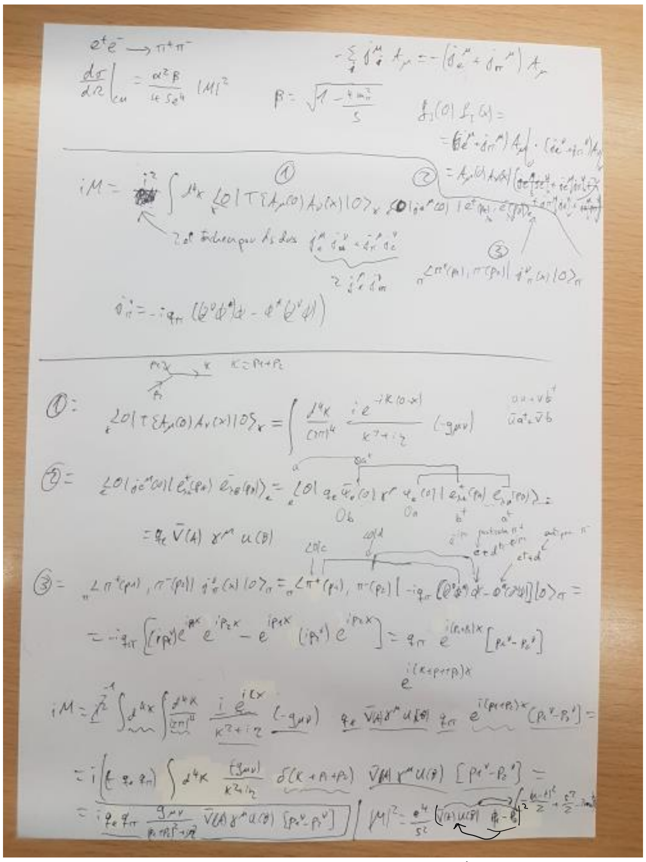


$e^+e^- \longrightarrow \pi^+\pi^-$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{\alpha^2\beta}{4 s e^4}\,|M|^2 \qquad \beta = \sqrt{1-\frac{4m_\pi^2}{s}}$$

$$-\sum_i j_i^\mu A_\mu = -\left(j_e^\mu + j_\pi^\mu\right)A_\mu$$

$\mathcal{L}_I(0)\,\mathcal{L}_I(x) =$

$= (j_e^\mu + j_\pi^\mu)A_\mu \cdot (j_e^\nu + j_\pi^\nu)A_\nu$

$$iM = \frac{i^2}{2}\int d^4x \; {}_\gamma\langle 0|T\{A_\mu(0)A_\nu(x)\}|0\rangle_\gamma \; {}_e\langle 0|j_e^\mu(0)|e^+(p_A)\,e^-(p_B)\rangle_e \; {}_\pi\langle \pi^+(p_1),\pi^-(p_2)|j_\pi^\nu(x)|0\rangle_\pi$$

2 al intercambiar los dos $j_e^\mu j_\pi^\nu + j_\pi^\mu j_e^\nu \approx 2 j_e^\mu j_\pi^\nu$

$$j_\pi^\nu = -iq_\pi\left((\partial^\nu\phi^*)\phi - \phi^*(\partial^\nu\phi)\right)$$

$K = p_1 + p_2$

$$①:\quad {}_\gamma\langle 0|T\{A_\mu(0)A_\nu(x)\}|0\rangle_\gamma = \int\frac{d^4k}{(2\pi)^4}\,\frac{i e^{-ik(0-x)}}{k^2+i\varepsilon}\,(-g_{\mu\nu})$$

$au + vb^\dagger$, $\bar{u}a^\dagger + \bar{v}b$

$$②:\quad {}_e\langle 0|j_e^\mu(0)|e^+_{s_A}(p_A)\,e^-_{s_B}(p_B)\rangle_e = {}_e\langle 0|\,q_e\,\bar{\psi}_e(0)\,\gamma^\mu\,\psi_e(0)\,|e^+_{s_A}(p_A)\,e^-_{s_B}(p_B)\rangle_e =$$

$$= q_e\,\bar{v}(A)\,\gamma^\mu\,u(B)$$

$$③:\quad {}_\pi\langle \pi^+(p_1),\pi^-(p_2)|j_\pi^\nu(x)|0\rangle_\pi = {}_\pi\langle \pi^+(p_1),\pi^-(p_2)|\left[-iq_\pi\left((\partial^\nu\phi^*)\phi - \phi^*(\partial^\nu\phi)\right)\right]|0\rangle_\pi =$$

$$= -iq_\pi\left[(ip_1^\nu)e^{ip_1x}e^{ip_2x} - e^{ip_1x}(ip_2^\nu)e^{ip_2x}\right] = q_\pi\, e^{i(p_1+p_2)x}\left[p_2^\nu - p_1^\nu\right]$$

$e^{i(k+p_1+p_2)x}$

$$iM = \frac{i^2}{2}\cdot 2\int d^4x\int\frac{d^4k}{(2\pi)^4}\,\frac{i e^{ikx}}{k^2+i\varepsilon}\,(-g_{\mu\nu})\; q_e\,\bar{v}(A)\gamma^\mu u(B)\; q_\pi\, e^{i(p_1+p_2)x}\,(p_2^\nu - p_1^\nu) =$$

$$= i\,(q_e q_\pi)\int d^4k\,\frac{(g_{\mu\nu})}{k^2+i\varepsilon}\,\delta(k+p_1+p_2)\;\bar{v}(A)\gamma^\mu u(B)\,\left[p_2^\nu - p_1^\nu\right] =$$

$$= \boxed{i\,q_e q_\pi\,\frac{g_{\mu\nu}}{(p_1+p_2)^2+i\varepsilon}\,\bar{v}(A)\gamma^\mu u(B)\,[p_2^\nu - p_1^\nu]}$$

$$|M|^2 = \frac{e^4}{s^2}\left(\bar{v}(A)\,u(B)\;(\not p_1 - \not p_2)\right)^2$$

$$\left(-\frac{(u-t)^2}{2} + \frac{s^2}{2} - 2m_\pi^2 s\right)$$

$$|M|^2 = \frac{e^4}{s^2}\left(\bar{V}(A)\,(\not p_1 - \not p_2)\,u(B)\right)^2$$

$$= \left(\left(\sqrt{p_a\sigma}\,\eta^{s\dagger}, -\sqrt{p_a\bar{\sigma}}\,\eta^{s\dagger}\right)\gamma^0\gamma^\mu(p_1-p_2)_\mu\begin{pmatrix}\sqrt{p_b\sigma}\,\xi^s\\ \sqrt{p_b\bar{\sigma}}\,\xi^s\end{pmatrix}\right)^2 =$$

$$\begin{pmatrix}0 & 1\!\!1\\ 1\!\!1 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^\mu\\ \bar{\sigma}^\mu & 0\end{pmatrix}$$

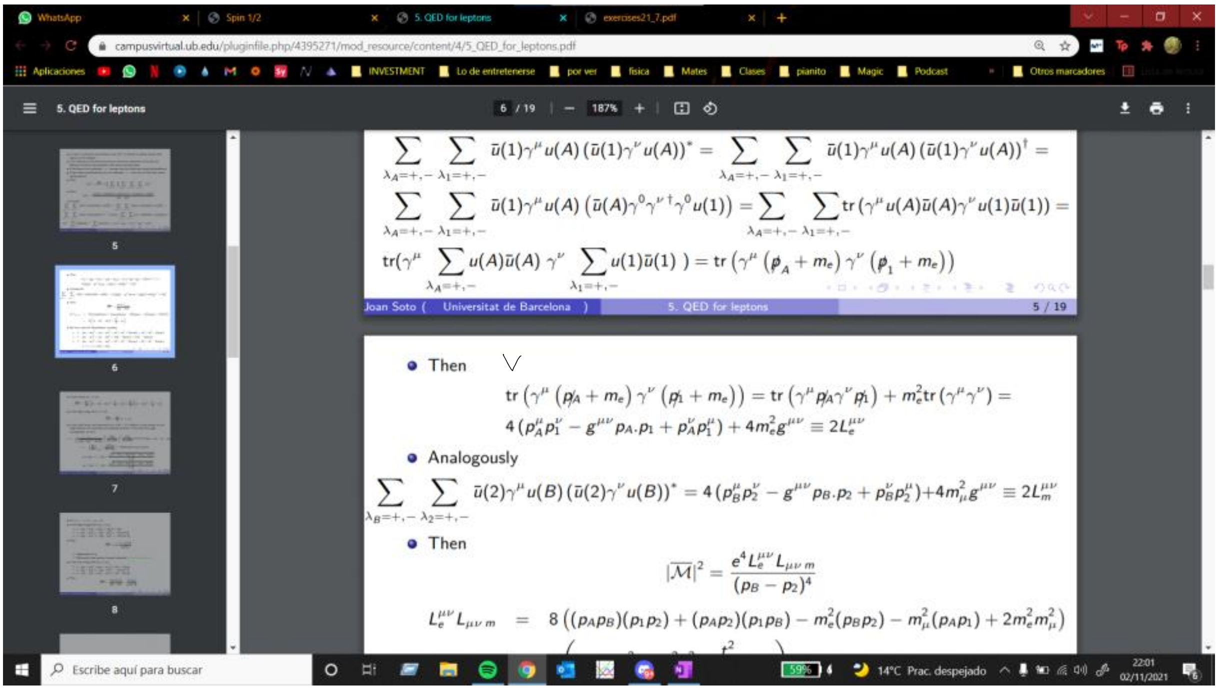
$$= \left((p_1-p_2)_\mu\,\eta^{s\dagger}\left(-\sqrt{p_a\bar{\sigma}},\ \sqrt{p_a\sigma}\right)\right)^2$$

$$= \left((p_1-p_2)_\mu\,\eta^{s\dagger}\left(\sqrt{p_a\sigma}\,\bar{\sigma}^\mu, -\sqrt{p_a\bar{\sigma}}\,\sigma^\mu\right)\begin{pmatrix}\sqrt{p_b\sigma}\\ \sqrt{p_b\bar{\sigma}}\end{pmatrix}\xi^s\right)^2 =$$

$$\left(\bar{\sigma}^\mu\sigma_\mu = 1\!\!1 - \sigma^{1^2} - \sigma^{2^2} - \sigma^{3^2} = -2\,1\!\!1\right)$$

$$= \left[(p_1-p_2)_\mu\,\eta^{s\dagger}\left(\sqrt{p_\alpha\sigma_\alpha}\,\bar{\sigma}^\mu\sqrt{p_b\sigma} - \sqrt{p_a\bar{\sigma}}\,\sigma^\mu\sqrt{p_b\bar{\sigma}}\right)\xi^s\right]^2 =$$

$$= (p_1-p_2)_\mu(p_1-p_2)_\nu\,\eta^{s\dagger}\left(\sqrt{p_\alpha\sigma_\alpha}\,\bar{\sigma}^\mu\sqrt{p_b\sigma} - \sqrt{p_a\bar{\sigma}}\,\sigma^\mu\sqrt{p_b\bar{\sigma}}\right)\underbrace{\xi^s\,\eta^{r\dagger}}_{\xi^{s r}}\left(\sqrt{p_\alpha\sigma_\alpha}\,\bar{\sigma}^\nu\sqrt{p_b\sigma} - \sqrt{p_a\bar{\sigma}}\,\sigma^\nu\sqrt{p_b\bar{\sigma}}\right)\xi^r$$



$$\sum_{\lambda_A=+,-}\sum_{\lambda_1=+,-}\bar u(1)\gamma^\mu u(A)\left(\bar u(1)\gamma^\nu u(A)\right)^* = \sum_{\lambda_A=+,-}\sum_{\lambda_1=+,-}\bar u(1)\gamma^\mu u(A)\left(\bar u(1)\gamma^\nu u(A)\right)^\dagger =$$

$$\sum_{\lambda_A=+,-}\sum_{\lambda_1=+,-}\bar u(1)\gamma^\mu u(A)\left(\bar u(A)\gamma^0\gamma^{\nu\,\dagger}\gamma^0 u(1)\right) = \sum_{\lambda_A=+,-}\sum_{\lambda_1=+,-}\mathrm{tr}\left(\gamma^\mu u(A)\bar u(A)\gamma^\nu u(1)\bar u(1)\right) =$$

$$\mathrm{tr}(\gamma^\mu \sum_{\lambda_A=+,-} u(A)\bar u(A)\,\gamma^\nu \sum_{\lambda_1=+,-} u(1)\bar u(1)\,) = \mathrm{tr}\left(\gamma^\mu\left(\not p_A + m_e\right)\gamma^\nu\left(\not p_1 + m_e\right)\right)$$

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- Then

$$\mathrm{tr}\left(\gamma^\mu\left(\not p_A + m_e\right)\gamma^\nu\left(\not p_1 + m_e\right)\right) = \mathrm{tr}\left(\gamma^\mu \not p_A \gamma^\nu \not p_1\right) + m_e^2\mathrm{tr}\left(\gamma^\mu\gamma^\nu\right) =$$

$$4\left(p_A^\mu p_1^\nu - g^{\mu\nu}p_A.p_1 + p_A^\nu p_1^\mu\right) + 4m_e^2 g^{\mu\nu} \equiv 2L_e^{\mu\nu}$$

- Analogously

$$\sum_{\lambda_B=+,-}\sum_{\lambda_2=+,-}\bar u(2)\gamma^\mu u(B)\left(\bar u(2)\gamma^\nu u(B)\right)^* = 4\left(p_B^\mu p_2^\nu - g^{\mu\nu}p_B.p_2 + p_B^\nu p_2^\mu\right) + 4m_\mu^2 g^{\mu\nu} \equiv 2L_m^{\mu\nu}$$

- Then

$$\overline{|\mathcal{M}|^2} = \frac{e^4 L_e^{\mu\nu} L_{\mu\nu\, m}}{(p_B - p_2)^4}$$

$$L_e^{\mu\nu}L_{\mu\nu\, m} = 8\left((p_A p_B)(p_1 p_2) + (p_A p_2)(p_1 p_B) - m_e^2(p_B p_2) - m_\mu^2(p_A p_1) + 2m_e^2 m_\mu^2\right)$$

$$\frac{1}{4}\sum_{\lambda_A,\lambda_B=+,-}\left(\bar v(A)\gamma^\mu u(B)\right)\left(\bar v(A)\gamma^\mu u(B)\right)^+ = \frac{\sum}{4}\bar v(A)\gamma^\mu u(B)\,\overline{u(B)}\,\gamma^\mu v(A) =$$

$$= \mathrm{Tr}\left(\gamma^\mu \frac{1}{2}\sum_{\lambda_B} u(B)\bar u(B)\,\gamma^\nu \frac{1}{2}\sum_{\lambda_A} v(A)\bar v(A)\right) = \frac{1}{4}\mathrm{Tr}\left[\gamma^\mu\left(\not p_B + m_e\right)\gamma^\nu\left(\not p_A - m_e\right)\right] =$$

$$\frac{1}{4}\left[\mathrm{Tr}\left(\gamma^\mu \not p_B \gamma^\nu \not p_A\right) - m_e^2\,\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right)\right] = \frac{\cancel{4}}{\cancel{4}}\left(p_B^\mu p_A^\nu - g^{\mu\nu}p_B p_A + p_B^\nu p_A^\mu\right) - \frac{\cancel{4}}{\cancel{4}}m_e^2 g^{\mu\nu} \equiv L^{\mu\nu}$$

$$|\bar{\mathcal{M}}|^2 = \frac{e^4}{s^2}L^{\mu\nu}(p_1-p_2)_\mu(p_1-p_2)_\nu = \frac{e^4}{s^2}\left[\left(p_B^\mu p_A^\nu - g^{\mu\nu}p_B p_A + p_B^\nu p_A^\mu\right) - \cancel{m_e^2}\, g^{\mu\nu}\right]$$

$$\cdot\left[p_{1\mu}p_{1\nu} + p_{2\mu}p_{2\nu} - p_{1\mu}p_{2\nu} - p_{2\mu}p_{1\nu}\right] =$$

$$= \frac{e^4}{s^2}\Big[\left((p_B p_1)(p_A p_1) + (p_B p_2)(p_A p_2) - (p_B p_1)(p_A p_2) - (p_B p_2)(p_A p_1) -\right.$$

$$- (p_B p_A)\left[(p_1 p_1) + (p_2 p_2) - (p_1 p_2) - (p_2 p_1)\right] +$$

$$+ (p_A p_1)(p_B p_1) + (p_A p_2)(p_B p_2) - (p_A p_1)(p_B p_2) - (p_A p_2)(p_B p_1)\Big] =$$

$$= \frac{e^4}{s^2}\Big[2(p_B p_1)(p_A p_1) + 2(p_B p_2)(p_A p_2) - 2(p_B p_1)(p_A p_2) - 2(p_B p_2)(p_A p_1)$$

$$+ (p_B p_A)\left[-(p_1 p_1) - (p_2 p_2) + (p_1 p_2) + (p_2 p_1)\right] =$$

$$= \frac{e^4}{s^2}\Big[2(\bar p_A p_1)(p_A p_1) + 2(\bar p_A \bar p_1)(p_A \bar p_1) - 2(\bar p_A p_1)(p_A \bar p_1) - 2(\bar p_A \bar p_1)(p_A p_1)$$

$$+ (\bar p_A p_A)\left[-(p_1 p_1) - (\bar p_1 \bar p_1) + (p_1 \bar p_1) + (\bar p_1 p_1)\right] =$$

$$= \frac{e^4}{s^2}\Big[4\underbrace{\left(E_1 E_A + \vec p_1\vec p_A\right)\left(E_1 E_A - \vec p_1 \vec p_A\right)}_{\cancel{(E_1 E_A)^2} - (\vec p_1 \vec p_A)^2} \underbrace{- 2\left(E_1 E_A + \vec p_1 \vec p_A\right)^2 - 2\left(E_1 E_A - \vec p_1 \vec p_A\right)^2}_{-2\left[\cancel{(E_1 E_A)^2} + (\vec p_1\vec p_A)^2\right] - 2\left[\cancel{(E_1 E_A)^2} + (\vec p_1\vec p_A)^2\right] - 2\left[\cancel{+X}\right] - 2\left[\cancel{-X}\right]} \oplus$$

$$-8\left(\vec p_1 \vec p_A\right)^2$$

$$\oplus\ \left(E_A^2 + |\vec p_A|^2\right)\Big[\underbrace{-2\left(\cancel{E_1^2} - |p_1|^2\right) + 2\left(\cancel{E_1^2} + |\vec p_1|^2\right)}_{4|p_1|^2}\Big]$$

$$4|p_A|^2$$

$$= \frac{e^4}{s^2}\left[-8(\vec{p}_1\,\vec{p}_A)^2 + 4E_A^2|\vec{p}_1|^2 + 4|\vec{p}_A|^2|\vec{p}_1|^2\right] =$$

$$\left(-\frac{(u-t)^2}{2} + \frac{s^2}{2} - 2m_\pi^2 s\right) =$$
$$\;\;\;(1)\qquad\quad (2)\qquad\quad (3)$$

$p_B p_1 + p_B p_2$

$$(1) = \frac{\left[(p_1-\bar{p}_A)^2 - (p_1-p_A)^2\right]^2}{-2} =$$

$$\frac{-\left[\begin{matrix}\cancel{p_1^2} + \cancel{\bar{p}_A^2} - 2p_1\bar{p}_A \\ -\cancel{p_1^2} + \cancel{p_A^2} - 2p_1p_A\end{matrix}\right]^2}{-2} =$$

$$= -2\left[p_1\bar{p}_A - p_1p_A\right]^2 =$$

$$= -2\left[2(\vec{p}_1\,\vec{p}_A)\right]^2 =$$

$$= -8(\vec{p}_1\,\vec{p}_A)^2$$

$$(2) = \frac{s^2}{2} = \frac{(p_A+\bar{p}_A)^4}{2} = \frac{(2E_A)^4}{2} = 8E_A^4$$

$$(3) = -2\,p_1^2(p_A+\bar{p}_A)^2 =$$

$$= -2(E_1^2 - |\vec{p}_1|^2)(2E_A)^2 =$$

$$= -8(E_1^2E_A^2 - |\vec{p}_1|^2E_A^2)$$

$$4E_A^2|\vec{p}_1|^2 - 8E_1^2E_A^2 + 4E_A^2|\vec{p}_1|^2$$

$$E_A^2 = \cancel{m_e^2}^{\,0} + \vec{p}_A^{\,2}$$

$$8|p_A|^2|p_A|^2 - 8E_1^2|p_A|^2 + 4|p_A|^2|p_1|^2 =$$

$$= 4|p_A|^2\left(2|E_A|^2 - 2\cancel{E_1^2} + |p_1|^2\right)$$

- $E_A = E_B$

$$E_A^2 = \frac{E_AE_B + E_A^2 + E_B^2}{3} = \frac{(E_A+E_B)^2}{3}$$

- $E_1 = E_2$

$$E_1^2 = \frac{E_1E_2 + E_2^2 + E_1^2}{3} = \frac{(E_1+E_2)^2}{3}$$

OK

$$\left(-\frac{(u-t)^2}{2} + \frac{s^2}{2} - 2m_\pi^2 s\right) =$$

$$= \left[-\frac{\left(-2E_A^2\cdot(\cancel{1}+\cos\theta) + 2E_A^2(\cancel{1}-\cos\theta)\right)^2}{2} + \frac{(4E_A^2)^2}{2} - 2m_\pi^2(4E_A^2)\right] =$$

$$= -\frac{(-4E_A^2\cos\theta)^2}{2} + \frac{(4E_A^2)^2}{2} - 2m_\pi^2(4E_A^2) =$$

$$= 8E_A^4\underbrace{(1-\cos\theta^2)}_{\sin\theta^2} - 8m_\pi^2E_A^2$$

$$1 = \cos^2\theta + \sin^2\theta$$

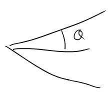
$$\hookrightarrow \frac{e^4}{16E_A^4}\,8E_A^2\left(E_A^2\sin^2\theta - m_\pi^2\right) = \frac{e^4}{2}\left(\sin^2\theta - \frac{m_\pi^2}{E_A^2}\right)$$

$$-\frac{4m_\pi^2}{s}$$

CM $\theta = \frac{\pi}{2}$

$$= \frac{e^4}{2}\sin^2\theta\left[1 - \frac{4m_\pi^2}{s}\left(1 + \frac{1}{\tan^2\theta}\right)\right] \qquad \approx 0$$

$$= \frac{e^4}{2}\sin^2\theta\,\underbrace{\left(1 - \frac{4m_\pi^2}{s}\right)}_{\beta^2}$$



$$= \boxed{\frac{e^4}{2}\sin^2\theta\,\beta^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4se^4}|M|^2 = \frac{\cancel{e^4}}{2}\sin^2\theta\,\beta^2\,\frac{\alpha^2\beta}{16s\cancel{e^4}} = \frac{\alpha^2\beta^3\sin^2\theta}{8s}}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{\alpha^2\beta}{4se^4}|M|^2$$

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$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2 \beta}{4 s e^4} |M|^2$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \cancel{\int_0^{\pi}\int \frac{\alpha^2 \beta^3 \sin^2\theta}{8s}} \sin\theta \, d\theta \, d\phi = \pi \frac{\alpha^2 \beta^3}{3s}$$