Homework 3

Due date: 21/05/2021

Analyse the production of three anti-k_t jets in e^+-e^- annihilation. Instead of using the anti-k_t distances defined for hadronic collisions (as described in the lecture notes), use a more suitable definition for spherical coordinates

$$d_{ij} = \min\left(\frac{1}{E_i^2}, \frac{1}{E_j^2}\right)\frac{1-\cos\theta_{ij}}{1-\cos R} \quad d_{iB} = \frac{1}{E_i^2} \tag{1}$$

with θ_{ij} the relative angle between particles i and j and $R < \pi$ the jet reconstruction parameter. While the distance measure changes, the steps for recombination of particles remain the same.

a) Show that the final three-particle phase space $d\Phi_3$ can be written as

$$d\Phi_3 = d\phi_2 d\phi_1 d\theta_1 \sin\theta_1 dx_1 dx_2 \tag{2}$$

with ϕ_1 and θ_1 the azimuthal and polar angle of one the quarks, ϕ_2 the revolution angle of the other quark around the direction of the first quark and $x_i = 2E_i/Q$ the fraction of energy of each quark with respect to the total available energy $Q = \sqrt{s}$

b) Use that the QCD matrix element for the production for the reaction $e^-(q_1) + e^+(q_2) \to q(p_1) + \bar{q}(p_2) + g(k)$ is

$$\frac{1}{4}\sum_{\text{pol}} |M|^2 = 24 C_F e^4 Q_f^2 g_s^2 \frac{(p_1\cdot q_1)^2 + (p_1\cdot q_2)^2 + (p_2\cdot q_1)^2 + (p_2\cdot q_2)^2}{q_1\cdot q_2 \; p_1\cdot k \; p_2\cdot k} \tag{3}$$

to express the differential cross section in terms of the variables of section a). Assume you will focus on observables insensitive to the angular variables and integrate over them. Analyse the divergences of the integrand in x_1 and x_2 and identify the collinear and soft regions.

Hint: You may want to use a rotation matrix to express the momentum of the second quark in cartesian coordinates. Use revolution symmetry around the collision axis to set $\phi_1 = 0$.

c) Show that independently of which jet is first removed from the particle listing, the condition to form 3 jets implies that the angles between each particle θ_{12}, θ_{1g}, θ_{2g}, are all larger than R, as long as $R < \pi/2$.

d) In the small R limit ($\cos R \approx 1 - R^2/2$) express the condition c) in terms of the variables of section a) and find the region for integration relevant for three jet events to leading order in R. **Without performing the integral** over the allowed (x_1, x_2), show that the exclusive production of three jets is collinear finite but it is not infrared finite. This example shows you that not all observables you can construct with anti-k_t jets are calculable in perturbation theory.

Hint: focus on the region $x_i \to 1$ and approximate the integrand and the region of integration to show that the cross section is soft-divergent.

e) Requiring that the energy of each jet is bigger than $E_{cut} = \varepsilon Q$ with ε a small parameter. Find the constraint that this condition imposes in the (x_1, x_2)-plane. In the small ε, small R limit analyse the integral **in the vicinity of** $x_i \to 1$ and show that this rate is now IR&Coll finite. Integrate the cross section **only in that region** to determine the leading $\log R$, $\log\varepsilon$ dependence of the three jet cross section. From this result, determine the leading logarithmic dependence of the inclusive two anti-k_t jet cross section of radius R and $E > E_{cut}$.