

# A different perspective on the vev insertion approximation for electroweak baryogenesis

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**ABSTRACT:** In the vev insertion approximation (VIA) the spacetime dependent part of the mass matrix is treated as a perturbation. We calculate the source terms for baryogenesis expanding both the self-energy and propagator to first order in mass insertions, which gives the same results as the usual approach of calculating the self-energy at second order and using zeroth order propagators. This procedure shows explicitly the equivalence between including the mass in the free or in the interaction Lagrangian. The VIA source then originates from the same term in the kinetic equation as the semi-classical source, but at leading order in the derivative expansion (the expansion in diamond operators). On top, another type of derivative expansion is done, which we estimate to be valid for a bubble width larger than the inverse thermal width. This cuts off the divergence in the VIA source in the limit that the thermal width vanishes.

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In electroweak baryogenesis the matter-antimatter asymmetry of the universe is created during the electroweak phase transition [1–4]. The scenario relies on new physics at the electroweak scale, which can be probed by the LHC [5–15], electric dipole moment measurements [16–22], and possibly future gravitational wave observatories [23–25]. This requires accurate theoretical predictions.

The dynamics of the phase transition is a non-equilibrium, non-perturbative, and finite-temperature process, and to arrive at transport equations describing the (phase space) densities of the particles in the plasma various approximations and expansions have to be made [26–43]. Different theoretical approaches have been shown to give predictions for the asymmetry that may vary by more than an order of magnitude [33, 37].

In this paper we take a closer look at the vev insertion approximation (VIA), in which the source term in the transport equations is derived expanding in the spacetime dependent part of the mass matrix [40–42]. It is, however, not clear how exactly the sources in VIA are related to the source terms derived in other approximation schemes. In the standard derivation the VIA source derives from the collision term in the kinetic equations, which is puzzling, as this contribution is absent in approaches that do not expand in mass perturbations. Moreover, it has been observed that the VIA source term diverges (in the degenerate mass limit) in the limit that the thermal width is taken to zero, while it is not clear what the physical origin of this enhancement is [37].

The VIA source can be derived from the Schwinger-Dyson equations, which in turn can be reformulated in terms of the Kadanoff-Baym (KB) equations. The spacetime dependent mass is treated as an interaction, resulting in non-zero self energies. In the standard derivation the self-energy is calculated to second order in VIA, that is, from diagrams with two insertions of the mass, while the propagators are those of the free theory (dressed with thermal corrections) [40–42]. Using these expressions in the collision term in the kinetic equation then gives the source term at second order in VIA. However, the self-energy diagram is not one-particle irreducible (1PI) in that it can be split by cutting a single line. In the usual resummation of self-energies in the Schwinger-Dyson equations one would only include the 1PI diagrams, to avoid double counting. At second order in VIA we still expect to get the correct results, but this approach may obscure the origin of the source term.

Instead, we will calculate both the self-energy and the propagator at first order in VIA to calculate the source term at 2nd order, and show that this gives the same final result for the source term as the standard approach. For simplicity we work with a scalar toy model, but we expect the results to be straightforwardly generalizable to fermions as well. At first order the (hermitian) self-energy is nothing but a mass insertion. This immediately makes it clear that it is equivalent to (1) include the mass term  $\delta m^2(z)$  in the free Lagrangian, and then expand the KB equations in  $\delta m^2(z)$ ; or (2) include the mass term in the interaction Lagrangian, and then expand the KB equations in first order self-energy  $\Pi_{(1)}^h$ . The VIA source thus derives from the same term  $\propto [-i\delta m^2 + \Pi^h, G^\lambda]$ , with  $G^\lambda$  the Wightman function, in the kinetic KB equation as the classical source term.

The VIA source is obtained at leading order in the derivative expansion, that is, at leading order in an expansion of the diamond operator of the Wigner space KB equation [40–42]. To evaluate the resulting integral expression, on top another type of derivative expansion is done, which assumes the mass insertion – and thus the bubble wall background – varies slowly compared to the typical scales that dominate the integral. We estimate this expansion to be valid for bubble wall widths  $L_w \gg \max(\Gamma_T^{-1}, \bar{m}_T^{-1})$ , with  $\Gamma_T$  thermal width and  $\bar{m}_T^2$  the constant diagonal mass term including thermal corrections. This cuts off the earlier mentioned divergence in the source term for  $\Gamma_T \rightarrow 0$ , as in this limit the approximations used are no longer valid.

This paper is organized as follows. In the next section 1 we briefly introduce our scalar toy model. All results are formulated in general terms, and can easily be adapted to other set-ups. Section 2 reviews the derivation of the KB equations from the Schwinger-Dyson equations, and the transformation to Wigner space. In section 3 we then focus on the derivation of the source in VIA. Section 3.1 discusses the self-energy at first order in VIA and the equivalence between absorbing the mass perturbation in the free or in the interaction Lagrangian. We calculate the source from the self-energy and Green’s functions both expanded to first order in section 3.2, and show that the results are equivalent to the usual approach in section 3.3. Finally, we estimate the validity of the derivative-like expansion done to arrive at the final VIA results in section 3.4. We end with concluding remarks.

## 1 Scalar toy model

For simplicity we will consider a scalar model in this paper, which avoids complication with spin projections, but we expect the results can be rather straightforwardly generalized to fermionic models as well.

Consider then a two-flavor scalar model with CP violating couplings to the bubble background [38, 39]. The quadratic Lagrangian for the flavor doublet  $\phi = (\phi_L \ \phi_R)^T$  is

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \phi^\dagger M^2(z) \phi \quad (1.1)$$

with mass matrix

$$M^2 = M_D^2 + \delta M^2(z) = \begin{pmatrix} m_L^2 & 0 \\ 0 & m_R^2 \end{pmatrix} + \begin{pmatrix} 0 & \delta m^2(z) \\ (\delta m^2(z))^* & 0 \end{pmatrix}. \quad (1.2)$$

The flavor-diagonal masses  $m_{L,R}^2$  are constant, while the off-diagonal mass  $\delta m^2 = |\delta m^2| e^{i\theta}$  depends on the bounce solution  $v_b = v_b(z)$  describing the bubble wall. In VIA the off-diagonal term is treated as an interaction, and the mass and flavor eigenstates coincide for the free Lagrangian. In addition,  $\phi$  interacts with the degrees of freedom in the plasma. These plasma effects are incorporated dressing the Green's functions with a non-zero self-energy. Essential for the VIA mechanism is that this self-energy correction is complex, and gives apart from a thermal mass also a thermal width correction [40, 41].

For future reference we define the Klein-Gordon operator  $\mathcal{D}(u)$  via

$$S = - \int d^4 z \phi(z)^\dagger \mathcal{D}(z) \phi(z) = - \int d^4 z \phi(z)^\dagger (\partial_z^2 + M^2(z)) \phi(z). \quad (1.3)$$

## 2 Kadanoff-Baym equations

In this section we briefly review the derivation of the Kadanoff-Baym (KB) equations that govern the quantum dynamics of the system. This also serves to set the notation.

### 2.1 CTP formalism and Green functions

The Closed Time Path (CTP) or Schwinger-Keldysh formalism can describe finite-temperature and non-equilibrium quantum systems [44–46]. The path integral is defined along a contour  $\mathcal{C}$  that starts from an initial time  $t_0 \rightarrow -\infty$  to time  $t = \infty$  and then back, which can be split in a  $\mathcal{C}_+$  and  $\mathcal{C}_-$  branch. The scalar Green function defined via

$$G(u, v) = \langle \Omega | T_{\mathcal{C}} \phi(u) \phi^\dagger(v) | \Omega \rangle, \quad (2.1)$$

with  $T_{\mathcal{C}}$  denoting time-ordering along the contour  $\mathcal{C}$ , can be split depending on which branch the time-arguments of the fields lie. We use the notation that  $G^t = G^{++}$ ,  $G^{\bar{t}} = G^{--}$ ,  $G^> = G^{-+}$ ,  $G^< = G^{+-}$ .

The retarded and advanced propagators, and hermitian and anti-hermitian propagators are

$$\begin{aligned} G^r &\equiv G^t - G^< = G^> - G^{\bar{t}}, & G^a &\equiv G^t - G^> = G^< - G^{\bar{t}}, \\ G_h &\equiv \frac{1}{2}(G^r + G^a) = \frac{1}{2}(G^t - G^{\bar{t}}), & G_{ah} &\equiv \frac{1}{2i}(G^a - G^r) = \frac{i}{2}(G^> - G^<). \end{aligned} \quad (2.2)$$

A further useful relation is

$$G^t(u, v) + G^{\bar{t}}(u, v) = G^>(u, v) + G^<(u, v). \quad (2.3)$$

The hermiticity properties are  $G^\lambda(u, v)^\dagger = G^\lambda(v, u)$  with  $\lambda = >, <$  and  $G^t(u, v)^\dagger = G^{\bar{t}}(v, u)$ .

Similarly, one can define the advanced/retarded and hermitian/anti-hermitian scalar self-energies in terms of  $\Pi^{ab}$ , with  $a, b = +, -$ .

## 2.2 From Schwinger-Dyson to Kadanoff-Baym

The starting point are the Schwinger-Dyson equations, derived from the 2PI effective action in [26, 32], which can be written in the form

$$\begin{aligned} \mathcal{D}(u)G^{ab}(u, v) &= -ai\delta_{ab}\delta^4(u - v) - i\sum_c c \int d^4w \Pi^{ac}(u, w)G^{cb}(w, v), \\ G^{ab}(u, v) \overleftarrow{\mathcal{D}}(v) &= -ia\delta_{ab}\delta^4(u - v) - i\sum_c c \int d^4w G^{ac}(u, w)\Pi^{cb}(w, v), \end{aligned} \quad (2.4)$$

with  $\mathcal{D}$  the Klein-Gordon operator eq. (1.3) and  $a, b = +, -$ . For  $\lambda = >, <$  they become

$$\begin{aligned} \partial_u^2 G_{uv}^\lambda &= -M_u^2 G_{uv}^\lambda - i \int d^4w \left( \Pi_{uw}^h G_{wv}^\lambda + \Pi_{uw}^\lambda G_{wv}^h + \frac{1}{2} (\Pi_{xz}^> G_{zy}^< - \Pi_{xz}^< G_{zy}^>) \right), \\ \partial_v^2 G_{uv}^\lambda &= -G_{uv}^\lambda M_v^2 - i \int d^4w \left( G_{uw}^h \Pi_{wv}^\lambda + G_{uw}^\lambda \Pi_{wv}^h + \frac{1}{2} (G_{xz}^> \Pi_{zy}^< - G_{xz}^< \Pi_{zy}^>) \right), \end{aligned} \quad (2.5)$$

where we introduced the shorthand notation  $A_{uw} = A(u, w)$ . Now add and subtract these equations to obtain the anti-hermitian and hermitian part, which correspond to the constraint and kinetic equation respectively:

$$(\partial_u^2 + \partial_v^2)G_{uv}^\lambda = -\{M^2, G^\lambda\}_{uv} - i \int d^4w \left( \{\Pi^h, G^\lambda\} + \{\Pi^\lambda, G^h\} + \mathcal{C}_{\text{coll}}^- \right)_{uv} \quad (2.6)$$

$$(\partial_u^2 - \partial_v^2)G_{uv}^\lambda = -[M^2, G^\lambda]_{uv} - i \int d^4w \left( [\Pi^h, G^\lambda] + [\Pi^\lambda, G^h] + \mathcal{C}_{\text{coll}}^+ \right)_{uv} \quad (2.7)$$

with the shorthand notation  $[\Pi, G]_{uv} = \Pi_{uw}G_{wu} - G_{uw}\Pi_{wu}$  and similar for the anti-commutators. The collision term is

$$(\mathcal{C}_{\text{coll}}^\pm)_{uv} = \frac{1}{2} ([\Pi^>, G^<]^\pm - [\Pi^<, G^>]^\pm)_{uv} \quad (2.8)$$

with  $[\dots]^+ = \{\dots\}$  anti-commutator and  $[\dots]^- = [\dots]$  commutator. Here we used eq. (2.2) to rewrite the right hand side

$$\begin{aligned}\Pi^>G^t - \Pi^{\bar{t}}G^> &= \Pi_h G^> + \Pi^>G_h + \frac{1}{2}(\Pi^>G^< - \Pi^<G^>), \\ \Pi^tG^< - \Pi^<G^{\bar{t}} &= \Pi_h G^< + \Pi^<G_h + \frac{1}{2}(\Pi^>G^< - \Pi^<G^>).\end{aligned}\quad (2.9)$$

Equations (2.6) and (2.7) are the position space Kadanoff-Baym (KB) equations.

### 2.3 Wigner representation

To do a gradient expansion in the slowly varying bubble background, it is useful to define the relative and center-of-mass coordinates

$$x = \frac{1}{2}(u + v), \quad r = u - v \quad \Leftrightarrow \quad u = x + \frac{1}{2}r, \quad v = x - \frac{1}{2}r. \quad (2.10)$$

We use the notation for a general function  $A(u, v)$

$$A(u, v) = A(x + \frac{1}{2}r, x - \frac{1}{2}r) \equiv \bar{A}(r, x). \quad (2.11)$$

The Wigner transform is defined as the Fourier transform with respect to the relative coordinate

$$A(k, x) = \int d^4(u - v) e^{ik \cdot (u - v)} A(u, v) = \int d^4r e^{ik \cdot r} \bar{A}(r, x). \quad (2.12)$$

Now integrate eq. (2.5) with a factor  $\int d^4(u - v) e^{ik \cdot (u - v)}$ , and add and subtract the two equations to extract the hermitian and anti-hermitian parts

$$\left(\frac{1}{2}\partial_x^2 - 2k^2\right) G^\lambda(k, x) = -ie^{-i\Diamond} \left(\{-iM^2 + \Pi^h, G^\lambda\} + \{\Pi^\lambda, G^h\} + \mathcal{C}_{\text{coll}}^-\right), \quad (2.13)$$

$$2ik \cdot \partial_x G^\lambda(k, x) = ie^{-i\Diamond} \left([-iM^2 + \Pi^h, G^\lambda] + [\Pi^\lambda, G^h] + \mathcal{C}_{\text{coll}}^+\right). \quad (2.14)$$

where all  $G^i$ ,  $\Pi^i$  are a function of  $(k, x)$ , and  $M^2$  a function of  $(x)$ . The collision term eq. (2.8) in Wigner space is

$$\mathcal{C}_{\text{coll}}^\pm(k, x) = \frac{1}{2}([\Pi^>(k, x), G^<(k, x)]^\pm - [\Pi^<(k, x), G^>(k, x)]^\pm). \quad (2.15)$$

The diamond operator is defined as

$$\Diamond(A(k, x)B(k, x)) = \frac{1}{2}(\partial_x A(k, x) \cdot \partial_k B(k, x) - \partial_k A(k, x) \cdot \partial_x B(k, x)). \quad (2.16)$$

Some useful relations involving the diamond operator, used in the above derivation, are collected in appendix A.

Equations (2.13) and (2.14) are the Wigner space Kadanoff-Baym equations, corresponding to the constraint and kinetic equation respectively.

### 3 VEV insertion approximation

In the vev insertion approximation (VIA) the off diagonal and space-time dependent parts of the mass matrix,  $\delta M^2$  in eq. (1.2), are treated as a perturbation. Flavor oscillations and coherence effects are neglected. The source terms are calculated expanding the KB equations in the number of mass insertions, which should converge for sufficiently small enough mass perturbations  $\delta m^2/(\bar{m}_T \Gamma_T) \ll 1$ , with  $\bar{m}_{T,i}^2$  the flavor diagonal mass including thermal corrections and  $\Gamma_T$  the thermal width [47].

In the usual derivation of the source term in VIA the self-energy is calculated for two mass insertions, while the Green's functions of the free theory (dressed with thermal corrections) are used [40–42]. As we will show in section 3.3 this gives the same result as calculating the both the self-energy and the Green's function to first order in VIA. We first follow the latter approach.

#### 3.1 Self-energy at first order in VIA

It is clear that we can either (1) include  $\delta M^2$  in the free Lagrangian, derive the KB equations, and expand in  $\delta M$ , or (2) include  $\delta M^2$  in the interaction Lagrangian which result in a non-zero self-energy, derive the KB equations and expand in powers of the self-energy. These are two different descriptions of the same expansion of the same physical system. We proceed with the latter approach and set  $\delta M^2 = 0$  in the KB equations, and incorporate the mass insertions via the self-energy.

At first order in VIA (one mass insertion) the non-zero self energies in position space are

$$\Pi_{ij,(1)}^{ab}(u, v) = f_{ij}^{ab}(u) \delta^4(u - v), \quad (3.1)$$

with

$$\begin{aligned} f_{LR}^{++}(u) &= -i\delta m^2(u), & f_{LR}^{--}(u, v) &= i\delta m^2(u), \\ f_{RL}^{++}(u) &= -i(\delta m^2(u))^*, & f_{RL}^{--}(u, v) &= i(\delta m^2(u))^*. \end{aligned} \quad (3.2)$$

The vertices on the  $\mathcal{C}^-$  branch pick up an additional minus sign. Throughout we will use  $a, b, .. = +, -$  for CTP contour indices and  $i, j, .. = L, R$  for flavor indices. The subscript (#) on  $\Pi_{ij}^{ab}$  and  $G_{ij}^{ab}$  refers to the number # of mass insertions. Flavor diagonal self-energies vanish at this order, as well as  $\Pi^\lambda$ , as vertices always are inserted on the same branch of the CTP contour. We note that the hermitian self-energy

$$\Pi_{LR,(1)}^h(u, v) = \frac{1}{2} \left( \Pi_{LR,(1)}^{++}(u, v) - \Pi_{LR,(1)}^{--}(u, v) \right) = -i\delta m^2(v) \delta^4(u - v) \quad (3.3)$$

is proportional to the mass perturbation, whereas the orthogonal combination  $\propto (\Pi_{LR,(1)}^{++} + \Pi_{LR,(1)}^{--}) = 0$  vanishes. If we insert these results in the position space KB eqs. (2.6) and (2.7), the integral over  $\Pi_h$  can be done with the delta-function, and using eq. (3.2), the result is

$$\begin{aligned} (\partial_u^2 + \partial_v^2) G_{uv}^\lambda &= -\{M_D^2 + \delta M^2, G^\lambda\}_{uv}, \\ (\partial_u^2 - \partial_v^2) G_{uv}^\lambda &= [M_D^2 + \delta M^2, G^\lambda]_{uv}. \end{aligned} \quad (3.4)$$

This indeed gives exactly the same equations as one would have obtained by absorbing  $\delta M^2$  in the free Lagrangian and setting  $\Pi_h = 0$ .

From the KB equations in Wigner space eqs. (2.13) and (2.14) it is even more transparent that both procedures give the same result, as these only depend on the combination  $(-i\delta M^2 + \Pi^h)$ . The Wigner space self-energy is

$$\Pi_{ij,(1)}^{ab}(k, x) = \int d^4r e^{ikr} f_{ij}^{ab}(x + \frac{1}{2}r) \delta(r) = f_{ij}^{ab}(x), \quad (3.5)$$

and thus  $\Pi_{LR,(1)}^h = -i\delta m^2(x)$ , which depends on the collective coordinate  $x = u = v$ , but not on the momenta conjugate to the relative coordinates.

The above implies that the source term in VIA can be equally derived from an expansion in  $(-i\delta M^2)$  as from an expansion in  $\Pi_h$ . The source term thus should arise from

$$2ik \cdot \partial_x G^\lambda(k, x) = ie^{-i\phi} [-i(M_D^2(x) + \delta M^2(x)) + \Pi^h(x), G^\lambda(k, x)] \quad (3.6)$$

in the kinetic equation. This is the same term that gives rise to the semi-classical force [28, 29, 48–50]. The KB equations resums all mass insertions; instead in VIA the kinetic equation is solved perturbatively expanding in mass insertions.

### 3.2 Source term from first order self-energy

To derive the transport equations at 2nd order in the VIA expansion, we perform a derivative expansion of the kinetic equations in Wigner space, and then transform the results back to position space. The propagators are most easily derived directly in position space from the Schwinger-Dyson equations. We will calculate the source at 2nd order expanding both the self-energy and Green's functions to first order in VIA. The self-energy was discussed in the previous section.

Setting  $\delta M^2 = 0$  in the free Lagrangian, the mass matrix is flavor diagonal, and the zeroth order in VIA constraint equation for the Green's function becomes

$$(\partial_u^2 + m_i^2) G_{ij,(0)}^{ab}(u - v) = -ia\delta_{ab}\delta^4(u - v) \quad (3.7)$$

for  $i = L, R$  flavors. The mass matrix is diagonal and the solutions are the usual free thermal propagators [51]. Coherence effects are neglected ( $f_{LR}$  distributions are set to zero) and the off-diagonal propagators vanish. In VIA the propagators are dressed with a non-zero self-energy to account for the interaction with the thermal plasma [52], which amounts to adopting the spectral function [40]

$$\rho(k) = i \left[ \frac{1}{(k^0 + i\epsilon + i\Gamma_T)^2 - \omega^2} - \frac{1}{(k^0 - i\epsilon - i\Gamma_T)^2 - \omega^2} \right], \quad (3.8)$$

with  $\omega^2 = \vec{k}^2 + m_i^2 + m_{T,i}^2$  and  $m_T^2, \Gamma_T$  the thermal mass and width respectively [40]. For vanishing plasma corrections  $m_T^2, \Gamma_T \rightarrow 0$  this reduces to the free spectral function.



At first order in VIA the Schwinger-Dyson equation reads

$$G_{(1),ij}^{ab}(u,v) = \sum_{cd} cd \int d^4 z_1 \int d^4 z_2 G_{(0),ik}^{ac}(u-z_1) \Pi_{(1),kl}^{cd}(z_1-z_2) G_{(0),lj}^{db}(z_2-v). \quad (3.9)$$

Using the non-zero first order self-energy eq. (3.2) and zeroth order Green's function this gives

$$\begin{aligned} G_{(1),ij}^{++}(u,v) &= \sum_{cd} \int d^4 z_1 \int d^4 z_2 G_{(0),ii}^{+-}(u-z_1) \Pi_{(1),ij}^{--}(z_1-z_2) G_{(0),jj}^{-+}(z_2-v) \\ &= \int d^4 z G_{(0),LL}^{+-}(u-z) f_{ij}^{--}(z) G_{(0),RR}^{-+}(z-v), \\ G_{(1),ij}^{--}(u,v) &= \int d^4 z G_{(0),ii}^{-+}(u-z) f_{ij}^{++}(z) G_{(0),jj}^{+-}(z-v), \end{aligned} \quad (3.10)$$

for  $i \neq j$ , and as before  $i, j = L, R$ .

The source term can be derived from the kinetic equation eq. (2.14), which at 2nd order becomes

$$\begin{aligned} k \cdot \partial_x (G^> + G^<)_{ij} &= \frac{1}{2} e^{-i\phi} \left( [\Pi^h, G^t + G^{\bar{t}}] + [\Pi_i^t + \Pi_i^{\bar{t}}, G^h] + \mathcal{C}_{\text{coll}}^+ \right)_{ij} \\ &\stackrel{(2)}{=} \frac{1}{2} e^{-i\phi} \left( [\Pi_{(1)}^h, G_{(1)}^t + G_{(1)}^{\bar{t}}] \right) \delta_{ij}. \end{aligned} \quad (3.11)$$

On the first line we used eq. (2.3), and on the 2nd line we gave the 2nd order VIA contribution originating from multiplying a 1st order 1PI self-energy with a 1st order propagator. Integrating over momenta gives the derivative of the number current (particles minus antiparticles) density for the left hand side. Using eq. (3.5) to write  $\Pi_{(1)}^h(k, x) = f^h(x)$ , which only depends on the collective coordinate  $x$ , this gives

$$\begin{aligned} \partial_\mu j_{(2),i}^\mu(x) &\equiv \int \frac{d^4 k}{(2\pi)^4} k^\mu \left( G_{(2)}^<(k, x) + G_{(2)}^>(k, x) \right)_{ii} \\ &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{-i\phi} \left( [f^h(x), G_{(1)}^t(k, x) + G_{(1)}^{\bar{t}}(k, x)] \right)_{ii}. \end{aligned} \quad (3.12)$$

The right hand side is known as the source term  $S$ , i.e., we write  $\partial_\mu j_{(2),i}^\mu(x) = S_{(2),i}$  at second order in VIA. The first two terms in the derivative expansion, corresponding to an expansion of  $e^{-i\phi} = 1 - i\phi + \mathcal{O}(\phi^2)$ , are

$$\begin{aligned} S_{(2),i}(x)|_{\text{LO}} &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} [f^h(x), G_{(1)}^t(k, x) + G_{(1)}^{\bar{t}}(k, x)]_{ii}, \\ S_{(2),i}(x)|_{\text{NLO}} &= -\frac{i}{2} \{ \partial_x f^h(x), \partial_k (G_{(1)}^t(k, x) + G_{(1)}^{\bar{t}}(k, x)) \}_{ii}, \end{aligned} \quad (3.13)$$

with LO and NLO denoting the leading and next-to-leading order in the derivative expansion. To transform back the results to coordinate space we use that

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} f^h(x) G(k, x) &= \int \frac{d^4 k}{(2\pi)^4} \int d^4 r e^{ikr} f^h \bar{G}(r, x) = f^h \bar{G}(0, x) = f^h(u) G(u, u), \\ \int \frac{d^4 k}{(2\pi)^4} \partial_x f^h \partial_k G(k, x) &= \int \frac{d^4 k}{(2\pi)^4} \int d^4 r (ir) \partial_x f^h \bar{G}(r, x) e^{ikr} = 0. \end{aligned} \quad (3.14)$$

Thus only the leading order term gives a non-zero result; all higher order terms in the derivative expansion vanish as they are proportional to  $r^n$  (and the  $k$ -integral gives a  $\delta(r)$ ). We thus only get a source term at leading order in the derivative expansion  $S_{(2),ii}(u) = S_{(2),ii}|_{\text{LO}}$  with

$$\begin{aligned} S_{(2),ii} &= \frac{1}{2} \left[ f^h(u), G_{(1)}^t(u, u) + G_{(1)}^{\bar{t}}(u, u) \right]_{ii} \\ &= \text{Re} \int d^4w \left( f_{ij}^{\bar{t}}(u) G_{(0),jj}^<(u, w) f_{ji}^t(w) G_{(0),ii}^>(w, u) \right. \\ &\quad \left. - f_{ij}^t(u) G_{(0),jj}^>(u, w) f_{ji}^{\bar{t}}(w) G_{(0),ii}^<(w, u) \right). \end{aligned} \quad (3.15)$$

On the 2nd line we rewrote the first order Green's function in terms of the lowest order Green's functions using eq. (3.10). This is the same result as in the standard calculation, as shown in the next subsection.

### 3.3 Source term from 2nd order self-energy

The standard approach the source is derived using the 0th order Green's functions and the 2nd order self-energy. In this subsection we briefly review this derivation, and show that the resulting source term is the same as derived in the previous subsection.

The kinetic equation eq. (2.7) for  $G^> + G^< = G^t + G^{\bar{t}}$  can be written in the form

$$(\partial_u^2 - \partial_v^2)(G^t + G^{\bar{t}})_{uv} = -i \int d^4w \left( [\Pi^{++}, G^{++}] - [\Pi^{--}, G^{--}] + \{\Pi^{+-}, G^{+-}\} - \{\Pi^{-+}, G^{-+}\} \right)_{uv} \quad (3.16)$$

where for notational convenience we suppressed flavor indices. The last two terms on the right hand side originate from the collision term. At second order this becomes

$$\begin{aligned} (\partial_u^2 - \partial_v^2)(G^t + G^{\bar{t}})_{uv} &\stackrel{(2)}{=} -i \int d^4w \left( [\Pi_{(1)}^{++}, G_{(1)}^{++}] - [\Pi_{(1)}^{--}, G_{(1)}^{--}] \right)_{uv} \\ &\stackrel{(2)}{=} -i \int d^4w \left( -\{\Pi_{(2)}^{+-}, G_{(0)}^{+-}\} + \{\Pi_{(2)}^{-+}, G_{(0)}^{-+}\} \right)_{uv}, \end{aligned} \quad (3.17)$$

where the first line is the approach of the previous subsection, and the 2nd line the standard derivation. Including both would be double counting. To show that both expressions are indeed the same, we start with the self-energy at 2nd order in VIA

$$\Pi_{(2),ii}^{ab}(u, v) = cd \int d^4z_1 \int d^4z_2 \Pi_{(1),ij}^{ac}(u, z_1) G_{(0),jj}^{cd}(z_1, z_2) \Pi_{(1),ji}^{db}(z_2, v). \quad (3.18)$$

It follows that

$$\begin{aligned} &\int d^4w \{ \Pi_{(2)}^{+-}(u, w), G_{(0)}^{-+}(w, v) \} \\ &= - \int d^4w \int d^4z_1 \int d^4z_2 \left( \Pi_{(1)}^{++}(u, z_1) G_{(0)}^{+-}(z_1, z_2) \Pi_{(1)}^{--}(z_2, w) G_{(0)}^{-+}(w, v) \right. \\ &\quad \left. + G_{(0)}^{-+}(u, w) \Pi_{(1)}^{++}(w, z_1) G_{(0)}^{+-}(z_1, z_2) \Pi_{(1)}^{--}(z_2, v) \right) \\ &= - \int d^4z_1 \left( \Pi_{(1)}^{++}(u, z_1) G_{(1)}^{++}(z_1, v) + G_{(1)}^{++}(z_1, v) \Pi_{(1)}^{++}(u, z_1) \right), \end{aligned} \quad (3.19)$$

$$\underbrace{-\Pi_{(1),ij}^{++} \odot G_{(0),jj}^{+-} \odot \Pi_{(1),ji}^{--}}_{\Pi_{(2),ii}^{+-}} \odot G_{(0),ii}^{+-} = -\Pi_{(1),ij}^{++} \odot \underbrace{G_{(0),jj}^{+-} \odot \Pi_{(1),ji}^{--} \odot G_{(0),ii}^{+-}}_{G_{(1),ji}^{++}}$$

**Figure 1.** The equivalence eq. (3.19) between  $\Pi_{(2),ii}^{+-} \odot G_{(0),ii}^{+-} = -\Pi_{(1),ij}^{--} \odot G_{(1),ji}^{--}$ , with  $\odot$  denoting integration over the internal variable, in terms of Feynman diagrams. A mass insertion is represented by a cross,  $\times$ , in the diagrams.

where the expression for the 1st order propagator in eq. (3.10) was used. The overall minus sign of  $\Pi_{(2)}^{+-}$  arises from the definition eq. (3.18), as the sum over the internal indices  $c, d = +, -$  gives an extra sign. This expression can be equally understood in terms of Feynman diagrams as shown in Figure 1, where we also reinstated flavor indices. Likewise,

$$\int d^4w \{ \Pi_{(2)}^{+-}(u, w), G_{(0)}^{+-}(w, v) \} = - \int d^4z_1 \left( \Pi_{(1)}^{--}(u, z_1) G_{(1)}^{--}(z_1, v) + G_{(1)}^{--}(z_1, v) \Pi_{(1)}^{--}(u, z_1) \right), \quad (3.20)$$

which together proof that the two expressions in eq. (3.17) above are the same.

Let's briefly review the derivation of the source in terms of the 2nd order self-energy. The non-zero self-energies are

$$\Pi_{(2),ii}^{+-} = -f_{ij}^{--}(u) G_{(0),jj}^{+-}(u, v) f_{ji}^{++}(v), \quad \Pi_{(2),ii}^{+-} = -f_{ij}^{++}(u) G_{(0),jj}^{+-}(u, v) f_{ji}^{--}(v), \quad (3.21)$$

which can be substituted in the kinetic equation eq. (3.17). For the left hand side we use that<sup>1</sup>

$$\frac{i}{2} (\partial_u^2 - \partial_v^2) \left( G^t(u, v) + G^{\bar{t}}(u, v) \right) = \partial_{x^\mu} \int \frac{d^4k}{(2\pi)^4} k^\mu e^{-ikr} (G^<(k, x) + G^>(k, x)) = \partial_\mu j^\mu(x) \quad (3.22)$$

with  $x = \frac{1}{2}(u + v)$ . Taking the leading order result in the diamond expansion, corresponding to setting  $v = u$  in position space, then gives the source  $\partial_\mu j_{(2)}^\mu(x)|_{\text{LO}} = S_{(2),ii}|_{\text{LO}}$  with

$$\begin{aligned} S_{(2),ii}|_{\text{LO}} &= \frac{1}{2} \int d^4w \left( -\{ \Pi_{(2)}^{+-}, G_{(0)}^{+-} \} + \{ \Pi_{(2)}^{+-}, G_{(0)}^{+-} \} \right)_{uu} = \int d^4w \text{Re} \left( \Pi_{(2)}^{+-} G_{(0)}^{+-} - \Pi_{(2)}^{+-} G_{(0)}^{+-} \right)_{uu} \\ &= \text{Re} \int d^4w \left( f_{ij}^{\bar{t}}(u) G_{(0),jj}^<(u, w) f_{ji}^t(w) G_{(0),ii}^>(w, u) \right. \\ &\quad \left. - f_{ij}^t(u) G_{(0),jj}^>(u, w) f_{ji}^{\bar{t}}(w) G_{(0),ii}^<(w, u) \right). \end{aligned} \quad (3.23)$$

This agrees with eq. (3.15).

<sup>1</sup>In [40–42] the current conservation is derived directly in position space in the limit  $v \rightarrow u$  via  $\lim_{u \rightarrow v} i (\partial_u^2 - \partial_v^2) (G^t(u, v) + G^{\bar{t}}(u, v)) = 2\partial_\mu j^\mu(x)$ .

### 3.4 CP even and odd sources

The integral in the source eq. (3.23) cannot be evaluated analytically for a generic background  $f(x)$ . To proceed it is assumed that the background is slowly varying, which in addition allows to identify the dominant contribution to the CP even and CP odd parts of the source. This expansion is not the usual derivative expansion, in the sense of an expansion in powers of the diamond operator in the Wigner space KB equations eq. (2.14); indeed eq. (3.23) is the leading order term, and – as follows from eq. (3.14) – the only contribution from the diamond expansion. What is done instead is that the mass insertion  $f(w)$  in the source eq. (3.23), with  $w$  the coordinate that is integrated over, is expanded as

$$f(w) = f(u) + f'(u)(w - u) + \mathcal{O}((w - u)^2). \quad (3.24)$$

The  $u$ -dependent functions can then be taken out of the integral, and the resulting integral becomes manageable. The leading and next-to-leading terms in this expansion give a CP even and CP odd contribution to the source. For the left flavor

$$\begin{aligned} S_L^{\text{CP}} &= 2|\delta m^2|^2 \int d^4w \operatorname{Re} \left( G_{(0),RR}^<(u, w) G_{(0),LL}^>(w, u) - G_{(0),RR}^>(u, w) G_{(0),LL}^<(w, u) \right) \quad (3.25) \\ S_L^{\text{CPV}} &= 2\operatorname{Im} \left[ \delta m^2 (\delta m^2)^{*'} \right] \times \\ &\quad \int d^4w \operatorname{Im} \left[ (w - u) \left( G_{(0),RR}^<(u, w) G_{(0),LL}^>(w, u) - G_{(0),RR}^>(u, w) G_{(0),LL}^<(w, u) \right) \right], \end{aligned}$$

where we used eq. (3.2) for  $f(u)$ . The source for the right flavor is  $S_R = -S_L$ .

The expansion eq. (3.24) is only valid if the background varies on much larger length scales than the length scales dominating the integral in the expressions above, otherwise the full spacetime dependence of the mass should be taken into account. The mass insertion is a function of the background bounce solution. For large bubbles curvature effects can be neglected, and the bubble is generically well approximated by a kink solution, which in the bubble wall rest frame takes the form [53]

$$v_b(u) \approx \frac{v_N}{2} \left( 1 + \tanh\left(\frac{u^0 - u^3}{L_w}\right) \right). \quad (3.26)$$

Here  $v_N$  is the vev in the bubble interior,  $L_w$  the bubble wall width,  $u^0 = v_w t$  with  $v_w$  the bubble wall speed, and  $u^3$  the distance from the center of the wall.

The background thus varies on length scales of the bubble wall width. The typical scales dominating the integral can be estimated from the explicit expressions for the source terms; here we look at the CP-odd source which has been calculated to give [40–42]

$$\begin{aligned} S_L^{\text{CPV}} &\propto \int \frac{k^2 dk}{\omega_L \omega_R} \operatorname{Im} \left[ \frac{(n_B(\mathcal{E}_L^*) - n_B(\mathcal{E}_R))}{(\mathcal{E}_L^* - \mathcal{E}_R)^2} + \frac{(n_B(\mathcal{E}_L) + n_B(\mathcal{E}_R))}{(\mathcal{E}_L + \mathcal{E}_R)^2} \right] \\ &\propto \int \frac{k^2 dk}{\omega^2} \operatorname{Re} \left[ \frac{e^{\omega/T}}{\Gamma_T(1 + e^{\omega/T})^2} + \mathcal{O}(\Gamma_T) \right]_{\bar{m}_{T,L}^2 = \bar{m}_{T,R}^2}, \quad (3.27) \end{aligned}$$

with  $n_B(\mathcal{E})$  is the Bose-Einstein distribution, and  $k = |\vec{k}|$  the modulus of the three-momenta. The complex energies are defined via  $\mathcal{E}_j = \omega_j - i\Gamma_j$  with  $\omega_j = \sqrt{k^2 + \bar{m}_{T,j}^2}$  with  $\bar{m}_{T,j}^2 = m_j^2 + m_{T,j}^2$  the sum of the diagonal zero-temperature mass and the thermal mass, and  $\Gamma_{T,j}$  the thermal decay width. On the 2nd line in eq. (3.27) we took the limit of degenerate masses  $\bar{m}_{T,L}^2 = \bar{m}_{T,R}^2 \equiv \bar{m}_T^2$ . This expression shows that the integral over three momentum is cutoff by the mass term in  $\omega$  and the integral varies over  $|\vec{k}| \sim \bar{m}_T$ , corresponding to a length scale  $\bar{m}_T^{-1}$ .

To find the typical  $k^0$  momenta, and the typical scale of the conjugate  $u^0$  variation, it is useful to see how the denominators in the first line of eq. (3.27) arise, namely from integrals of the form

$$\int_0^\infty du^0 u^0 e^{ik^0 u^0} = -\frac{1}{k_0^2}, \quad k_0 = \mathcal{E}_L^* - \mathcal{E}_R, \mathcal{E}_L + \mathcal{E}_R, \quad (3.28)$$

with the specific  $k_0$  values determined from contour integrals and the poles of the propagators. The  $u^0$ -integral is cutoff by the thermal width  $\text{Im}(k_0) \propto \Gamma_T$ .

We thus conclude that the background changes slowly compared to the length scales dominating the intergal, and can be safely approximated as nearly constant, if<sup>2</sup>

$$L_w \gg \max [\Gamma_T^{-1}, \bar{m}_T^{-1}], \quad (3.29)$$

where the two terms on the right hand side come from the constraints on the temporal and spatial variation respectively. If eq. (3.29) is violated the expansion eq. (3.24) breaks down, and the resulting expressions for the source eq. (3.27) cannot be trusted.

The source term eq. (3.27) diverges for degenerate flavor-diagonal masses in the limit  $\Gamma_T \rightarrow 0$ . This has been noted before, and questions on the physical origin of this divergence were raised [37]. We see now that the expansion eq. (3.24) used to derive this result breaks down when the inverse thermal width becomes larger than the bubble wall width  $\Gamma_T^{-1} > L_w$ . This cuts off the divergence for any finite bubble size.

Although for simplicity we have concentrated on the scalar model, the derivation in VIA of the source for fermionic systems is very similar, and the resulting source terms have a very similar structure to eq. (3.27). In particular, the same expansion eq. (3.24) is made, and the same bound on the validity eq. (3.29) applies. We can thus apply this limit on the use of the VIA source also for fermionic systems. As an explicit example, consider that the CP violation originates from modified Yukawa interactions

$$\mathcal{L}_{\text{int}} = -\delta m(z) \bar{\psi}_L \psi_R - \delta m(z)^* \bar{\psi}_R \psi_L, \quad (3.30)$$

with  $\psi_i$  the left- and right-hand chiralities of a Standard Model (SM) fermion.<sup>3</sup> In the Standard Model  $\delta m$  is just the mass term arising from the Yukawa coupling to the Higgs,

<sup>2</sup>Refs. [40, 41] cite the milder bound  $L_w \gg v_w \Gamma_T^{-1}$ .

<sup>3</sup>The left and right chiralities are the equivalent of the left and right flavors of the scalar model, with the mass insertion off-diagonal in left-right ‘flavor’ space. For top quarks the vev insertion expansion breaks down as the expansion parameter  $\sim \delta m^2/T^2 \gtrsim 1$ , and the vev insertions must be resummed [47].

but to obtain sufficient CP violation requires new physics corrections to this, which we leave unspecified. Including thermal corrections, crucial for the VIA source term, the fermion propagator is dressed with a flavor diagonal thermal mass and thermal width  $m_{T,i}^2$  and  $\Gamma_{T,i}$  with  $i = L, R$ . The thermal corrections are of the order  $m_T^2 \sim \alpha T^2$  and  $\Gamma_T \sim \alpha T$ , with  $\alpha \sim \alpha_3$  the QCD coupling for quarks and  $\alpha = \alpha_2$  the electroweak coupling for leptons. The VIA results are then valid, see eq. (3.29), for

$$L_w T \gg \alpha^{-1} \sim \begin{cases} 100, & \text{for quarks,} \\ 30, & \text{for leptons,} \end{cases} \quad (3.31)$$

which requires thick bubble walls.

## 4 Conclusion

In this paper we have calculated the source term at second order in the vev insertion approximation (VIA) from the self-energy and Green's functions both calculated at first order. Our approach agrees with the known results, but has the advantage that it shows clearly the equivalence of expanding the transport equations in terms of mass perturbations or in self-energies. The source term then derives, as expected from this equivalence, from the same term in the kinetic Kadanoff-Baym equation as the classical force term, but at zeroth order in derivative expansion.

To evaluate the integral in the source term another derivative-type expansion is performed, which assumes the background varied slowly compared to the length scales dominating the source integral. We estimate that this expansion is valid, and the VIA results can be trusted, for bubble widths  $L_w \gg \max[\Gamma_T^{-1}, \bar{m}_T^{-1}]$ , with  $\bar{m}_T^2$ , the diagonal mass including thermal corrections, and  $\Gamma_T$  the thermal width. The thermal corrections arise from the plasma interactions. This bound cuts off the divergence in the source term (for degenerate flavor diagonal masses) in the limit that the thermal width vanishes.

To further assess and improve the above estimate on the validity of the VIA results, it will be useful to explicitly calculate the next term in the series expansion eq. (3.24). In addition, one could try to devise alternative methods to evaluate the source term for small bubble wall widths, to bypass the expansion completely. This is both left for future work.

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## A Diamond operator

To rewrite the KB equations in Wigner space we use the following identities for the diamond operator

$$\begin{aligned}
\int d^4(u-v) e^{ik \cdot (u-v)} \int d^4 w A(u, w) B(w, v) &= e^{-i\Diamond} (A(k, x) B(k, x)) \\
\int d^4(u-v) e^{ik \cdot (u-v)} A(u) B(u, v) &= e^{-i\Diamond} (A(x) B(k, x)) \\
\int d^4(u-v) e^{ik(u-v)} (\partial_u^A + 2\partial_u^B) A(u) B(u, v) &= e^{-i\Diamond} (\partial_x^B - 2ik) A(x) B(k, x) \\
\int d^4(u-v) e^{ik(u-v)} (\partial_v^A + 2\partial_v^B) B(u, v) A(v) &= e^{-i\Diamond} (\partial_x^B + 2ik) A(x) B(k, x) \quad (A.1)
\end{aligned}$$

To derive the first identity above, we rewrite the left hand side in terms of the collective coordinate  $x = \frac{1}{2}(u+v)$  and relative coordinate  $r = u-v = r_1 + r_2$  with  $r_1 = u-w$ , and  $r_2 = w-v$ .

$$\begin{aligned}
A(u, w) &= \bar{A}(r_1, x + \frac{1}{2}r_2) = e^{\frac{1}{2}r_2 \partial_x^A} \bar{A}(r_1, x) \\
B(w, v) &= \bar{B}(r_2, x - \frac{1}{2}r_1) = e^{-\frac{1}{2}r_1 \partial_x^B} \bar{B}(r_2, x) \quad (A.2)
\end{aligned}$$

where in the last step we used that  $\partial_x$  is the generator of translation, and the superscript denotes the term the derivative is acting on. Then

$$\begin{aligned}
&\int d^4(u-v) e^{ik(u-v)} \int d^4 w A(u, w) B(w, v) \\
&= \int d^4 r_1 e^{(ik - \frac{1}{2}\partial_x^B)r_1} \bar{A}(r_1, x) \int d^4 r_2 e^{(ik + \frac{1}{2}\partial_x^A)r_2} \bar{B}(r_2, x) \\
&= A(k + \frac{i}{2}\partial_x^B, x) B(k - \frac{i}{2}\partial_x^A, x) = e^{-i\frac{1}{2}(\partial_x^A \partial_k^B - \partial_x^B \partial_k^A)} A(k, x) B(k, x) \\
&= e^{-i\Diamond} A(k, x) B(k, x). \quad (A.3)
\end{aligned}$$

where in the penultimate step we used that  $\partial_k$  is the generator of translations in momentum space. The derivation of the other identities in eq. (A.1) can be done in a similar way.

The derivative expansion  $e^{-i\Diamond} = 1 - i\Diamond + \dots$  transformed back to position space becomes

$$\begin{aligned}
\int \frac{d^4 k}{(2\pi)^4} e^{-i\Diamond} A(k, x) B(k, x) &= \int \frac{d^4 k}{(2\pi)^4} \int d^4 r \int d^4 r' e^{-\frac{i}{2}(\partial_x^A \cdot \partial_k^B - \partial_k^A \cdot \partial_x^B)} e^{ik(r+r')} \bar{A}(r, x) \bar{B}(r', x) \\
&= \int d^4 r e^{r \cdot (\partial_x^A + \partial_x^B)} \bar{A}(r, x) \bar{B}(-r, x) \\
&= \int d^4 w [1 + (u-w) \cdot (\partial_u + \partial_w) + \mathcal{O}(\partial^2)] A(u, w) B(w, u). \quad (A.4)
\end{aligned}$$

Note that the leading order derivative expansion corresponds to the limit  $v \rightarrow u$  in position space

$$\int d^4w A(u, w) B(w, v) \Big|_{\text{LO}} = \int \frac{d^4k}{(2\pi)^4} e^{-i\phi} A(k, x) B(k, x) \Big|_{\text{LO}} = \int d^4A(u, w) B(w, u). \quad (\text{A.5})$$

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