Week 4

1. Consider the Lagrangian of N free Dirac fermions with the same mass.

   (a) Show that it is invariant under internal $U(N) = U(1) \otimes SU(N)$ transformations.

   (b) Calculate the conserved charges associated to $SU(N)$ according to Noether's theorem.

   (c) Show that the conserved charges Q^a can be choosen in such a way that they fulfil the usual $SU(N)$ commutation relations $\left[Q^a, Q^b\right] = i f^{abc} Q^c$, where f^{abc} are the $SU(N)$ structure constants. (Use that $\{\psi^\dagger_{k\,\alpha}(x),\, \psi_{j\,\beta}(y)\} = \delta(\mathbf{x}-\mathbf{y})\delta_{kj}\delta_{\alpha\beta}$, for $x^0 = y^0$; $i\,, j$ and $\alpha\,, \beta$ are $SU(N)$ and Dirac indices respectively, and $[A, BC] = B\,[A, C] + [A, B]\,C$, $[A, BC] = -B\,\{A, C\} + \{A, B\}\,C$, which hold for any operator A, B, C).