Spin 1

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Gauge Principle

We have seen that the electromagnetic Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu j^\mu \tag{1}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$\Rightarrow$ Maxwell equations

$$\partial_\nu F^{\nu\mu} = j^\mu \tag{2}$$

$\Rightarrow$ gauge invariant action:

$$A^\mu(x) \to A^\mu + \partial^\mu \Lambda(x) \tag{3}$$

as long as j^μ is a conserved current: $\partial_\mu j^\mu = 0$.

$\Rightarrow$ the $U(1)$ currents of the complex Klein-Gordon field, and the Dirac field are good candidates for the right-hand-side of the Maxwell equations (2).

$\Rightarrow$ **But is there another reason?**

Dirac or complex Klein-Gordon Lagrangians are invariant under the **global** $U(1)$ symmetry:

$$\phi(x) \to \phi'(x) = e^{-i\alpha}\phi(x) \tag{4}$$

α is a constant for all space-time.

$\Rightarrow$ **BUT** relativistic theory $\Rightarrow$ no sense to change at the same time the phases of two fields which have space-like separation.

$\Rightarrow$ the phase in eq. (4) could be different at each space-time point:

$$\phi(x) \to \phi'(x) = e^{-i\alpha(x)}\phi(x) \tag{5}$$

$\Rightarrow$ derivative terms $\Rightarrow$ particles' Lagrangians not invariant!!

$$\begin{aligned}
\phi^\dagger(x)\partial_\mu\phi(x) &\to \phi^\dagger(x)e^{i\alpha(x)}\partial_\mu(e^{-i\alpha(x)}\phi(x)) \\
&= \phi^\dagger(x)e^{i\alpha(x)}e^{-i\alpha(x)}\partial_\mu\phi(x) + \phi^\dagger(x)e^{i\alpha(x)}\phi(x)\partial_\mu e^{-i\alpha(x)} \\
&= \phi^\dagger(x)\partial_\mu\phi(x) - i(\partial_\mu\alpha(x))\phi^\dagger(x)\phi(x) \qquad (6)
\end{aligned}$$

unless an extra term in the Lagrangian with an $A_\mu(x)$ field:

$$i\phi^\dagger(x)A_\mu(x)\phi(x) \to \phi^\dagger(x)(iA_\mu(x)+i\partial_\mu\alpha(x))\phi(x) = i\phi^\dagger(x)(A_\mu(x)+\partial_\mu\alpha(x))\phi(x) \tag{7}$$

$\Rightarrow$ **gauge transformation** for the A_μ field (3) with $\Lambda = \alpha$!

Terms in eq. (7) included **for any field derivative**

- In the Klein-Gordon or Dirac Lagrangian of a ϕ_i field
  $\Rightarrow$ substitute the derivative by the **covariant derivative**:

$$\partial_\mu\phi_i(x) \to D_\mu\phi_i(x) \equiv (\partial_\mu + iq_iA_\mu(x))\phi_i(x) \tag{8}$$

  $\Rightarrow$ **minimal coupling**,
  $\Rightarrow$ coupling strength q_i different for each field ϕ_i $\Rightarrow$ **electric charge**
- Lagrangian **invariant under local gauge transformations** $U(1)$

$$\phi_i(x) \to \phi_i'(x) = e^{-iq_i\Lambda(x)}\phi_i(x)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\Lambda(x)$$

- Kinetic part of the A_μ field: free-field Maxwell Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

  $\Rightarrow$ gauge-invariant by itself.

Introduction of the minimal coupling (8)

$\Rightarrow$ presence of an interaction term in the Lagrangian

$$\mathcal{L}_{int} = q_i A_\mu j_i^\mu$$

where j_i^μ is the $U(1)$ conserved current.

$U(1)$ symmetry + relativity $\Longrightarrow$ electromagnetism

Quantum Electrodynamics: Dirac + Electromagnetism:

$$\begin{aligned}\mathcal{L}_{QED} &= \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi \qquad (9)\end{aligned}$$

Scalar Quantum Electrodynamics: Klein-Gordon + Electromagnetism:

$$\begin{aligned}\mathcal{L}_{SQED} &= (D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= ((\partial_\mu + ieA_\mu)\phi)^\dagger((\partial^\mu + ieA^\mu)\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + ieA_\mu((\partial^\mu\phi^\dagger)\phi - \phi^\dagger\partial^\mu\phi) + e^2A^\mu A_\mu\phi^\dagger\phi \\ &= (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - ieA_\mu(\phi^\dagger\overleftrightarrow{\partial^\mu}\phi) + e^2A^\mu A_\mu\phi^\dagger\phi \qquad (10)\end{aligned}$$

$$f\overleftrightarrow{\partial^\mu}g \equiv f\partial^\mu(g) - (\partial^\mu f)g$$

Classical field: Covariant theory

$$\mathcal{L}_{Maxwell} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{11}$$

Free-field equations: vacuum Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0 \quad ; \quad \Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0 \tag{12}$$

$$\Pi_{A_0} = 0 \text{ (see exercises!)}$$

⇒ not suitable to carry out quantization

⇒ canonical momenta of the other components: electric field:

$$\Pi_{A_i} = F^{0i} = -F^{i0} = \partial^0 A^i - \partial^i A^0 = -E^i$$

Problem?

- $A^\mu \Rightarrow 4$ degrees of freedom
- but light has only 2 degrees of freedom (classical electromagnetism, polarization)
  ⇒ We have added extra degrees of freedom!

- gauge symmetry

$$A^\mu \to A'^\mu = A^\mu + \partial^\mu \Lambda \tag{13}$$

  freedom to choose some gauge.
- Quantization ⇒ need to **fix the gauge**
  ⇒ fix some condition on the gauge fields.
  Many possibilities:
  - Lorentz-covariant (R_ξ gauges, Lorenz-gauge[1], Feynman gauge, ...)
  - not Lorentz-covariant (Coulomb gauge, Radiation gauge, ...).

Lorenz-gauge

Covariant gauge condition:

$$\partial_\mu A^\mu = 0 \tag{14}$$

⇒ Always possible to obtain from gauge freedom (13)

⇒ residual gauge freedom. We can choose a Λ in eq. (13) such that:

$$\Box \Lambda = 0$$

and A'^μ will still fulfill the Lorenz equation (14).

[1] Do not confuse Ludvig Lorenz with Hendrik Lorentz

- To break this residual gauge freedom
  $\Rightarrow$ need to use a non-covariant gauge, like the radiation gauge:

$$A^0 = 0 \quad ; \quad \nabla \cdot \boldsymbol{A} = 0$$

- not necessary to break covariance to quantize the theory
  $\Rightarrow$ Lorenz condition (14) is sufficient.

Maxwell equations (12) $\oplus$ Lorenz condition (14):

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0 \quad \oplus \quad \partial_\nu A^\nu = 0$$

$$\Rightarrow \Box A^\mu = \partial^\nu \partial_\nu A^\mu(x) = 0$$

$\Rightarrow$ Klein-Gordon equations for a massless field: $A^\mu \in \mathbb{R}$

$$A^\mu(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3 \sqrt{2E_k}} \sum_{\lambda=0}^{3} (\epsilon^\mu_{(\lambda)}(\boldsymbol{k}) a_{(\lambda)\boldsymbol{k}} e^{-ikx} + \epsilon^{\mu *}_{(\lambda)}(\boldsymbol{k}) a^\dagger_{(\lambda)\boldsymbol{k}} e^{ikx}) \quad ; \quad E_k = k^0 = |\boldsymbol{k}| \quad (15)$$

- 4 polarization vectors $\epsilon^\mu_{(\lambda)}(\boldsymbol{k})$, corresponding to each (formal) degree of freedom.

We choose the normalization and completeness relations:

$$\epsilon^\mu_{(\lambda)}(\boldsymbol{k}) \epsilon^*_{(\sigma)\mu}(\boldsymbol{k}) = g_{\lambda\sigma} \quad ; \quad \sum_{\lambda=0}^{3} \xi_\lambda \epsilon^\mu_{(\lambda)}(\boldsymbol{k}) \epsilon^{\nu *}_{(\lambda)}(\boldsymbol{k}) = -g^{\mu\nu} \qquad (16)$$

$$\xi_0 = -1 \quad ; \quad \xi_i = 1 \, ; \, i = 1,2,3$$

for a given 3-momenta $\boldsymbol{k}$ $\Rightarrow$ explicit values for the polarization vectors

E.g.: $\boldsymbol{k} = (0, 0, k)$

$$\begin{aligned}
\epsilon^\mu_{(0)}(\boldsymbol{k}) &= n^\mu = (1,0,0,0) \text{ scalar or time-like polarization, non-physical} \\
\epsilon^\mu_{(3)}(\boldsymbol{k}) &= (0,0,0,1) = (0, \frac{\boldsymbol{k}}{|\boldsymbol{k}|}) \text{ longitudinal polarization, non-physical} \\
\epsilon^\mu_{(1)}(\boldsymbol{k}) &= (0,1,0,0) \text{ transverse polarization, physical} \\
\epsilon^\mu_{(2)}(\boldsymbol{k}) &= (0,0,1,0) \text{ transverse polarization, physical} \qquad (17)
\end{aligned}$$

Covariant form of longitudinal polarization:

$$\epsilon^\mu_{(3)}(\boldsymbol{k}) = \frac{k^\mu - (kn)n^\mu}{[(kn)^2 - k^2]^{1/2}}$$

The Lorenz condition (14) translates to:

$$\sum_{\lambda=0}^{3} k_\mu \epsilon^\mu_{(\lambda)}(\boldsymbol{k}) = 0 \tag{18}$$

- transverse polarizations $\Rightarrow$ directly satisfied:

$$k_\mu \epsilon^\mu_{(1,2)}(\boldsymbol{k}) = -\boldsymbol{k} \cdot \boldsymbol{\epsilon}_{(1,2)}(\boldsymbol{k}) = 0 \tag{19}$$

- scalar and longitudinal polarizations
  $\Rightarrow$ not individually satisfied, but the sum:

$$k_\mu \epsilon^\mu_{(0)}(\boldsymbol{k}) + k_\mu \epsilon^\mu_{(3)}(\boldsymbol{k}) = k_0 - |\boldsymbol{k}| = 0 \tag{20}$$

Linear polarizations of eq. (17) are real ($\mathbb{R}$)
$\Rightarrow$ circular or elliptic polarizations $\Rightarrow$ complex polarization vectors ($\mathbb{C}$)

Covariant Quantization

Maxwell Lagrangian (11) is not suitable for quantization $\Rightarrow$ need another approach.

Gupta-Bleuler quantization

- use a **modification** of the Maxwell Lagrangian
- impose a given **gauge-fixing condition**, like the one in eq. (14),
  $\Rightarrow$ selects the **physical states**.

Modified Lagrangian for the Maxwell field:

$$\mathcal{L} = \mathcal{L}_{Maxwell} \underbrace{-\frac{\lambda}{2}(\partial_\mu A^\mu)^2}_{\text{gauge fixing term}} \tag{21}$$

For fields fulfilling the Lorenz gauge condition (14): $\mathcal{L} = \mathcal{L}_{Maxwell}$

For $\lambda = 1$: Equivalent to (from Fermi):

$$\mathcal{L}_F = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) \tag{22}$$

e.o.m.:

$$\partial_\mu \partial^\mu A^\nu = 0 \tag{23}$$

$\Rightarrow$ equivalent to the Maxwell Lagrangian (11) **only** if the Lorenz-gauge condition (14) is fulfilled.

Conjugate momenta:

$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu} = -\dot{A}^\mu(x)$$

$\Rightarrow$ all the fields have non-zero momenta

$\Rightarrow$ NOTE index position!

$\Rightarrow$ perform canonical quantization, as in the Klein-Gordon field, using the normal modes expansion (15).

Canonical equal-time-commutation relations

$$\begin{aligned}
[A^\mu(t,\boldsymbol{x}), A^\nu(t,\boldsymbol{y})] &= 0 \\
[\Pi^\mu(t,\boldsymbol{x}), \Pi^\nu(t,\boldsymbol{y})] &= 0 \Rightarrow [\dot{A}^\mu(t,\boldsymbol{x}), \dot{A}^\nu(t,\boldsymbol{x})] = 0 \\
[A_\mu(t,\boldsymbol{x}), \Pi^\nu(t,\boldsymbol{y})] &= i\delta_\mu^\nu \delta^3(\boldsymbol{x}-\boldsymbol{y}) \Rightarrow \\
[A_\mu(t,\boldsymbol{x}), \dot{A}^\nu(t,\boldsymbol{y})] &= -i\delta_\mu^\nu \delta^3(\boldsymbol{x}-\boldsymbol{y}) \Rightarrow \\
[A^\mu(t,\boldsymbol{x}), \dot{A}^\nu(t,\boldsymbol{y})] &= -ig^{\mu\nu}\delta^3(\boldsymbol{x}-\boldsymbol{y})
\end{aligned} \tag{24}$$

- $1-3$ components: same e.t.c. relations as hermitic Klein-Gordon field
- 0 component has a $-$ sign

Commutation relations of the a operators in (15)

$$
\begin{aligned}
[a_{(\lambda)\boldsymbol{k}}, a^\dagger_{(\sigma)\boldsymbol{p}}] &= -g_{\lambda\sigma}(2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{k}) \\
[a_{(\lambda)\boldsymbol{k}}, a_{(\sigma)\boldsymbol{p}}] &= [a^\dagger_{(\lambda)\boldsymbol{k}}, a^\dagger_{(\sigma)\boldsymbol{p}}] = 0
\end{aligned} \tag{25}
$$

$\lambda = \sigma = 0$ has an extra $-$ sign

- The vacuum is defined as:

$$a_{(\lambda)\boldsymbol{p}}|0\rangle = 0 \quad \forall \boldsymbol{p}, \lambda$$

- or, equivalently, by defining the positive and negative-energy part of the A^μ field:

$$A^\mu = A^{\mu+} + A^{\mu-}$$

$$A^{\mu+}(x)|0\rangle = 0 \quad \forall x$$

- A particle (photon) with a given momentum $\boldsymbol{k}$ and polarization λ is created:

$$|1_{\lambda,\boldsymbol{k}}\rangle = \sqrt{2E_k}\, a^\dagger_{(\lambda)\boldsymbol{k}}|0\rangle$$

  $\Rightarrow$ same normalization as for the Klein-Gordon field.

- The normalization of the one-particle states is:

$$\langle 0|a_{(\lambda)\boldsymbol{k}} a^\dagger_{(\sigma)\boldsymbol{p}}|0\rangle = \langle 0|a^\dagger_{(\sigma)\boldsymbol{p}} a_{(\lambda)\boldsymbol{k}} + [a_{(\lambda)\boldsymbol{k}}, a^\dagger_{(\sigma)\boldsymbol{p}}]|0\rangle = -g_{\lambda\sigma}(2\pi)^3\delta^3(\boldsymbol{k}-\boldsymbol{p})\langle 0|0\rangle$$

$\Rightarrow$ **the scalar state $\lambda = 0$** has a **negative norm!**

$\Rightarrow$ scalar product has no definite sign

$\Rightarrow$ does not admit the probabilistic interpretation of Quantum Mechanics

- However, we still have not applied the Lorenz gauge condition (14)

Gupta-Bleuler solution:

if $|\Psi\rangle$ is a **physical state** then:

$$\boxed{\partial_\mu A^{\mu +}(x)|\Psi\rangle = 0 \text{ for physical states}} \tag{26}$$

- Expected value of the Lorenz condition for physical states is:

$$\langle\Psi|\partial_\mu A^\mu(x)|\Psi\rangle = 0$$

- going to the momentum-space,
  taking into account the transversality of $\epsilon_{1,2}$ (18),(19),(20)

$$(a_{(3)\boldsymbol{k}} - a_{(0)\boldsymbol{k}})|\Psi\rangle = 0 \quad \forall \boldsymbol{k} \tag{27}$$

Hamiltonian

$$H = \int \mathrm{d}^3x : \Pi^\mu A_\mu - \mathcal{L}_F := \int \frac{\mathrm{d}^3k}{(2\pi)^3} k^0 \left(\sum_{\lambda=1}^{3} a^\dagger_{(\lambda)\boldsymbol{k}} a_{(\lambda)\boldsymbol{k}} - a^\dagger_{(0)\boldsymbol{k}} a_{(0)\boldsymbol{k}} \right) \tag{28}$$

⇒ Scalar photons contribute to negative energy.

⇒ for a physical state fulfilling the subsidiary Lorenz condition (27):

$$a_{(3)\boldsymbol{k}}|\Psi\rangle = a_{(0)\boldsymbol{k}}|\Psi\rangle \Rightarrow \langle\Psi|a^\dagger_{(0)\boldsymbol{k}} = \langle\Psi|a^\dagger_{(3)\boldsymbol{k}}$$

the contribution of the longitudinal and scalar photons to the energy is:

$$\langle\Psi|a^\dagger_{(3)\boldsymbol{k}} a_{(3)\boldsymbol{k}} - a^\dagger_{(0)\boldsymbol{k}} a_{(0)\boldsymbol{k}}|\Psi\rangle = \langle\Psi|a^\dagger_{(3)\boldsymbol{k}}(a_{(3)\boldsymbol{k}} - a_{(0)\boldsymbol{k}})|\Psi\rangle = 0$$

⇒ scalar and longitudinal photons **do not** contribute to the total energy of the system for physical states, due to a cancellation between their contributions.

Photon Fock space

Allowed photon state:

$$|\Psi\rangle = |\Psi_T\rangle + |\Psi_{SL}\rangle \tag{29}$$

- Transverse part contains only transverse photons:

$$|\Psi_T\rangle \propto a^\dagger_{(1)\boldsymbol{k_1}} a^\dagger_{(2)\boldsymbol{k_2}}|0\rangle$$

- scalar-longitudinal part contains a state fulfilling (27), it can be written as:

$$|\Psi_{SL}\rangle \propto (a^\dagger_{(3)\boldsymbol{k}} - a^\dagger_{(0)\boldsymbol{k}})|0\rangle$$

  - choosing different values for Ψ_{SL} $\Rightarrow$ different states Ψ which correspond to the same physical state (since they have the same Ψ_T).
  - Residual gauge freedom.
  - Choosing different Ψ_{SL} means choosing different residual gauge-fixing terms.

It can be shown:

- the norm of a $|\Psi_{SL}\rangle$ state is:

$$\langle\Psi_{SL}|\Psi_{SL}\rangle = 0$$

- the Ψ_{SL} and Ψ_T states are orthogonal

$$\langle\Psi_{SL}|\Psi_T\rangle = 0$$

- the scalar product in the Fock space is:

$$\langle\Psi|\Psi\rangle = \langle\Psi_T|\Psi_T\rangle$$

$\Rightarrow$ has a definite sign

A probabilistic interpretation of Quantum Mechanics is possible

Propagators

The commutation relations (24), (25)

- same as for the real Klein-Gordon field ϕ and $\dot{\phi}$, except for the $-$ sign in the A^0 component,

$\Rightarrow$ generic commutators and propagators will be the same as for the Klein-Gordon field (except for the $-$ sign),

$\Rightarrow$ with a zero mass

$$
\begin{aligned}
D^{\mu\nu}(x-y) &= [A^\mu(x), A^\nu(y)] = -g^{\mu\nu}\Delta(x-y) = -g^{\mu\nu}\int \frac{\mathrm{d}^3p}{(2\pi)^3 2E_p}(e^{-ip(x-y)} - e^{ip(x-y)}) \\
&= \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{ig^{\mu\nu}}{p^2}e^{-ip(x-y)}
\end{aligned}
$$

- p^0 integration around the proper circuit in the plane $p^0 \in \mathbb{C}$.

Retarded propagator

$$
D_R^{\mu\nu}(x-y) = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{-ig^{\mu\nu}}{p^2}e^{-ip(x-y)}
$$

integration circuit *above* the poles (in the positive side of the imaginary

Feynman propagator

$$
\begin{aligned}
D_F^{\mu\nu}(x-y) &= \langle 0|T\{A^\mu(x)A^\nu(y)\}|0\rangle = -g^{\mu\nu}\Delta_F(x-y) \\
&= \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{-ig^{\mu\nu}}{p^2+i\varepsilon}e^{-ip(x-y)} \qquad (30)
\end{aligned}
$$

Alternative:

- construct the propagators as the gauge-field equations of motion (23) Green's function
- with the $+i\varepsilon$ prescription.
- The numerator contains the polarization vector completeness relations (16).
- Choosing different gauge-fixing terms in the modified Lagrangian (21)
  - $\Rightarrow$ different conditions for the polarization vectors (17)
  - $\Rightarrow$ different completeness relations (16)
  - $\Rightarrow$ different numerators for the gauge-boson propagators

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Massive gauge fields

Maxwell Lagrangian: only contains derivative terms,

$\Rightarrow$ describes a massless field.

Add a mass-term to the Lagrangian in the form:

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A^\mu A_\mu \qquad (31)$$

$\Rightarrow$ Mass term obviously breaks gauge-invariance

It is interesting however for:

- Electroweak theory, where the gauge-invariance is broken through the Higgs mechanism
- Vector mesons of QCD

The e.o.m. of this field is:

$$\partial_\mu F^{\mu\nu} + M^2 A^\nu = 0$$

by taking the 4-divergence ∂_ν of it, one obtains:

$$M^2 \partial_\nu A^\nu = 0$$

$\Rightarrow$ if $M \neq 0$, the Lorenz condition is fulfilled,

$\Rightarrow$ there are only three degrees of freedom.

Use Lorenz condition to simplify the e.o.m. to:

$$(\Box + M^2) A^\nu = 0$$

$\Rightarrow$ Klein-Gordon equation for a field of mass M.

The three independent polarization vectors, for a particle of momentum $k^\mu = (E, 0, 0, k)$ can be chosen to be:

$$\begin{aligned}
\epsilon^\mu_{(1)}(\boldsymbol{k}) &= (0,1,0,0) \text{ Transverse} \\
\epsilon^\mu_{(2)}(\boldsymbol{k}) &= (0,0,1,0) \text{ Transverse} \\
\epsilon^\mu_{(3)}(\boldsymbol{k}) &= \frac{1}{M}(k,0,0,E) \text{ Longitudinal}
\end{aligned}$$

$\Rightarrow$ now the longitudinal vector is physical.

The normalization and completeness relations are:

$$\epsilon^\mu_{(\lambda)}(\boldsymbol{k}) \epsilon^*_{(\sigma)\mu}(\boldsymbol{k}) = -\delta_{\lambda\sigma} = g_{\lambda\sigma} \quad ; \quad \sum_{\lambda=1}^{3} \epsilon^\mu_{(\lambda)}(\boldsymbol{k}) \epsilon^{\nu *}_{(\lambda)}(\boldsymbol{k}) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}$$