Contents

1 Introduction

Thirty-four years after the discovery of asymptotic freedom [1], Quantum Chromodynamics (QCD), the theory of the strong interactions between quarks and gluons, remains a challenge. There exist no analytic, truly systematic methods with which to analyse its non-perturbative properties. Some of these, for example its thermodynamic properties, can be studied by means of the lattice formulation of QCD. However, other more dynamical ones, for example the transport properties of the quark-gluon plasma (QGP), are very hard to study on the lattice because of the inherent Euclidean nature of this formulation. Even if in the future most features of QCD can be addressed on the lattice, as good as possible a theoretical understanding will still be desirable.

A long-standing hope is that a reformulation of QCD in terms a of a new set of string-like degrees of freedom would shed light on some of its mysterious properties. The purpose of these lectures is to review recent progress in the implementation of this idea in the context of the so-called 'gauge/gravity' correspondence.

We will begin by explaining why one ought to expect a stringy reformulation of QCD, or more generally of any gauge theory, to exist. We will then review one of the simplest examples of a gauge/gravity correspondence, namely the AdS/CFT correspondence between type IIB string theory on $AdS_5 \times S^5$ and four-dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM) theory. After that we will consider this correspondence at finite temperature, and we will see that this suffices to make contact with the physics of the deconfined QGP created in heavy ion collision experiments. In the following chapter we will consider a simple example of a confining theory with a gravity dual, and study the confinement/deconfinement phase transition that occurs as a function of the temperature. We will finish with a brief discussion of present limitations and challenges for the future.

The emphasis of this review is on conciseness and conceptual aspects rather than calculational details. It is also not exhaustive but rather the opposite, since the goal is to be able to discuss some of the most recent developments with as little technology as possible.

2 Why QCD ought to have a string dual

The expectation that it ought to be possible to reformulate QCD as a string theory can be motivated at different levels. Heuristically, the motivation comes from the fact that QCD is believed to contain string-like objects, namely the flux tubes between quark-antiquark pairs responsible for their confinement. Modelling these tubes by a string leads to so-called Regge behaviour, that is, the relation $M^2 \sim J$ between the mass and the angular momentum of the tube. The same behaviour is observed in the spectrum of mesons, *i.e.*, quark-antiquark bound states, in the real world. This argument, however, would not apply to non-confining gauge theories.

A more precise motivation for the existence of a string dual of QCD, or more generally of any gauge theory, comes from consideration of the 't Hooft's large-$N_{\rm c}$ limit [2] (see [3] for a beautiful review). QCD is a gauge theory with gauge group $SU(3)$ and, because of dimensional transmutation, it possesses no expansion parameter. 't Hooft's idea is to consider a generalisation of QCD obtained by replacing the gauge group by $SU(N_{\rm c})$, to take the limit $N_{\rm c} \to \infty$, and to perform an expansion in $1/N_{\rm c}$.

The degrees of freedom of this generalised theory are the gluon fields $A^i_{\mu j}$ and the quark fields q^i_a, where $i, j = 1, \ldots, N_{\rm c}$ and $a = 1, \ldots, N_{\rm f}$, with $N_{\rm f}$ the number of quark flavours. The number of independent gauge fields is $N_{\rm c}^2 - 1$ because of the fact the gauge group is

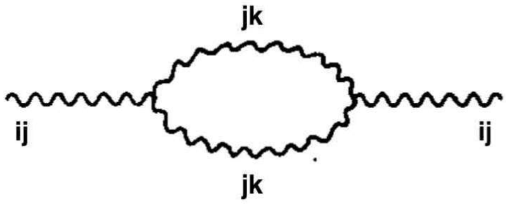


Figure 1: One-loop gluon self-energy Feynman diagram.

$SU(N_c)$ and not $U(N_c)$, but since we will be working in the limit $N_c \to \infty$ we will ignore this difference. We will thus take the number of gluons to be $\sim N_c^2$. This is much larger than the number of quark degrees of freedom, $N_f N_c$, so we may expect (correctly) that the dynamics is dominated by the gluons in the large-N_c limit. We will therefore start by studying the theory in this limit as if no quarks were present, and then examine the effect of their inclusion.

To start with, consider the one-loop gluon self-energy Feynman diagram of fig. 1. There are two vertices and one free colour index, so this scales as $g_{\rm YM}^2 N_c$. This means that for this diagram to possess a smooth limit in the limit $N_c \to \infty$, we must take at the same time $g_{\rm YM} \to 0$ while keeping the so-called 't Hooft coupling $\lambda \equiv g_{\rm YM}^2 N_c$ fixed. This is equivalent to demanding that the confinement scale, $\Lambda_{\rm QCD}$, remain fixed in the large-N_c limit. This can be seen by noting that, with the scaling above, the one-loop β-function,

$$\mu \frac{d}{d\mu} g_{\rm YM}^2 \propto -N_c \, g_{\rm YM}^4 \,, \tag{1}$$

becomes independent of N_c when written in terms of λ:

$$\mu \frac{d}{d\mu} \lambda \propto -\lambda^2 \,. \tag{2}$$

The determination of the N_c-scaling of Feynman diagrams is simplified by the so-called double-line notation. This consists of drawing the line associated to a gluon as a pair of lines associated to a quark and an anti-quark, as indicated in fig. 2. In fig. 3 the three-gluon vertex (left) and the quark-antiquark-gluon vertex (right) are drawn in double-line notation. Fig. 4 displays the one-loop gluon self-energy diagram of fig. 1 in this notation. We note that the factor of N_c is associated to the 'free' internal line carrying the index 'k' in the figure.

We will now see that Feynman diagrams naturally organise themselves in a double-series expansion in powers of $1/N_c$ and λ. For this purpose it suffices to consider a few vacuum diagrams (with no quarks for the time being). Some one-, two- and three-loop diagrams are shown in fig. 5. We see that they all scale with the same power of N_c but a

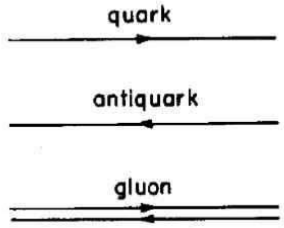


Figure 2: Double-line notation.

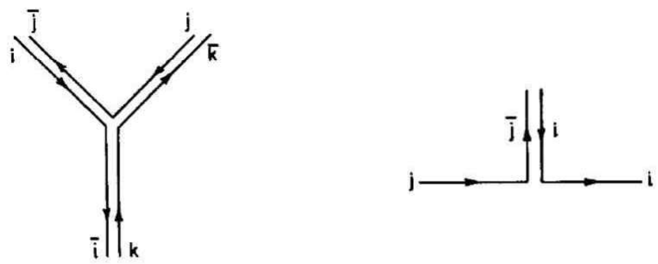


Figure 3: Vertices in double-line notation.

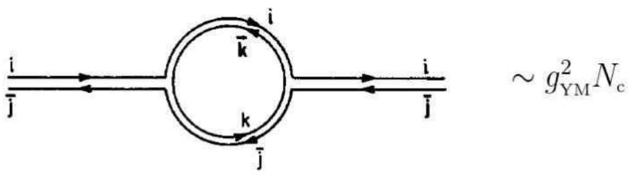


Figure 4: Gluon self-energy diagram in double-line notation.

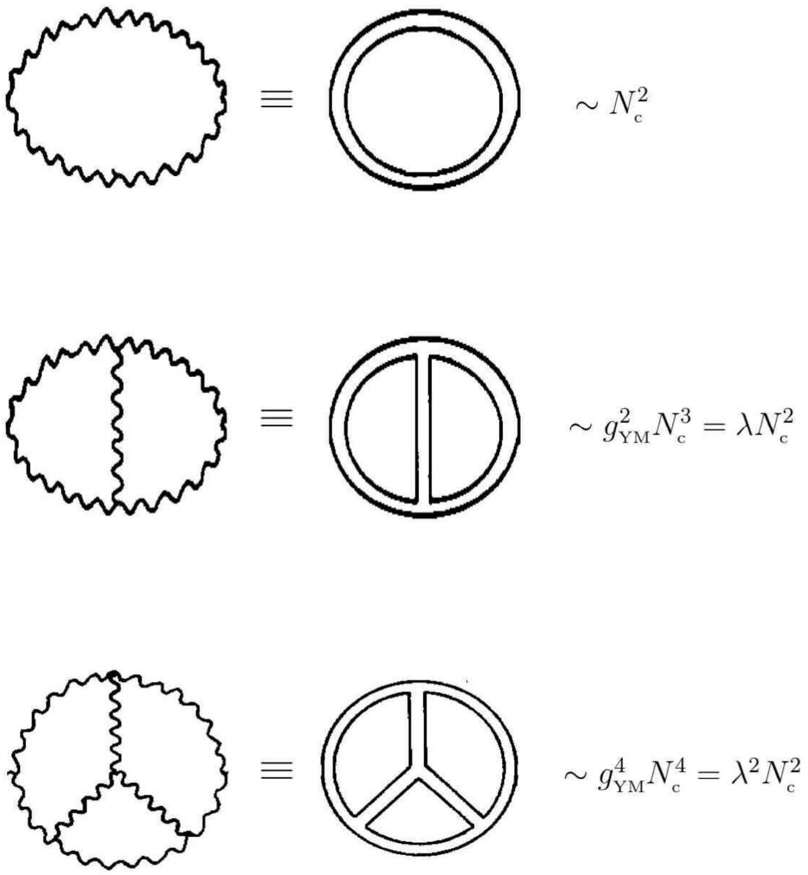
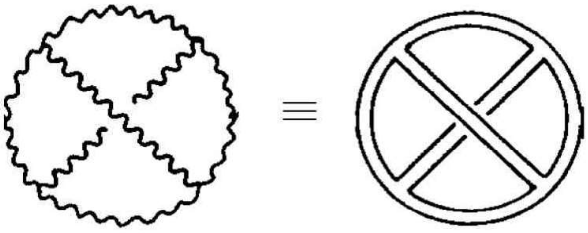


Figure 5: Some planar diagrams.

power of $\lambda^{\ell-1}$, with ℓ the number of loops. The N_c^2 scaling is not the same for all diagrams, however. For example, the three-loop diagram in fig. 6 scales as λ^2, and is thus suppressed with respect to those in fig. 5 by a power of $1/N_c^2$; the reader is invited to draw other diagrams suppressed by higher powers of $1/N_c^2$. The difference between the diagrams in fig. 5 and that of fig. 6 is that the former are planar, *i.e.,* can be 'drawn without crossing lines', whereas the latter is not. We thus see that diagrams are classified by their topology, and that non-planar diagrams are suppressed in the large-N_c limit.

The topological classification of diagrams, which leads to the connection with string theory, can be made more precise by associating a Riemann surface to each Feynman diagram, as follows. In double-line notation, each line in a Feynman diagram is a closed loop that we think of as the boundary of a two-dimensional surface or 'face'. The Riemann



$\sim g_{\rm YM}^4 N_c^2 = \lambda^2$

Figure 6: A non-planar diagram.

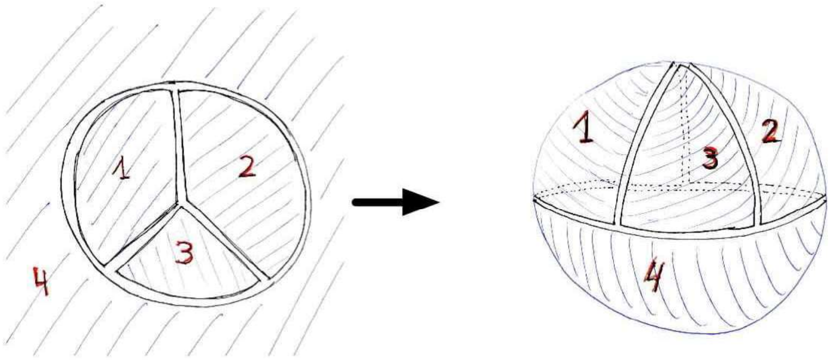


Figure 7: A Riemann surface associated to a planar diagram.

surface is obtained by gluing together these faces along their boundaries as indicated by the Feynman diagram. In order to obtain a compact surface we add 'the point at infinity' to the face associated to the external line in the diagram. This procedure is illustrated for a planar diagram in fig. 7, and for a non-planar diagram in fig. 8. In the first case we obtain a sphere, and the same is true for any planar diagram. In the second case we obtain a torus. It turns out that the power of N_c associated to a given Feynman diagram is precisely N_c^χ, where χ is the Euler number of the corresponding Riemann surface. For a compact, orientable surface of genus g with no boundaries we have $\chi = 2 - 2g$. Thus for the sphere $\chi = 2$ and for the torus $\chi = 0$. We therefore conclude that the expansion of any gauge theory amplitude in Feynman diagrams takes the form

$$\mathcal{A} = \sum_{g=0}^{\infty} N_c^\chi \sum_{n=0}^{\infty} c_{g,n} \lambda^n \,, \tag{3}$$

where $c_{g,n}$ are constants. We recognise the first sum as the loop expansion in Riemann

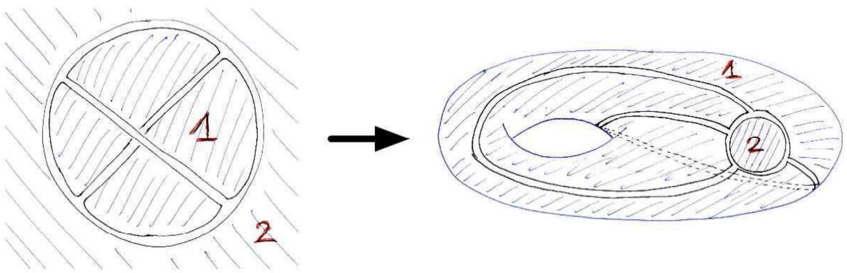


Figure 8: A Riemann surface associated to a non-planar diagram.

surfaces for a closed string theory with coupling constant $g_s \sim 1/N_c$. Note that the expansion parameter is therefore $1/N_c^2$. As we will see later, the second sum is associated to the so-called α'-expansion in the string theory.

The above analysis holds for any gauge theory with Yang-Mills fields and possibly matter in the adjoint representation, since the latter is described by fields with two colour indices. In order to illustrate the effect of the inclusion of quarks, or more generally of matter in the fundamental representation, which is described by fields with only one colour index, consider the two diagrams in fig. 9. The bottom diagram differs from the top diagram solely in the fact that a gluon internal loop has been replaced by a quark loop. This leads to one fewer free colour line and hence to one fewer power of N_c. Since the flavour of the quark running in the loop must be summed over, it also leads to an additional power of N_f. Thus we conclude that internal quark loops are suppressed by powers of N_f/N_c with respect to gluon loops. In terms of the Riemann surface associated to a Feynman diagram, the replacement of a gluon loop by a quark loop corresponds to the introduction of a boundary, as illustrated in fig. 10 for the diagrams of fig. 9. The power of N_c associated to the Feynman diagram is still N_c^χ, but in the presence of b boundaries the Euler number is $\chi = 2 - 2g - b$. This means that in the large-N_c expansion (3) we must also sum over the number of boundaries, and so we now recognise it as an expansion for a theory with both closed and open strings. The open strings are associated to the boundaries, and their coupling constant is $g_{\rm op} \sim N_f \, g_s^{1/2} = N_f/N_c$.

The main conclusion of this section is therefore that the large-N_c expansion of a gauge theory can be identified with the genus expansion of a string theory. Through this identification the planar limit of the gauge theory corresponds to the classical limit of the string theory. However, the analysis above does not tell us how to construct explicitly the string dual of a specific gauge theory. We will see that in some cases this can be 'derived' by thinking about the physics of D-branes.

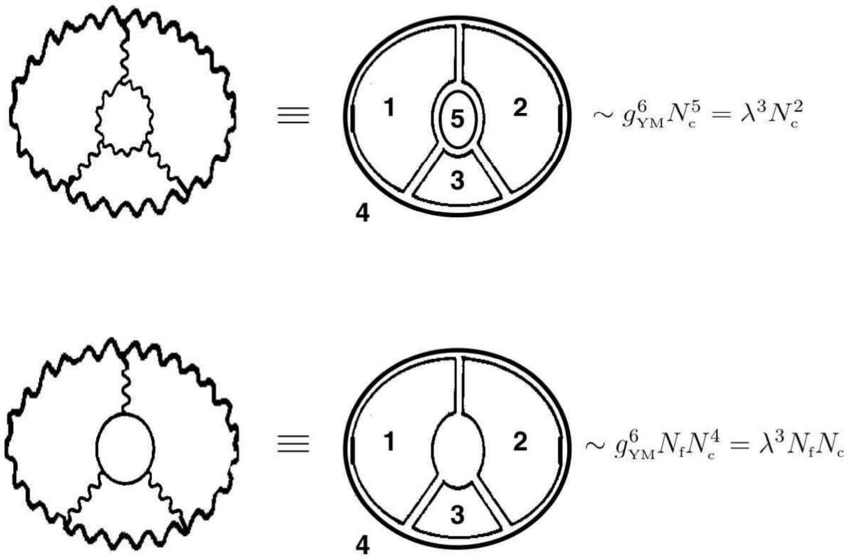


Figure 9: Two planar diagrams, one without quark loops (top diagram), and one with an internal quark loop (bottom diagram).

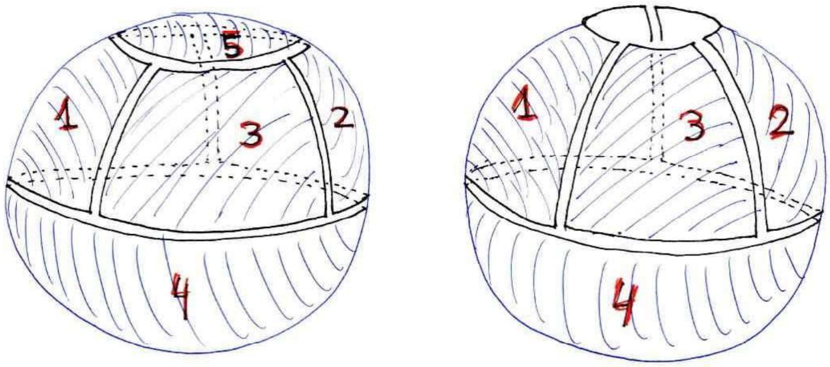


Figure 10: Riemann surfaces associated to the planar diagrams of fig. 9. The surface on the left (right) corresponds to the top (bottom) diagram.

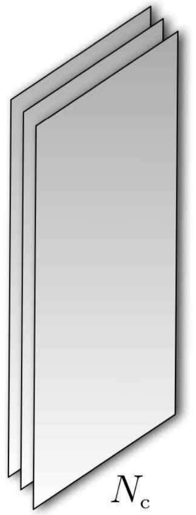


Figure 11: A set of N_c D3-branes.

3 The AdS/CFT correspondence

In this section we will study one of the simplest examples of a gauge/gravity duality: The equivalence between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills (SYM) theory on four-dimensional Minkowski space. Since this gauge theory is conformally invariant, this duality is an example of an AdS/CFT correspondence. We will see in later sections how non-AdS/non-conformal examples can be constructed.

3.1 The decoupling limit

To motivate the duality, let us consider the 'ground-state' of type IIB string theory in the presence of N_c D3-branes, as depicted in fig. 11. Although the picture may suggest that the spacetime around the branes is flat, this is not true. D-branes carry mass and charge, and therefore curve the spacetime around them, as indicated in fig. 12.

Far away from the branes the spacetime is flat, ten-dimensional Minkowski space, whereas close to them a 'throat' geometry of the form $AdS_5 \times S^5$ develops. Although this is not the way this spacetime is constructed in practice, conceptually it could be obtained by resumming an infinite number of tadpole-like diagrams with boundaries, of the form depicted in fig. 13, for a closed string propagating in the presence of the D3-branes.

We would like to compare the gravitational radius R of the D3-branes with the string length. On general grounds we expect (some power of) R to be proportional to Newton's

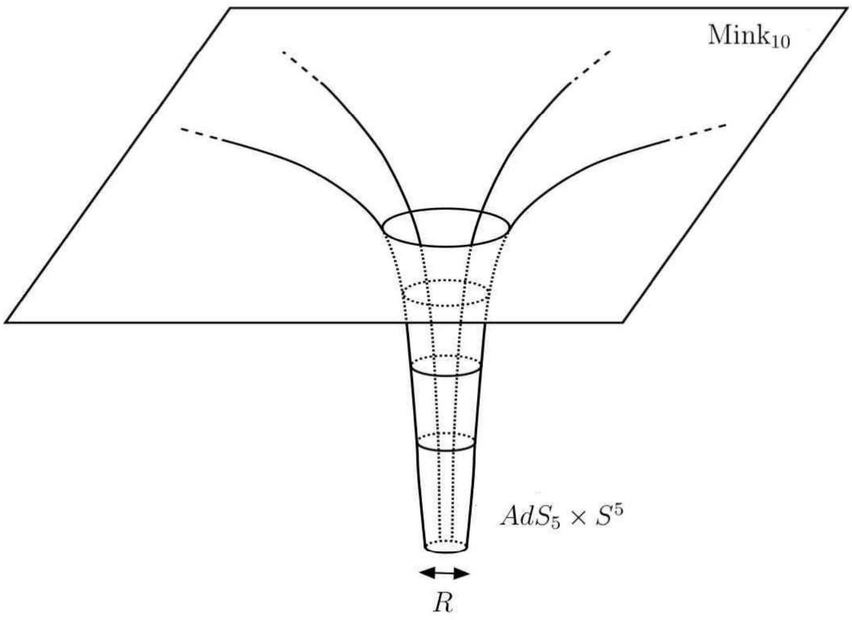


Figure 12: Spacetime around D3-branes.

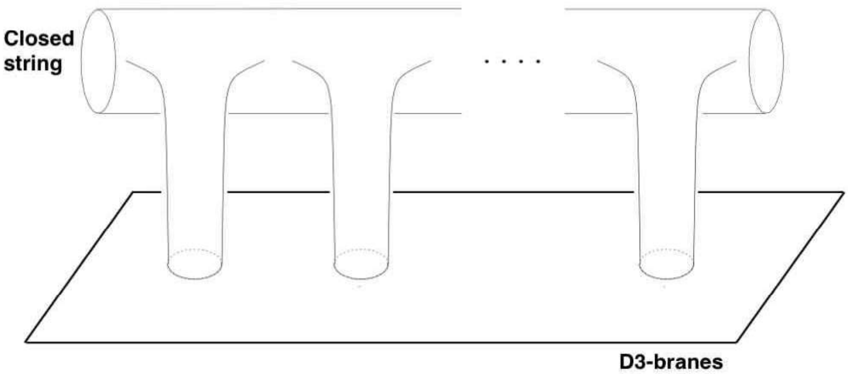


Figure 13: Tadpole-like diagrams whose sum leads to an effective geometry for closed strings.

constant, to the number of D3-branes and to their tension. Newton's constant is given by

$$16\pi G = (2\pi)^7 g_s^2 \ell_s^8 \,, \tag{4}$$

with ℓ_s the string length. We note that it is proportional to g_s^2, and that in ten dimensions it has dimensions of length^8. The D3-branes are solitonic objects whose tension scales as an inverse power of the coupling, $T_{\rm D3} \sim 1/g_s \ell_s^4$. It follows that the gravitational radius in string units must scale as $g_s N_{\rm c}$. The precise relation turns out to be

$$\frac{R^4}{\ell_s^4} = 4\pi g_s N_{\rm c} \,. \tag{5}$$

This means that if $g_s N_{\rm c} \ll 1$ then the description suggested in fig. 11 in terms of essentially zero-thickness objects in an otherwise flat spacetime is a good description. In this limit the the D3-branes are well described as a defect in spacetime, or more precisely as a boundary condition for open strings. Note that this conclusion relies crucially on the fact that the tension of D-branes scales as $1/g_s$ and not as $1/g_s^2$, as is typically the case for field theory solitons and as it would be the case for NS5-branes in string theory.

In the opposite limit, $g_s N_{\rm c} \gg 1$, the backreaction of the branes on a finite region of spacetime cannot be neglected, but fortunately in this case the description in terms of an effective geometry for closed strings becomes simple, since in this limit the size of the near-brane $AdS_5 \times S^5$ region becomes large in string units.

Now we are ready to motivate the AdS/CFT correspondence, by considering excitations around the ground-state in the two descriptions above and taking a low-energy or 'decoupling' limit. In the first description the excitations of the system consist of open and closed strings, as displayed in fig. 14, in interaction with each other. At low energies we may focus on the light degrees of freedom. Quantisation of the open strings leads to a spectrum consisting of a massless $\mathcal{N} = 4$ $SU(N_{\rm c})$ SYM multiplet plus a tower of massive string excitations. Since the open string endpoints are constrained to lie on the D3-branes, all these modes propagate in 3+1 flat dimensions – the worldvolume of the branes. Similarly, quantisation of the closed strings leads to a massless graviton supermultiplet plus a tower of massive string modes, all of which propagate in flat ten-dimensional spacetime. The strength of interactions of closed string modes with each other is controlled by Newton's constant G, so the dimensionless coupling constant at an energy E is GE^8. This vanishes at low energies and so in this limit closed strings become non-interacting, which is essentially the statement that gravity is infrared free. Interactions between closed and open strings are also controlled by the same parameter, since gravity couples universally to all forms of matter. Therefore at low energies closed strings decouple from open strings. In contrast, interactions between open strings are controlled by the $\mathcal{N} = 4$ SYM coupling constant in four dimensions, which is given by $g_{\rm YM}^2 \sim g_{\rm op}^2 \sim g_s$. Note that this relation

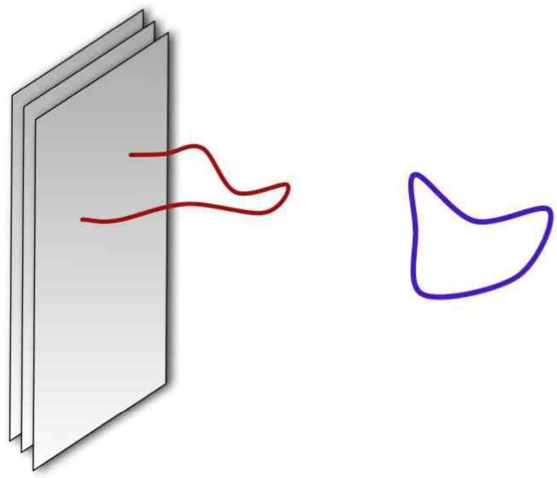


Figure 14: Excitations of the system in the first description.

is consistent with the fact that $g_{\rm YM}$ is dimensionless in four dimensions, and it can be derived, for example, by expanding the low-energy effective action for the D3-branes, the so-called Dirac-Born-Infeld (DBI) action:

$$S_{\rm D3} \sim -T_{\rm D3} \int d^4x \sqrt{-\det(\eta_{\mu\nu} + \alpha'^2 F_{\mu\nu})} \sim -\frac{1}{g_s} \int d^4x \, F_{\mu\nu}^2 \,, \tag{6}$$

where $\alpha' = \ell_s^2$. We thus conclude that at low energies the first description of the system reduces to an interacting $\mathcal{N}=4$ SYM theory in four dimensions plus free gravity in ten dimensions.

Let us now examine the same limit in the second description. In this case the low-energy limit consists of focusing on excitations that have arbitrarily low energy with respect to an observer in the asymptotically flat Minkowski region. As above, we now have two distinct sets of degrees of freedom, those propagating in the Minkowski region and those propagating in the throat – see fig. 15. In the Minkowski region the only modes that remain are those of the massless ten-dimensional graviton supermultiplet. Moreover, at low energies these modes decouple from each other, since their interactions are governed by GE^8, as above. They also decouple from modes in the throat region, since at low energies the wave-length of these modes becomes much larger than the size of the throat. In the throat region, however, the whole tower of massive string excitations survives. This is because a mode in the throat must climb up a gravitational potential in order to reach the asymptotically flat region. Consequently, a closed string of arbitrarily high proper

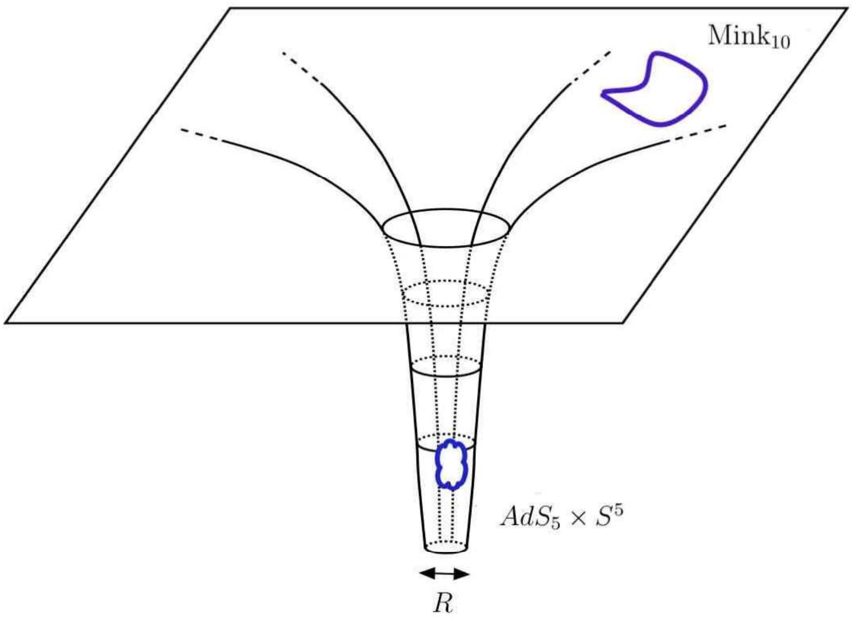


Figure 15: Excitations of the system in the second description.

energy in the throat region may have an arbitrarily low energy as seen by an observer at asymptotic infinity, provided the string is located sufficiently deep down the throat. As we focus on lower and lower energies these modes become supported deeper and deeper in the throat as so they decouple from those in the asymptotic region. We thus conclude that at low energies the second description of the system reduces to interacting closed strings in $AdS_5 \times S^5$ plus free gravity in flat ten-dimensional spacetime.

Comparing the results of the low-energy limits above it is reasonable to conjecture that four-dimensional $\mathcal{N} = 4$ $SU(N_c)$ SYM and type IIB string theory on $AdS_5 \times S^5$ are two apparently different descriptions of the same underlying physics [4], and we will say that the two theories are 'dual' to each other.

3.2 Matching of parameters

Let us examine more closely the parameters that enter the definition of each theory, and the map between them. The gauge theory is specified by the rank of the gauge group, N_c, and the 't Hooft coupling constant, $\lambda = g^2_{\mathrm{YM}} N_c$. The string theory is determined by the string coupling constant g_s and by the size of the AdS_5 and S^5 spaces. Both of these are maximally symmetric spaces which are completely specified by a single scale, their radius of curvature R. It turns out that the two spaces in the string solution sourced by D3-branes have equal radii. As we argued above, this is related to the parameters in the gauge theory through

$$\frac{R^2}{\alpha'} \sim \sqrt{g_s N_{\rm c}} \sim \sqrt{\lambda}\,. \tag{7}$$

This means that the so-called α'-expansion on the string side, which controls corrections associated to the finite size of the string as compared to the size of the spacetime it propagates in, corresponds to a strong-coupling, $1/\sqrt{\lambda}$ expansion in the gauge theory.

It follows from (7) that a necessary condition in order for the particle or supergravity limit of the string theory to be a good approximation we must have $\lambda \to \infty$. Note, however, that this condition is not sufficient: It must be supplemented by the requirement that $g_s \to 0$ (which then implies $N_{\rm c} \to \infty$) in order to ensure that additional degrees of freedom such as D-strings, whose tension scales as $1/g_s$, remain heavy.

The string coupling is related to the gauge theory parameters through

$$g_s \sim g^2_{\rm YM} \sim \frac{\lambda}{N_{\rm c}}\,, \tag{8}$$

which means that, for a fixed-size $AdS_5 \times S^5$ geometry (*i.e.,* for fixed λ), the string loop expansion corresponds precisely to the $1/N_{\rm c}$ expansion in the gauge theory. Equivalently, one may note that the radius in Planck units is precisely

$$\frac{R^4}{\ell_p^4} \sim \frac{R^4}{\sqrt{G}} \sim N_{\rm c}\,, \tag{9}$$

so quantum corrections on the string side are suppressed by powers of $1/N_{\rm c}$. In particular, the classical limit on the string side corresponds to the planar limit of the gauge theory.

3.3 Matching of symmetries

The metric on AdS_5, in the so-called 'Poincare patch', may be written as

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2}dr^2\,. \tag{10}$$

The coordinates x^μ may be thought of as the coordinates along the worldvolume of the original D3-branes, and hence may be identified with the gauge theory coordinates. The coordinate r, and those on the S^5, span the directions transverse to the branes. As displayed in fig. 16, the coordinates used in (10) provide a very simple geometric picture of AdS_5 as a foliation by constant-r slices, each of which is isometric to four-dimensional Minkwoski spacetime. As $r \to \infty$ we approach the so-called 'boundary' of AdS_5. This is not a boundary in the topological but in the conformal sense of the word. Although this

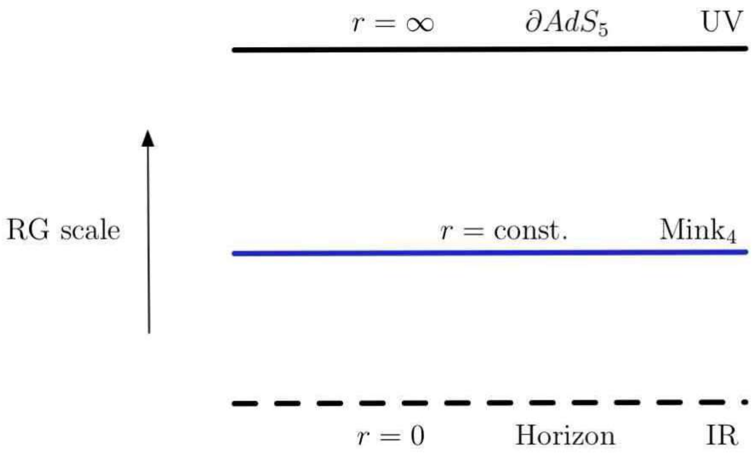


Figure 16: A geometric picture of AdS_5.

concept can be given a precise mathematical meaning, we will not need these details here. Since the norm of $\partial/\partial t$ vanishes at $r = 0$, we will refer to this surface as 'the horizon'. Note, however, that the determinant of the induced three-metric on a constant-time slice vanishes at $r = 0$, so this is not a finite-area horizon.

$\mathcal{N} = 4$ SYM is a conformal field theory (CFT). In particular, this means that it is invariant under the action of the dilatation operator

$$D: \quad x^\mu \to \Lambda x^\mu \,, \tag{11}$$

where Λ is a constant. As one would expect, this transformation is also a symmetry on the gravity side: Indeed, the metric (10) is invariant under (11) provided this is accompanied by the rescaling $r \to r/\Lambda$. This means that short-distance physics in the gauge theory is associated to physics near the AdS boundary, whereas long-distance physics is associated to physics near the horizon. In other words, r is identified with the renormalisation group (RG) scale in the gauge theory. Since a quantum field theory is defined by an ultraviolet (UV) fixed point and an RG flow, one may think of the $\mathcal{N} = 4$ gauge theory as residing at the boundary of AdS_5. The fact that D acts on the AdS_5 metric as an exact isometry merely reflects the fact the RG flow is trivial for this gauge theory. For non-conformal theories with an UV fixed point D is not an exact isometry of the dual geometry but only an asymptotic isometry.

Let us examine more closely the matching of global symmetries on both sides of the correspondence. The $\mathcal{N} = 4$ SYM theory is invariant not only under dilatations but under $\text{Conf}(1, 3) \times SO(6)$. The first factor is the conformal group of four-dimensional Minkowski space, which contains the Poincaré group, the dilatation symmetry generated by D, and four special conformal transformations whose generators we will denote by K_μ. The second factor is the R-symmetry of the theory. In addition, the theory is invariant under sixteen ordinary or 'Poincare' supersymmetries, the fermionic superpartners of the translation generators P_μ, as well as under sixteen special conformal supersymmetries, the fermionic superpartners of the special conformal symmetry generators K_μ.

The string side of the correspondence is of course invariant under the group of diffeomorphisms, which are gauge transformations. The subgroup of these consisting of large gauge transformations that leave the asymptotic (*i.e.,* near the boundary) form of the metric invariant is precisely $SO(2,4) \times SO(6)$. The first factor, which is isomorphic to $\mathrm{Conf}(1,3)$, corresponds to the isometry group of AdS_5, and the second one to the isometry group of S^5. As usual, large gauge transformations must be thought of as global symmetries, so we see that the bosonic global symmetry groups on both sides of the correspondence agree. An analogous statement can be made for the fermionic symmetries. $AdS_5 \times S^5$ is a maximally supersymmetric solution of type IIB string theory, and so it possesses thirty-two Killing spinors which generate fermionic isometries. These can be split into two groups that match those of the gauge theory.

We therefore conclude that the global symmetries are the same on both sides of the duality. It is important to note, however, that on the gravity side the global symmetries arise as large gauge transformations. In this sense there is a correspondence between global symmetries in the gauge theory and gauge symmetries in the dual string theory. This is an important general feature of all known gauge/gravity dualities, to which we will return below after discussing the field/operator correspondence. It is also consistent with the general belief that the only conserved charges in a theory of quantum gravity are those associated to global symmetries that arise as large gauge transformations.

3.4 The field/operator correspondence

So far we have not provided a precise prescription for the map between observables in the two theories. The technical details will not be needed in these lectures, but we will now sketch the main idea [5]. This can be motivated by recalling that the SYM coupling constant g^2_{YM} is identified with the string coupling constant g_s. In string theory this is given by $g_s = e^{\Phi_\infty}$, where Φ_∞ is the value of the dilaton at the AdS boundary. This suggests that deforming the gauge theory by changing the value of a coupling constant corresponds to changing the value of a string field at ∂AdS. More generally, one may imagine deforming the gauge theory action as

$$S \to S + \int d^4x\, \phi(x)\mathcal{O}(x)\,, \qquad (12)$$

where $\mathcal{O}(x)$ is a gauge-invariant, local operator and $\phi(x)$ is a possibly point-dependent coupling, namely a source. It is then reasonable to expect that to each such possible operator there corresponds a dual string field $\Phi(x,r)$ such that its value at the AdS boundary may be regarded as a source for the above operator, *i.e.,* we identify $\phi = \Phi|_{\partial AdS}$. For example, the dilaton field is dual to (roughly) the operator $\mathrm{Tr}F^2$. A natural conjecture is then that the partition functions of the two theories agree upon this identification, namely that

$$Z_{\text{CFT}}\left[\phi\right] = Z_{\text{string}}\left[\Phi|_{\partial AdS}\right] . \tag{13}$$

The left-hand side encodes all the physical information in the gauge theory, since it allows the calculation of correlation functions of arbitrary gauge-invariant operators.[1] The right-hand side is in general not easy to compute, but it simplifies dramatically in the large-N_{c}, large-λ limit, in which it reduces to

$$Z_{\text{string}} \simeq e^{-S_{\text{sugra}}} , \tag{14}$$

where S_{sugra} is the on-shell supergravity action.

An especially important set of operators in a gauge theory are conserved currents associated to global symmetries. Given the correspondence between these and gauge symmetries on the string side, we expect the field dual to a conserved current J^μ to be a gauge field A_μ. This is indeed true, and is consistent with the fact that the coupling

$$\int d^4x \, A_\mu(x) J^\mu(x) \tag{15}$$

is invariant under gauge transformations $\delta A_\mu = \partial_\mu f$ by virtue of the fact that $\partial_\mu J^\mu = 0$.

A particular set of currents that are conserved in any translationally invariant theory are those in the energy-momentum tensor operator $T_{\mu\nu}$. This must couple to a symmetric, spin-two gauge field, namely to a graviton, in the form

$$\int d^4x \, g^{\mu\nu}(x) T_{\mu\nu}(x) \, . \tag{16}$$

Thus we reach the general conclusion that the dual of a translationally invariant gauge theory must involve dynamical gravity.

3.5 Remarks

Let us close this section with a few general remarks. First, the AdS/CFT correspondence described here is not proven, but it has passed an large number of tests. In these lectures we will assume that it holds in its strongest form, *i.e.,* for all values of λ and N_{c}. Second,

[1] The prescription may be extended to non-local operators, such as Wilson loops [6].