



Brane-intersection dynamics from branes in brane backgrounds

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Abstract

We derive the dynamics of M-brane intersections from the worldvolume action of one brane in the background supergravity solution of another one. In this way we obtain an effective action for the self-dual string boundary of an M2-brane in an M5-brane, and show that the dynamics of the 3-brane intersection of two M5-branes is described by a Dirac-Born-Infeld action. © 1998 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Intersecting M-branes are becoming increasingly important in many aspects of non-perturbative QFT and quantum gravity. Our interest here will be with 1/4-supersymmetric orthogonal intersections of two M-branes. These have been investigated, and classified, by a variety of methods. From the perspective of the worldvolume of one of the participating M-branes the intersection with the other one appears as a 1/2-supersymmetric soliton-type solution of its worldvolume field theory. A notable example is the self-dual string soliton in the M5-brane [1] which can be interpreted as the boundary of an M2-brane [2,3] (this is the M-theory version of the Dirac-Born-Infeld ‘BIons’ [4,5] which can be interpreted as the

endpoints of ‘fundamental’ strings on D-branes). Another example is the 3-brane soliton in the M5-brane [6], which can be interpreted as the intersection with another M5-brane [7]. All of these worldvolume solitons saturate a Bogomolnyi-type bound in terms of a central charge appearing in the worldvolume supersymmetry algebra [8], and consideration of these charges suffices to classify all possible 1/4 supersymmetric intersections [9].

Here we address the issue of the effective actions describing the dynamics of the self-dual string and 3-brane solitons of the M5-brane. These are expected to be κ -symmetric string and 3-brane actions in the 6-dimensional Minkowski background provided by an infinite static planar M5-brane. The worldvolume fields are in correspondence with the zero modes in an analysis of fluctuations about the worldvolume soliton solutions, and this type of analysis has been carried out in [10,11]. Here we take a different

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approach. We consider the worldvolume field theory of a ‘test’ M-brane in the background spacetime of an M5-brane. This is a justifiable approximation if the source of the ‘supergravity M5-brane’ is actually a large number of coincident M5-branes. The approach is similar to one adopted in a number of recent works in which a brane of M-theory or string theory is put into the background of a large number of parallel branes of the same type [12,13]. The ‘test’ brane feels no force in this background because it is parallel to the ‘source’ brane. Our work exploits the fact that there are various other embeddings of test M-branes in the same background for which the test brane again feels no force. In fact, such embeddings correspond precisely with the possible 1/4 supersymmetric intersections of an M5-brane [14].

While an M2-brane boundary on an M5-brane appears as a worldvolume string soliton of the M5-brane’s worldvolume field theory, there is no similar interpretation of this ‘intersection’ from the M2-brane’s point of view, essentially because boundaries are determined by imposing boundary conditions rather than by solving field equations. One motivation for the approach taken here is that it circumvents this difficulty. When the M5-brane is replaced by its supergravity solution the M2-brane actually has no boundary, it just disappears down the infinite M5-brane ‘throat’. There is therefore no need to impose boundary conditions on the M2-brane equations. Nevertheless, on scales that are large compared to that determined by the M5-brane tension the supergravity solution can be replaced by an effective 5-brane source, and it will then appear that the M2-brane has a boundary on the M5-brane. We can therefore study the dynamics of this boundary by considering fluctuations of the membrane in the M5-brane background. In this way, we are able to derive an effective action for the string boundary. The string tension is formally infinite, but this is to be expected of an infinite membrane. By considering a membrane stretched between two M5-branes the tension is made finite. As we shall see, this is true even though the *proper* length of the membrane in the direction separating the M5-branes is infinite.

The 3-brane intersection of two M5-branes can be treated in the same way. In this case we consider the fluctuations of a test M5-brane embedded in an appropriate way in the background of a source M5-

brane. The resulting effective action, for which the (partially gauge-fixed) fields are those of a $D = 6$ vector supermultiplet [6], is of Dirac-Born-Infeld (DBI) type. From the work of [4,5] it then follows that this 3-brane has its own worldvolume ‘bions’ which can be interpreted as the endpoints of self-dual strings. We therefore confirm the claim of [9] that the 3-brane soliton is a D-brane for the self-dual string soliton.

2. The M2-brane ending on the M5-brane

Our starting point will be the action for the supermembrane in a $D = 11$ supergravity background [15]. To specify the latter we must, in principle, choose a background supervielbein E_M^A and 3-form super-space gauge-potential $C_{(3)}$ satisfying the on-shell superfield constraints of $D = 11$ supergravity. The field equations of this action are the M2 ‘branewave’ equations. We shall choose a purely bosonic background for which the fermion equations are trivially solved by setting them to zero. Equivalently, we can start by discarding the worldvolume fermions, in which case the action is

$$S_{M2} = - \int d^3\xi \sqrt{-\det g} + \int_W \mathcal{E}_{(3)}, \quad (1)$$

where g is the metric induced from the spacetime 11-metric and $\mathcal{E}_{(3)}$ is now the pullback of the spacetime 3-form potential to the worldvolume W (with coordinates ξ^I , $I = 0, 1, 2$). We shall take the background to be that of the M5-brane solution. This is a purely bosonic background with 11-metric and 4-form field strength $F = dC_{(3)}$ given by [16]

$$ds_{(11)}^2 = U^{-1/3} \eta_{\mu\nu} dY^\mu dY^\nu + U^{2/3} dX \cdot dX, \quad (2)$$

$$F_{mnpq} = \epsilon_{mnpqr} \partial_r U,$$

where $\eta_{\mu\nu}$ is the metric on the 6-dimensional Minkowski space with Y coordinates, and U is a harmonic function on the transverse 5-space \mathbb{E}^5 with cartesian coordinates X^m and euclidean metric $dX \cdot dX$. To begin with we choose

$$U = 1 + \frac{q}{r^3}, \quad (3)$$

where r is the radial distance from the origin in \mathbb{E}^5 .

We shall first seek a static solution of the membrane field equations in this background that can be interpreted as the linear orthogonal intersection of an M2-brane with an M5-brane. Setting $\xi^I = (\sigma^i, \rho); (i = 0, 1)$, we are thus led to make the partial gauge choice³

$$X^1 = \rho, \quad (4)$$

combined with the ansatz

$$Y^0 = \sigma^0, \quad Y^1 = \sigma^1, \quad (5)$$

with all other worldvolume fields vanishing. It is straightforward to verify that this membrane configuration solves the branewave equations. The solution represents a membrane that ‘disappears’ down the infinite M5-brane throat at $X = 0$. On the surface $X^2 = X^3 = X^4 = X^5 = 0$, the proper distance to $X^1 = 0$, i.e. $\rho = 0$, is infinite. This means that $\rho = 0$ does not correspond to any points of the membrane; the coordinate ρ therefore takes values in the open interval $(0, \infty)$.

Although the membrane has no boundary it will appear to end on the M5-brane on length scales for which the M5-brane background can be replaced by an effective M5-brane source. It should therefore be possible to determine the dynamics of this effective membrane boundary from the dynamics of the membrane itself. To do so we must consider the (not-necessarily small) fluctuations about the above solution of the branewave equations. To proceed, we restrict the oscillations of the M2-brane to those obeying the following conditions:

$$Y^\mu = Y^\mu(\sigma), \quad X^1 = \rho, \quad X^2 = X^3 = X^4 = X^5 = 0. \quad (6)$$

These restrictions force the membrane oscillations to be uniform in the X^1 direction. On sufficiently large length scales this will be interpretable as a membrane oscillating rigidly with its boundary in an M5-brane, the boundary oscillations being unrestricted. The restrictions (6) also constitute a consistent truncation. In particular, the branewave equa-

tions for the X fields are automatically satisfied. To verify this it is crucial to observe that the pull-back 3-form $\mathcal{E}_{(3)}$ vanishes for worldvolume fields satisfying the above conditions⁴. Because the M5-brane supergravity solution is such that F is a 4-form on the 5-space with coordinates X , one can choose the 3-form potential $C_{(3)}$ such that it too is a form on this 5-space. It follows that the pullback of $C_{(3)}$ to the worldvolume involves derivatives of at least three different X coordinates, only one of which can be non-zero for fields satisfying the ansatz (6).

We now note that the induced 3-metric g takes the block diagonal form

$$g = \begin{pmatrix} U^{-1/3} \tilde{g} & \\ & U^{2/3} \end{pmatrix}, \quad (7)$$

from which it can be seen that the branewave equations for Y reduce to

$$\partial_i \left[\sqrt{-\det \tilde{g}} \tilde{g}^{ij} \partial_j Y^\mu \right] = 0, \quad (8)$$

where

$$\tilde{g}_{ij} = \eta_{\mu\nu} \partial_i Y^\mu \partial_j Y^\nu. \quad (9)$$

These are the field equations of the Nambu-Goto (NG) action for a string moving in a $D = 6$ Minkowski spacetime. Thus, the string boundary of the M2-brane in the M5-brane is governed by the NG string action.

Of course, we could have obtained this result by substituting (6) directly into the M2-brane action (1). Indeed, it follows from (7) that $\det g = \det \tilde{g}$ and therefore that the M2 action collapses to the NG action:

$$S_{M2} \rightarrow -T \int d^2\sigma \sqrt{-\det \tilde{g}}, \quad (10)$$

where the tension T is given by

$$T = \left[\int_0^\infty d\rho \right]. \quad (11)$$

The tension is infinite because the string is the boundary of an infinite membrane, but this can be remedied by considering a membrane stretched be-

³ The choice $X^1 = f(\rho)$ for any monotonic function f would be equally good, but the range of the membrane coordinate ρ will depend on the choice, as discussed below. Here we make the simplest choice.

⁴ And hence for our ‘vacuum’ solution of these equations; this fact was implicitly used earlier.

tween two parallel M5-branes. Let the two M5-branes (with charges q and q' and worldvolumes aligned with the Y axes) be separated by a distance L along the X^1 axis. This can be achieved by choosing the harmonic function U to be

$$U = 1 + \frac{|q|}{|X|^3} + \frac{|q'|}{|X - \bar{X}|^3}, \quad (12)$$

where $\bar{X} = (L, 0, 0, 0, 0)$. Most of the previous discussion still applies because the explicit form of the harmonic function U was not used. However, on the surface $X^2 = X^3 = X^4 = X^5 = 0$ both $X^1 = 0$ and $X^1 = L$ are now at infinite proper distance, so the membrane coordinate ρ must now be restricted to take values in the open interval $(0, L)$. In this case $T = L$, which is finite⁵.

3. The 3-brane intersection of two M5-branes

Two M5-branes can have a $1/4$ supersymmetric 3-brane intersection. We shall derive the dynamics of this 3-brane within one of the M5-branes by replacing the latter by its supergravity solution, given above in terms of the harmonic function U . Let ξ^I ($I = 0, 1, \dots, 5$) be coordinates for the M5-brane's worldvolume W . The M5-brane's Lorentz covariant effective action [17] is

$$S_{M5} = \int d\xi^6 \left[\sqrt{-\det(g + i\tilde{H})} + \frac{1}{4} \frac{1}{\sqrt{(\partial a)^2}} \tilde{\mathcal{H}}^{IJ} H_{IJK} \partial^K a \right] + \int_W (\mathcal{E}_{(6)} + \frac{1}{2} H \wedge \mathcal{E}_{(3)}), \quad (13)$$

where $\mathcal{E}_{(6)}$ is the pull-back to the worldvolume of the *on-shell* 6-form dual $C_{(6)}$ of the 3-form potential $C_{(3)}$. The field a is the non-dynamical ‘PST’ gauge field; it can be eliminated by a choice of gauge. The

worldvolume 3-form H is a ‘modified’ field-strength for a worldvolume 2-form potential A :

$$H = dA - \mathcal{E}_{(3)}. \quad (14)$$

The worldvolume tensor density $\tilde{\mathcal{H}}$ is defined by⁶

$$\tilde{\mathcal{H}}^{IJ} \equiv \frac{1}{6\sqrt{(\partial a)^2}} \epsilon^{IJKLMN} \partial_K a H_{LMN}, \quad (15)$$

while the worldvolume 2-form \tilde{H} has components

$$\tilde{H}_{IJ} = \frac{1}{\sqrt{-\det g}} g_{IK} g_{JL} \tilde{\mathcal{H}}^{KL}. \quad (16)$$

We now set $\xi^I = (\sigma^i, \rho, \lambda)$ ($i = 0, 1, 2, 3$) and choose a gauge for which $a = \lambda$. Note that all gauge choices for a appear to break some symmetry that we wish to keep, but this will not show up in the final result. Following the previous M2-M5 case, we now seek a vacuum solution of the M5-brane's branewave equations that can be interpreted as representing the intersection on a 3-brane with the five-brane source of the background. The appropriate vacuum solution is

$$\begin{aligned} Y^0 &= \sigma^0, & Y^1 &= \sigma^1, & Y^2 &= \sigma^2, & Y^3 &= \sigma^3, \\ Y^4 &= Y^5 = 0, \\ X^1 &= \rho, & X^2 &= \lambda, & X^3 &= X^4 = X^5 = 0, \\ H &= 0. \end{aligned} \quad (17)$$

We shall consider fluctuations about this solution satisfying

$$\begin{aligned} Y^\mu &= Y^\mu(\sigma), \\ X^1 &= \rho, & X^2 &= \lambda, & X^3 &= X^4 = X^5 = 0, \\ i_\rho A &= \frac{1}{2} V(\sigma), & i_\lambda A &= 0, & A_{ij} &= 0, \end{aligned} \quad (18)$$

where i_ρ and i_λ indicate the contraction with the vector fields $\partial/\partial\rho$ and $\partial/\partial\lambda$, respectively. Note that the only non-zero component of H is $(i_\rho H)_{ij} = -(dV)_{ij}$. These conditions constitute a consistent truncation of the full M5-brane degrees of freedom.

⁵ The $L \rightarrow 0$ limit cannot be taken because the singularities of U in this limit are genuine curvature singularities of the M5-brane solution.

⁶ This and the following definition differ slightly from those of [17].

An immediate implication is that the induced world-volume 6-metric takes the block diagonal form

$$g = \begin{pmatrix} U^{-1/3} \tilde{g} & & \\ & U^{2/3} & \\ & & U^{2/3} \end{pmatrix}, \quad (19)$$

where

$$\tilde{g}_{ij} = \eta_{\mu\nu} \partial_i X^\mu \partial_j X^\nu. \quad (20)$$

Note that $\det g = \det \tilde{g}$.

A further implication of (18) is that the pull-backs of the space-time forms $C_{(3)}$ and $C_{(6)}$ vanish. The worldvolume 3-form $\mathcal{C}_{(3)}$ vanishes for essentially the same reasons as before. To see that $\mathcal{C}_{(6)}$ also vanishes we recall that it is defined, up to a gauge transformation, by the relation (see [18] for a review)

$$dC_{(6)} = \star dC_{(3)} - \frac{1}{2} C_{(3)} \wedge dC_{(3)}. \quad (21)$$

In our case this reduces to $dC_{(6)} = \star dC_{(3)}$ because $C_{(3)} \wedge dC_{(3)}$ is a 7 form on \mathbb{E}^5 . One solution of this equation is

$$C_{(6)} = U dY^0 \wedge \dots \wedge dY^5. \quad (22)$$

Any other solution will be a gauge transform of this one, so we may assume that $C_{(6)}$ is of this form. The pullback to the worldvolume of this form vanishes because it contains (for example) a factor of $\partial Y^\mu / \partial \rho$, which vanishes for the ansatz (18).

We are nearly ready to extract the 3-brane action. The second term in (13) vanishes upon imposition of (18), so we just need the 2-form \tilde{H} . First note that the only non-zero components of the tensor density \mathcal{H} are

$$\mathcal{H}^{ij} = \frac{1}{2} U^{1/3} \epsilon^{ijkl45} (dV)_{kl}, \quad (23)$$

and therefore that the only non-zero components of the 2-form are

$$\begin{aligned} \tilde{H}_{ij} &= U^{-1/3} \tilde{B}_{ij}, \\ \tilde{B}_{ij} &\equiv \frac{1}{2} \frac{1}{\sqrt{-\det \tilde{g}}} \tilde{g}_{ik} \tilde{g}_{jl} \epsilon^{klmn} (dV)_{mn}. \end{aligned} \quad (24)$$

This immediately implies the following block diagonal form of the matrix appearing in the first term of the M5-brane action:

$$g + i\tilde{H} = \begin{pmatrix} U^{-1/3} [\tilde{g} + i\tilde{B}] & & \\ & U^{2/3} & \\ & & U^{2/3} \end{pmatrix}. \quad (25)$$

It follows that

$$\det[g + i\tilde{H}] = \det[\tilde{g} + i\tilde{B}] = \det[\tilde{g} + dV]. \quad (26)$$

To obtain the last equality one uses firstly that, for any antisymmetric 4×4 matrix D ,

$$\det(\tilde{g} + D) = (\det \tilde{g}) \left[1 + \frac{1}{2} D^2 + \frac{1}{8} (D^2)^2 - \frac{1}{4} D^4 \right], \quad (27)$$

where $D^2 = D^{ij} D_{ij}$ and $D^4 = D_{ij} D^{jk} D_{kl} D^{li}$, and then that $\tilde{B}^2 = -(dV)^2$ and $\tilde{B}^4 = (dV)^4$.

Given these results, the M5-brane action reduces to

$$S_{M5} \rightarrow T \int d^4 \sigma \sqrt{-\det[\tilde{g} + dV]}, \quad (28)$$

where the tension T is formally infinite, as expected. Apart from this, we conclude that the dynamics of the 3-brane living in the orthogonal intersection of two M5-branes is governed by the Dirac-Born-Infeld action, at least in the M5-brane Minkowski vacuum.

4. Discussion

In this paper we have used the bosonic sector of the M2-brane and M5-brane worldvolume actions to derive actions describing the dynamics of M5-brane intersections corresponding to the 1-brane and 3-brane solitons of the M5-brane worldvolume field theory. Essentially, we have obtained the latter by a consistent truncation of the former. In principle, our method could be used to derive the full supersymmetric action for the 1-brane and 3-brane in the M5-brane (in a vacuum background) by the simple expedient of retaining fermions from the beginning. Although we have not done this, we expect that the resulting actions will be κ -symmetric extensions of those found here.

The $D = 6$ NG string action is presumably to be interpreted as a special case of a self-dual string in a more general background that would include a coupling to the 2-form potential on the M5-brane. It seems likely, in analogy to branes in spacetime, that κ -symmetry will require that the background solve the M5 brane equations. One solution of these equations is $D = 6$ Minkowski space with vanishing 2-form potential, i.e. the M5-brane vacuum. Our method yields the action for the self-dual string in this vacuum background. The 3-brane action found here should be similarly interpreted.

The fact that the 3-brane action is of Dirac-Born-Infeld type means that it has its own worldvolume solitons, which can be interpreted as endpoints of strings [4,5]. It is natural to interpret these strings as the self-dual strings in the M5-brane. A $D = 11$ spacetime interpretation of this possibility was given in [9]. Thus, the 3-brane is very likely the D-brane of a new intrinsically non-perturbative self-dual $D = 6$ superstring theory.

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