QUANTUM FIELD THEORY

Fall 2021

One-loop vacuum polarization in scalar QED

Consider the following theory of an Abelian gauge field A_μ coupled to a free complex scalar ϕ,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*\,(D^\mu\phi) - m_0^2\phi^*\phi \tag{1}$$

where $D_\mu\phi = (\partial_\mu + ie_0A_\mu)\phi$. The fields that appear in this Lagrangian density are bare fields, and m_0, e_0 are bare parameters.

1. Set up renormalized perturbation theory for this model: *i.e.*, introduce renormalized fields, renormalized mass and charge. Write the full Lagrangian density. **(2 points)**

2. Write the momentum space Feynman rules, including the ones for counterterms. Argue that some of these counterterms must be related. **(2 points)**

3. Compute the relevant one-loop diagrams for the photon self-energy in this theory. Working in the on-shell renormalization scheme, compute the full one-loop contribution $\hat{\Pi}_2^{\mu\nu}(q)$. In particular, show that the Ward identity $q_\mu\hat{\Pi}_2^{\mu\nu}(q) = 0$ is satisfied (do not use Ward's identity to simplify your intermediate computations). **(6 points)**