Consider a discrete-time signal $s[n]$ which propagates through a multi-path channel and arrives at the receiving end as the signal

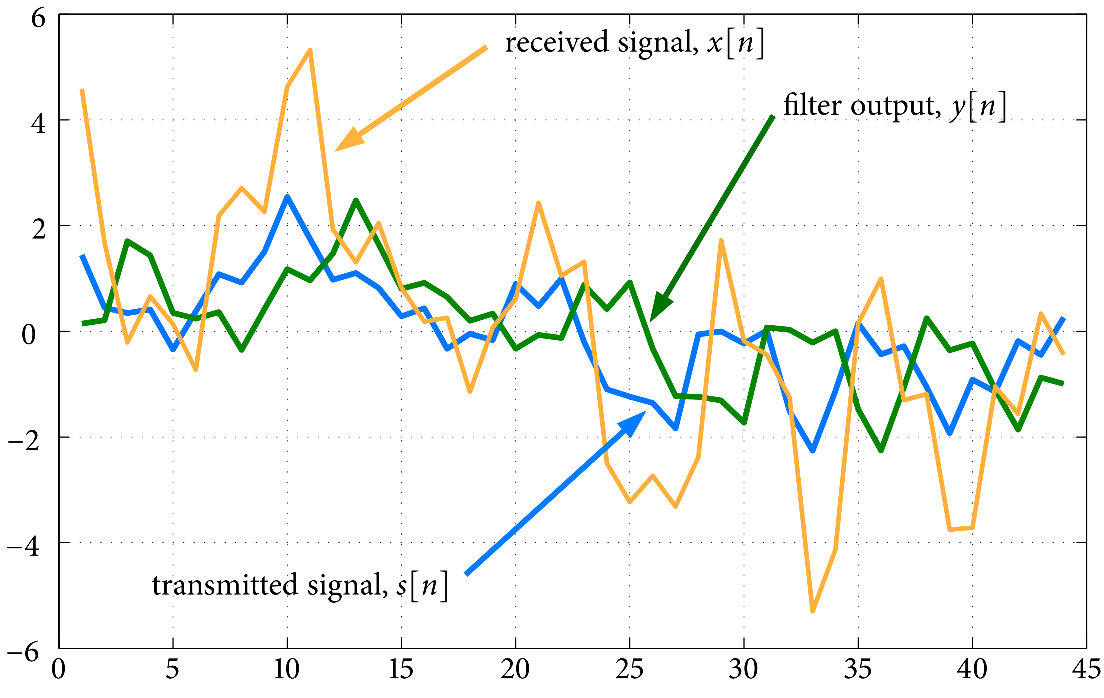
$$x[n] = s[n] + 2s[n-1] - s[n-2].$$

This signal is passed through a linear receive-side filter at which output the signal

$$y[n] = \frac{1}{17}x[n] - \frac{1}{7}x[n-1] + \frac{6}{17}x[n-2] + \frac{1}{7}x[n-3] + \frac{1}{17}x[n-4]$$

appears. It can serve as an estimate of the *delayed* transmitted signal $s[n-3]$:

$$y[n] = \underbrace{\frac{118}{119}s[n-3]}_{\substack{\text{delayed}\\ \text{\& scaled signal}}} + \underbrace{\frac{7s[n]-3s[n-1]+s[n-2]-s[n-4]-3s[n-5]-7s[n-6]}{119}}_{\text{intersymbol interference}}.$$



The filter used here is a *least-squares*[1] filter. In the example from the Figure above, the signal has got a zero mean and an auto-correlation of $\mathrm{E}\big[s[n]s[n-m]\big] = 0.7^{|m|}$. From this fact follows that the signal to intersymbol-interference ratio is about 119, or about 20.8 dB. This is

1 It tries its best to equalize the channel perfectly, that is, to make the combined impulse response of the channel and the filter equal to $\delta[n-3]$, but it simply hasn't got enough degrees of freedom to achieve this perfectly. So it makes the sum of the squares of the difference of the actual combined impulse response and the ideal $\delta[n-3]$ to its absolutue minimum.

not bad, but it could be improved if the knowledge about the signal's auto-correlation was actually used in the design of the filter. But the least-squares filter does not care about it! Its design solely depends on the coefficients of the multi-path channel. However, the *linear minimum mean square error* filter does care for the signal correlation and thereby may produce better results. In our example case, the output of such a filter equals

$$y[n] = \frac{39570}{855703}x[n] - \frac{1}{7}x[n-1] + \frac{551}{1553}x[n-2] + \frac{1}{7}x[n-3] + \frac{70630}{855703}x[n-4],$$

and you shall learn in this course how and why this is so. If this $y[n]$ is taken now as an estimation of $s[n-3]$ (scaled down by the factor 0.99531), the signal to intersymbol-interference ratio becomes about 181, or some 22.6 dB. Comparing to the least-squares filter from before, this is about 1.8 dB better! This gain in performance comes about because the linear minimum mean square error filter uses information about the auto-correlation function of the signal,[1] while the least-squares filter does not.[2]

1 It also uses the auto-correlation of the noise and the cross-correlation between the noise and the signal. In the example studied here, there is no noise, though.

2 However, one could also make the point that the least-squares filter has the *advantage* that it does not *need* to know the auto-correlation of the signal!