

PERTURBATIVE SOLUTION OF THE SCHWINGER MODEL

CHRISTOPH ADAM^{*)}

*Institut für theoretische Physik, Universität Bern,
Sidlerstraße 5, CH-3012 Bern, Switzerland*

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One interesting question for the exactly solvable Schwinger model is how to infer the exact solution from perturbation theory. We give a systematic procedure of deriving the exact solution from Feynman diagrams of arbitrary order for arbitrary n -point functions. As a byproduct, we derive from perturbation theory exact integral equations that the n -point functions have to obey.

1 Introduction

The Schwinger model [1], which is QED₂ with one massless fermion, has a long history of extensive study. The reason for this is that the Schwinger model combines two features that make it an ideal test laboratory for field theory studies and phenomenological methods. On one hand, it is exactly solvable and solutions within the operator formalism [2,3] or the path integral framework ([3–8] are well known. On the other hand, it shares some nontrivial features with more realistic field theories: An anomaly is present and may be used for quantization of the fermion [8–14]). Further, instantons are present ([3–8,15]) and cause the formation of the fermion condensate and of a nontrivial vacuum (θ -vacuum) ([2–8,15]). Besides, confinement of the fermion is realized in a precise manner: the fermionic n -point functions do not tend to free ones for large separations ([2,3,16–20]).

One interesting question in this context is whether or how far it is possible to derive the exact solution and its nontrivial — partly nonperturbative — features from perturbation theory.

This is the question we investigate, and we find that by a systematic procedure we may transform all graphs of a given order into the corresponding contribution to the exact n -point function at hand. Further, from an investigation of Feynman graphs of arbitrary order we prove some exact integral equations to hold for specific n -point functions. These are related to the Dyson-Schwinger equations of the Schwinger model (see e.g. [6]).

The organization of the paper is as follows: first we review the exact path integral solution of the Schwinger model in Euclidean space and show how even contributions from nontrivial vacuum sectors may be derived from the trivial sector, being thereby accessible to perturbation theory.

Then we present the perturbative approach to the Schwinger model and derive the rules of how to reduce Feynman graphs to the exact solution. These computations turn out especially simple in a graphical fashion.

^{*)} Permanent address: *Institut für theoretische Physik, Universität Wien, Boltzmannngasse 5, 1090 Wien, Austria*; e-mail: adam@pap.univie.ac.at

At last, we derive from perturbation theory, some exact integral equations for the n -point functions and comment on their meaning.

2 The exact path integral solution

As a starting point we use the exact vacuum functional after integration of the fermions (which can be found e.g. in [7,6]),

$$\begin{aligned}
 Z[\lambda, \eta, \bar{\eta}] &= \sum_{k=-\infty}^{\infty} Z_k[\lambda, \eta, \bar{\eta}], \\
 Z_k[\lambda, \eta, \bar{\eta}] &= N \int D\beta^{[k]} \prod_{i_0=0}^{k-1} (\bar{\eta} \Psi_{i_0}^{[\beta]}) (\bar{\Psi}_{i_0}^{[\beta]} \eta) \\
 &\quad \times e^{\int dx dy \bar{\eta}(x) G^{[\beta]}(x, y) \eta(y)} e^{\int dx (\frac{1}{2} \beta \mathcal{D} \beta + \frac{1}{2} \beta \lambda)}, \quad (1)
 \end{aligned}$$

where the gauge field is parametrized by the prepotential β

$$\begin{aligned}
 A_\mu &= \frac{1}{e} \epsilon_{\mu\nu} \partial^\nu \beta, \quad A_\mu J^\mu =: \beta \lambda \\
 \Rightarrow \frac{\delta}{\delta J_\mu} &= \epsilon^{\mu\nu} \partial_\nu \frac{\delta}{\delta \lambda} \quad (2)
 \end{aligned}$$

corresponding to Lorentz gauge. $G^{[\beta]}$ is the exact fermion propagator

$$\begin{aligned}
 G^{[\beta]}(x, y) &= e^{i(\beta(x) - \beta(y)) \gamma_5} G_0(x - y), \\
 G_0(z) &= \frac{z^\mu \gamma_\mu}{2\pi(z^2 - i\epsilon)}. \quad (3)
 \end{aligned}$$

\mathcal{D} is the operator of the effective action of the prepotential β , with Green's function \mathcal{G} ,

$$\begin{aligned}
 \mathcal{D} &= \frac{1}{\pi \mu^2} \square (\square - \mu^2), \quad \mu^2 = \frac{e^2}{\pi}, \\
 \mathcal{G}(x) &= \pi (D_\mu(x) - D_0(x)), \quad (4) \\
 \mathcal{D}_x \mathcal{G}(x - y) &= \delta(x - y),
 \end{aligned}$$

where μ is the "photon" mass and $D_\mu(x)$, $D_0(x)$ are the massive and massless scalar propagators in two dimensions:

$$D_0(x) = \frac{1}{4\pi} \ln(x^2 - i\epsilon) + \text{const.}, \quad D_\mu(x) = -\frac{1}{2\pi} K_0(\mu|x|), \quad (5)$$

$K_0(z)$ being the McDonald function (for details see e.g. [6-8]).

k is the instanton number and $\Psi_{i_0}^{[\beta]}$ are the zero modes corresponding to the gauge field β with instanton number k . (Their precise form can be found in [6-8]).

Due to the Gaussian nature of the path integral (1) and the knowledge of the exact fermion propagator in an external field (3) all n -point functions of the theory may be computed explicitly. E.g. for the fermionic two-point function we find (for the sectors $k = 0, \pm 1$; higher sectors do not contribute)

$$\langle T\bar{\Psi}^\beta(y)\Psi^\alpha(x) \rangle^{k=0} = G_0^{\alpha\beta}(y-x)e^{\mathcal{G}(0)-\mathcal{G}(x-y)}, \quad (6)$$

$$\langle T\bar{\Psi}^\beta(y)\Psi^\alpha(x) \rangle^{k=\pm 1} = \frac{1}{2\pi} P_\pm^{\alpha\beta} e^{\mathcal{G}(0)+\mathcal{G}(x-y)} \quad (7)$$

($P_\pm = \frac{1}{2}(1 \pm \gamma_5)$). We observe a feature that persists to hold for higher n -point functions: for vector-like fermionic bilinears only the zero sector contributes, whereas for chiral bilinears the contributing instanton sector is fixed by chirality. Therefore for an n -point function consisting of an arbitrary number of vectors, n_+ positive chirality bilinears ($\bar{\Psi}(y)P_+\Psi(x)$) and n_- negative chirality bilinears only the sector $k = n_+ - n_-$ contributes.

General n -point functions always may be decomposed into a sum of n -point functions of definite chirality. In fact, this may be used to compute all n -point functions from the trivial vacuum sector via clustering ([21]). E.g. for the four-point function we find ($S_\pm(y, x) \equiv \bar{\Psi}(y)P_\pm\Psi(x)$)

$$\begin{aligned} \langle TS_+(y_2, y_1)S_-(x_2, x_1) \rangle &\equiv \langle T\bar{\Psi}(y_2)P_+\Psi(y_1)\bar{\Psi}(x_2)P_-\Psi(x_1) \rangle^{k=0} = \\ &= \frac{(x_1 - y_2)^\mu}{2\pi(x_1 - y_2)^2} \frac{(x_2 - y_1)_\mu}{2\pi(x_2 - y_1)^2} e^{2\mathcal{G}(0)+\mathcal{G}(x_1-x_2)+\mathcal{G}(y_1-y_2)-\sum_{i,j} \mathcal{G}(x_i-y_j)}. \end{aligned} \quad (8)$$

Performing now the limit $y_j \rightarrow y_j + a$, $a \rightarrow \infty$, $y_1 - y_2$ fixed, we find that the free fermion propagators just cancel against the large range part of $\exp \sum \mathcal{G}(x_i - y_j)$ (the massless propagator D_0 ; the massive one, D_μ , vanishes exponentially). Comparing with the expression (7) for the two-point function we therefore find

$$\lim_{a \rightarrow \infty} \langle TS_+(y_2 + a, y_1 + a)S_-(x_2, x_1) \rangle^{k=0} = \langle TS_+(y_2, y_1) \rangle^{k=1} \langle TS_-(x_2, x_1) \rangle^{k=-1}, \quad (9)$$

which is the cluster property.

Generally, any VEV can be computed from the trivial sector by multiplying the operators of given chiralities by their opposite chirality counterparts and by the use of the cluster property.

3 Schwinger model perturbation theory

In our perturbative calculations we will always use the exact photon propagator, which consists of a sum of all possible vacuum polarization insertions,

$$\text{-----} = \text{wavy} + \text{wavy} \text{ with bubble} + \text{wavy} \text{ with two bubbles} + \dots$$

and all other possible contributions vanish in the Schwinger model (for details see e.g. [8,9,22,23]). In momentum space it is

$$D^{\mu\nu}(p) = (g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}) \frac{1}{p^2 + \mu^2} = -\epsilon^{\mu\lambda} p_\lambda \epsilon^{\nu\rho} p_\rho \frac{1}{p^2(p^2 + \mu^2)} \quad (10)$$

as can be easily computed from the vacuum functional (1) or within perturbation theory ([9]). The propagator looks like being in "Landau gauge", however this is the unique and gauge invariant result. When inserted into a Feynman diagram, this propagator gives rise to

$$-e\gamma_\mu\epsilon^{\mu\lambda}p_\lambda\cdots\frac{1}{p^2(p^2+\mu^2)}\cdots e\gamma_\nu\epsilon^{\lambda\rho}p_\rho=-\not{p}\gamma_5\cdots\not{p}\gamma_5\tilde{\mathcal{G}}(p^2), \quad (11)$$

$\tilde{\mathcal{G}}(p^2)$ being the Fourier transform of $\mathcal{G}(x)$, eq. (4).

In actual computations, when an exact photon line begins and ends at the same fermion line, there is always an even number of γ matrices between the two γ_5 , therefore we may omit the two γ_5 in the computations (in more general cases we will reinsert them when it is necessary).

Further we will frequently use the identity

$$\frac{1}{\not{p}}\not{k}\frac{1}{\not{p}+\not{k}}=\frac{1}{\not{p}}-\frac{1}{\not{p}+\not{k}}. \quad (12)$$

Let us take the fermionic two-point function as a first example. Up to second order in the exact photon propagator we have

For the first order graph

we obtain, using (11) and (12)

$$\begin{aligned} & -\int\frac{dk}{(2\pi)^2}\mathcal{G}(k)\frac{1}{\not{p}}\not{k}\frac{1}{\not{p}+\not{k}}\not{k}\frac{1}{\not{p}}= \\ & -\int\frac{dk}{(2\pi)^2}\mathcal{G}(k)\frac{1}{\not{p}}\not{k}\frac{1}{\not{p}}+\int\frac{dk}{(2\pi)^2}\mathcal{G}(k)\frac{1}{\not{p}+\not{k}}\not{k}\frac{1}{\not{p}}= \\ & -\text{diagram} + \text{diagram} \end{aligned} \quad (13)$$

and the first term vanishes. The final result is

$$\int\frac{dk}{(2\pi)^2}\mathcal{G}(k)\frac{1}{\not{p}}-\int\frac{dk}{(2\pi)^2}\mathcal{G}(k)\frac{1}{\not{p}+\not{k}}= \text{diagram} - \text{diagram} \quad (14)$$

which, after Fourier transformation, is obviously the first order (in \mathcal{G}) expansion of the exact solution (6).

From this simple example we derive the following graphical rules: starting at any vertex, one obtains two graphs by leaving out the inner or outer fermion propagator (the latter one with a minus sign). Photon lines get a dot at that vertex that has been changed (however photon lines running to the very begin or end always have been changed, so it is not necessary to draw a dot there). Closed loops vanish when only one of the two vertices has been changed. Otherwise they decouple from all momenta and may be drawn to the begin or end of the fermion line. Whenever a changed vertex coincides with an unchanged one, the unchanged one cannot be removed further.

By applying these rules, we may reduce each individual graph to a final form. Then we have to sum all graphs of a given order, and only the sum will lead to the exact solution.

Applying these rules to the second order graphs we find

$$\begin{aligned}
 & \text{---} \text{---} \text{---} = - \text{---} \text{---} \text{---} = + \text{---} \text{---} \text{---} \\
 & = \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} \\
 & = \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} \\
 & \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} = \\
 & \dots = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\
 & \quad - \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\
 & \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} = \\
 & \dots = - \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\
 & \quad - 2 \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\
 & = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\
 & \quad - \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}
 \end{aligned}$$

where we used

$$\text{---} \text{---} \text{---} = - \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---}$$

$$\int d\mu(k, l) \frac{1}{\not{p} + \not{k} + \not{l}} \not{k} \frac{1}{\not{p}} =$$

$$= - \int d\mu(k, l) \frac{1}{p+k+l} I \frac{1}{p} + \int d\mu(k, l) \left(\frac{1}{p} - \frac{1}{p+k+l} \right) \quad (15)$$

in the last step (the symmetric integration measure $d\mu(k, l)$ we display in a moment). For the sum of all three graph we get

$$\text{---} \bullet \text{---} \bigcirc \text{---} - \bigcirc \text{---} \text{---} + \text{---} \bigcirc \text{---}$$

In a next step we have to use the symmetry with respect to the two momentum integrations,

$$\Rightarrow \text{Diagram} = 1/2(\text{Diagram} - \text{Diagram})$$

to obtain the final result

$$1/2 \text{ (loop) } - \text{ (triangle) } + 1/2 \text{ (box) }$$

$$\int d\mu(k, l) \left(\frac{1}{2} \frac{1}{\not{p}} - \frac{1}{\not{p} + \not{k}} + \frac{1}{2} \frac{1}{\not{p} + \not{k} + \not{l}} \right), \quad (16)$$

$$d\mu(k, l) = \frac{dk}{(2\pi)^2} \frac{dl}{(2\pi)^2} \tilde{\mathcal{G}}(k) \tilde{\mathcal{G}}(l).$$

This result, after a Fourier transformation, obviously is the second order (in \mathcal{G}) of the exact solution (6).

So the general way of obtaining the exact solution is: reduce all graphs of a given order according to the graphical rules, rearrange some dotted graphs (like in (15)) in order to cancel them, and when some dotted graphs are left, rewrite them by using symmetric momentum integration. This recipe remains the same for higher n -point functions. Only when translating back into formulae you have to insert one γ_5 matrix at the beginning of each fermion line that carries an *odd* number of vertices.

E.g. for the exchange part of the four-point function

one obtains

$$\begin{array}{|c|} \hline \text{---} \\ \hline \end{array} - \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} - \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array}$$

for the first order and

$$\begin{aligned}
& \left(\text{Diagram 1} + \text{Diagram 2} \right) = \\
& 1/2 \left(\text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} \right) \\
& - \left[\text{Diagram 7} - \text{Diagram 8} + \text{Diagram 9} \right] \\
& + \left[\text{Diagram 10} - \text{Diagram 11} - \text{Diagram 12} \right]
\end{aligned}$$

in second order. These results coincide with the first and second order expansions of the exact result (8).

So we found systematic rules to derive the exact solution order by order, but the question remains if there is a way to derive the whole exact solution at once from perturbation theory; and indeed there is. More precisely, for any n -point function we may derive an exact integral equation from perturbation theory. This we discuss in the next section.

4 Integral equations for n -point functions

Again, we start with the two-point function (for the two-point function an analogous computation in light cone gauge has been done in [24]). Let us look at a graph of arbitrary order where the first photon runs from vertex 1 to some vertex i :

whereas when running from 1 to $(i-1), (i+1)$ (all other lines being unchanged) we find

and the diagram with $1 \dots i$ is cancelled by contributions from $1 \dots (i - 1)$ and $1 \dots (i + 1)$ (the diagrams that cancel are indeed identical although we indexed the

$$\begin{aligned}
 T_n^{\alpha\beta\gamma\delta}(p, q, p-k, q+k) = & \\
 = \int \frac{dk_2}{(2\pi)^2} dk_1 \delta(k-k_1-k_2) \tilde{G}(k_2) \gamma_5^{\alpha\alpha'} \gamma_5^{\beta\beta'} & \\
 \times [T_{n-1}^{\alpha'\beta'\gamma\delta}(p-k_2, q+k_2, p-k, q+k) - T_{n-1}^{\alpha'\beta'\gamma\delta}(p-k_2, q, p-k, q+k_1) & \\
 - T_{n-1}^{\alpha'\beta'\gamma\delta}(p, q+k_2, p-k_1, q+k) + T_{n-1}^{\alpha'\beta'\gamma\delta}(p, q, p-k_1, q+k_1)], & \quad (20)
 \end{aligned}$$

where however this operation only rises the number of *exchanging* photons. The “self energy” part is, of course, computed like for the two-point function, eq. (17).

In order to derive an equation analogous to (18), we again multiply by \not{p} , amputating thereby the upper incoming fermion line. As a consequence, the first upper vertex is fixed, and we obtain (including now the self energy):

(the slash indicates amputation) and therefore the equation

$$\begin{aligned}
 \not{p} T_n^{\alpha\beta\gamma\delta}(p, q, p-k, q+k) = & \\
 = \delta^{\alpha\gamma} S^{\beta\delta}(q) \delta(k) + (\gamma_\mu \gamma_5)^{\alpha\alpha'} \gamma_5^{\beta\beta'} \int \frac{dk_2}{(2\pi)^2} dk_1 \delta(k-k_1-k_2) k_2^\mu \tilde{G}(k_2) & \\
 \times [-T_{n-1}^{\alpha'\beta'\gamma\delta}(p-k_2, q+k_2, p-k, q+k) + T_{n-1}^{\alpha'\beta'\gamma\delta}(p-k_2, q, p-k, q+k_1)] & \\
 - \gamma_\mu^{\alpha\alpha'} \delta^{\beta\beta'} \int \frac{dk_2}{(2\pi)^2} dk_1 \delta(k_1-k-k_2) k_2^\mu \tilde{G}(k_2) T_{n-1}^{\alpha'\beta'\gamma\delta}(p-k_2, q, p-k_1, q+k) & \quad (21)
 \end{aligned}$$

(for the inhomogenous part we used our knowledge of the two-point function). Equation (21) is the momentum space version of the Dyson-Schwinger equation for the four-point function, and, of course, the exact four-point function obeys eq. (21). For convenience, we display the exact four-point function in x space in the appendix.

The generalization to higher n -point functions is straightforward.

5 Summary

We have given a systematic procedure of finding the exact n -point functions from a resummed perturbation theory order by order. We have explicitly demonstrated the procedure for the fermionic two- and four-point functions, but generalizations are straightforward. Further we have derived integral equations for the exact n -point functions from perturbative recursion relations. By solving these equations the n -point functions can be found in principle, which corresponds to a complete summation of the perturbation series. By the way, these equations are just the Dyson-Schwinger equations (in momentum space) which the exact n -point functions have to obey.

By these methods, and having in mind that the trivial instanton sector suffices to

obtain the complete solution of the Schwinger model, all features of the Schwinger model in principle can be treated by perturbative methods.

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Appendix

The exact fermionic four-point function of the Schwinger model in the $k = 0$ instanton sector and for arbitrary spinor indices reads:

$$\begin{aligned}
 & \langle \bar{\Psi}_\delta(y_2) \Psi_\gamma(y_1) \bar{\Psi}_\beta(x_2) \Psi_\alpha(x_1) \rangle^{k=0} = \\
 & = \frac{1}{2} G_0(x_1 - x_2) \otimes G_0(y_1 - y_2) \int D\mu[\beta] \\
 & \times [\cosh(\beta(x_1) - \beta(x_2) + \beta(y_1) - \beta(y_2)) + \cosh(\beta(x_1) - \beta(x_2) - \beta(y_1) + \beta(y_2))] \\
 & + \frac{1}{2} \gamma_5 G_0(x_1 - x_2) \otimes \gamma_5 G_0(y_1 - y_2) \int D\mu[\beta] \\
 & \times [\cosh(\beta(x_1) - \beta(x_2) + \beta(y_1) - \beta(y_2)) - \cosh(\beta(x_1) - \beta(x_2) - \beta(y_1) + \beta(y_2))] \\
 & - \frac{1}{2} G_0(x_1 - y_2) \otimes G_0(y_1 - x_2) \int D\mu[\beta] \\
 & \times [\cosh(\beta(x_1) - \beta(x_2) + \beta(y_1) - \beta(y_2)) + \cosh(\beta(x_1) + \beta(x_2) - \beta(y_1) - \beta(y_2))] \\
 & - \frac{1}{2} \gamma_5 G_0(x_1 - y_2) \otimes \gamma_5 G_0(y_1 - x_2) \int D\mu[\beta] \\
 & \times [\cosh(\beta(x_1) - \beta(x_2) + \beta(y_1) - \beta(y_2)) - \cosh(\beta(x_1) + \beta(x_2) - \beta(y_1) - \beta(y_2))],
 \end{aligned}$$

where

$$G_0(x_1 - y_2) \equiv G_0^{\alpha\delta}(x_1 - y_2)$$

etc. and

$$D\mu[\beta] \equiv D\beta e^{-\int d^2x \beta \mathcal{D}\beta}$$

(here β is assumed real, whereas in (1) it is pure imaginary).

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