1 Schwinger model

Consider QED in 1+1 dimensions. In 2d, the coupling e has dimensions of mass, so it is relevant. F has only one component F^{01}, an electric field, there is no magnetic field. The vacuum equations only allow for a solution with constant electric field. Placing a charge somewhere in the line generates constant fields to each side, it is like an infinite plate in 3+1 dimensions. Since the Coulomb potential grows linearly with distance in one spatial dimension, there is classical confinement.

When the mass of the fermion is $m = 0$, the theory has classically two conserved currents, the vector and the axial one,

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi \qquad j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

At the quantum level, the axial current is not conserved (ABJ anomaly)

$$\partial_\mu j_A^\mu = \frac{e}{\pi}F^{01}$$

The theory can be solved exactly [1, 2]. The photon develops a mass

$$m_\gamma^2 = \frac{e^2}{\pi}$$

This can be seen using bosonization, which maps the fermions to a compact boson (e.g. [3]). It is convenient to study the model on a spatial circle S^1 [4, 5]. If the circle has length L, there is a new dimensionless parameter eL, and we can consider various regimes $eL \ll 1$ and $eL \gg 1$.

Even in this massless limit, there is a non-zero chiral condensate. The theory does not display chiral symmetry. This does not contradict the Coleman-Mermin-Wagner theorem, because this is not spontaneous breaking; rather, it is due to the axial ABJ anomaly.

The model with $N \geq 2$ massless fermions is qualitatively different [6]. Now it has a global chiral symmetry $SU(N)_L \times SU(N)_R$. The chiral condensate is zero. There is one massive boson of mass

$$m_\gamma^2 = N\frac{e^2}{\pi}$$

and $N - 1$ massless bosons.

Finally, one can also consider a generalization where the fermions have charge $q > 1$ [7, 8, 9].

When the fermion is massive, the model is no longer exactly soluble, but there are approximate results when $m \ll e$ [10]. The massive multi-flavor model on a circle is discussed in [11]. When $m > 0$ there is an additional physical parameter, the θ angle [10]. It is periodic, and it can be understood as a background electric field. At $\theta = \pi$, there is a critical point as we vary m/e, at $m/e \simeq 0.3335$ [12]. The critical point is in the universality class of the Ising model.

1.1 Finite temperature and/or chemical potential

See [13] for a pedagogical review of the phase diagram for 2d field theories (although they don't discuss much of the Schwinger model).

The Schwinger model at finite temperature is considered in [14]. At finite temperature and chemical potential in [15].

At high temperatures, one generically expects restoration of broken symmetries, but since in the Schwinger model chiral symmetry is broken by the anomaly, it is not restored at any temperature, see section VI in [16].

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