# Homework 1

Due date: 03/05/2021

**Problem 1:**

Use the one-loop expression for the electron self energy

$$\begin{aligned}\Sigma^{1\,\mathrm{Loop}} &= -\frac{\alpha}{2\pi}C_F\left\{\left(\frac{1}{2}\not{p}-2m_{\mathrm{R}}\right)\left[\frac{1}{\varepsilon}+\log\left(\frac{4\pi\mu^2}{p^2}\right)-\gamma_E\right]\right. \\ &\left. -\frac{1}{2}\not{p}+m_{\mathrm{R}}-\int_0^1 dx\left[\not{p}(1-x)-2m_{\mathrm{R}}\right]\log\left[-x(1-x)+x\frac{m_{\mathrm{R}}{}^2}{p^2}\right]\right\}, \end{aligned} \tag{1}$$

to renormalise QED in the $\overline{\mathrm{MS}}$-scheme

a) Find the regularised self energy $\Sigma_R$.

b) Find the relation between $m_R$ and the mass of the electron, $m_e$, at one-loop precision. Define the mass of the electron as the pole of the renormalised propagator

$$S_F(p^2, g_R(\mu), m_R(\mu), \mu) = \frac{i}{\not{p}-m_R(\mu)-\Sigma_R(p^2, g_R(\mu), m_R(\mu), \mu)} \tag{2}$$

By using the expression of $\Sigma_R$ at one loop order, determine the leading $g_R^2$ shift between these two masses

$$m_e = m_R - \alpha_R^{\mathrm{em}}\delta m \tag{3}$$

c) Determine the residue of this pole. Is it the same as in on-shell renormalisation?

d) Compute the variation of $m_e$ with the scale $\mu$ at one loop order. Interpret the result.

**Note:**

$$\int_0^1 dx\log\left(R^2x-(1-x)x\right) = 2R^2\log(R)-\left(R^2-1\right)\log\left(R^2-1\right)-2 \tag{4}$$

$$\int_0^1 dx\,(1-x)\log\left(R^2x-(1-x)x\right) = \frac{1}{2}\left(2R^4\log(R)-R^2-\left(R^4-1\right)\log\left(R^2-1\right)-2\right) \tag{5}$$