Guillem Sánchez, Guillermo Abad

Week 8

1. For the process $e^- p \to e^- X$ at $\sqrt{s} \gg m_p$, show that in the parton model

$$\frac{d\sigma}{dx\,dy} = \left(\sum_i x f_i(x) Q_i^2\right) \frac{2\pi\alpha^2 s}{q^4}\left[1 + (1-y)^2\right]$$

where

$$x = -\frac{q^2}{2m_p\nu} \quad , \nu = \frac{p_B . q}{m_p} \quad , y = \frac{p_B . q}{p_B . p_A},$$

$s = (p_A + p_B)^2$, $q = p_A - p_1$, m_p is the proton mass, p_B is the momentum of the proton, and p_A and p_1 are the momenta of the incoming and outgoing electron respectively. $f_i(x)$ is the parton distribution function of the parton i and Q_i its charge in units of e.

$$\frac{d\sigma}{dx dy} = \int d\Omega \int dE_1 \left(\frac{d\sigma}{dE_1 d\Omega}\right)_{LAB} \delta\left(x + \frac{q^2}{2m_p \nu}\right)\delta\left(y - \frac{p_B q}{p_B p_A}\right) =$$

$$= \int d\Omega \int dE_1 \frac{4\alpha^2 E_1^2}{q^4} F_2(x) \left(\frac{\cos^2(\theta/2)}{\nu} + \frac{\sin^2(\theta/2)}{x\, m_p}\right)\delta\left(x + \frac{q^2}{2m_p \nu}\right)\delta\left(y - \frac{p_B q}{p_B p_A}\right) =$$

$$= F_2(x) \frac{4\alpha^2}{q^4} 2\pi \int d\theta \int dE_1 \; E_1^2 \left(\frac{\cos^2(\theta/2)}{\nu} + \frac{\sin^2(\theta/2)}{x\, m_p}\right) \sin\theta \; \delta\left(x + \frac{q^2}{2m_p \nu}\right)\delta\left(y - \frac{p_B q}{p_B p_A}\right)$$

$$\delta\left(\frac{2 \overset{p_B \cdot q}{m_p \nu}\, x}{4E_A E_1} + \sin^2\left(\frac{\theta}{2}\right)\right)$$

$\delta($

- $E_1 = |p_1|^2$
- $q = p_A - p_1$
- $q^2 = (p_A - p_1)^2 = \underbrace{p_A^2 + p_1^2}_{2m_e^2 \simeq 0} - 2p_A p_1 = 4E_A E_1 \sin^2\left(\frac{\theta}{2}\right)$
- $\nu = \dfrac{p_B\, q}{m_p}$

$d\Omega = d\theta \sin\theta\, d\phi$, $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$

$$-2(E_A \cdot E_1 - E_A \cdot E_1 \underbrace{\cos\theta}_{1 - 2\sin^2\left(\frac{\theta}{2}\right)})$$

$$-2\left(+2E_A E_1 \sin^2(\theta/2)\right)$$

$$-4 E_A E_1 \sin^2\left(\frac{\theta}{2}\right)$$

$$\boxed{\frac{s}{4}\left(1 + (1-y)^2\right) = \int d\theta \int dE_1 \left(\quad \right) \sin\theta\, \delta(\,)\, d(\,)}$$

LAB $p_B = 0$ → θ

$$y = \frac{p_B \cdot q}{p_B \cdot p_A} \overset{?}{=} \frac{E_A - E_1}{E_A}$$

- $p_B \cdot q = p_B \cdot (p_A - p_1) = p_B p_A - p_B p_1$
- $p_B \cdot p_A = p_B p_A$

$$1 - \frac{p_B p_1}{p_B p_A} = 1 - \frac{E_B E_1 - \cancel{p_B p_1}}{E_B E_A - \cancel{p_B p_A}} = 1 - \frac{E_1}{E_A} = \frac{E_A - E_1}{E_A}$$

$$\frac{d\sigma}{dx dy} = \int F_2(x) \frac{2\pi\alpha^2 s}{q^4}\left(1 + \left(1 - \frac{p_B q}{p_B p_A}\right)^2\right)\delta\left(y - \frac{p_B q}{p_B p_A}\right) dE_1$$