Particle creation by an external field

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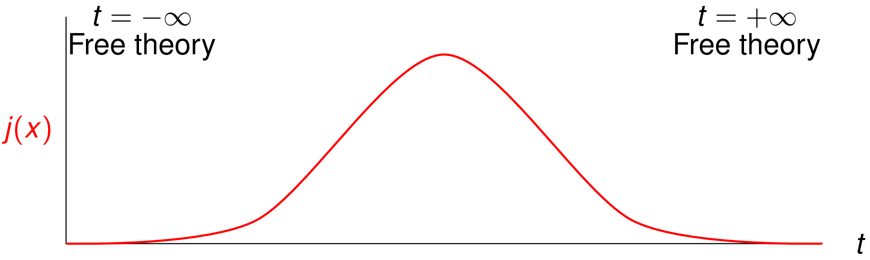
In the presence of an external source field:

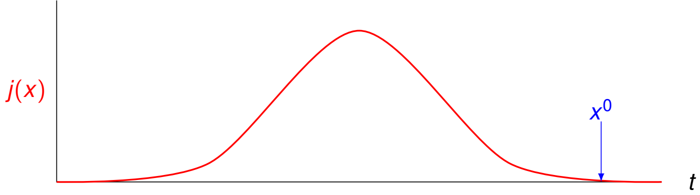
$$(\partial_\mu \partial^\mu + m^2)\phi(x) = j(x) \tag{1}$$

which derives from the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi \partial_\mu \phi - m^2 \phi^2) + j(x)\phi(x)$$

If $j(x)$ is active for a finite time:





- $t \to \infty$ and $t \to -\infty$: free Lagrangian $\mathcal{L}_0 = \frac{1}{2}(\partial^\mu \phi \partial_\mu \phi - m^2 \phi^2)$
  - $\Rightarrow$ free-field solution: $\phi_0(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 \sqrt{2E_p}} (a_{\boldsymbol{p}} e^{-ipx} + a^\dagger_{\boldsymbol{p}} e^{ipx})$
  - $\Rightarrow$ usual particle interpretation
- Non-homogeneous differential equation (1)
  - $\Rightarrow$ Use retarded Green's function:

$$
\begin{aligned}
\phi(x) &= \phi_0(x) + i \int \mathrm{d}^4 y D_R(x-y) j(y) \\
&= \phi_0(x) + i \int \mathrm{d}^4 y \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_p} \Theta(x^0 - y^0)(e^{-ip(x-y)} - e^{ip(x-y)}) j(y)
\end{aligned}
$$

- Wait until a time x^0 in which $j(x^0, \boldsymbol{x}) = 0$
  - $\Rightarrow$ $j(x)$ is all in the past $\Rightarrow$ $x^0 > y^0$: $\Theta(x^0 - y^0) = 1$

- Define: Fourier transform of $j(x)$: $\tilde{j}(p) = \int \mathrm{d}^4 y \, e^{ipy} j(y)$; $[p^2 = m^2]$

$$\phi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 \sqrt{2E_p}} \left\{ (a_{\mathbf{p}} + \frac{i}{\sqrt{2E_p}} \tilde{j}(p)) e^{-ipx} + (a_{\mathbf{p}}^\dagger - \frac{i}{\sqrt{2E_p}} \tilde{j}^\dagger(p)) e^{ipx} \right\}$$

- Hamiltonian after $j(x)$:

$$H = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p (a_{\mathbf{p}}^\dagger - \frac{i}{\sqrt{2E_p}} \tilde{j}^\dagger(p))(a_{\mathbf{p}} + \frac{i}{\sqrt{2E_p}} \tilde{j}(p))$$

- vacuum energy:

$$\langle 0 | H | 0 \rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{2} |\tilde{j}(p)|^2$$

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Interpretation

- The source $j(x)$ has created particles with momentum $\boldsymbol{p}$ with a probability density

$$\frac{|\tilde{j}(p)|^2}{2E_p}$$

- The total number of created particles is:

$$\int \mathrm{d}N = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_p} |\tilde{j}(p)|^2$$