UNIVERSITAT DE BARCELONA

Facultat de Física

PREHEATING

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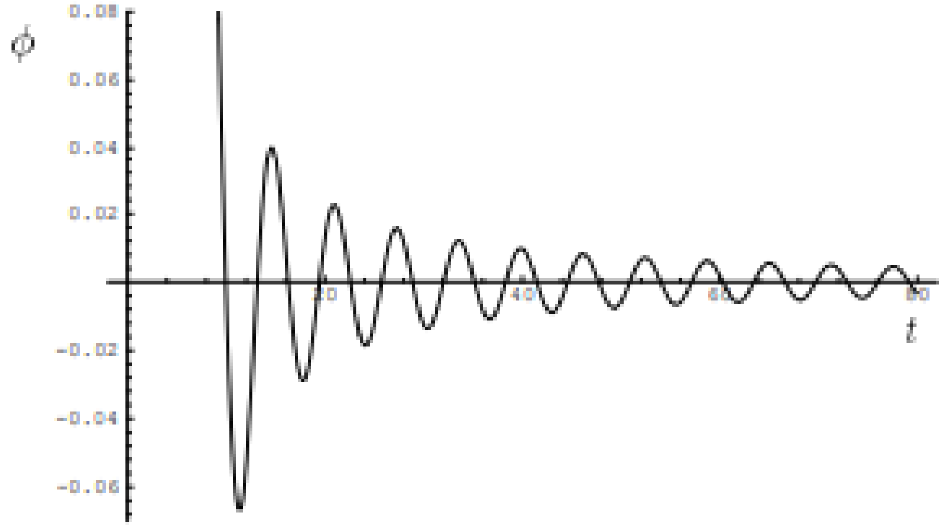
Evolution of the inflation field

$$L(\phi) = \tfrac{1}{2}\phi_{,i}\phi^{,i} - V(\phi) \qquad\qquad H^2 = \frac{8\pi}{3M_p^2}\left(\tfrac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi} + 3H\,\dot{\phi} + V_{,\phi} = 0$$

$$\Big\downarrow\; V(\phi) = \tfrac{1}{2}m\phi^2$$

$$\phi(t) = \frac{M_p}{\sqrt{3\pi}mt}\cdot \sin mt \longrightarrow$$



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Oscillations and decay of the scalar field

$$L = \tfrac{1}{2}\phi_{,i}\phi^{,i} - V(\phi) + \tfrac{1}{2}\chi_{,i}\chi^{,i} - \tfrac{1}{2}m_\chi^2(0)\chi^2 + \tfrac{1}{2}\xi R\chi^2$$
$$+ \bar{\psi}\left(i\gamma^i\partial_i - m_\psi(0)\right)\psi - \tfrac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\phi$$

$$\left(m \gg m_\chi,\, m_\psi\right) \quad \left(V(\phi) \sim \tfrac{1}{2}m^2(\phi-\sigma)^2\ ,\ \phi - \sigma \to \phi\right)$$

$$\downarrow$$

$$\ddot{\phi} + 3H(t)\,\dot{\phi} + \left(m^2 + \Pi(\omega)\right)\phi = 0$$

$$\downarrow \left(m^2 \gg H^2,\ m^2 \gg \operatorname{Im}\Pi\right)$$

$$\phi \approx \phi_0\,\exp(imt)\cdot\exp\left[-\frac{1}{2}\left(3H + \frac{\operatorname{Im}\Pi(m)}{m}\right)t\right]$$

$$\left(\begin{gathered}\operatorname{Im}\Pi = m\Gamma\ ,\\ \Gamma = \Gamma(\phi\to\chi\chi) + \Gamma(\phi\to\psi\psi)\\ \Gamma(\phi\to\chi\chi) = \frac{g^4\sigma^2}{8\pi m}\ , \qquad \Gamma(\phi\to\psi\psi) = \frac{h^2 m}{8\pi}\end{gathered}\right)$$

$$\downarrow$$

Oscillations and decay of the scalar field

$$\frac{1}{a^3}\ \frac{d}{dt}(a^3\Phi^2) = -\Gamma\Phi^2$$

$$\left(n_\phi = \tfrac{1}{2}m\Phi^2 \right)$$

$$\frac{d}{dt}(a^3 n_\phi) = -\Gamma \cdot a^3\, n_\phi$$

The density of particles exponientally decreases with the decay rate, which is constant.

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Perturbation theory vs narrow resonance

if many χ-particles have already been produced, $n_k > 1$, then the probability of decay becomes greatly enhanced due to effects related to Bose-statistics. This may lead to explosive particle production.

$$L = \tfrac{1}{2}\phi_{,i}\phi^{,i} - V(\phi) + \underline{\tfrac{1}{2}\chi_{,i}\chi^{,i}} - \underline{\tfrac{1}{2}m_\chi^2(0)\chi^2} + \underline{\tfrac{1}{2}\xi R\chi^2} + \bar{\psi}(i\gamma^i\partial_i - m_\psi(0))\psi - \underline{\tfrac{1}{2}g^2\phi^2\chi^2} - h\bar{\psi}\psi\phi$$

$$\hat{\chi}(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left(\hat{a}_k\chi_k(t)\,e^{-i\mathbf{kx}} + \hat{a}_k^+\chi_k^*(t)\,e^{i\mathbf{kx}}\right)$$

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{\mathbf{k^2}}{a^2} + m_\chi^2(0) - \xi R + g^2\phi^2\right)\chi_k = 0$$

[the effective mass of the field χ vanishes for $\phi = 0$]

[$V(\phi) \sim \frac{1}{2}m^2(\phi-\sigma)^2$, $\phi - \sigma \to \phi$]

$$\ddot{\chi}_k + \left(k^2 + g^2\sigma^2 + 2g^2\sigma\Phi\,\sin mt\right)\chi_k = 0$$

Oscillations and decay of the scalar field

Mathieu equation $$\chi_k'' + \left(A_k - 2q\cos 2z\right)\chi_k = 0 \qquad \left(A_k = 4\frac{k^2+g^2\sigma^2}{m^2},\ q = \frac{4g^2\sigma\Phi}{m^2},\ z = \frac{mt}{2} \right)$$

The solutions of these equations have an important feature, which is it's instability, the existence of an exponential instability within the set of resonance bands of frequencies $\Delta k^{(n)}$

$$\chi_k \propto \exp(\mu_k^{(n)} z)$$

This instability correspond to an exponential growth of occupation numbers of quantum fluctuations $n_k(t) \propto \exp(2\mu_k^{(n)} z)$

That may be interpreted as particle production

Oscillations and decay of the scalar field

In the case under consideration, $g\Phi \ll g\sigma \ll m$, the theory of parametric resonance is well known

The widest and more important band is: $A_k \sim 1 \pm q = 1 \pm \frac{4g^2\sigma\Phi}{m^2}$

$$\left(m^2 \gg g^2\sigma^2 \right)$$

Which has a factor of exponential growth: $\mu_k = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{2k}{m} - 1\right)^2}$

So, resonance occurs for: $k = \frac{m}{2}(1 \pm \frac{q}{2})$

With a maximum value: $\mu_k = \frac{q}{2} = \frac{2g^2\sigma\Phi}{m^2}$ at $k = \frac{m}{2}$

Oscillations and decay of the scalar field

So, the corresponding mode χ_k grow at a maximal rate: $\exp(\frac{qz}{2})$

$$\left(n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2} \right)$$

When the modes χ_k grow as $\exp(\frac{qz}{2})$, the number of χ-particles grows as $\exp(qz)$, which in our case is equal to $\exp(\frac{qmt}{2}) = \exp(\frac{2g^2\sigma\Phi t}{m})$.

Finally say, that the fact that the resonance occurs near $k = \frac{m}{2}$, has a simple explanation:

one decaying ϕ-particle creates two χ-particles with $k \sim m/2$