First Part: Space-time and the expansion of the Universe.

Lecture 1:

Fundamental observations: Hubble's law and the scale factor.

Introduction: Course: First part taught by me will be graded based on two homeworks, a presentation, and one exam on November 16. An additional homework is shared with Alessio's Notari part. Textbooks: Ryden, easiest one. Other books by Peacock, Mukhanov, Dodelson... Office hours: Friday 3:30 - 5. Participation requested.

Q: What is cosmology? When did it start?

L: History of basic ideas:

1. Earliest civilizations: Flat Earth, sky dome.
2. Spherical Earth, geocentric model, celestial sphere.
3. Heliocentric model, the Earth is a planet, Newton's laws.
4. The Sun is a star, stars are in a disk-shaped, dusty Galaxy.
5. Universe of many galaxies which is expanding.

Q: What were the most important physical laws and principles to advance the science of cosmology from the beginning to the present state?

A: There were observations, and there was our growing understanding of the laws of physics that govern the universe. In particular, our understanding of gravity has played a dominant role in supporting our description of the universe. Our modern understanding of cosmology is based on the new theory of gravity: General Relativity.

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L: Around 1900, the "Victorian universe": the universe is our Galaxy, distances and velocities of stars were being measured. The vision was of a static Newtonian universe of a finite Galaxy surrounded by infinite, empty space.

L: Discovery of galaxies: nebulae had been known for a long time, but it took time to realise they belong to a very diverse set of objects. Some nebulae are gaseous clouds in our own Galaxy, as was proven when the first spectra were taken and were shown to be emission-line spectra. But what we know today as galaxies have a continuum spectrum that arises from stars. Still, this was not clear in the early twentieth century, when the issue of the nature of the "spiral nebulae" was being debated. Two discoveries provided the essential proof that they were galaxies like the Milky Way: the period-luminosity relation of Cepheids in the Magellanic Clouds by Henrietta Leavitt, and the discovery of Cepheid stars and measurement of their periods in M31 by Edwin Hubble.

L: Henrietta Leavitt, Cepheid lightcurve, Hubble, M31 Cepheids.

Q: What is flux f of a star? How do we calculate it from the luminosity L and distance r to a star?

$$f = \frac{L}{4\pi r^2} \ . \tag{1}$$

Q: Once you know the period-luminosity relation, how was the distance to M31 known?

Answer: The key is to estimate the luminosity L by some means other than the flux f, in this case by comparing to other Cepheids of known distance and using the period-luminosity relation. Then with the known flux the distance r is obtained.

L: One may try to find what are known as standard candles, objects where the luminosity is predicted from some other observation. The most recent useful example of this are Supernovae Type Ia. A relation is also used for SNIa to estimate their luminosity by comparing to others.

Q: Could this universe of galaxies be infinite, both in space and in time? In other words, could the Universe be infinite and have existed forever?

L: Olbers' paradox: consider an infinite universe of stars. Let us measure the flux from each star.

Q: What is the contribution to the total intensity received from the sky, if there is a constant number density n of stars?

L: Intensity (or surface brightness): Flux per unit solid angle (solid angle can be measured in units of sterradian, or square degrees, square arc seconds...)

$$dI = \frac{df}{d\Omega}; \quad df = \frac{L}{4\pi r^2}\, n\, r^2\, dr\, d\Omega = \frac{n\,L}{4\pi} dr\, d\Omega\ . \tag{2}$$

$$dI = \frac{n\,L}{4\pi}\, dr\ . \tag{3}$$

Q: So what is the total intensity? Each spherical shell of thickness dr contributes the same intensity. So the total is infinite!

Q: What possible solutions to the problem can you think of?

1. The stars are not uniform, they are present only in galaxies.
2. Stars have finite angular size, and they should obstruct each other.
3. Dust may obscure the more distant stars.
4. The universe might not be infinitely large, or might have a decreasing density of stars at great distances from us.
5. The universe might not be infinitely old.
6. The universe might be expanding instead of being static, then light from distant stars would get redshifted.

Q: Which of the above possible solutions can and cannot explain Olbers' paradox?

L: Possibilities 1 to 4 do not work at all as solutions. Solution number 6 is the most important one: it alone solves the problem in the Steady State model. Solution number 5 is also relevant for the Big Bang model.

L: Final conclusion: the Universe cannot be static, infinitely large with a uniform population of stars, and infinitely old.

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Hubble's Law

L: Hubble measured redshift and distances of several galaxies.

L: Redshift: this measures how much the wavelength of an absorption or emission line in the spectrum of a galaxy has been shifted owing to the velocity of the galaxy relative to us.

$$z = \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}} \ . \tag{4}$$

L: Hubble found a linear relation between distance measured from Cepheids and other luminosity indicators, and redshift. Very few galaxies (only a few among the most nearby ones) had blueshifts.

L: Hubble's law:

$$z = \frac{H_0}{c} r; \quad v = H_0 r \ . \tag{5}$$

L: Hubble obtained the value $H_0 = 500\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. However he had made several errors in the distance measurements which were only corrected over a long period of time. The best current measurement from Cepheids, maser in NGC4258 and Detached Eclipsing Binaries is $H_0 = 73.2 \pm 1.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Riess et al. 1604.01424).

L: Hubble's law implies that we can write the position of a galaxy relative to us at any time as: $\mathbf{r}_1(t) = a(t)\,\mathbf{x}_1$, where $\mathbf{x}_1$ is the *comoving* coordinate.

Q: What is the velocity of the galaxy?

$$\mathbf{v}(t) = \dot{a}(t)\mathbf{x}_1 = \frac{\dot{a}(t)}{a(t)} \mathbf{r}_1(t) \ . \tag{6}$$

This implies Hubble's law. Hubble's constant at time t is $H(t) = [\dot{a}(t)/a(t)]$.

Important: The quantity $a(t)$ is called the *scale factor*. The expansion of the universe is like a stretching of space in proportion to the scale factor.

L: *Cosmological principle*: the universe is homogeneous and isotropic on sufficiently large scales.

Q: What does homogeneous mean? What does isotropic mean?

Q: If the expansion of the Universe is homogeneous and isotropic, what is the most general law relating velocity to distance? It's only Hubble's law!

Important: Hubble's law does not mean that we are at the center of an expansion of all the galaxies away from us. Hubble's law is observed the same way from any point. It is the only expansion law that is compatible with a homogeneous and isotropic Universe.

L: Before Hubble's discovery, Albert Einstein, Alexander Friedman and others discovered models of the universe based on the new theory of gravity of General Relativity, and on the assumption of homogeneity and isotropy of the universe. This assumption was then not suggested by any observation, but it was assumed because of its simplicity and beauty. It is interesting that the universe actually turned out to be that way!

L: Hubble's law showed that the universe is expanding, and it was the first indication of the validity of the cosmological principle. Other evidence for the cosmological principle came only much later: the uniform distribution of distant radio sources on the sky, or quasars. More recently, galaxy surveys show them to be homogeneously distributed. We'll see later about the isotropy of the Cosmic Microwave Background.

L: Homogeneity and isotropy start being good approximations on scales larger than $\sim 50\,\mathrm{Mpc}$.

Q: Since the universe is expanding now, what does that say about its past?

A: Galaxies were closer together than they are now.

Q: If there were no change in the velocity of a fixed galaxy, currently at distance r, how long has the universe been expanding?

A: $t_0 = 1/H_0 = 13.4 \pm 0.4\,\mathrm{Gyr}$.

L: All galaxies were increasingly close as we go back to the past. If matter was conserved, the density had to be a lot higher:

Q: How did the density of matter depend on the scale factor?

A: $\rho = \rho_0 (a/a_0)^{-3}$.

L: This leads to the Big Bang idea. In the Big Bang, all matter was indeed created a finite time ago, even though the age is not given exactly by $1/H_0$ because of the acceleration by gravity.

Q: In this Big Bang model, once the Hubble constant is measured, what prediction can we make that can test the model?

A: Stars and any other objects in the universe should all be younger than the age of the universe.

L: Is the Big Bang the only possibility? Not if matter can be created continuously. The Steady State model makes the hypothesis of the perfect cosmological principle: the properties of the universe do not change with time. As the universe expands, more matter is created so that the density is constant. Since the Hubble constant is then constant in time, the scale factor changes as $a(t) \propto e^{H_0 t}$. There is no beginning or end.

Q: How would you test the Steady-State model?

A: Look at the universe at great distances, where it is seen in the past. Is it exactly the same as the present universe?

L: In the 1960s there was increasing evidence that the radio source population has evolved. Today we can see this directly in the galaxies at high redshift. The discovery of the Cosmic Microwave Background discarded the Steady State model, but it is still conceptionally useful to understand the alternative possibilities.

Summary: The observation of the expansion of the universe led to two models: the Big Bang and the Steady State theory. The Steady State theory was eliminated observationally. The Big Bang predicts the age of the universe, and a horizon. The expansion solves Olbers' paradox.