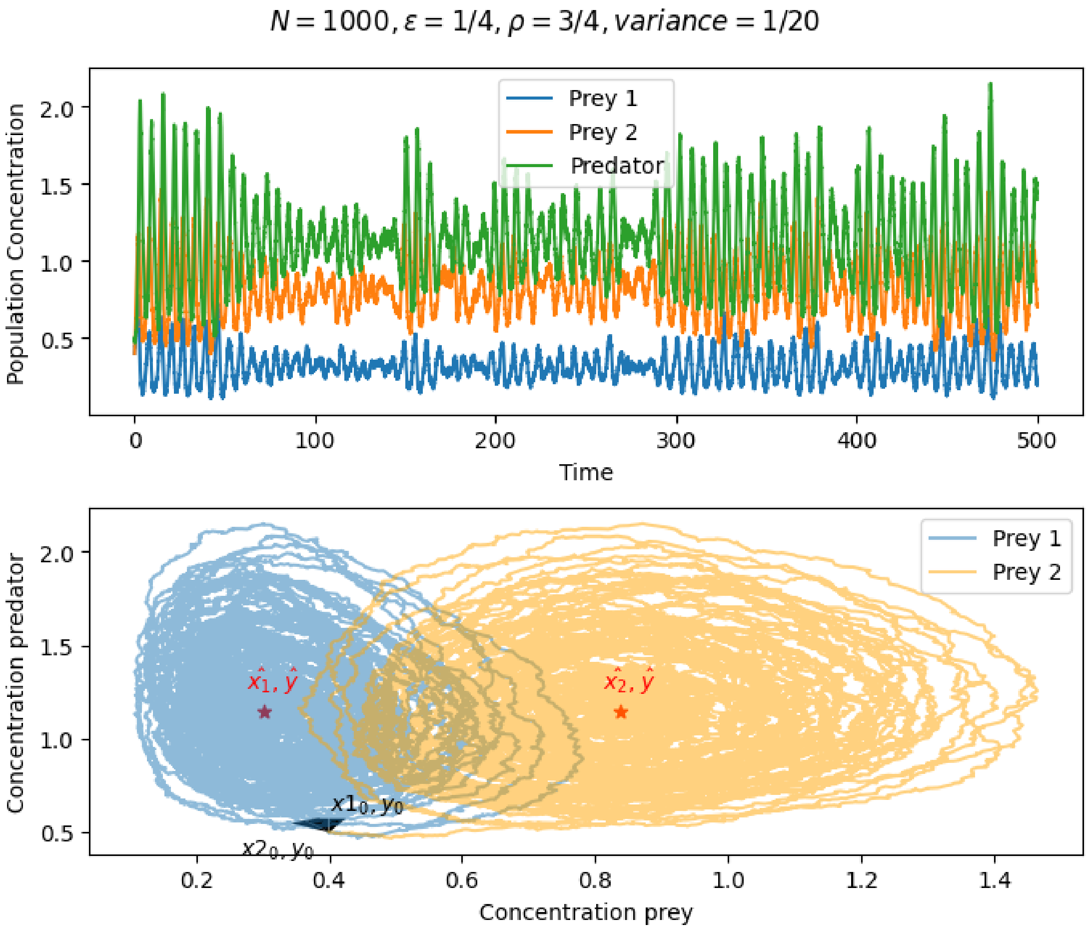
The results are contrary to the ones before!



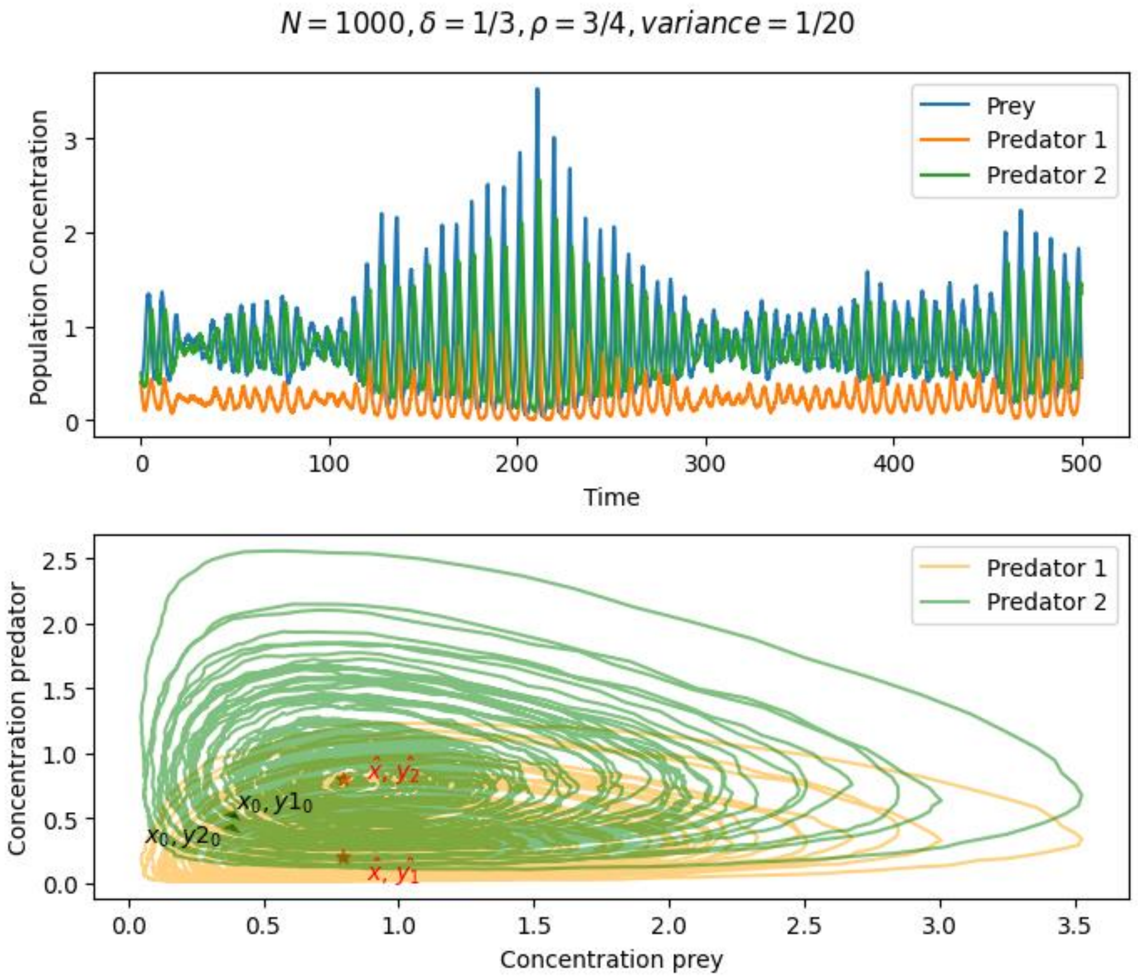
ALL TOGETHER?

- Add enviromental noise to our ecoevolutionary **stochastic** model

TWO PREY



TWO PREDATOR



CONCLUSIONS

- Both kind of noises induce sustained oscillations to a deterministically damped system.
- Enviromental and demographic noises have contrary effects when changing the ρ parameter,