

# CONSTITUENT QUARKS FROM QCD: PERTURBATION THEORY AND THE INFRA-RED <sup>a</sup>

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Systematic approaches to building up gauge invariant descriptions of charged fields, such as electrons or quarks, are described. Physically relevant descriptions must then be singled out from a multiplicity of possibilities and to this end we give a physical interpretation of one description. Perturbative calculations which back up this interpretation are outlined. A non-perturbative obstruction to observing an isolated quark is reported. This sets the limits of the constituent quark model.

The physical content of our theories of the fundamental interactions is profoundly affected by the gauge symmetries that lie at their heart. Such theories are in fact examples of systems with constraints and it is well known that a consequence of this for QED is that only two of the four initial  $A^\mu$  potentials are actually physical. The implications of the gauge symmetries of QED for charged fields (such as electrons) are less well understood and for non-abelian theories, such as QCD, the extraction of the physical degrees of freedom has not been performed. This talk reports recent progress in understanding these fundamental issues. In particular a gauge invariant description of charged fields in electrodynamics and a physical interpretation is provided. This leads to predictions which are then tested in perturbation theory. The gauge structure of scalar QED is so similar to that of standard QED that exactly the same predictions may be made for it. They are also verified here.

Any description of a physical charge must be gauge invariant and Gauß's law implies an intimate link between charges and a chromo-(electro-)magnetic cloud. Such a description in terms of a so-called dressed field,  $\psi_f$ , was proposed by Dirac in the 50's:  $\psi_f(x) = \exp \{ ie \int d^4z f_\mu(z, x) A^\mu(z) \} \psi(x)$ . This can be seen to be gauge invariant if  $f_\mu(z, x)$  satisfies  $\partial_\mu^z f^\mu(z, x) = \delta^{(4)}(z - x)$ .

Not all gauge invariant descriptions are, however, physically relevant. In Fig.1 two such possible clouds are shown. Clearly, (a) is not stable in QED and

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Figure 1: Two gauge invariant configurations of a static field surrounded by an electromagnetic field. A stronger field is represented by a whiter shading. The very singular string-like configuration in (a) can be shown to decay into the Coulombic one in (b) which is stable<sup>1</sup>.

for a static charge it will decay into the Coulomb cloud in (b). Our claim<sup>2,3,4</sup> is that the latter, and a more general version corresponding to a charge moving with velocity  $\vec{v}$ , are suitable for constructing physical asymptotic states. This is true in either QED or scalar QED since the magnetic field associated with the magnetic moment of an electron falls off rapidly away from the charge and is thus infra-red safe. To be explicit, for an electron moving with velocity  $\vec{v}$  we dress the fermion as follows

$$\psi_v = \exp \left\{ ie \frac{g^{\mu\nu} - (\eta + v)^\mu (\eta - v)^\nu}{\partial^2 - (\eta \cdot \partial)^2 + (v \cdot \partial)^2} \partial_\nu A_\mu \right\} \psi, \quad (1)$$

where  $v = (0, \vec{v})$ , and  $\eta = (1, \vec{0})$ . Rather than giving the arguments underlying this statement, we now report a perturbative calculation<sup>3,4</sup> which verifies the following. Recall that the usual gauge-dependent fermion propagator in QED or QCD is plagued with infra-red divergences in an on-shell scheme. These reflect the fact that the fermion is not a good physical state —the chromo-(electro-)magnetic field it generates is missing. If our dressing has a physical significance, we should be able to perform an *infra-red finite* on-shell renormalization for the propagator of the dressed charge defined by (1).

For this non covariant description, we find<sup>4</sup> a multiplicative *matrix* renormalization

$$\psi_v^{(\text{bare})} = \sqrt{Z_2} \exp \left\{ -i \frac{Z'}{Z_2} \sigma^{\mu\nu} \eta_\mu v_\nu \right\} \psi_v \quad \text{and} \quad m^{(\text{bare})} = m - \delta m \quad (2)$$

necessary. This is reminiscent of a naive Lorentz boost upon a fermion. The mass shift renormalization from demanding that the pole is at  $m$  yields the standard gauge-invariant result found in any textbook. The residue renormalization condition is where the infra-red divergences are usually found and we find here for our non covariant case three equations for only two unknowns,  $Z'$  and  $Z_2$ . It is highly gratifying that at the expected physical momentum,  $p = m\gamma(1, \vec{v})$ , we can consistently solve these three equations and further that *the renormalization constants are gauge-invariant and infra-red finite*.

The matrix renormalization (2) is forced upon us by the fermion structure of QED. We have checked that in scalar QED where such a scheme is not possible, a straightforward multiplicative renormalization also yields infra-red finite results — this despite exactly the same non-covariant dressing also being used in the scalar theory.

The next step is the non-abelian theory. Quarks and gluons are believed to be confined inside colourless hadrons and yet the success of the constituent quark model and the jet structures observed in experiments show that it must be possible to attach some physical meaning to quarks and gluons. The Lagrangian fields are, however, gauge dependent — like their QED counterparts — and we need to find the physical degrees of freedom of such a non-abelian gauge theory. We note here that the action of the gauge dependent colour charge operator is only gauge invariant on locally gauge invariant objects and so the colour statistics of the constituent quark model require a gauge invariant description<sup>2</sup>. Perturbative calculations are possible here, but, it can be shown that a non-perturbative obstruction, the Gribov ambiguity, prevents quarks and gluons from being true observables. This is because the QCD equivalent of the dressing function  $f_\mu(z, x)$  can be used to fix the gauge. This obstruction sets the fundamental hadronic scale and the limits of the quark model.

## References

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