9.3 Particle Accelerators

1

Two basic types of accelerator

linear

circular

2) Two types of beam

electrons

protons

3) Two types of collisions

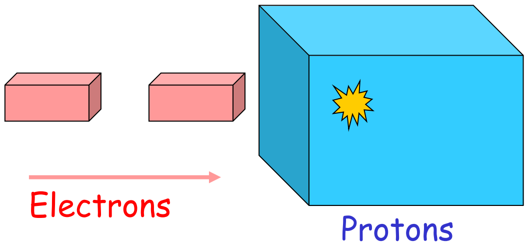
fixed target

colliding beam

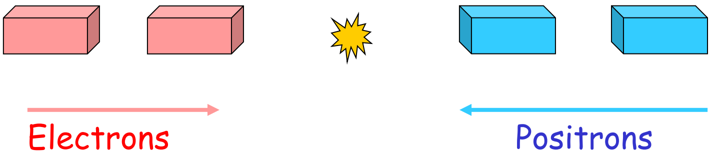
2

The two most important accelerator properties are:
Luminosity (L)
&
Center-of-Mass Energy (E_{CM})

Fixed target: L large, E_{CM} small



Colliding beams: L small, E_{CM} large



3

Luminosity (Flux) & cross-section

Number of events/sec = σ (cm^2) x Luminosity (cm^{-2} s^{-1})

Measured in the detectors

$$\sigma = \frac{\text{Number of events}}{\text{Luminosity}}$$

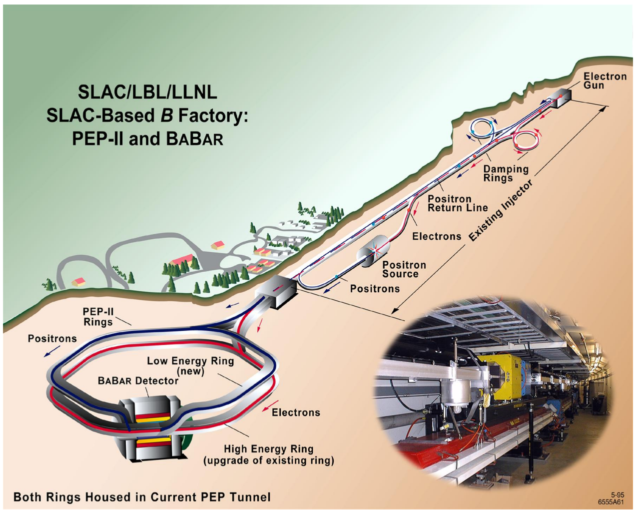
Measured in the accelerator

To be compared with the theory

4

Example: PEPII @ SLAC

Produced about a billion B – $\overline{B}$ meson pairs in e^+e^- collisions at the PEPII storage ring

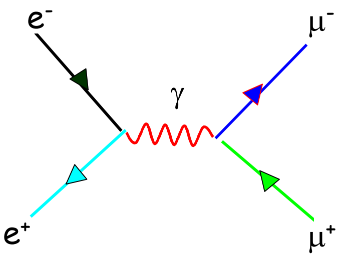


5

How probable are the various physics processes?

<u>$e^+e^- \to \mu^+\mu^-$ at PEP II</u>

$1/GeV^2 = (0.197\ fm)^2 = 388\ \mu b$



$$\sigma = \frac{4\pi\alpha^2}{3s} \quad = \quad 87\ nb\ /\ s\ (GeV^2)$$

$\alpha = 1/137$ $\qquad s = (E_{CM})^2$

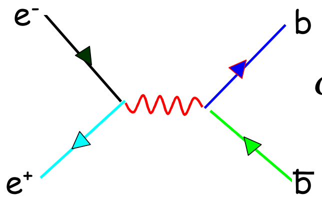
At $s = (10.58\ GeV)^2$ (PEP II): $\sigma(e^+e^- \to \mu^+\mu^-) = 0.78\ nb$

$(1\ nb = 10^{-9} \times 10^{-24}\ cm^2)$

6

e^+e^- -> bb at PEP II

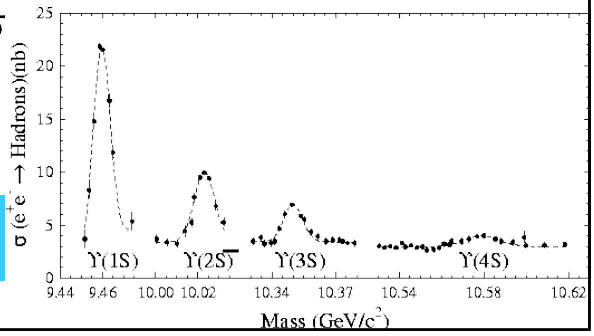
At SLAC we wanted to make as many $b\bar{b}$ pairs as possible.



$$\sigma = 3Q_b^2 \frac{4\pi\alpha^2}{3s} = 3/3^2\, 0.78 \text{ nb} = 0.26 \text{ nb}$$

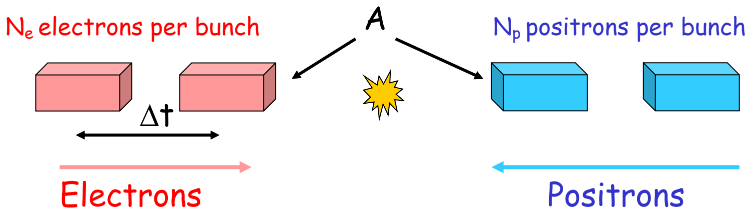
But for b in B meson there is a resonance, the $\Upsilon(4S)$.

At $s = (10.58 \text{ GeV})^2$:
$\sigma(e^+e^- \to b\bar{b}) = 1.05$ nb



7

Luminosity



$$L = \frac{fN_eN_p}{A}$$

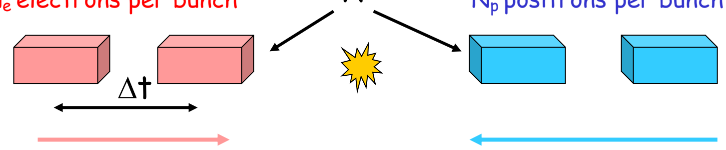
Frequency of collisions: $f = 1/\Delta t$
Area of beam = A

For large L need large N, f and small A (focusing)

8

PEP II Luminosity

N_e electrons per bunch A N_p positrons per bunch



Electrons Positrons

$2\pi R$ = 2200m, 1658 bunches -> 0.0075 bunches/cm
-> f = 2.3 x 10^8 bunches/sec

$$L = \frac{fN_eN_p}{A} = \frac{(2.3\times10^8)(2.7\times10^{10})(5.9\times10^{10})}{150\mu m\times15\mu m}$$

L = 1.6 x 10^{34} cm^{-2} s^{-1}

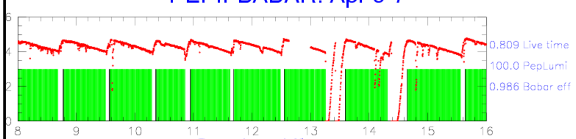
9

Luminosity (Flux)

Number of events/sec = σ (cm^2) x Luminosity (cm^{-2} s^{-1})

$\sigma(e+e- \to b\bar{b})$ = 1.05 nb

PEPII-BABAR: Apr 6-7



Sun day shift

PEP II Luminosity
about 4 x 10^{33} cm^{-2} s^{-1}
= 4 nb^{-1} s^{-1}

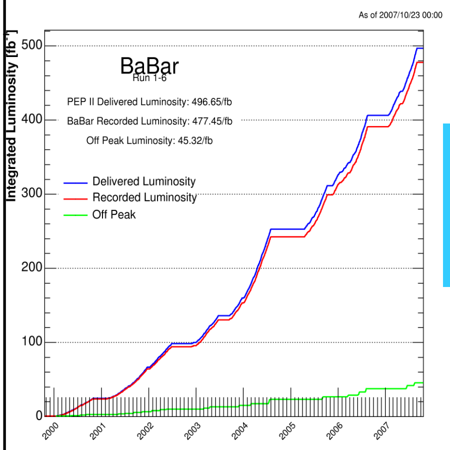
Produced about 5 $b\bar{b}$ pairs per second

10

Integrated Luminosity

$$L_{\text{int}} = \int L dt \quad (\text{cm}^{-2})$$

1 nb = 10^{-33} cm^2 1 nb^{-1} = 10^{33} cm^{-2}
1 pb = 10^{-36} cm^2 1 pb^{-1} = 10^{36} cm^{-2}
1 fb = 10^{-39} cm^2 1 fb^{-1} = 10^{39} cm^{-2}



Total BaBar events = $L_{int} \cdot \sigma$

= 477 x 10^6 nb^{-1} x 1.05 nb
= 500 million bb pairs

11

Example of luminosity measurement: Bhabha Scattering

For e+e- colliders, the Bhabha scattering process is normally used for luminosity measurements (γ = photon):

$$e^+e^- \rightarrow e^+e^-(\gamma)$$

- Large cross-section.
- Almost purely electromagnetic (QED) interaction.
- Can be computed to high accuracy. LEP : < 0.05%.

At small angle from the beam the cross-section for a detector covering the angular range (wrt beam axis) from θ_{min} to θ_{max} is :

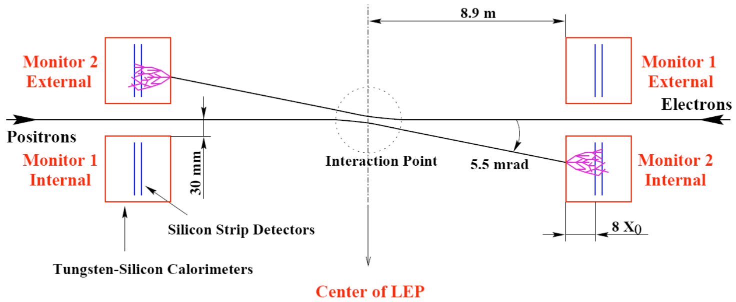
$$\sigma = k\left(\frac{1}{\theta^2_{\min}} - \frac{1}{\theta^2_{\max}}\right)$$

For high rates, aim for the smallest possible θ_{min}

12

Bhabha Scattering : LEP Example

- At LEP each experiments had a luminosity monitor covering the angular region of 30 to 50-100 mrad. They achieved measurements with accuracies of ~0.1%.
- The LEP machine had its own detectors, based on a ‚sandwich' of Tungsten plates and Silicon trip detectors:
  - Installed at smaller angles (2-5 mrad) for faster update rates, but also more background !
  - Poor(er) accuracy, calibration wrt to the monitors of the experiments.
  - The machine monitors were used for luminosity optimization (see later).



13

13

Basic Accelerator Components

a) Source → b) Acceleration → c) Steering → d) Focusing

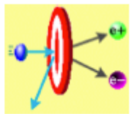
14

a) Particle sources:

- electrons: by heating metallic plates

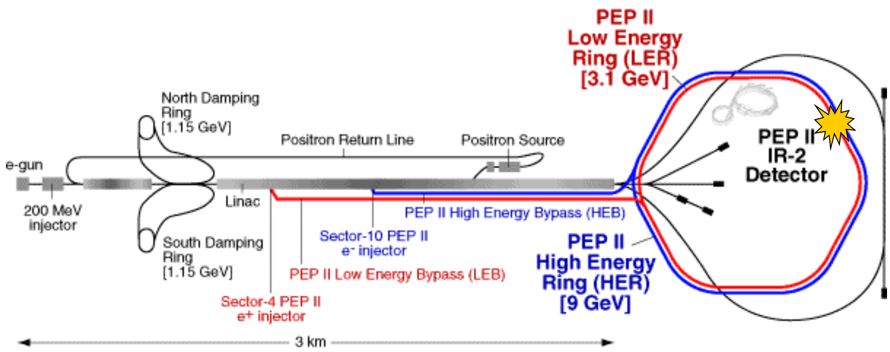
- protons: by ionization of the hydrogen atom

- antiparticles: from particle reactions



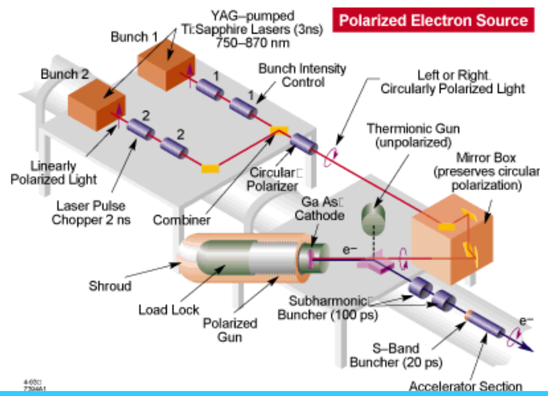
15

The SLAC accelerator complex



16

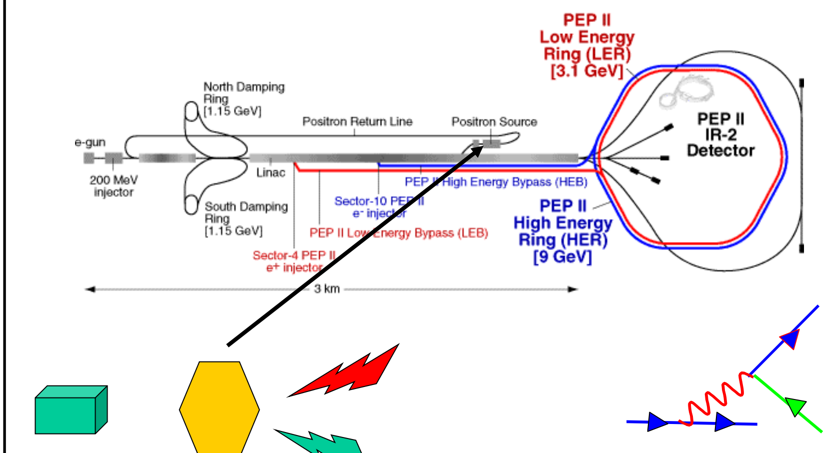
Electron Source



Thermionic (hot high voltage cathode)
or photo-electric emission to create electrons.
With polarized laser light can create polarized electrons.
Pulse the high voltage
→ bunches created for insertion into linear accelerator.

17

Positron Source



electrons

tungsten target

Electrons+ positrons

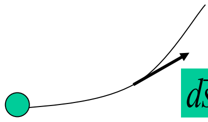
bremsstrahlung in target
followed by pair production

18

b) Particle Acceleration

$$\vec{F} = e\vec{E} + e(\vec{v} \times \vec{B})$$

$$\Delta\vec{p} = \int \vec{F}dt \qquad \Delta E = \int \vec{F} \cdot d\vec{s}$$



$$d\vec{s} = \vec{v}dt$$

$$\Delta E = \int e\vec{E} \cdot \vec{v} + e(\vec{v} \times \vec{B}) \cdot \vec{v}dt \qquad (\vec{v} \times \vec{B}) \cdot \vec{v} = 0$$

$$\Delta E = \int e\vec{E} \cdot \vec{v}dt$$

$\vec{E}$ accelerate

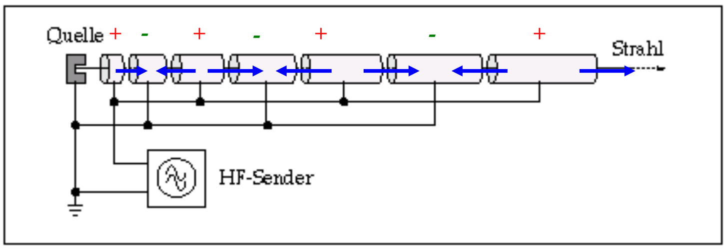
$\vec{B}$ steer/focus

19

The first RF-Accelerator: Drift tube acceleratots

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



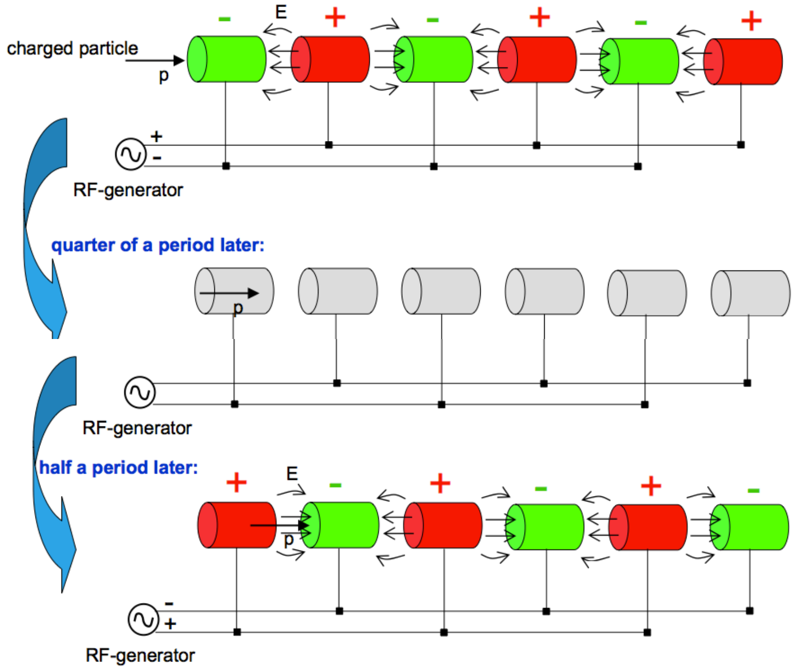
Energy gained after n acceleration gaps

$$E_n = n * q * U_0 * \sin\psi_s$$

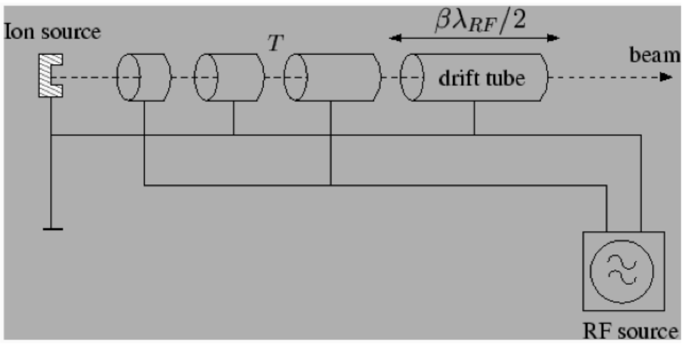
n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

20



21



original Wideroe drift-tube principle

Limitations of drift tube accelerators:

- only low freq. (<10 MHz) can be used

$$L_{tube} = \beta \frac{\lambda}{2} = \beta \frac{c}{2f} \quad \rightarrow \quad \text{30 m for } \beta=1 \text{ and } f=10 \text{ MHz}$$

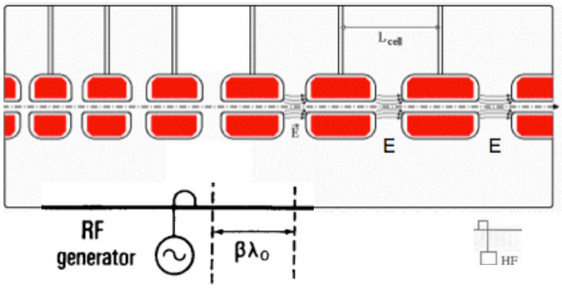
→ drift tubes are impracticable for ultra-relativistic particles (β=1)

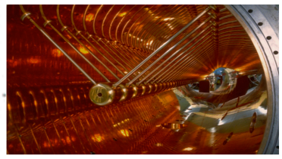
→ only for very low β particles

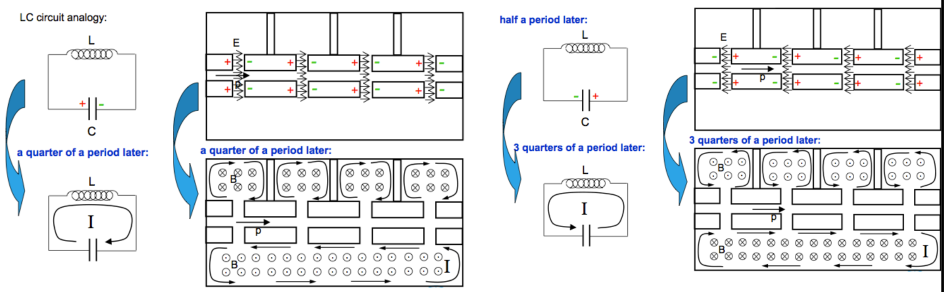
22

Resonant cavities

Alvarez drift-tube structure:



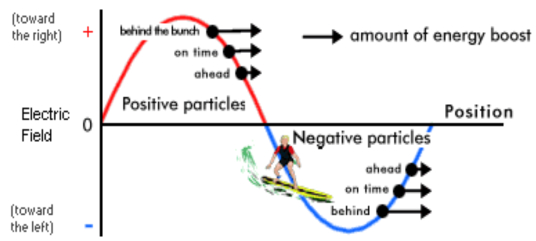




23

RF acceleration

Particle "surfs" an electromagnetic wave



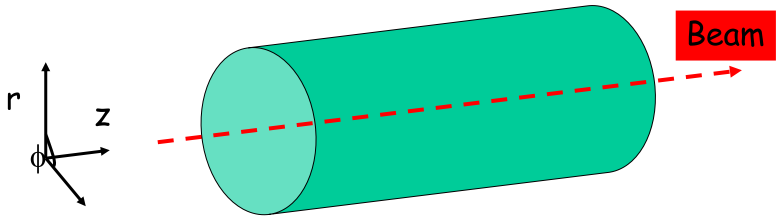
1. Need E field in direction of motion
(i.e. longitudinal E-field - not free space EM wave)

2. Need particles locked in phase with the wave

24

Creating longitudinal E for acceleration

Use waveguides and resonant cavities.
Geometry of waveguide can create the correct field.
(Free space EM waves are transverse; need walls.)



Solve Maxwell's equations with appropriate boundary conditions for a cylindrical cavity.

25

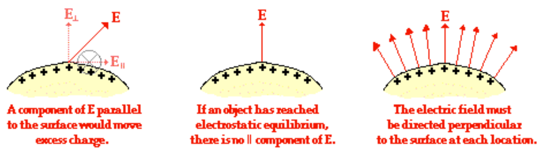
Maxwell's Equations and boundary conditions

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}=\frac{-1}{c}\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B}=\frac{\mu\varepsilon}{c}\frac{\partial\vec{E}}{\partial t}$$

From these equations can derive:

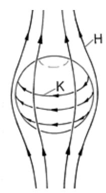
Solve $\nabla^2\vec{E}=\frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$

In cylindrical coordinates



With boundary conditions for surface of conductor

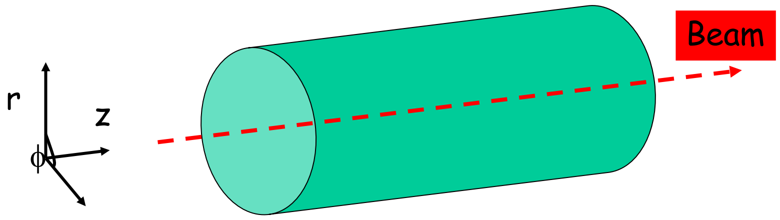
$\vec{E}_{\parallel}=0 \qquad \vec{B}_{\perp}=0$



26

Creating longitudinal E for acceleration

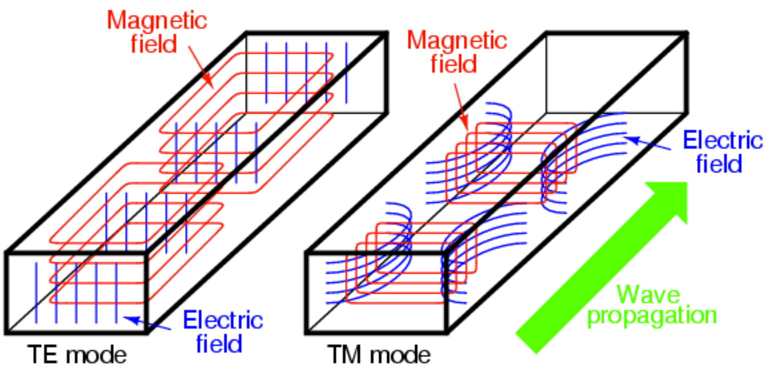
Use separation of variables to solve.
Two sets of solutions:
TE_{klm} Transverse E (longitudinal B) modes
TM_{klm} Transverse B (longitudinal E) modes
klm is the periodicity of the solution in ϕrz



The TM modes are useful for particle acceleration

27

TE and TM modes

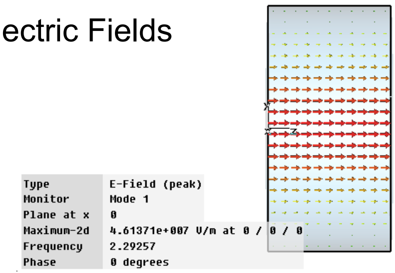


Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

28

TM$_{010}$ Accelerating mode

Electric Fields

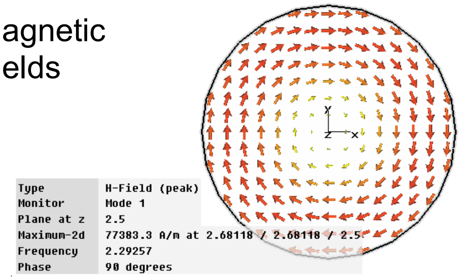


Almost every RF cavity operates using the TM$_{010}$ accelerating mode.

This mode has a longitudinal electric field in the centre of the cavity which accelerates the electrons.

The magnetic field loops around this and caused ohmic heating.

Magnetic Fields



29

TM$_{010}$ Mode

$$E_z = E_0 J_0\left(\frac{2.405r}{R}\right)e^{-i\omega t}$$

$$H_z = 0$$

$$H_r = 0$$

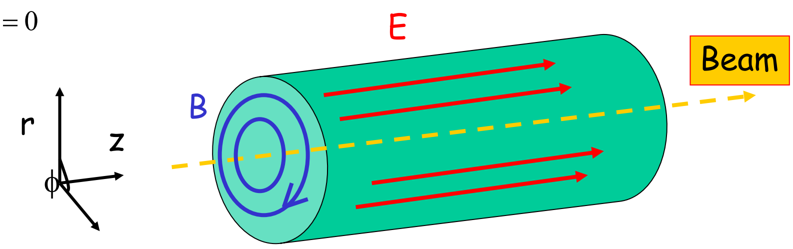
$$H_\varphi = \frac{-i}{Z_0} E_0 J_1\left(\frac{2.405r}{R}\right)e^{-i\omega t}$$

$$E_\varphi = 0$$

$$E_r = 0$$

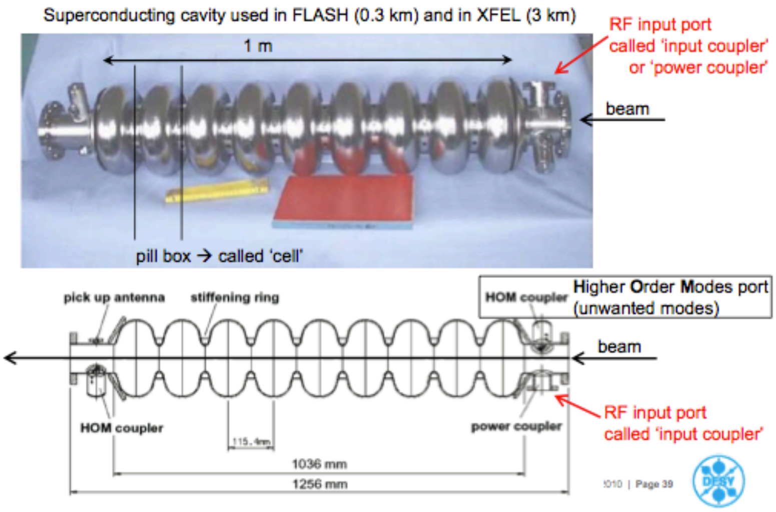
J$_0$(r/R) is a Bessel function, with *J$_0$(R)* = 0.

$$w = 2.405c / R$$



30

In reality...



31

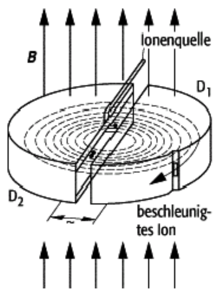
c) Steering

The Cyclotron: (Livingston / Lawrence ~1930)

Idea: Bend a Linac on a Spiral

Application of a constant magnetic field

keep B = const, RF = const



➔ Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p / q$$

increasing radius for
increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$Bq = \frac{P}{R} \Rightarrow \omega_z = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency
is independent of the momentum

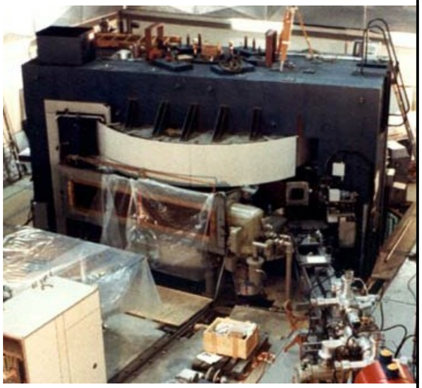
32

Cyclotron

! *ω is constant for a given q & B*

!! *B*R = p/q*
large momentum → huge magnet

!!!! *ω ~ 1/m ≠ const works properly only for non relativistic particles*



PSI Zurich

Application:
Work for medium energy protons
Proton / Ion Acceleration up to ≈ 60 MeV (proton energy)
nuclear physics
radio isotope production, proton / ion therapy

33

The Betatron: Wideroe 1928/ Kerst 1940

...apply the transformer principle to an electron beam: no RF system needed, changing magnetic B field

Idea: a time varying magnetic field induces a voltage that will accelerate the particles

Farady induction law

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\int_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

$$\oint E\,dl = -\int_A \dot{B} ds = -\dot{\Phi}$$

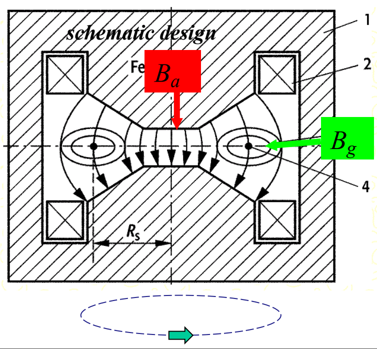
circular orbit

$$\frac{mv^2}{r} = e * v * B$$

$$\rightarrow \quad p = e * B * r$$

magnetic flux through this orbit area

$$\Phi = \int B\,ds = \pi\, r^2 * B_a$$



34

induced electric field

$$\oint E\,dl = E * 2\pi r = -\dot{\Phi} \quad \Rightarrow \quad E = \frac{-\pi r^2 * \dot{B}_a}{2\pi r} = -\frac{1}{2}\dot{B}_a r$$

force acting on the particle: $\dot{p} = -|\vec{E}|e = \frac{1}{2}\dot{B}_a r\, e$

The increasing momentum of the particle has to be accompanied by a rising magnetic guide field:

$\dot{p} = e * \dot{B}_g r$ $\quad$ $B_g = \frac{1}{2} B_a$

robust, compact machines,
Energy ≤ 300...500 MeV,
limit: Synchrotron radiation

35

Synchrotrons / Storage Rings / Colliders:

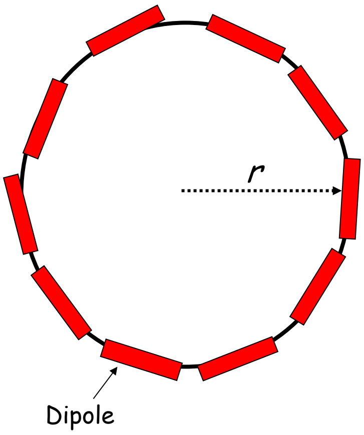
Wideroe 1943, McMillan, Veksler 1944, Courant, Livingston, Snyder 1952

Idea: define a circular orbit of the particles,
keep the beam there during acceleration,
put magnets at this orbit to guide and focus

Advanced Photon Source, Berkley

36

The Synchrotron: Steering and accelerating (I)



$$p = mv\gamma = eBr \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

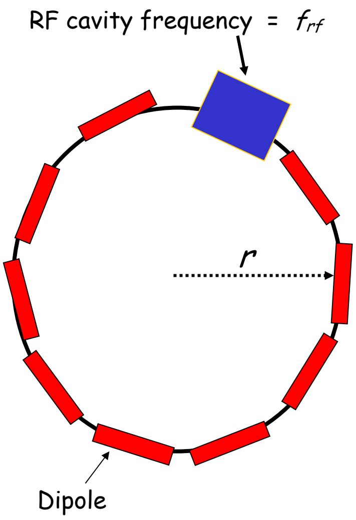
$$f_{rev} = \frac{1}{t_{rev}} = \frac{v}{2\pi r} = \frac{eB}{2\pi\gamma m}$$

assuming the ring is completely filled with dipoles of strength B

If one accelerates a beam in a ring, B must increase in time ("ramp") as p does.

37

The Synchrotron: Steering and accelerating (II)



$$f_{rf} = f_{rev} = \frac{v}{2\pi r} \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

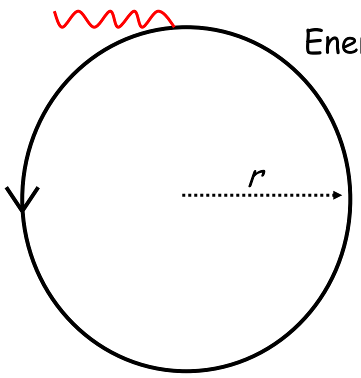
Circular particle accelerator (1 rf cavity) with rf frequency (and B) changing as v increases to keep in phase

$$f_{rev} = \frac{1}{t_{rev}} = \frac{v}{2\pi r} = \frac{eB}{2\pi\gamma m}$$

38

Synchrotron Radiation

As particle circles, it loses energy through EM radiation



Energy loss per turn: $$\Delta E_{loss} = \frac{4\pi\alpha}{3}\frac{E^4}{m^4 r}$$

Limits maximum E
for circular electron accelerator
because m_e is small

39

Proton Synchrotrons

$$\Delta E_{loss} = \frac{4\pi\alpha}{3}\frac{E^4}{m^4 r}$$

1) At LEP (Large Electron-Positron Collider):
- 100 GeV electron beam: Energy loss per turn = 2 GeV

1) At LHC (Large Hadron Collider):
- 7000 GeV proton beam: Energy loss per turn = 2 keV

$$\frac{\Delta E_{lossLHC}}{\Delta E_{lossLEP}} = \left(\frac{E_{LHC}}{E_{LEP}}\right)^4 \left(\frac{m_e}{m_p}\right)^4 = 70^4 \frac{1}{1837^4} \approx 10^{-6}$$

Energy loss per turn much more favorable for protons,
because m_p large (= 1837 m_e)

40

d) Focusing

To decrease beam area at a collision point, we need to focus the beam.

Also a number of effects in the accelerator cause the beam to defocus (space charge repulsion, residual transverse E fields, etc.)

All accelerators need focusing throughout the accelerator to maintain a stable beam.

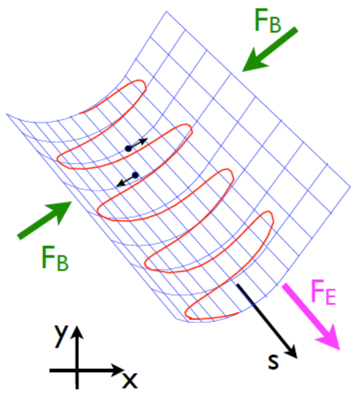
To study the beam behavior we use beam optics.

41

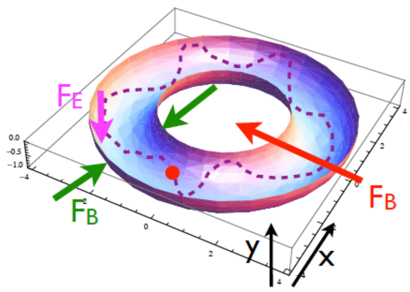
Focusing Properties – Transverse Beam Optics

$$\overline{F(t)} = q\left(\underbrace{\overline{E(t)}}_{F_E} + \underbrace{\overline{v(t)} \otimes \overline{B(t)}}_{F_B}\right)$$

Linear Accelerator

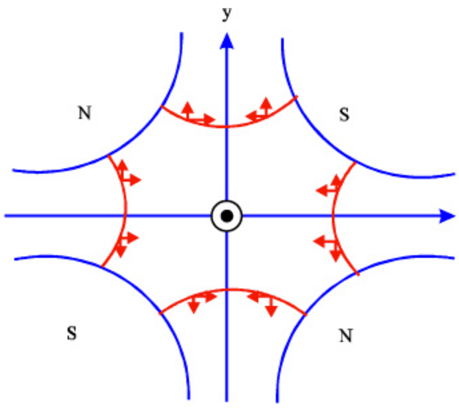


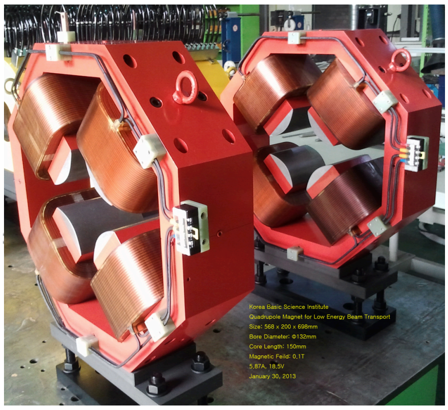
Circular Accelerator



42

Solution: a quadrupole





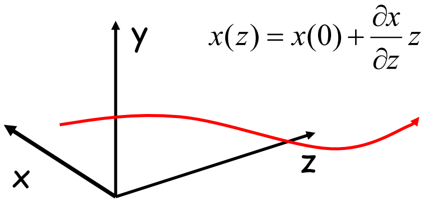
The blue lines mark the magnet poles (polarity indicated as 'North' or 'South'); the red lines mark the magnetic field lines. The red arrows indicate the force applied to a positively-charged particle beam travelling out of the page: this quadrupole magnet focuses in x and defocuses in y

We will see that **two consecutive quadrupoles will focus in both directions: BEAM OPTICS**

43

Beam Optics

Consider a particle moving in the z direction



$$x(z) = x(0) + \frac{\partial x}{\partial z} z \qquad x' = \frac{\partial x}{\partial z}$$

If the x and y motions are decoupled then we can describe the motion by a transfer matrix M which acts in transverse phase space: x and ~p_x.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_z = M \begin{pmatrix} x \\ x' \end{pmatrix}_0 = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

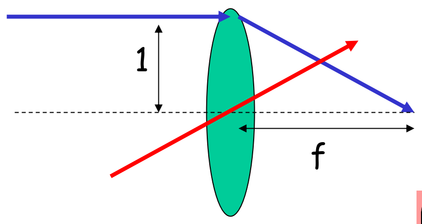
The M shown here describes a drift region of empty space.

44

Transfer matrix of a thin lens

An off-axis parallel ray is focused

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1/f \end{pmatrix}$$



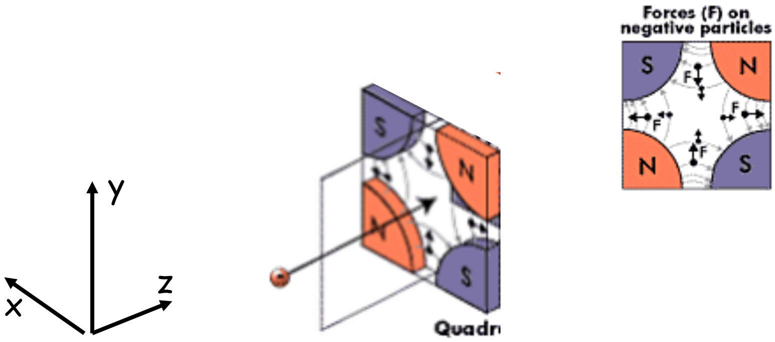
Ray through center is unaffected

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$M_{convergent} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \qquad M_{divergent} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

45

Quadrupole Focusing



$\vec{\nabla} \times \vec{B} = 0$ and $B_z = 0$ so $\dfrac{\partial B_x}{\partial y} = \dfrac{\partial B_y}{\partial x}$

then $B_x = by$ $\quad B_y = bx$

46

Quadrupole focusing

In the x dimension the particle executes simple harmonic motion since

$$m\gamma^2 \frac{d^2x}{dt^2} = e\gamma(\vec{v}\times\vec{B})_x = -e\frac{p}{m}bx \qquad dz = \frac{p}{m\gamma}dt$$

$$\frac{d^2x}{dz^2} = -k^2 x \qquad k = \sqrt{\frac{eb}{p}}$$

The solution for x is then $\quad x = A\cos kz + B\sin kz$

so
$$x(z) = x(0)\cos kz + x'(0)k^{-1}\sin kz$$
$$x'(z) = -x(0)k\sin kz + x'(0)\cos kz$$

If magnet length is l, then

$$M_x = \begin{pmatrix} \cos kl & k^{-1}\sin kl \\ -k\sin kl & \cos kl \end{pmatrix} \approx \begin{pmatrix} 1 & l \\ -k^2 l & 1 \end{pmatrix}$$

47

Quadrupole focusing

Can show that M_x is equivalent to $M_{convergent}$ plus drifts

$$M_x \approx \begin{pmatrix} 1 & l \\ -k^2 l & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} = M_{l/2} M_{conv} M_{l/2}$$

$$f = \frac{1}{k^2 l}$$

Similarly, can show that M_y is equivalent to $M_{divergent}$ plus drifts

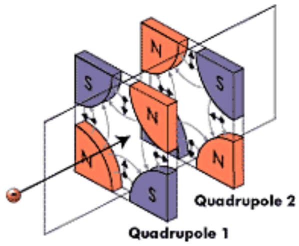
$$M_y \approx \begin{pmatrix} 1 & l \\ k^2 l & 1 \end{pmatrix} \approx M_{l/2} M_{div} M_{l/2}$$

If focus in x then defocus in y !

48

Quadrupole focusing

However, two opposite polarity quads separated by a drift distance cause focusing in both x and y



$$M_x M_z M_y = M_{l/2} M_{conv} M_{z+l} M_{div} M_{l/2}$$

where is easy to prove that

$$M_{conv} M_{z+l} M_{div} \approx M_{l/2} M_{conv,f'} M_{l/2}$$

with $$f' = \frac{f^2}{(z+l)} = \frac{1}{k^4 l^2 (z+l)}$$

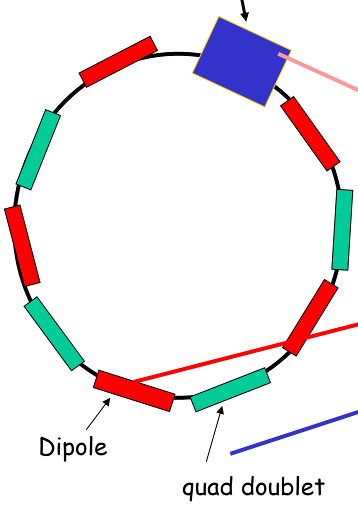
And $M_{div} M_{z+l} M_{conv}$ as before with $f \rightarrow -f$ so it also focuses

49

Elements of an Accelerator

RF cavity frequency $=f_{rf}$

Acceleration - rf cavity
steering - dipoles
focusing - quad doublet

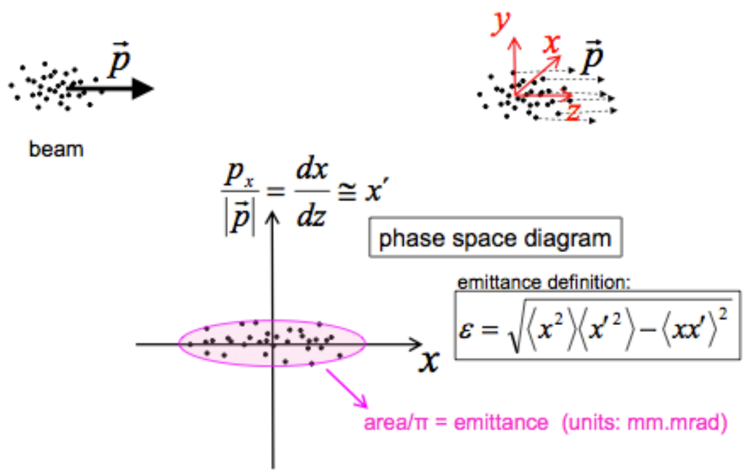


50

Emittance
and
Beta function
of the beam

51

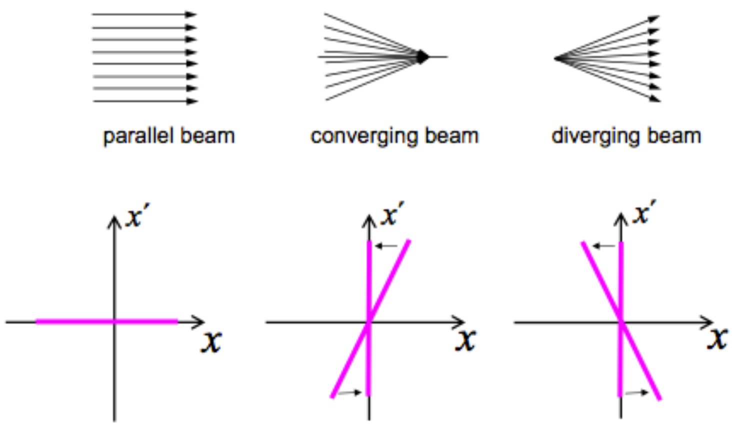
Concept of beam emittance



(Similar for y and z)

52

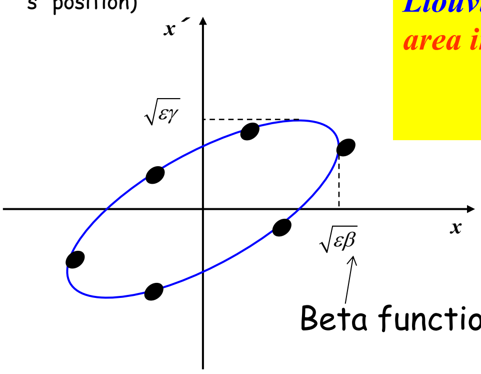
Concept of beam emittance



53

Beam Emittance and Phase Space Ellipse

(at a given longitudinal "s" position)



***Liouville:* in reasonable storage rings area in phase space is constant.**

$A = \pi * \varepsilon = const$

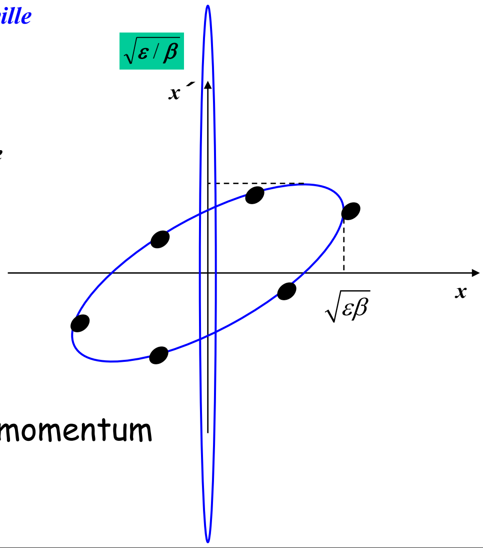
ε beam emittance = the volume of the phase space of the particle ensemble, intrinsic beam parameter, cannot be changed by the foc. properties, only ist shape.
Scientifiquely spoken: area covered in transverse x, x´ phase space is constant !!!

54

Increasing Luminosity: reduce Beta.
(Mini-β Insertions)

A mini-β insertion is always a kind of special symmetric drift space.
→greetings from Liouville

the smaller the beam size
the larger the bam divergence



Price: dispersion on momentum

55

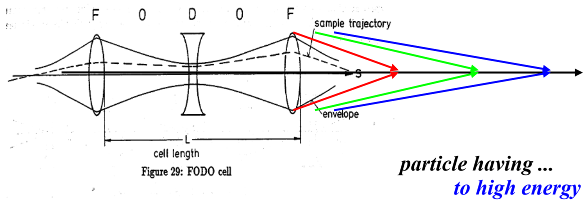
Are there Any other Problems ???

sure there are

56

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

focusing lens $k = \frac{g}{p/e}$

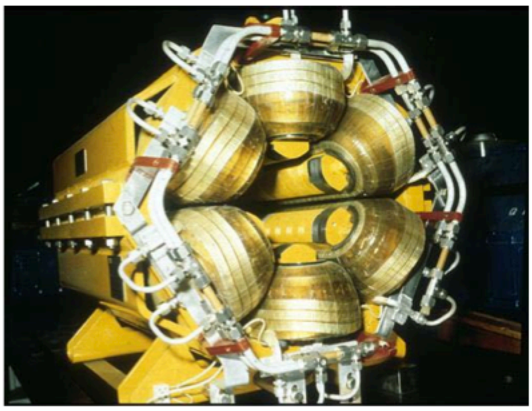


particle having ...
to high energy
to low energy
ideal energy

57

Sextupoles:

SPS



58

What is left?

Vacuum
Power converters
Radio Protection
Transfer lines
Beam Diagnostics
....

59