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Infinitely many rigid symmetries of kappa-invariant D-string actions

Friedemann Brandt [a], Joaquim Gomis [b], David Mateos [b], Joan Simón [b]

[a] *Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany*
[b] *Departament ECM, Facultat de Física, Universitat de Barcelona and Institut de Física d'Altes Energies, Diagonal 647, E-08028 Barcelona, Spain*

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Abstract

We show that each rigid symmetry of a D-string action is contained in a family of infinitely many symmetries. In particular, kappa-invariant D-string actions have infinitely many supersymmetries. The result is not restricted to standard D-string actions, but holds for any two-dimensional action depending on an abelian world-sheet gauge field only via the field strength. It applies thus also to manifestly $SL(2,Z)$ covariant D-string actions. Furthermore, it extends analogously to d-dimensional actions with $(d-1)$-form gauge potentials, such as brane actions with dynamical tension. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction and conclusion

In [1–3] a complete classification of the rigid symmetries of bosonic D-string actions was given and several examples were worked out explicitly, both in flat and curved backgrounds. In particular it was shown that each rigid symmetry is contained in a family of infinitely many rigid symmetries. All these families together form a loop (or loop-like) symmetry algebra. Two important examples were those of a D-string in the near horizon geometries of D3 and D1 + D5 branes [3]. The near horizon metrics involve AdS factors whose isometry groups are $SO(2,4)$ and $SO(2,2)$ respectively, and thus the symmetries of the D-string action in these backgrounds contain an infinite loop generalization of these conformal symmetries.

In this paper we show that the above structures extend to supersymmetric and, in particular, to kappa-invariant D-string actions. Hence, these actions have actually infinitely many supersymmetries, forming infinite dimensional loop-generalizations of the familiar supersymmetry algebras (in flat or curved backgrounds).

As in the purely bosonic case, the infinite symmetry structure is a direct consequence of the presence of the Born-Infeld gauge field A_μ. This will become particularly clear from the way in which we shall derive the result. Namely we shall use a simple general argument which neither makes use of the

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particular form of the action nor of any specific properties of the target space or its symmetries. Rather, the argument uses solely that the Lagrangian depends on A_μ only via the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and that the world-volume is two-dimensional.

Our result is thus not restricted to D-string actions of the Born-Infeld type but applies actually to a much larger class of two dimensional actions containing a $U(1)$ gauge field. Moreover, if such an action contains several $U(1)$ gauge fields only via their field strengths, then the argument applies to each of these gauge fields separately, yielding an even larger symmetry structure. In particular this applies to the manifestly $SL(2,Z)$-covariant D-string actions constructed in [6,7] which contain two $U(1)$ gauge fields.

Furthermore we shall show that the argument is actually not restricted to two-dimensional actions. Rather, it extends analogously to d-dimensional actions containing $(d-1)$-form gauge potentials only via their (abelian) field strengths. In the context of branes, such actions have been discussed in [6–12] where the $(d-1)$-form gauge potentials serve to implement the brane tension dynamically (as an integration constant). The two-dimensional case appears to be somewhat special in the context of D-branes, as only in this case the Born-Infeld gauge field itself serves as a $(d-1)$-form gauge potential.

Therefore the paper focuses mainly on the existence and construction of infinite families of symmetries of D-string actions. We do not provide a complete characterization of all these families of symmetries. From the results in the bosonic case, we expect that such a characterization can be given in terms of generalized super-Killing vector equations. Here we just remark that the families of symmetries of D-string actions do not necessarily correspond one-to-one to the target space (super-) isometries. For instance, in the bosonic case there are backgrounds which admit the presence of dilatational symmetries in addition to families of symmetries arising from the target space isometries [1–3]. We shall provide a supersymmetric version of these dilatational symmetries in a flat background which however does not seem to extend (at least not straightforwardly) to the kappa-invariant case as the Wess-Zumino term breaks these dilatational symmetries.

One interesting case would be that of the kappa-invariant D-string action in a D1 + D5 supersymmetric background, which could be constructed along the lines of [4,5]. It follows from our results that such an action should contain among its rigid symmetries an infinite loop generalization of the background isometry supergroup $SU(1,1 \mid 2) \times SU(1,1 \mid 2)$.

Finally, we wish to stress that the nature of the infinite symmetry structure described here differs from the infinite conformal symmetry of gauge fixed two dimensional sigma models discussed in [13,14]. Namely, these conformal symmetries of sigma models are a mixture of finitely many target space symmetries and infinitely many (conformal) world-sheet symmetries which arise as residual symmetries from world-sheet diffeomorphisms in appropriate gauges of the latter. In contrast, the infinitely many symmetries of D-string actions discussed here exist in addition to the world-sheet diffeomorphisms and are thus present even before gauge fixing the latter.

2. The general argument in the two-dimensional case

We consider a two-dimensional action $S = \int d^2\sigma L$ with a Lagrangian L which depends on the gauge field A_μ only via its field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and, possibly, derivatives thereof (or which can be brought into such a form by means of a partial integration). We shall denote by $\{Z^M\}$ all the other fields which occur in the action. For instance, in the case of standard bosonic D-string actions $\{Z^M\}$ contains only the target space coordinates x^m, while it contains in addition the fermionic fields θ^α in the supersymmetric or kappa-invariant case. If the action contains additional abelian gauge fields (as, e.g., in [6,7]), the latter count also among the Z^M, and A_μ can be any of those gauge fields which enter the action only via their field strengths.

As L depends by assumption on A_μ only via $F_{\mu\nu}$ and derivatives thereof, its Euler Lagrange derivative $\hat{\partial} L / \hat{\partial} A_\mu$ with respect to A_μ takes the form

$$\frac{\hat{\partial} L}{\hat{\partial} A_\mu} = \epsilon^{\mu\nu} \partial_\nu \varphi \tag{1}$$

where $\varphi = \epsilon_{\nu\mu}\,\partial L/\partial F_{\mu\nu}$ (using $\epsilon^{01} = \epsilon_{10} = 1$). Note that we have used here that we are dealing with a two dimensional theory, as we took advantage of the fact that $F_{\mu\nu}$ is proportional to $\epsilon_{\mu\nu}$.

We shall now show that any rigid symmetry of S is actually contained in a family of infinitely many rigid symmetries. Let us therefore assume that there are infinitesimal transformations ΔZ^M and ΔA_μ which generate a symmetry of the action, i.e., by assumption the Δ-variation of the Lagrangian is a total derivative, $\Delta L = \partial_\mu k^\mu$. This invariance property is equivalent to

$$(\Delta Z^M)\,\frac{\hat{\partial} L}{\hat{\partial} Z^M} + (\Delta A_\mu)\,\frac{\hat{\partial} L}{\hat{\partial} A_\mu} = \partial_\mu j^\mu_\Delta . \tag{2}$$

Here j^μ_Δ is of course nothing but the Noether current associated with Δ. We claim that the following transformations $\tilde{\Delta}$ generate further rigid symmetries of the action,

$$\tilde{\Delta} Z^M = \lambda(\varphi)\,\Delta Z^M \tag{3}$$

$$\tilde{\Delta} A_\mu = \lambda(\varphi)\,\Delta A_\mu - \frac{d\lambda(\varphi)}{d\varphi}\,\epsilon_{\mu\nu} j^\nu_\Delta \tag{4}$$

where $\lambda(\varphi)$ is an arbitrary function of the quantity φ occurring in Eq. (1). Indeed, using Eqs. (1) and (2) one easily verifies that

$$(\tilde{\Delta} Z^M)\,\frac{\hat{\partial} L}{\hat{\partial} Z^M} + \left(\tilde{\Delta} A_\mu\right)\frac{\hat{\partial} L}{\hat{\partial} A_\mu} = \partial_\mu\left[\lambda(\varphi)\, j^\mu_\Delta\right]. \tag{5}$$

This implies $\tilde{\Delta} L = \partial_\mu \tilde{k}^\mu$ and thus $\tilde{\Delta}$ generates a symmetry of the action. Furthermore Eq. (5) shows that the Noether current associated with $\tilde{\Delta}$ arises from the one associated with Δ simply through multiplication with $\lambda(\varphi)$,

$$j^\mu_{\tilde{\Delta}} = \lambda(\varphi)\, j^\mu_\Delta . \tag{6}$$

Hence, given a symmetry Δ of the action, any choice $\lambda(\varphi)$ yields another symmetry $\tilde{\Delta}$, and thus gives indeed rise to a family of infinitely many symmetries. Notice that if Δ is a linear combination of a set of independent rigid symmetries, $\Delta = \epsilon^i \Delta_i$, each of the symmetries Δ_i yields a corresponding family of symmetries $\tilde{\Delta}_i$ through functions $\lambda^i(\varphi)$.

3. Kappa invariant D-string

As a first example of the above statement, we consider the kappa-invariant D-string action in a flat ten-dimensional background with two target space Majorana-Weyl fermions $\theta^\alpha_1, \theta^\alpha_2$ of the same chirality (type IIB case). Using the notation and conventions of [15,16] (in particular $\theta = \theta_1 + \theta_2$ with $\theta_1 = \frac{1}{2}(1+\tau_3)\theta$ and $\theta_2 = \frac{1}{2}(1-\tau_3)\theta$), the action reads

$$S = -T\int d^2\sigma\sqrt{-\det\left(\mathscr{G}_{\mu\nu} + \mathscr{F}_{\mu\nu}\right)} + T\int \Omega_{(2)}(\tau_1) \tag{7}$$

where

$$\mathscr{G}_{\mu\nu} = \Pi_\mu{}^m \Pi_\nu{}^n \eta_{mn}\,, \quad \Pi_\mu^m = \partial_\mu X^m - \bar{\theta}\Gamma^m \partial_\mu \theta\,,$$

$$\mathscr{F} = dA - \Omega_{(2)}(\tau_3)\,,$$

$$\Omega_{(2)}(\tau_i) = -\bar{\theta}\Gamma_m \tau_i\, d\theta\left(dx^m + \tfrac{1}{2}\bar{\theta}\Gamma^m d\theta\right). \tag{8}$$

The above action is known to be invariant up to a total derivative under super-Poincaré transformations $a^m \Delta_m + \frac{1}{2}a^{mn}\Delta_{mn} + \epsilon^\alpha \Delta_\alpha$, where a^m, $a^{mn} = -a^{nm}$ and $\epsilon^\alpha = \epsilon^\alpha_1 + \epsilon^\alpha_2$ are constant infinitesimal parameters associated with Poincaré and supersymmetry transformations, respectively, while Δ_m, Δ_{mn}, Δ_α are the corresponding generators. They act as follows

$$\Delta_n x^m = \delta^m_n\,, \quad \Delta_n \theta^\alpha = \Delta_n A_\mu = 0 \tag{9}$$

$$\Delta_{pq} x^m = \left(\delta^m_p \eta_{qr} - \delta^m_q \eta_{pr}\right) x^r\,, \Delta_{pq}\theta^\alpha = \tfrac{1}{2}\left(\Gamma_{pq}\theta\right)^\alpha,$$

$$\Delta_{pq} A_\mu = 0 \tag{10}$$

$$\Delta_\beta x^m = (\bar{\theta}\Gamma^m)_\beta\,, \quad \Delta_\beta \theta^\alpha = \delta^\alpha_\beta\,,$$

$$\Delta_\alpha A_\mu = \left(\bar{\theta}\tau_3\Gamma_m\right)_\alpha \partial_\mu x^m - \tfrac{1}{6}\Big[\left(\bar{\theta}\tau_3\Gamma_m\right)_\alpha \bar{\theta}\Gamma^m \partial_\mu \theta + \left(\bar{\theta}\Gamma_m\right)_\alpha \bar{\theta}\tau_3 \Gamma^m \partial_\mu \theta)\Big] \tag{11}$$

Up to the irrelevant factor T, (1) yields in this case

$$\varphi = \frac{\bar{\varphi}}{\sqrt{1-\bar{\varphi}^2}}\,, \quad \bar{\varphi} = \frac{\epsilon^{\mu\nu}\mathscr{F}_{\mu\nu}}{2\sqrt{-\mathscr{G}}} \tag{12}$$

where $\mathscr{G} = \det(\mathscr{G}_{\mu\nu})$. It is now straightforward to apply Eqs. (3) and (4) to any $\Delta \in \{\Delta_m, \Delta_{mn}, \Delta_\alpha\}$, using (9)–(11) and the corresponding Noether currents. The latter are given by

$$j^\mu_{\Delta_m} = \hat{\Pi}^\mu_m - \epsilon^{\mu\nu}\bar{\theta}\hat{\Gamma}_m \partial_\nu \theta \tag{13}$$

$$\begin{aligned} j^\mu_{\Delta_{mn}} &= \hat{\Pi}^\mu_p \Big(2\,\delta^p_{[m}\eta_{n]q}\,x^q - \tfrac{1}{2}\bar{\theta}\Gamma^p\Gamma_{mn}\theta \Big) \\ &\quad - \epsilon^{\mu\nu}\bar{\theta}\hat{\Gamma}_p \partial_\nu\theta \Big(2\,\delta^p_{[m}\eta_{n]q}\,x^q - \tfrac{1}{4}\bar{\theta}\Gamma^p\Gamma_{mn}\theta \Big) \\ &\quad + \tfrac{1}{2}\epsilon^{\mu\nu}\bar{\theta}\hat{\Gamma}_p\Gamma_{mn}\theta \Big(\partial_\nu x^p - \tfrac{1}{2}\bar{\theta}\Gamma^p\partial_\nu\theta \Big) \end{aligned} \tag{14}$$

$$\begin{aligned} j^\mu_{\Delta_\alpha} &= (\bar{\theta}\Gamma^m)_\alpha \Big(2\,\hat{\Pi}^\mu_m - \tfrac{4}{3}\epsilon^{\mu\nu}\bar{\theta}\hat{\Gamma}_m\partial_\nu\theta \Big) \\ &\quad - \epsilon^{\mu\nu}\Big(\bar{\theta}\hat{\Gamma}_m\Big)_\alpha \Big(2\,\partial_\nu x^m - \tfrac{2}{3}\bar{\theta}\Gamma^m\partial_\nu\theta \Big) \end{aligned} \tag{15}$$

where

$$\hat{\Pi}^\mu_m = \sqrt{-\mathscr{G}(1+\varphi^2)}\;\mathscr{G}^{\mu\nu}\eta_{mn}\Pi^n_\nu \tag{16}$$

$$\hat{\Gamma}_m = \Gamma_m(\varphi\tau_3 - \tau_1)\,. \tag{17}$$

Notice that each Δ has its corresponding arbitrary function $\lambda(\varphi)$, which can be expanded in an appropriate basis (e.g. in powers of φ) to get the loop version of the corresponding super-Poincaré algebra, cf. [1,3].

4. Purely supersymmetric D-string

By purely supersymmetric D-string, we mean a supersymmetric D-string with no coupling to the RR-potentials and NS-NS two form. Again we consider an action in a flat background,

$$S = -T\int d^2\sigma \sqrt{-\det\big(\mathscr{G}_{\mu\nu} + F_{\mu\nu}\big)} \tag{18}$$

with $\mathscr{G}_{\mu\nu}$ as in (8) and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This example illustrates that Born-Infeld type actions can have more symmetries than those associated with background (super-) isometries, as was already pointed out in [1,3]. Namely, in addition to the super-Poincaré symmetries [1], the action has a dilatational invariance generated by

$$\Delta_d\, x^m = x^m\,, \quad \Delta_d\,\theta = \tfrac{1}{2}\,\theta \tag{19}$$

$$\Delta_d\, A_\mu = 2\big(1-\varphi^{-2}\big)A_\mu \tag{20}$$

where

$$\varphi = \frac{\bar{\varphi}}{\sqrt{1-\bar{\varphi}^2}}\,, \quad \bar{\varphi} = \frac{\epsilon^{\mu\nu}F_{\mu\nu}}{2\sqrt{-\mathscr{G}}}\,. \tag{21}$$

Indeed, the Lagrangian is invariant under Δ_d up to a total derivative,

$$\Delta_d\, L = \partial_\mu\big(4\varphi^{-1}A_\nu\epsilon^{\nu\mu}\big). \tag{22}$$

The corresponding symmetries (3), (4) read as follows

$$\tilde{\Delta}_d\, x^m = \lambda(\varphi)\, x^m\,, \quad \tilde{\Delta}_d\,\theta = \tfrac{1}{2}\lambda(\varphi)\,\theta \tag{23}$$

$$\begin{aligned} \tilde{\Delta}_d A_\mu &= 2\,\lambda(\varphi)\big(1-\varphi^{-2}\big)A_\mu \\ &\quad + \frac{d\lambda(\varphi)}{d\varphi}\Big\{ 2\big(\varphi+\varphi^{-1}\big)A_\mu \\ &\quad - \sqrt{-\mathscr{G}(1+\varphi^2)}\;\epsilon_{\mu\nu}\mathscr{G}^{\nu\varrho}\Pi^n_\varrho\,\eta_{nm} \\ &\quad \times\Big[x^m - \tfrac{1}{2}\bar{\theta}\Gamma^m\theta \Big]\Big\}. \end{aligned} \tag{24}$$

5. The general argument in higher dimensions

The argument given in the two-dimensional case can be easily generalized to higher dimensions. Consider a $(p+1)$-dimensional action $S = \int d^{p+1}\sigma L$ which depends on a p-form gauge field $A_{\mu_1\ldots\mu_p}$ only through its field strength $F_{\mu_1\mu_2\ldots\mu_{p+1}} = (p+1)\partial_{[\mu_1}A_{\mu_2\ldots\mu_{p+1}]}$ and derivatives thereof. Again we denote by $\{Z^M\}$ all other fields in the action. Simi-

[1] In this case, the gauge field does not transform at all under the super-Poincaré transformations due to the non-appearance of the NS-NS two form (in particular, $\Delta_\alpha A_\mu = 0$). Furthermore, in contrast to the kappa symmetric case, the Lagrangian itself is exactly supersymmetric, not just up to a total derivative.

larly to the two-dimensional case, the Euler-Lagrange derivatives of L with respect to $A_{(p)}$ are then

$$\frac{\hat{\partial} L}{\hat{\partial} A_{\mu_1 \dots \mu_p}} = \epsilon^{\mu_1 \dots \mu_p \nu} \partial_\nu \varphi . \tag{25}$$

Note that in this case we have used that $F_{\mu_1 \dots \mu_{p+1}}$ is proportional to $\epsilon_{\mu_1 \dots \mu_{p+1}}$, since we are considering a $(p+1)$-dimensional theory.

In order to show that any rigid symmetry of S is contained in a family of infinitely many rigid symmetries, we proceed along the lines of the two-dimensional case. Thus, let ΔZ^M and $\Delta A_{\mu_1 \dots \mu_p}$ be infinitesimal transformations which generate a symmetry of S. This means that

$$(\Delta Z^M) \frac{\hat{\partial} L}{\hat{\partial} Z^M} + \left(\Delta A_{\mu_1 \dots \mu_p}\right) \frac{\hat{\partial} L}{\hat{\partial} A_{\mu_1 \dots \mu_p}} = \partial_\mu j^\mu_\Delta . \tag{26}$$

If this is so, then the transformations

$$\tilde{\Delta} Z^M = \lambda(\varphi)\, \Delta Z^M \tag{27}$$

$$\tilde{\Delta} A_{\mu_1 \dots \mu_p} = \lambda(\varphi)\, \Delta A_{\mu_1 \dots \mu_p} \tag{28}$$

$$- \frac{1}{p!} \frac{d\lambda(\varphi)}{d\varphi} \epsilon_{\mu_1 \dots \mu_p \nu} j^\nu_\Delta \tag{29}$$

where $\lambda(\varphi)$ is an arbitrary function of the quantity φ that appears in Eq. (25), also generate rigid symmetries of the action (we have used $\epsilon^{01 \dots p} = -\epsilon_{01 \dots p} = 1$). Indeed, making use of Eqs. (25) and (26) it is easily checked that

$$(\tilde{\Delta} Z^M) \frac{\hat{\partial} L}{\hat{\partial} Z^M} + \left(\tilde{\Delta} A_{\mu_1 \dots \mu_p}\right) \frac{\hat{\partial} L}{\hat{\partial} A_{\mu_1 \dots \mu_p}} = \partial_\mu [\lambda(\varphi)\, j^\mu_\Delta] \tag{30}$$

This shows that $\tilde{\Delta}$ generates a symmetry of the action, and also that the associated Noether current is simply $j^\mu_{\tilde{\Delta}} = \lambda(\varphi)\, j^\mu_\Delta$.

6. Examples: D-Branes and M-Branes

The previous argument for the occurrence of infinite families of rigid symmetries for $(p+1)$-dimensional actions depending on p-form gauge potentials $A_{(p)}$ only through their field strengths $G_{(p+1)} = dA_{(p)}$ applies readily to different brane actions. We will consider (super)D-branes and (super)M-branes. Both types of objects can be described by Lagrangian densities in which the tension of the brane is generated dynamically as an integration constant of the field equations for the p-form gauge potential. Their form is [11,12], see also [10],

$$L = \frac{1}{2v} \left[L_K^2 + \left(* G_{(p+1)} \right)^2 \right] \tag{31}$$

where v is an independent worldvolume density and $*$ denotes the worldvolume Hodge dual. For instance, for a Dp-brane in a general $D=10$ supergravity background one has [12]

$$L_K^2 = e^{-2\phi} \det\left(g_{\mu\nu} + \mathscr{F}_{\mu\nu} \right) \tag{32}$$

$$\mathscr{F} = dV - B \tag{33}$$

$$G_{(p+1)} = dA_{(p)} - C e^{\mathscr{F}}, \quad C = \oplus_k C_k \tag{34}$$

where g is the induced metric, V is the Born-Infeld gauge field, B is the pull-back of the NS-NS two-form and C_k are the pull-backs of the R-R gauge potentials. The corresponding expressions for the M2-brane and the M5-brane in a $D=11$ supergravity background can be found in [9] and [12] respectively.

In all these cases (for $p>1$) the 'source' of an infinite number of symmetries of the action is the worldvolume p-form gauge potential $A_{(p)}$. The quantity φ occurring in Eq. (25) in these cases takes the form

$$\varphi \propto \frac{* G_{(p+1)}}{v} . \tag{35}$$

For every rigid symmetry Δ of these actions, the construction explained in the previous section yields an infinite family $\{\tilde{\Delta}\}$ of symmetries in which the original one is included. For instance, this applies to all (super)isometries of the supergravity background, which were shown in [11,12] to yield rigid symmetries of the corresponding brane action. It also applies to space-time scale transformations under which (31) in a flat background is invariant. For the particular case of Dp-branes they take the form [2]:

$$
\begin{aligned}
&x^m \to k x^m \\
&\theta \to k^{1/2}\,\theta \\
&V \to k^2\, V \\
&A_{(p)} \to k^{p+1}\, A_{(p)} \\
&\upsilon \to k^{2(p+1)}\,\upsilon
\end{aligned}
\tag{36}
$$

The corresponding transformations for the M5-brane are obtained from the previous ones by setting $p=5$ and replacing the abelian one form gauge potential V by a self-dual two form $V^+_{(2)}$ that transforms with weight three, i.e. $V^+_{(2)} \to k^3\, V^+_{(2)}$.

The above discussion does not imply that the actions for these branes in their ‘usual’ form, i.e., without the fields υ and $A_{(p)}$, have infinitely many rigid symmetries too. The reason is that $A_{(p)}$ cannot be eliminated algebraically from the action. Rather, one eliminates it by solving its field equation through an integration constant [3]. Hence, φ turns into a constant once $A_{(p)}$ has been eliminated in this manner. Accordingly, after eliminating $A_{(p)}$, Δ and $\tilde{\Delta}$ are not independent symmetries anymore, but simply proportional to one another (so are the corresponding Noether currents). In contrast, in the two-dimensional (D-string) case the ‘source’ of the infinite number of symmetries is the Born-Infeld gauge field itself [4]. Of course, the argument above does not disprove the existence of an infinite set of symmetries for Dp-branes ($p>1$) and M-branes. This is an issue which remains open.

[2] For p-branes, this set of transformations has already been written in [9].

[3] Therefore the ‘usual’ actions do not arise from the Lagrangian (31) simply by substituting a solution to the field equations of $A_{(p)}$. Rather, before doing so, a term proportional to $* dA_{(p)}$ must be added to the Lagrangian (see [8]).

[4] Note that in the case of the $SL(2,Z)$-covariant formulation [6,7] of the IIB superstring, both sources of infinite symmetry are present, since the action contains two $U(1)$ gauge fields, one of which can be considered as auxiliary and the other one as the Born-Infeld field.

7. Note added

Our result about the presence of infinitely many rigid symmetries extends also to the particle case, $p=0$, if the lagrangian depends on some variable X only through its derivative. One possible example is a particle moving in a curved background which possesses some isometries.

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