Lecture 7:
The Cosmic Microwave Background dipole.

L: If the Cosmic Microwave Background were perfectly isotropic, it could only be so in some special reference frame. When we change our velocity, the Doppler effect implies we should see a dipole. The dipole is in fact observed, and we also measure its time variation because of the motion of the Earth around the Sun (and other detailed complications due to the motion of the observatory, from the rotation of the Earth, etc.).

L: Review of Doppler effect and its impact on a diffuse background intensity:

Imagine an observer moving along the x-axis with velocity v relative to a source. From the frame of the observer, the source moves along the x-axis toward the negative direction. The observer sees the source along a line of sight forming an angle θ from the x-axis. The source emits light with an intrinsic period P', frequency ν'. The period and frequency observed by the observer is:

$$P = P'\gamma(1 - v/c\cos\theta) \ ; \qquad \nu = \frac{\nu'}{\gamma(1 - v/c\cos\theta)} \ ; \tag{1}$$

The light ray has a momentum in the frame of the observer with components: $p_x = -p\cos\theta$, $p_y = -p\sin\theta$ (p can be, for example, the momentum of one photon, $p = h/(cP)$, in the observer frame).

In the source frame, Doppler transformation of the momentum vector tells us the momentum of the light ray is (primes refer to quantities measured in the source frame):

$$p'_x = \gamma(p_x + vp/c) = -\gamma p(\cos\theta - v/c) \ ; \qquad p'_y = -p\sin\theta \ . \tag{2}$$

The modulus of the momentum transforms of course like the frequency: $p' = p\gamma(1 - v/c\cos\theta)$. The angle the light ray direction forms with the x-axis suffers aberration, with the relation

$$\sin\theta' = p'_y/p' = \frac{\sin\theta}{\gamma(1 - v/c\cos\theta)} \ . \tag{3}$$

When we measure an intensity of a diffuse background, we measure flux per unit solid angle. We need to know how an element of solid angle transforms.

$$\frac{d\Omega}{d\Omega'} = \frac{\sin\theta\, d\theta}{\sin\theta'\, d\theta'} = \frac{d(\cos\theta)}{d(\cos\theta')} \ . \tag{4}$$

$$d\cos\theta' = d\left(\frac{\cos\theta - v/c}{1 - v/c\cos\theta}\right) = \frac{1 - v/c\cos\theta + (\cos\theta - v/c)v/c}{(1 - v/c\cos\theta)^2} = \frac{d(\cos\theta)}{\gamma^2(1 - v/c\cos\theta)^2} \ . \tag{5}$$

This implies also $d\theta = d\theta'\gamma(1 - v/c\cos\theta)$.

How does intensity transform? Flux is $dF = Id\Omega$, and transforms as $dF = dF'/\gamma^2/(1-v/c\cos\theta)^2$, because each photon is blueshifted by $\nu = \nu'/\gamma/(1 - v/c\cos\theta)$, and the arrival rate of photons is blueshifted by the same factor. So intensity $I = dF/d\Omega$ transforms as:

$$I = \frac{I'}{\gamma^4(1 - v/c\cos\theta)^4} \ . \tag{6}$$

For intensity per unit frequency, noting that $d\nu = d\nu'/\gamma/(1 - v/c\cos\theta)$, we have

$$I_\nu = \frac{I'_{\nu'}}{\gamma^3(1 - v/c\cos\theta)^3} \ . \tag{7}$$

In general, and remembering the derivation of blackbody radiation, the intensity of light is related to the quantum occupation number of photons by:

$$I = \frac{2h\nu^3}{c^2}\, N_{QO} \ . \tag{8}$$

In the particular case of blackbody or radiation at equilibrium at temperature T, we have $N_{QO} = (e^{h\nu/(kT)} - 1)^{-1}$, but the above is a general relation. Obviously, when we look at radiation from different frames, the photon occupation of any quantum state does not change, it is an invariant. So, I_ν/ν^3 is a Lorentz invariant, in agreement with the result above. When we change frame, the temperature we measure for the CMB changes according to

$$T = \frac{T'}{\gamma(1 - v/c\cos\theta)} \ . \tag{9}$$

This implies the correct change in the shape and intensity at all frequencies, so that the blackbody spectrum is maintained under a Doppler shift, simply with a change of temperature as a function of the direction. There is a special frame, that of the CMB, where the CMB blackbody is isotropic, and in other frames moving with respect to the CMB, the temperature is given by the above equation as a function of the direction, or the angle θ formed with the axis of our motion with respect to the CMB frame. This gives rise to the CMB dipole.

Q: What is the velocity that causes us to observe a dipole on the CMB?

1. The velocity of the Earth around the Sun: $v \simeq 30\,\mathrm{km\,s^{-1}}$, this velocity varies in direction over the year so the CMB dipole changes. This has been measured (we also measure the Doppler effect caused by the Earth motion when we measure radial velocities of any star).
2. The Sun moves around the center of the Milky Way, at $v \simeq 240\,\mathrm{km\,s^{-1}}$.
3. The Milky Way moves relative to the mean frame of the Local Group: mainly, it is falling toward M31.
4. The Local Group moves owing to the acceleration by all the large-scale structure around us. For example, the Virgo cluster attraction has induced a motion of the Local Group towards Virgo.

All of these motions add up to a dipole measured on the CMB corresponding to a velocity $v = 369\,\mathrm{km\,s^{-1}}$, in the direction of Galactic longitude $\ell = 264^o$ and latitute $b = 48^o$. The direction of the motion of the Sun around the Milky Way center, and the fall toward M31, are such that the velocity of the Local Group from the CMB is larger, $v_{\rm LG} \simeq 600\,\mathrm{km\,s^{-1}}$.