8. Electroweak unification

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8.1 Weinberg-Salam model of electroweak interactions

- Let us first focus on a single family

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \; , \; Q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- We have ($F = L, Q$; $f = \nu_e, e, u, d$)

$$J^\mu_{cc} = \bar{\nu}_e \gamma^\mu P_L \, e + \bar{u} \gamma^\mu P_L \, d = \sum_F \bar{F} \tau^+ \gamma^\mu P_L F \quad , \quad J^\mu_{em} = \sum_f Q^f \bar{f} \gamma^\mu f$$

$$J^\mu_{nc} = \sum_f \bar{f} \gamma^\mu \frac{c_V^f - c_A^f \gamma^5}{2} f \equiv \sum_f c_L^f \bar{f} \gamma^\mu P_L f + \sum_f c_R^f \bar{f} \gamma^\mu P_R f$$

$$C_V^f = C_A^f - 2Q^f x \quad , \quad x \simeq 0.23 \quad , \quad C_A^\nu = C_A^u = \frac{1}{2} \quad , \quad C_A^e = C_A^d = -\frac{1}{2}$$

$$\Longrightarrow \quad C_L^f = C_A^f - Q^f x \quad , \quad C_R^f = -Q^f x$$

- If $Q^f = 0$ there is no right-handed current $\Longrightarrow$ a right handed neutrino does not interact
- Note that

$$J^\mu_{nc} = \sum_F \bar{F} \frac{\tau^3}{2} \gamma^\mu P_L F - x J^\mu_{em}$$

- The left component of the e.m. current can be written as

$$\sum_f Q^f \bar{f}\gamma^\mu P_L f = \sum_F \left(\bar{F}\frac{\tau^3}{2}\gamma^\mu P_L F + \bar{F}\frac{Y_F}{2}\gamma^\mu P_L F \right)$$

$Y_L = -1/2,\ Y_Q = 1/6$

- Note that the second term is proportional to the identity in isospin space $\Longrightarrow$ commutes with the first term and with the charged current
- The full e.m. current then reads

$$J^\mu_{em} = \sum_F \bar{F}\frac{\tau^3}{2}\gamma^\mu P_L F + \sum_F \bar{F}\frac{Y_F}{2}\gamma^\mu P_L F + \sum_f Q^f \bar{f}\gamma^\mu P_R f$$

- Hence the neutral current reads

$$J^\mu_{nc} = (1-x)\sum_F \bar{F}\frac{\tau^3}{2}\gamma^\mu P_L F - x\left(\sum_F \bar{F}\frac{Y_F}{2}\gamma^\mu P_L F + \sum_f Q^f \bar{f}\gamma^\mu P_R f \right)$$

- We have the following quantum numbers

	$SU_L(2)$	$T^3 = \frac{\tau^3}{2}$	Q^f	$\frac{Y}{2} \equiv Q^f - T^3$
$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
		$-\frac{1}{2}$	-1	$-\frac{1}{2}$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
		$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
u_R	0	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
e_R	0	0	-1	-1

$Y \equiv$ weak hypercharge

- Let us build a local gauge theory (Yang-Mills, 54) invariant under $SU_L(2) \times U_Y(1)$

$$\mathcal{L} = \sum_{F=L_L,Q_L} \bar{F} i \not{D}^F F + \sum_{f=u_R,d_R,e_R} \bar{f} i \not{D}^f f - \frac{1}{2} \mathrm{tr}(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$D_\mu^F = \partial_\mu + igW_\mu + ig' \frac{Y_F}{2} B_\mu \quad , \quad D_\mu^f = \partial_\mu + ig' \frac{Y_f}{2} B_\mu$$

$$W_\mu = T^a W_\mu^a \quad , \quad W_{\mu\nu} = T^a W_{\mu\nu}^a \quad , \quad W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc} W_\mu^b W_\mu^c$$

$$[T^a, T^b] = i\epsilon^{abc} T^c \quad , \quad T^a = \frac{\tau^a}{2} \quad , \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

- g and g' are the electroweak coupling constants
- The transformations under $SU_L(2) \times U_Y(1)$ read as follows

$$F(x) \to e^{i \frac{Y_F}{2} \theta(x)} g_L(x) F(x) \quad , \quad f(x) \to e^{i \frac{Y_f}{2} \theta(x)} f(x)$$

$$W_\mu(x) \to g_L(x) W_\mu(x) g_L^\dagger(x) - \frac{i}{g} g_L(x) \partial_\mu g_L^\dagger(x) \quad , \quad B_\mu(x) \to B_\mu(x) - \partial_\mu \theta(x)$$

$$\Longrightarrow \quad W_{\mu\nu}(x) \to g_L(x) W_{\mu\nu}(x) g_L^\dagger(x) \quad , \quad B_{\mu\nu}(x) \to B_{\mu\nu}(x)$$

- The physical vector fields are choosen as

$$
\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \\
A_\mu &= \cos\theta_W\, B_\mu + \sin\theta_W\, W_\mu^3 \quad , \; B_\mu = \cos\theta_W\, A_\mu - \sin\theta_W\, Z_\mu \\
Z_\mu &= -\sin\theta_W\, B_\mu + \cos\theta_W\, W_\mu^3 \quad , \; W_\mu^3 = \sin\theta_W\, A_\mu + \cos\theta_W\, Z_\mu
\end{aligned}
$$

$s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, $\theta_W \equiv$ Weinberg angle

- Charged currents

$$
\begin{aligned}
\mathcal{L}_{cc} &= -\sum_{F=L_L,Q_L} g\bar{F}\gamma^\mu\left(T^1 W_\mu^1 + T^2 W_\mu^2\right) F = -\sum_{F=L_L,Q_L} \frac{g}{\sqrt{2}}\bar{F}\gamma^\mu\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) F \\
&= -\frac{g}{\sqrt{2}}\left(J_{cc}^\mu W_\mu^+ + J_{cc}^{\dagger\,\mu} W_\mu^-\right)
\end{aligned}
$$

- Neutral and electromagnetic currents

$$
\mathcal{L}_{nc} + \mathcal{L}_{em} = -\sum_{F=L_L,Q_L} \bar{F}\gamma^\mu\left(gT^3 W_\mu^3 + g'\frac{Y_F}{2}B_\mu\right) F - \sum_{f=u_R,d_R,e_R} \bar{f}\gamma^\mu g'\frac{Y_f}{2}B_\mu f
$$

$$
\mathcal{L}_{em} = -\sum_{F=L_L,Q_L} \bar{F}\gamma^\mu\left(gT^3 s_W A_\mu + g'\frac{Y_F}{2}c_W A_\mu\right) F - \sum_{f=u_R,d_R,e_R} \bar{f}\gamma^\mu g'\frac{Y_f}{2}c_W A_\mu f
$$

$$
\mathcal{L}_{nc} = -\sum_{F=L_L,Q_L} \bar{F}\gamma^\mu\left(gT^3 c_W Z_\mu + g'\frac{Y_F}{2}(-s_W)Z_\mu\right) F - \sum_{f=u_R,d_R,e_R} \bar{f}\gamma^\mu g'\frac{Y_f}{2}(-s_W)Z_\mu f
$$

- For the e.m. current, we know that left and right components couple in the same way

$$eQ^f = g'\frac{Y_f}{2}c_W = gT^3 s_W + g'\frac{Y_F}{2}c_W \underset{\frac{Y_F}{2}=Q^f-T^3}{=} T^3(gs_W - g'c_W) + g'c_W Q^f$$

$$\Longrightarrow \quad gs_W = g'c_W = e$$

- Then, the weak neutral current reads,

$$\mathcal{L}_{nc} = -\frac{e}{c_W s_W} Z_\mu \left(\sum_{F=L_L,Q_L} \bar{F}\gamma^\mu \left(T^3 c_W^2 + \frac{Y_F}{2}(-s_W^2) \right) F + \sum_{f=u_R,d_R,e_R} \bar{f}\gamma^\mu \underbrace{\frac{Y_f}{2}}_{Q^f}(-s_W^2) f \right)$$

$$\underset{\frac{Y_F}{2}=Q^f-T^3}{=} -\frac{e}{c_W s_W} Z_\mu \left(\sum_{F=L_L,Q_L} \bar{F}\gamma^\mu T^3 F - s_W^2 \sum_{f=u,d,e} \bar{f}\gamma^\mu Q^f f \right)$$

$$\underset{\text{if } x=s_W^2}{=} -g_Z Z_\mu J^\mu_{nc} \quad , \quad g_Z \equiv \frac{e}{c_W s_W}$$

The Higgs mechanism

- A major problem with this theory is that the vector bosons are massless, thus producing long range interactions, rather than extremelly short range ones, as observed.
- Putting masses by hand is not an option: they break the $SU_L(2) \times U_Y(1)$ local gauge symmetry
- We have already seen a mechanism that produces a spectrum with less symmetry without breaking it in the Lagrangian: spontaneous symmetry breaking
- Recall the $SU_L(2) \times SU_R(2) \to SU(2)$ pattern of the linear sigma-model
- We need a scalar field multiplet that takes a non-zero vacuum expectation value
    - For this to be so at least one field in the multiplet must be neutral
    - Since $Q = Y/2 + T^3 \implies Y/2 = \pm 1/2$. Let us take $Y/2 = 1/2$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad , \quad \phi^0, \phi^+ \in \mathbb{C}$$

$$\phi(x) \underset{SU_L(2)\times U_Y(1)}{\longrightarrow} g_L(x) e^{i\frac{\theta(x)}{2}} \phi(x)$$

- Suppose we have a potential $V = V(\phi^\dagger\phi)$ such that it has the minimum at $\phi^\dagger\phi = v^2/2 \in \mathbb{R}^+$
- Then, we may write,

$$\phi(x) = \frac{U(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \quad , \quad U(x) \in SU(2)$$

  - Note that we keep having four degrees of freedom
  - Like in the linear sigma-model, $h(x) = 0$ corresponds to configurations that keep the potential at its minimum
  - $U(x)$ would contain the Goldstone bosons if the spontaneously broken symmetry was global (independent of the space-time)
  - However, since our gauge symmetry is local, U(x) can be removed by a gauge transformation
  - This is called taking the unitary gauge. In this gauge

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

  - Note that the vacuum configuration $h(x) = 0$ is invariant if $g_L(x) = e^{i\theta(x)T^3}$

$$e^{i\theta(x)T^3} e^{i\frac{\theta(x)}{2}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} e^{i\theta(x)} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

- Hence the unbroken symmetry is a $U_{em}(1)$, and the symmetry breaking pattern $SU_L(2)\times U_Y(1)\to U_{em}(1)$
- The kinetic term of the scalar multiplet reads

$$(D_\mu\phi)^\dagger D^\mu\phi \quad , \quad D_\mu=\partial_\mu+igW_\mu+ig'\frac{1}{2}B_\mu$$

$$\begin{aligned} D_\mu &= \partial_\mu+i\frac{g}{\sqrt{2}}\left(\tau^+W_\mu^++\tau^-W_\mu^-\right)+igT^3W_\mu^3+ig'\frac{1}{2}B_\mu \\ &= \partial_\mu+i\frac{g}{\sqrt{2}}\left(\tau^+W_\mu^++\tau^-W_\mu^-\right)+\begin{pmatrix} ieA_\mu+ig_Z(c_W^2-s_W^2)Z_\mu & 0 \\ 0 & -ig_Z\frac{1}{2}Z_\mu \end{pmatrix} \end{aligned}$$

Let us take ϕ in the vacuum configuration ($h(x)=0$), then

$$\phi(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \quad , \quad D_\mu\phi=\begin{pmatrix}\frac{igv}{2}W_\mu^+\\ -\frac{ig_Zv}{2\sqrt{2}}Z_\mu\end{pmatrix}$$

$$(D_\mu\phi)^\dagger D^\mu\phi=\frac{g^2v^2}{4}W^{\mu\,-}W_\mu^++\frac{g_Z^2v^2}{8}Z^\mu Z_\mu$$

- Hence, the vector bosons get masses

$$m_W=\frac{gv}{2} \quad , \quad m_Z=\frac{g_Zv}{2} \quad , \quad g=g_Zc_W \quad \Longrightarrow \quad m_Z>m_W$$

- The procedure above by which the massless vector bosons get masses is called the Higgs mechanism (64)
- Note that we originally had 3 massless vector bosons and 2 complex scalars $\Longrightarrow$ $3 \times 2 + 2 \times 2 = 10$ degrees of freedom
- And finally we have 3 massive vector bosons and 1 real scalar $\Longrightarrow$ $3 \times 3 + 1 \times 1 = 10$ degrees of freedom
- The degrees of freedom have just been reshuffled: the would-be Goldstone bosons (if the symmetry was global) have been eaten up (by a gauge transformation) by the massless vector bosons, which acquaire a mass and a longitudinal polarization
- The remaining real scalar field, describes a neutral scalar particle called the Higgs boson
- Note that the basic ingredients for the vector bosons to get a mass are the $SU(2)$ matrix $U(x)$ and the vacuum expectation value v
  - Some physicist speculated that the field $h(x)$ is superfluous and that the Higgs particle would not exist
  - Others generalized the model by introducing several scalar multiplets leading to several Higgs bosons
- The Higgs boson was found at LHC (CERN) in 2013, and so far no other fundamental scalar particle has showed up

Matching to the Fermi Theory

- At low energy $E \ll m_W \, , m_Z$, the amplitudes calculated in the Fermi theory must coincide with the ones calculated in the electroweak theory

$$\Longrightarrow \qquad \frac{g^2}{2m_W^2} = 2\sqrt{2}G \quad , \quad \frac{g_Z^2}{m_Z^2} = 4\sqrt{2}\rho G$$

- From these equalities we obtain

$$\rho = \frac{g_Z^2}{m_Z^2}\frac{m_W^2}{g^2} = 1 \quad , \quad v = \frac{1}{\sqrt{\sqrt{2}G}} \simeq 246\,\mathrm{GeV}$$

- Since we also know $s_W^2 \simeq 0.23$ from low energy experiments, we obtain

$$g = \frac{e}{s_W} \simeq 0.63 \quad , \quad g' = \frac{e}{c_W} \simeq 0.34 \quad , \quad g_Z = \frac{e}{s_W c_W} \simeq 0.72$$

$$m_W \simeq 78\,\mathrm{GeV} \quad , \quad m_Z \simeq 89\,\mathrm{GeV}$$

- This was a prediction for m_W and m_Z that were found at CERN in 1983
- The current experimental values are

$$m_W \simeq 80.379(12)\,\mathrm{GeV} \quad , \quad m_Z \simeq 91.1876(21)\,\mathrm{GeV}$$

Fermion masses

- We have managed to get massive vector bosons ($\Longrightarrow$ short range interactions) respecting the symmetry in the Lagrangian
- However, the fermions are still massless
- The scalar multiplet allows to write more terms in the Lagrangian, the so called Yukawa terms:

$$\mathcal{L}_{Yuk} = -\lambda_e \bar{L}_L \phi e_R - \lambda_d \bar{Q}_L \phi d_R - \lambda_u \bar{Q}_L \phi^c u_R + \text{H.c.}$$

$$\phi^c \equiv i\tau^2 \phi^* \quad , \quad \phi^c(x) \underset{SU_L(2)\times U_Y(1)}{\longrightarrow} g_L(x) e^{-i\frac{\theta(x)}{2}} \phi(x)$$

- In the unitary gauge $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$, $\phi^c(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{Yuk} &= -\left(\lambda_e \bar{e}_L e_R + \lambda_d \bar{d}_L d_R + \lambda_u \bar{u}_L u_R\right) \frac{1}{\sqrt{2}} (v + h) + \text{H.c.} \\ &= -\left(m_e \bar{e}_L e_R + m_d \bar{d}_L d_R + m_u \bar{u}_L u_R + \text{H.c.}\right) \\ &\quad -\left(m_e \bar{e}_L e_R + m_d \bar{d}_L d_R + m_u \bar{u}_L u_R + \text{H.c.}\right) \frac{h}{v} \end{aligned}$$

$m_f \equiv \frac{\lambda_f v}{\sqrt{2}}$, $f = e, d, u$

The Higgs interactions

- The field $h(x)$ describes a neutral scalar particle called the Higgs boson
- We have seen that the interaction with a fermion field is proportional to the mass of the fermion
- This is expected to be a general feature since the Higgs field enters the Lagrangian in the combination
$$v + h(x) = v\left(1 + \frac{h(x)}{v}\right)$$
and the v in front together with the suitable coupling constant becomes a mass
- Consider the interaction with the vector bosons
$$D_\mu \phi = \begin{pmatrix} \frac{ig}{2} W^+_\mu (v+h) \\ -\frac{ig_Z}{2\sqrt{2}} Z_\mu (v+h) + \frac{\partial_\mu h}{\sqrt{2}} \end{pmatrix}$$
$$(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2}\partial_\mu h \partial^\mu h + m_W^2 W^{\mu\,-} W^+_\mu \left(1+\frac{h}{v}\right)^2 + \frac{m_Z^2}{2} Z^\mu Z_\mu \left(1+\frac{h}{v}\right)^2$$
- Since $v \simeq 246$ GeV, the Higgs boson interactions with the remaining particles will be small, except for those with the top quark ($m_t \simeq 173$ GeV) and the $W^\pm$ and Z^0

The Higgs self-interaction

- The potential $V(\phi^\dagger\phi)$ that has a minimum at $\phi^\dagger\phi \neq 0$ produces a mass term for the Higgs field and self-interaction terms
- $V(\phi^\dagger\phi) = V(v+h)$, this is the same situation we had in the linear sigma model, so we can take the results from there ($m_S \to m_h$ and $f_\pi = \sqrt{\rho_0} \to v$)

$$\mathcal{L}_h = -\frac{m_h^2}{2}h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4$$

- m_h was a free parameter, hardly constrained by other observables
- Once m_h is known the Higgs self-interactions are fixed
- The current value of the Higgs boson mass is

$$m_h = 125.10(14)\,\text{GeV}$$

5.2 More families

- So far we have dealt with a single family
- In nature, at least three families exist
- Let us assume that we have an arbitrary number of families N_f

$$L_i = \begin{pmatrix} \nu_{l_i} \\ l_i \end{pmatrix} , \; Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \qquad i = 1, \dots, N_f$$

$$L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} , \; Q_1 = \begin{pmatrix} u \\ d \end{pmatrix} , \; L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} , \; Q_2 = \begin{pmatrix} c \\ s \end{pmatrix} , \; L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} , \; Q_3 = \begin{pmatrix} t \\ b \end{pmatrix} , \; \dots$$

- The parts of the Lagrangian that do not involve fermions are not modified by the inclusion of more families:
  - Yang-Mills and Maxwell terms of the vector boson fields
  - Kinetic and potential terms of the scalar fields, including interactions with vector boson fields
- The kinetic terms of the fermions, including the interaction with the vector boson, must be written for all the families:

$$\mathcal{L} = \sum_{i=1}^{N_f} \left(\sum_{F_i = L_{iL}, Q_{iL}} \bar{F}_i i \not{D}^F F_i + \sum_{f_i = u_{iR}, d_{iR}, e_{iR}} \bar{f}_i i \not{D}^f f_i \right)$$

- The Yukawa terms, however, may have a more general form:

$$\mathcal{L}_{Yuk} = \sum_{i,j=1}^{N_f} -\lambda_l^{ij} \bar{L}_{i\,L} \phi l_{j\,R} - \lambda_d^{ij} \bar{Q}_{i\,L} \phi d_{j\,R} - \lambda_u^{ij} \bar{Q}_{i\,L} \phi^c u_{j\,R} + \text{H.c.}$$

  - This leads to off-diagonal mass terms $\Longrightarrow$ the fields with well-defined $SU_L(2) \times U_Y(1)$ quantum numbers do not have well-defined masses
  - In order to find the fields with well defined masses we must diagonalize the mass matrices using unitary field redefinitions in order not to spoil the standard normalization of the kinetic terms
  - Consider for instance

$$\bar{d}_L \Lambda_d d_R = \sum_{i,j=1}^{N_f} \bar{d}_{i\,L} \Lambda_d^{ij} d_{j\,R} \quad , \quad \Lambda_d^{ij} \equiv \lambda_d^{ij} \frac{v}{\sqrt{2}}$$

  - Λ_d ($\det\Lambda_d \neq 0$) can be written as the product of a Hermitian matrix M_d and a unitary matrix U_d

$$\Lambda_d = U_d M_d \quad , \quad M_d = S_d D_d^{\frac{1}{2}} S_d^\dagger \quad , \quad U_d = \Lambda_d M_d^{-1}$$

    where S_d, a unitary matrix, and D_d, a diagonal positive definite matrix, are obtained from the diagonalization of the Hermitian matrix $\Lambda_d^\dagger \Lambda_d$

$$\Lambda_d^\dagger \Lambda_d = S_d D_d S_d^\dagger$$

- $D_d^{\frac{1}{2}}$ is also a diagonal matrix, the positive definite square root of D_d
- We can verify that U_d is indeed unitary

$$\begin{aligned} U_d U_d^\dagger &= \Lambda_d M_d^{-1} M_d^{-1\,\dagger} \Lambda_d^\dagger = \Lambda_d S_d D_d^{-\frac{1}{2}} S_d^\dagger S_d D_d^{-\frac{1}{2}} S_d^\dagger \Lambda_d^\dagger \\ &= \Lambda_d S_d D_d^{-1} S_d^\dagger \Lambda_d^\dagger = \Lambda_d \left(\Lambda_d^\dagger \Lambda_d \right)^{-1} \Lambda_d^\dagger = 1 \end{aligned}$$

- Hence

$$\Lambda_d = U_d S_d D_d^{\frac{1}{2}} S_d^\dagger$$

- The mass term of the d-type quarks becomes diagonal upon

$$d_R \to S_d d_R \quad , \quad d_L \to U_d S_d d_L$$

  We omit the family indices so d_R and d_L are vectors in family space
- We proceed analogously for the mass terms of the u-type quarks and the charged leptons

- Since the transformations are unitary, they do not have any effect on the terms diagonal in isospin space, namely in the neutral current and in the electromagnetic current
- However, since the unitary transformations need not be the same for u-type quarks as for d-type quarks, they do have an effect in the charged currents

$$\bar{u}_L \gamma^\mu d_L \to \bar{u}_L \gamma^\mu S_u^\dagger U_u^\dagger U_d S_d d_L \equiv \bar{u}_L \gamma^\mu V_{CKM} d_L$$

- In the case of the leptons, we have

$$\bar{\nu}_{l\,L}\gamma^{\mu} l_L \to \bar{\nu}_{l\,L}\gamma^{\mu} U_l S_l l_L \underset{\nu_{l\,L}\to U_l S_l \nu_{l\,L}}{=} \bar{\nu}_{l\,L}\gamma^{\mu} l_L$$

  Since there is no mass term for the neutrinos, we make unitary transformation to the neutrino field that diagonalizes the charged current
- If we include a right-handed neutrino, then a mass term is possible and we would be in a case totally analogous to the quarks: the PMNS matrix arises

The GIM mechanism

- Flavor changing neutral currents (FCNC) are very suppressed

$$\frac{\Gamma(K_L^0 \to \mu^+ \mu^-)}{\Gamma(K_L^0 \to \text{all})} \simeq 9\ 10^{-9}$$

- Naively one would expect a suppression of α_W^2

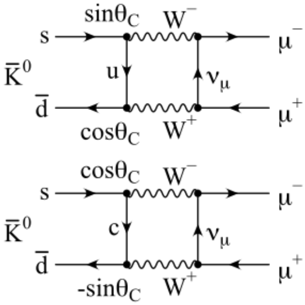
$$\alpha_W \equiv \frac{g^2}{4\pi} \simeq 0.032 \quad , \quad \alpha_W^2 \simeq 10^{-3}$$

- The reason of such a suppression is the unitarity of the CKM matrix, also known as the GIM mechanism (Glashow,Iliopoulos,Maiani, 1970)
- Let us illustrate it with two families

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J^\mu = \bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu + \bar{u} \gamma^\mu P_L d' + \bar{c} \gamma^\mu P_L s'$$

- There are two diagrams contributing



  - If we neglect the quark masses $m_u, m_c \ll m_W$, the contributions of these diagrams cancel each other $\Longrightarrow$
  - There is an extra suppression $\sim m_c^2/m_W^2 \sim 3.2\,10^{-4}$ in the amplitude $\Longrightarrow$
  - There is an extra suppression in the decay width

$$\sim \frac{m_c^4}{m_W^4} \sim 10^{-7}$$

- If we include the third family, there is an extra contribution due to the top quark
  - Since $m_t > m_W$ it could give a potentially large contribution
  - However
$$|V_{st}| \sim \lambda^2 \quad , \quad |V_{ut}| \sim \lambda^3 \quad , \quad \lambda \sim 0.22$$
  - Hence this diagram is suppressed by $\sim \lambda^5 \sim 5\,10^{-4}$, a similar suppression to the one of the two diagrams considered before
- Similar (but not identical) suppressions occur in $D^0 \to \mu^+ \mu^-$, $B^0 \to \mu^+ \mu^-$ and non-leptonic FCNC decays
- Since FCNC are very suppressed in the electroweak theory $\Longrightarrow$ a good place to find effects of physics beyond the standard model (LHCb, Supe-B factories)

8.3 Electroweak phenomenology

$W^{\pm}$ decay

Leptonic decays

$$\underset{A}{W^+} \to \underset{1}{l^+}\, \underset{2}{\nu_l}$$

- The relevant interaction Lagrangian reads

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}} J^{\mu}_{cc} W^{+}_{\mu} \quad , \quad J^{\mu}_{cc} \simeq \bar{\nu}_l \gamma^{\mu} P_L l$$

- Hence, the matrix element reads

$$\mathcal{M} = -\frac{g}{\sqrt{2}} \bar{u}(2)\gamma^{\mu} P_L v(1)\epsilon_{\mu}(A)$$

- Neglecting lepton masses and averaging over the polarizations of the W^+ we obtain

$$0.22\,\text{GeV} \underset{\text{Exp}}{\simeq} \Gamma(W^+ \to l^+ \nu_l) \underset{\text{Th}}{=} \frac{\alpha_W m_W}{12} \simeq 0.21\,\text{GeV}$$

- The result is the same for any (light) lepton family (lepton universality)

Hadronic W decays

- The duality hypothesis tells us $\Gamma(W^+ \to \text{hadrons}) = \Gamma(W^+ \to \text{quarks})$

$$\begin{aligned}\Gamma(W^+ \to \text{quarks}) &= \Gamma(W^+ \to u\,\bar{d}) + \Gamma(W^+ \to u\,\bar{s}) + \Gamma(W^+ \to u\,\bar{b}) \\ &+ \Gamma(W^+ \to c\,\bar{d}) + \Gamma(W^+ \to c\,\bar{s}) + \Gamma(W^+ \to c\,\bar{b})\end{aligned}$$

  - $\Gamma(W^+ \to u\,\bar{s}) \sim |V_{su}|^2 \sim \lambda^2$
  - $\Gamma(W^+ \to u\,\bar{b}) \sim |V_{bu}|^2 \sim \lambda^6$
  - $\Gamma(W^+ \to c\,\bar{d}) \sim |V_{dc}|^2 \sim \lambda^2$
  - $\Gamma(W^+ \to c\,\bar{b}) \sim |V_{bc}|^2 \sim \lambda^4$
- QCD corrections are $\sim \alpha_{\rm s}(m_Z) \sim 0.11 > \lambda^2 \sim 0.048$
- Hence at leading order, neglecting quark masses, we have

$$\begin{aligned}1.4\,\text{GeV} &\underset{\text{Exp}}{\simeq} \Gamma(W^+ \to \text{hadrons}) \underset{\text{Th}}{\simeq} \Gamma(W^+ \to u\,\bar{d}) + \Gamma(W^+ \to c\,\bar{s}) \\ &\simeq 2 \times N_c \times \Gamma(W^+ \to l^+\,\nu_l) = \frac{\alpha_W m_W N_c}{6} \simeq 1.3\,\text{GeV}\end{aligned}$$

$N_c = 3$ is the number of colors

Z^0 decays

- The relevant interaction Lagrangian reads

$$\mathcal{L}_{int} = -g_Z J^\mu_{nc} Z_\mu \quad , \quad J^\mu_{nc} \simeq C^f_L \bar{f}\gamma^\mu P_L f + C^f_R \bar{f}\gamma^\mu P_R f$$

$$g_Z = g/c_W \quad , \quad C^f_L = T^3 - Q^f s^2_W \quad , \quad C^f_R = -Q^f s^2_W$$

- Since $m_f \ll m_Z$ for all fermions (except for the top quark $m_t > m_Z$), we can neglect the masses $\Longrightarrow$
- The decay to left-handed and right-handed fermions decouples, and can be read of the decay of W^+ to leptons

$$\Gamma(Z^0 \to l_L \bar{l}_R) = \frac{\alpha_W m_Z {C^l_L}^2}{6c^2_W} \quad , \quad \Gamma(Z^0 \to l_R \bar{l}_L) = \frac{\alpha_W m_Z {C^l_R}^2}{6c^2_W}$$

$$\Gamma(Z^0 \to q_L \bar{q}_R) = \frac{\alpha_W m_Z {C^q_L}^2 N_c}{6c^2_W} \quad , \quad \Gamma(Z^0 \to q_R \bar{q}_L) = \frac{\alpha_W m_Z {C^q_R}^2 N_c}{6c^2_W}$$

- Then

$$\Gamma(Z^0 \to \text{all}) = \frac{\alpha_W m_Z}{6c_W^2}\left[\underbrace{N_\nu \left(\frac{1}{2}\right)^2}_{\text{neutrinos}} + \underbrace{3\left(\left(-\frac{1}{2}+s_w^2\right)^2 + s_w^4\right)}_{\text{charged leptons}}\right.$$

$$\left. + \underbrace{2N_c\left(\left(\frac{1}{2}-\frac{2s_w^2}{3}\right)^2 + \left(-\frac{2s_w^2}{3}\right)^2\right)}_{u\text{-type quarks}} + \underbrace{3N_c\left(\left(-\frac{1}{2}+\frac{s_w^2}{3}\right)^2 + \left(\frac{s_w^2}{3}\right)^2\right)}_{d\text{-type quarks}}\right]$$

N_ν is the number of neutrinos with mass $m_\nu < m_Z/2 \implies$

- An accurate measurement of $\Gamma(Z^0 \to \text{all})$ provides the number of light neutrinos $\implies$ the number of families in the electroweak (EW) theory
- Taking $m_Z \simeq 91.2$ GeV, $\alpha_W \simeq 0.0336$ and $s_W^2 \simeq 0.23$ we get (in GeV)

N_ν	$\Gamma(Z^0 \to \text{all})$ EW	$\Gamma(Z^0 \to \text{all})$ EW + QCD	$\Gamma(Z^0 \to \text{all})$ Exp
2	2.26	2.38	
3	2.42	2.55	2.4952(23)
4	2.59	2.71	LEP (00)