Homework 2

Due date: 10/05/2021

Problem 1:

Consider QCD with only one flavour $n_f = 1$.

a) Use Grassmann integration identities to express the partition function as

$$\mathscr{Z} = \mathscr{C} \int \mathscr{D}A \ \det \left\{ 1 - \frac{i}{i\not{\partial} - aM_R} (-ig_R)\, b \not{A}^c t^c \right\} e^{i \int d^4x \mathscr{L}_R(A_R, g_R, \mu)} \tag{1}$$

with $\mathscr{C}$, a and b constants and $\mathscr{L}_R$ the renormalised pure glue ($n_f = 0$) Lagrangian. Express a and b in terms of the field strength constants of the theory. Moments of this partition function generates all correlators with no external fermions.

b) The fermionic determinant in the partition function leads to correction to the pure glue correlation functions, as determined from the pure gauge Lagrangian. Using the matrix identity

$$\log(\det \mathscr{M}) = \mathrm{Tr}\{\log \mathscr{M}\} \tag{2}$$

exponentiate the determinant and interpret it as the modification of the pure glue Lagrangian induced by the heavy quark, ΔS. By expanding in powers of g_R, determine the leading order correction to the pure glue Lagrangian which is quadratic in the gauge field and express it in the form

$$\Delta S = g_R^2 \int \frac{d^4q}{(2\pi)^4} A_\mu^a(-q) A_\nu^b(q) \mathscr{V}_{ab}^{\mu\nu}(q, M_R, \mu) \tag{3}$$

Define first $\mathscr{V}(q, M_R, \mu)$ by an integral. Discuss the diagrammatic interpretation of the corresponding integral.

c) Perform the integral $\mathscr{V}(q, M_R, \mu)$. Regularise any divergence in the MS-scheme. Annalise the Lorentz structure of the divergent contribution. Interpret its effect when this modified action is combined with the counter-terms of the $n_f = 1$ theory.

d) Annalise the finite part of the integral. Assume that you are only interested in correlation function with momentum $q << M_R$. Expand the finite piece to leading order in q/M_R. Annalise the Lorentz structure of this leading order contribution and interpret its form. Identify the leading order contribution to the decoupling parameter ξ_A.