Lecture 14: The Cold Dark Matter power spectrum and galaxy formation

L: The Cold Dark Matter theory assumes that the dark matter is made of collisionless particles or objects, which did not have any significant primordial (or initial) velocity dispersion. In other words, before non-linear collapse and shell crossing, cold dark matter has a single velocity at every spatial location and moves following geodesics or orbits in the gravitational potential of the evolving density fluctuations, without any other interactions among the dark matter or with baryons or radiation.

The transfer function of a model such as the Cold Dark Matter theory is the function we use to transform the original power spectrum of the perturbation as they come out of the horizon, to the final perturbations that evolve in the non-linear regime and are measured at the present epoch.

If $P_i(k)$ is the power spectrum of mass density fluctuations as they appear on the horizon when the universe expands, and $P(k)$ is the power spectrum, then the transfer function $T(k)$ transforms from one to the other according to:

$$P(k) = P_i(k) \left[k^2 T(k)\right]^2 \ . \tag{1}$$

The reason we introduce the k^2 factor is because $T(k)$ is defined by convention to relate the original to final potential perturbation.

The transfer function $T(k)$ can be precisely computed for any theory of dark matter, which tells us how dark matter behaves during the gravitational evolution of fluctuations. Cold dark matter assumes that the dark matter is pressureless and collisionless, so it does not interact with baryons and radiation or with itself other than through gravity, and that it is made of objects or particles that are initially cold, meaning that they do not have any primordial velocity dispersion. Neutrinos with mass, on the other hand, are hot dark matter, because they are initially at the temperature they were left at when they stopped interacting with radiation.

For scales that are large enough to have evolved in the matter dominated regime from the time they entered the horizon, and ignoring the effect of dark energy in slowing the growth of the fluctuations close to the present time,

$$k^2 T(k) \simeq \frac{a}{a_{\rm hor}(k)} \ , \tag{2}$$

where $a_{\rm hor}(k)$ is the scale factor when the horizon scale was the scale of the perturbation, $\sim \pi/k$.

There is a special scale in the transfer function: the comoving horizon size at the epoch of equalization, at redshift $1 + z_{\rm eq} = \Omega_{m0}/\Omega_{r0}$:

$$r_{\rm eq} \simeq \frac{c(1+z_{\rm eq})}{H(z_{\rm eq})} \simeq \frac{c}{H_0 \Omega_{m0}^{1/2} (1+z_{\rm eq})^{1/2}} = \frac{c\Omega_{r0}^{1/2}}{H_0 \Omega_{m0}} \simeq 130\,{\rm Mpc} \ . \tag{3}$$

At comoving scales $r \gg r_{\rm eq}$, fluctuations have grown in the mass-dominated regime. In the matter dominated epoch, the horizon is

$$r_{\rm hor} = \frac{2c\, a^{1/2}}{H_0 \Omega_{m0}^{1/2}} \ , \tag{4}$$

so the scale factor $a_{\rm hor}$ at which $r = r_{\rm hor}$ is

$$a_{\rm hor} = \frac{H_0^2 \Omega_{m0}\, r^2}{4c^2} \ . \tag{5}$$

If the initial power spectrum is a power-law of the form $P_i(k) \propto k^{-3-\epsilon}$, where $\epsilon = 0$ corresponds to constant amplitude of fluctuations as they enter the horizon ($\Delta_i^2 \sim k^3 P_i \sim$ constant), then since $k^2 T(k) \propto a_{\rm hor}^{-1} \propto r^{-2} \propto k^2$,

$$\frac{P(k)}{a^2} \propto k^{-3-\epsilon} k^4 \propto k^{1-\epsilon} \propto k^{n_s} \ . \tag{6}$$

Observational determinations from the CMB fluctuations and the present large-scale structure have shown that $\epsilon \simeq 0.035$, so n_s is very close to one. This small value of ϵ is an important information for the *inflation scenario* for the origin of the Universe and its fluctuations.

On the other hand, in the limit of small scales, the fluctuations have grown by the same factor from the equalization epoch to the present, except for the logarithmic growth of the cold dark matter fluctuations that took place during the radiation-dominated epoch. So, the growth $k^2T(k)$ was practically constant except for this logarithmic increase with k, which leads to $P(k)/a^2 \propto k^{-3-\epsilon}$.

Note that the growth of $P(k)$ in the linear regime as a^2 will slow down as the present time is approached, when dark energy becomes dominant over matter.

L: This is the basic shape of the Cold Dark Matter power spectrum: $P(k) \propto k^{n_s}$ on very large scales, with n_s very close to one (except for the effects of dark energy on near-horizon scales today which have been affected by the slowed growth factor due to dark energy), and $P(k) \propto k^{-4+n_s}$ on very small scales with a slow logarithmic increase of $P(k)k^3$, with the change of slope occuring over a very broad range around the comoving scale $r_{\rm eq}$. On top of this, there are also effects of *baryon acoustic oscillations*, the acoustic waves that were propagating before the recombination epoch which have affected the mass power spectrum in addition to the CMB fluctuations; furthermore, the fact that a small fraction of the dark matter should be neutrinos which are known to have a small mass from various experiments, which act as hot dark matter, also implies small changes in the power spectrum. The precise shape of $P(k)$ is being measured with a variety of methods in cosmology, and is one of our probes to understand the composition and the origin of fluctuations in the Universe.