9.1 Interaction of particles and radiation with matter

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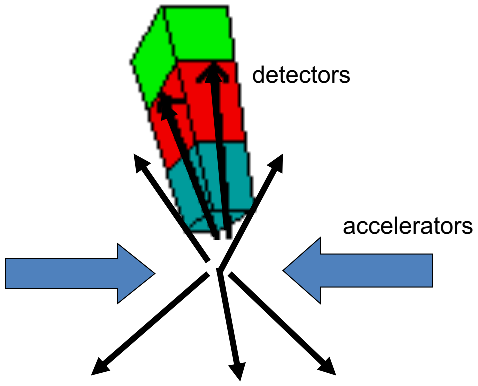
Particle Physics Event



Ideally, want to measure 4-vector and identify all particles produced

Many different particles are produced but all decay rapidly to

electrons, photons, protons, neutrons, pions, kaons, muons, and neutrinos

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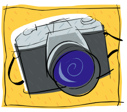
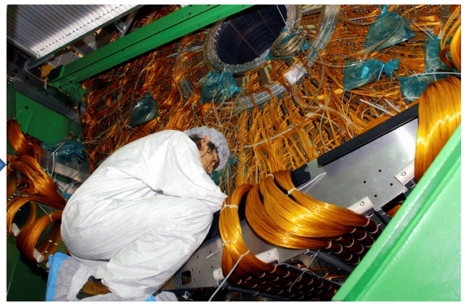
HEP detectors





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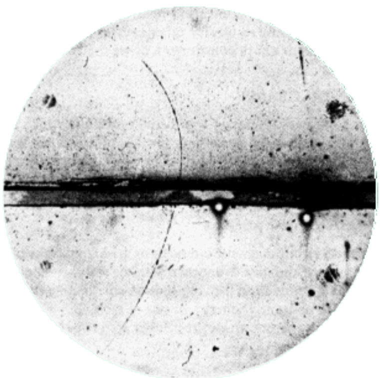
Cloud Chamber

A cloud chamber is a sealed environment containing a supersaturated vapor of water or alcohol.

When a charged particle (for example, an alpha or beta particle) interacts with the mixture, it ionizes it. The resulting ions act as condensation nuclei, around which a mist will form



Positron discovery,
Carl Andersen 1933

Nobel Prize 1936

- Magnetic field allows to measure p
- Drop density is proportional to particle's energy lost -> allows to measure v

Magnetic field 15000 Gauss, chamber diameter 15cm.

The ionization of the particle, and its behavior in passing through the foil are the same as those of an electron.

A 63 MeV positron passes through a 6mm lead plate, leaving the plate with energy 23MeV.

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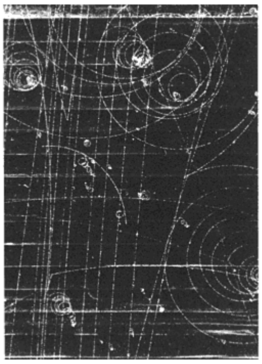
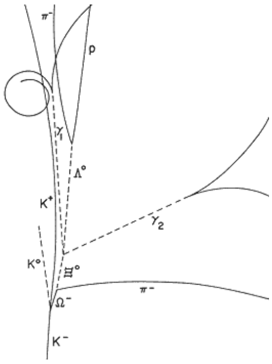
Bubble Chamber

A bubble chamber is a vessel filled with a superheated transparent liquid (most often liquid hydrogen) used to detect electrically charged particles moving through it.

It was invented in 1952 by Donald A. Glaser, for which he was awarded the 1960 Nobel Prize in Physics





The 80-inch Bubble Chamber

BNL, First Pictures 1963, 0.03s cycle

Discovery of the Ω^- in 1964

An incoming K- meson interacts with a proton in the liquid hydrogen of the bubble chamber and produces an omega-minus, a K° and a K+ meson which all decay into other particles. Neutral particles which produce no tracks in the chamber are shown by dashed lines. The presence and properties of the neutral particles are established by analysis of the tracks of their charged decay products and application of the laws of conservation of mass and energy.

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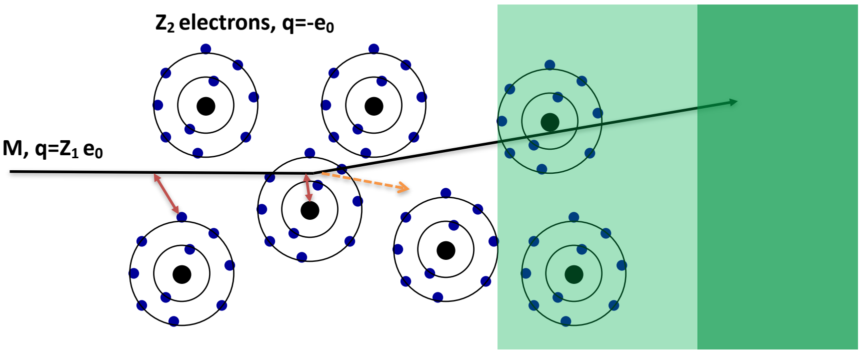
Electromagnetic Interaction of Particles with Matter (*)



1) Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

2) Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

3) In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation.

(*) strong interaction with nucleus later

From W. Riegler, Particle Detectors

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1) Ionization and Excitation

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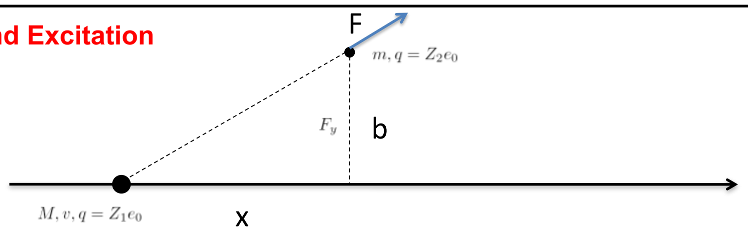
Ionization and Excitation



While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer

$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 (b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}} \qquad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter

The transferred energy is then $\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2}$

$$\Delta E(electrons) = Z_2 \frac{1}{m_e} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \Delta E(nucleus) = \frac{Z_2^2}{2 Z_2 m_p} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2}$$

$$\boxed{\frac{\Delta E(electrons)}{\Delta E(nucleus)} = \frac{2 m_p}{m_e} \approx 4000}$$

→ The incoming particle transfer energy only (mostly) to the atomic electrons !

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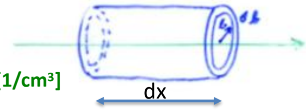
Ionization and Excitation

Target material: mass A, Z_2, density ρ [g/cm³], Avogadro number N_A



A grams → N_A Atoms: Number of atoms/cm³ $n_a = N_A \rho / A$ [1/cm³]

Number of electrons/cm³ $n_e = N_A \rho Z_2 / A$ [1/cm³]

$$\Delta E(electron) = \frac{2.\ \ Z_1^2 m_e c^2}{\beta^2 b^2} \frac{e_0^4}{(4\pi\varepsilon_0 m_e c^2)^2} = \frac{2 \ \ Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$

$$dE = -\int_{b_{min}}^{b_{max}} n_e \Delta E dx 2b\pi db = -\frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2} \frac{N_A \rho}{A} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

With $\Delta E(b)$ → $db/b = -1/2\ dE/E$ → $E_{max} = \Delta E(b_{min})$ $E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \int_{E_{min}}^{E_{max}} \frac{dE}{E} \quad = \quad -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

$E_{min} \approx I$ (Ionization Energy)

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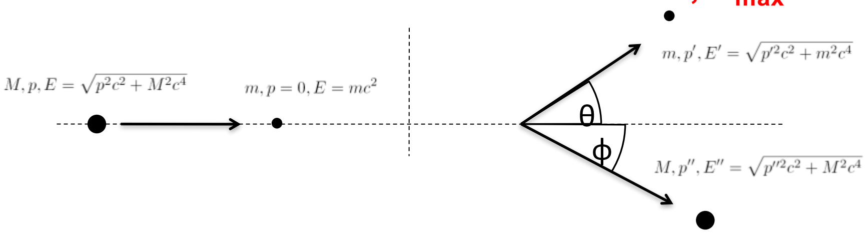
Relativistic Collision Kinematics, E_{max}



1) $\sqrt{p^2c^2 + M^2c^4} + mc^2 = \sqrt{p'^2c^2 + m^2c^4} + \sqrt{p''^2c^2 + M^2c^4}$

2) $$p = p' \cos\theta + p'' \cos\phi \qquad p''^2 = p'^2 + p^2 - 2pp'\cos\theta$$
$$0 = p' \sin\theta + p'' \sin\phi$$

1+2) $$E^{k'} = \sqrt{p'^2c^2 + m^2c^4} - mc^2 = \frac{2mc^2\, p^2c^2 \cos^2\theta}{\left[mc^2 + \sqrt{p^2c^2 + M^2c^4}\right]^2 - p^2c^2\cos^2\theta}$$

$$E^{k'}_{max} = \frac{2mc^2p^2c^2}{(m^2+M^2)c^4 + 2m\sqrt{p^2c^2 + M^2c^4}} = 2mc^2\beta^2\gamma^2 F \qquad F = \left(1 + \frac{2m}{M}\sqrt{1+\beta^2\gamma^2} + \frac{m^2}{M^2}\right)^{-1}$$

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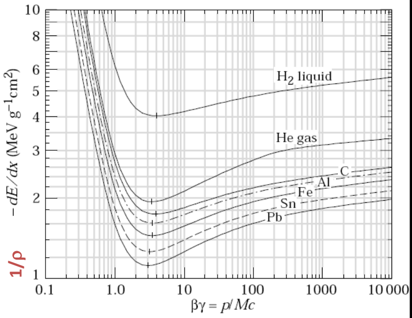
Classical Scattering on Free Electrons

$$\frac{1}{\rho}\frac{dE}{dx} = -2\pi r_e^2\, m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \ln\frac{2m_e c^2\beta^2\gamma^2 F}{I}$$



This formula is up to a factor 2 and the density effect identical to the precise QM derivation →

Bethe Bloch Formula

$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2\, m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \left[\ln\frac{2m_e c^2\beta^2\gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

$\delta(\beta\gamma) = \ln h\omega_p/I + \ln\beta\gamma - \frac{1}{2}$

Electron Spin

Density effect. Medium is polarized Which reduces the log. rise.

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Bethe Bloch Formula

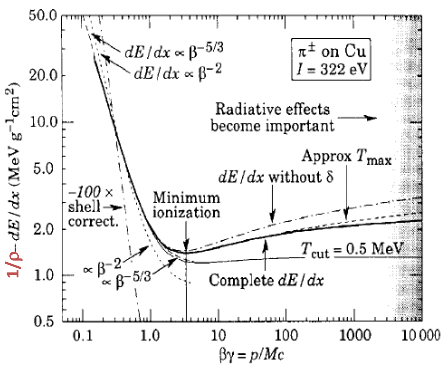
$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2\, m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \left[\ln\frac{2m_e c^2\beta^2\gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

For $Z>1$, $I \approx 16Z^{0.9}$ eV

Characteristics of the energy loss as a function of the particle velocity ($\beta\gamma$)

The specific Energy Loss $1/\rho\ dE/dx$

- first decreases as $1/\beta^2$
- increases with $\ln\gamma$ for $\beta=1$
- is ≈ independent of M ($M \gg m_e$)
- is proportional to Z_1^2 of the incoming particle.
- is ≈ independent of the material ($Z/A \approx$ const)
- shows a plateau at large $\beta\gamma$ (>>100)
- $dE/dx \approx 1\text{-}2 \times \rho$ [g/cm^3] MeV/cm



- is ≈ independent of M ($M \gg m_e$)
- first decreases as $1/\beta^2$
- $dE/dx \approx 1\text{-}2 \times \rho$ [g/cm^3] MeV/cm

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Bethe Bloch Formula

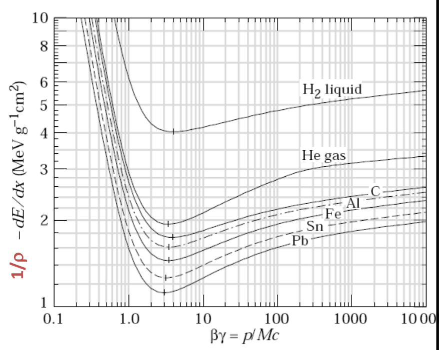
Bethe Bloch Formula, a few Numbers:

For $Z \approx 0.5$ A and $\beta\gamma \approx 3$

$1/\rho \; dE/dx \approx 1.4$ MeV cm^2/g

The energy lost traversing dx is:

$dE \approx (1.4 \text{ MeV cm}^2/\text{g}) \, \rho \, dx$



Example :
Iron: Thickness = 100 cm; ρ = 7.87 g/cm³
dE ≈ 1.4 * 100* 7.87 = 1102 MeV

This number must be multiplied with ρ [g/cm³] of the Material → dE/dx [MeV/cm]

→ A 1 GeV Muon can traverse 1m of Iron

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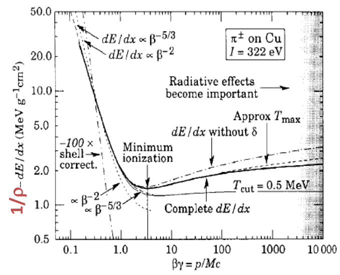
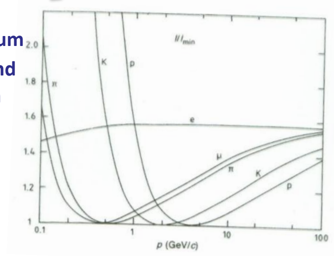
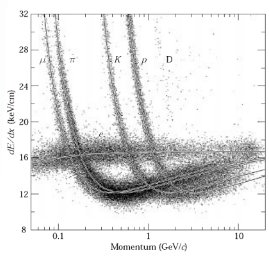
Energy Loss as a Function of the Momentum

Energy loss depends on the particle velocity (βγ) and is ≈ independent of the particle's mass M.



The energy loss as a function of Particle Momentum (Mγ βc) is however depending on the particle's mass





By measuring the particle momentum (deflection in the magnetic field) and measurement of the energy loss on can measure the particle mass

→ Particle Identification !

$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2 \, m_e c^2 \, Z_1^2 \frac{p^2 + M^2c^2}{p^2} N_A \frac{Z}{A}\left[\ln\frac{2m_e c^2 F}{I}\frac{p^2}{M^2c^2} - \frac{p^2}{p^2+M^2c^2}\right]$$

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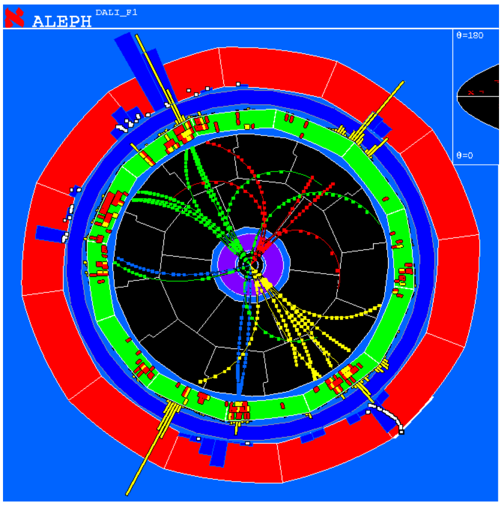
Energy Loss as a Function of the Momentum



Measure momentum by curvature of the particle track.

Find dE/dx by measuring the deposited charge along the track.

→Particle ID

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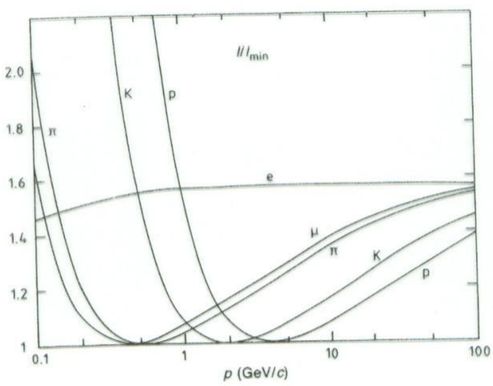
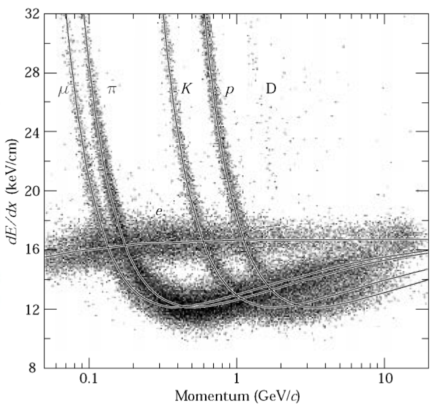
Particle Identification

'average' energy loss



Measured energy loss



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Range of Particles in Matter

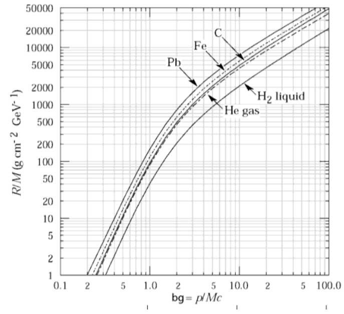
Particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{-1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0\gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

≈Independent of the material

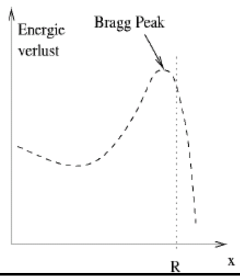


Bragg Peak:

For βγ>3 the energy loss is ≈ constant (Fermi Plateau)

If the energy of the particle falls below βγ=3 the energy loss rises as $1/\beta^2$

Towards the end of the track the energy loss is largest → **Cancer Therapy**.



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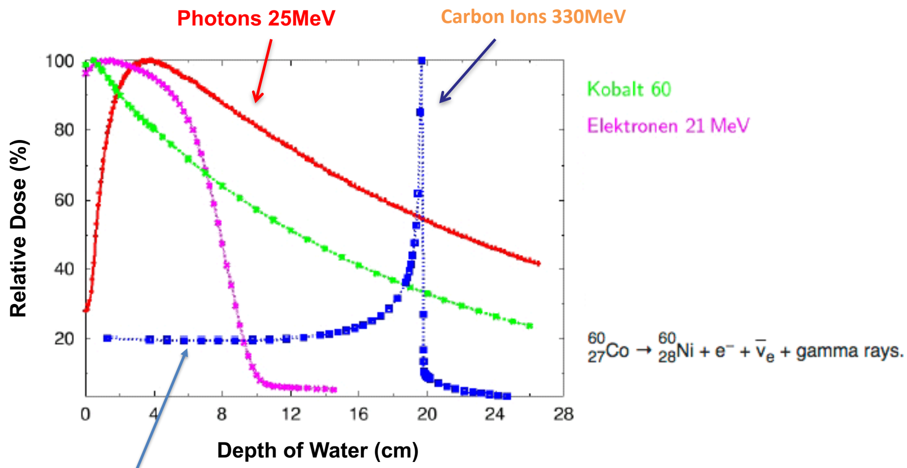
Range of Particles in Matter

Average Range:
Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



dE/dx ≈ 1.4 (MeV cm^2/g) ρ

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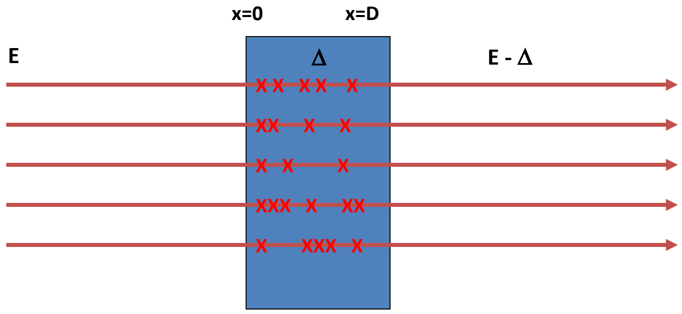
Fluctuations of the Energy Loss

Up to now we have calculated the average energy loss. The energy loss is however a statistical process and will therefore fluctuate from event to event.



P(Δ) = ? Probability that a particle loses an energy Δ when traversing a material of thickness D

The number of interactions follows a Poisson distribution, the probability of an interaction occurring between distance x and x+dx is exponentially distributed and the energy lost follows a Landau distribution

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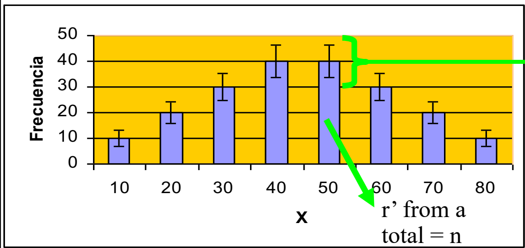
Binomial

- Experiment: n independent Bernoulli experiments
- Random variable: count the times "1" is obtained on the n Bernoulli experiments
  - P(X=r;n,p): probability to observe r times "1" while making the n Bernoulli experiments.

$$\Omega = \{0,1,\ldots,n\}, \quad P(r;n,p) = \binom{n}{r} p^r (1-p)^{n-r}$$

$$E(r) = \sum_{r=0}^{n} rP(r;n,p) = np, \quad V(r) = npq$$

Example: errors in a histogram of frequencies



$$error = \sqrt{r'}$$

$$E(r) = np \Rightarrow p' = r'/n$$

$$V(r) = np(1-p) \approx r'(1-r'/n)$$

$$\sigma(r) \approx \sqrt{r'(1-r'/n)} \xrightarrow[p\to 0]{} \sqrt{r'}$$

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Poisson and Gaussian distributions

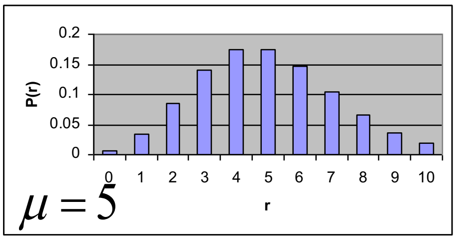
- **Poisson:**

$$P(r;n,p) \Rightarrow P(r;\mu) = \frac{1}{r!}\mu^r e^{-\mu}$$

$$p \to 0, n \to \infty, but ... np \to \mu$$

$$E(r) = \sum_{r=0}^{\infty} rP(r;\mu) = \mu$$

$$V(r) = \mu$$



- **Gaussian**

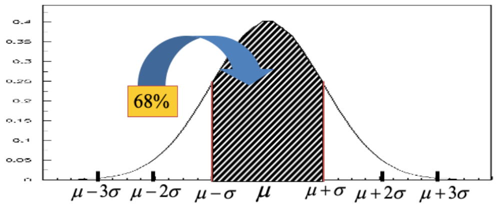
$$P(r;\mu) = \frac{1}{r!}\mu^r e^{-\mu} \xrightarrow{\mu > 5} N(\mu,\mu)$$

$$N(\mu,\sigma^2) \equiv f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \quad -\infty \le x \le \infty$$

$$E(x) = \mu \quad V(x) = \sigma^2$$

$$P(\mu - \sigma \le x \le \mu + \sigma) = 0.6827$$

$$P(\mu - 2\sigma \le x \le \mu + 2\sigma) = 0.9545$$



frequent notation in the physics results: $\mu \pm \sigma$

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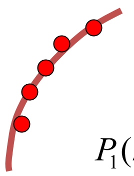
Example of Poisson distribution

- number of bubbles throughout a track in a bubble chamber.

Let us assume: :



- The average density of bubbles "g" is constant
- There is as maximum one bubble in the interval $[l, l+\Delta l]$
- The probability to find a bubble in the interval is proportional to Δl
- The show up of bubbles is independent

$$P_1(\Delta l) = g\Delta l, \quad P_0(\Delta l) = 1 - g\Delta l, \quad P_0(l+\Delta l) = P_0(l)P_0(\Delta l)$$

$$P_0(l+\Delta l) = P_0(l)(1-g\Delta l) \Rightarrow P_0(l) = e^{-gl}$$

- Probability of finding r bubbles in $l + \Delta l$

$$P_r(l+\Delta l) = P_r(l)P_0(\Delta l) + P_{r-1}(l)P_1(\Delta l) = P_r(l)(1-g\Delta l) + P_{r-1}(l)g\Delta l$$

$$\frac{dP_r(l)}{dl} = -gP_r(l) + gP_{r-1}(l) \Rightarrow P_r(l) = \frac{1}{r!}(gl)^r e^{-gl}$$

Is a Poisson with $\mu = gl$ (expected value of bubbles in l)

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Exponencial distribution

- probability of observing the first bubble at a distance l of a certain origin:

$$W(l)\Delta l = P_0(l)P_1(\Delta l) = e^{-gl} g\Delta l \Rightarrow$$

$$W(l) = ge^{-gl} \quad 0 \leq l \leq \infty$$

Is a exponential distribution. It has no memory.

Other examples: the decay of particles:

$$f(t;\tau) = \frac{1}{\tau} e^{-t/\tau} \qquad E(t) = \tau \qquad Var(t) = \tau^2$$

the probability of an interaction occurring at distance x

$$P(x)dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

mean free path λ

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Probability for n Interactions in D

If the distance between interactions is exponentially distributed with an mean free path of λ→ the number of interactions on a distance D is Poisson distributed with an average of D/λ.

$$P(n) = \frac{1}{n!}\left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}} = \frac{\overline{n}^n}{n!} e^{-\overline{n}} \qquad \overline{n} = \frac{D}{\lambda} \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

How do we find the energy loss distribution ?

If f(E) is the probability to lose the energy E in an interaction,

$$f(E) = \frac{1}{\sigma}\frac{d\sigma}{dE}$$

the probability p(E) to lose an energy E over the distance D ?

$$p(E) = P(1)f(E) + P(2)\int_0^E f(E-E')f(E')dE' + P(3)\int_0^E \int_0^{E'} f(E-E'-E'')f(E'')f(E')dE''dE' + \ldots$$

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Landau Distribution (simple parameterization)

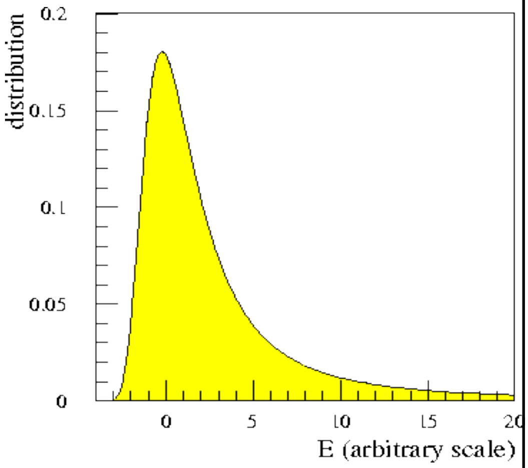
$$f(\lambda)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\lambda+e^{-\lambda})\right)$$

$$\text{with } \lambda=\frac{\Delta E-\overline{\Delta E}}{C\frac{m_e c^2}{\beta^2}\frac{Zz}{A}\rho\Delta x}$$

Probability for energy loss ΔE in matter of thickness Δx.

Landau distribution is very asymmetric.

Average and most probable energy loss must be distinguished !



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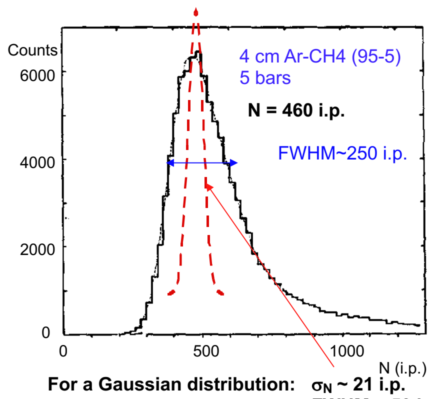
Landau Distribution

Energy lost proportional to the number of ion pairs (ip) produced

Example: peaked at 460 ip do not follow a gaussian with sigma = sqrt(460)=21

LANDAU DISTRIBUTION OF ENERGY LOSS:



For a Gaussian distribution: **σ_N ~ 21 i.p.**
FWHM ~ 50 i.p.

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2) Bremsstrahlung

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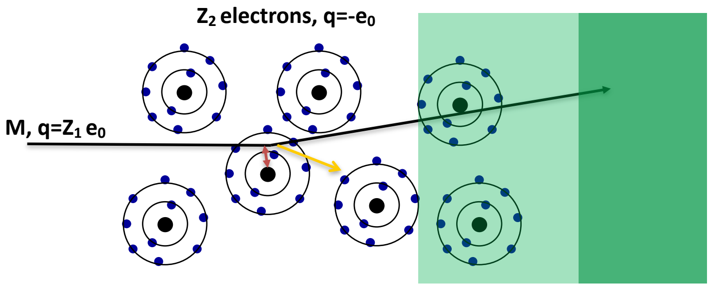
Bremsstrahlung

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated → Bremsstrahlung.



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Intermezzo: electromagnetic radiation

The electric and magnetic field produced by a charge q in point P is:

$$\vec{E}(t) = \frac{-q}{4\pi\epsilon_0}\left[\frac{\hat{e}_{r'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{e}_{r'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2}{dt^2}\hat{e}_{r'}\right]$$

$$\vec{B}(t) = -\hat{e}_{r'} \times \vec{E}/c$$

where e_r' is the unit vector from the retarded position of the charge (at t-r'/c) to P.
A very good approximation is

$$\vec{E}(t) = \frac{-q}{4\pi\epsilon_0 c^2 r}\vec{a}_\perp(t - r/c)$$

Where a_T(t-r/c) is the "observed" (now at t on P) of the perpendicular acceleration of the charged particle.

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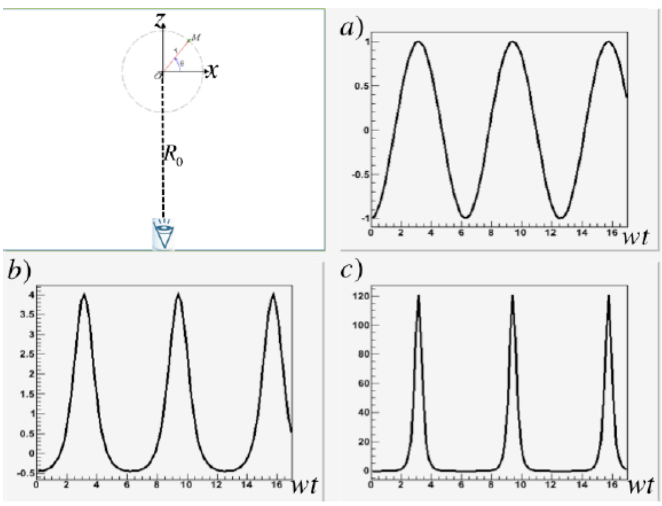
Intermezzo: synchrotron radiation

$$\frac{d^2}{dt^2}\hat{e}_{r'} = \left(\frac{\ddot{x}}{R_0}\hat{e}_N\right)\underbrace{\left(\frac{1}{1+\dot{z}(\tau)/c}\right)}_{\text{(relativistic effect)}} = -\frac{r\omega^2}{R_0}\cos(\omega t)\left(\frac{1}{1+\frac{v}{c}\cos(\omega t)}\right)\hat{e}_N$$

v << c

v=0.5 c

v=0.9 c



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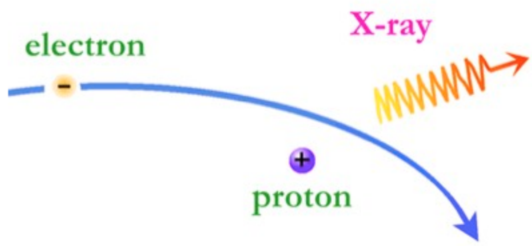
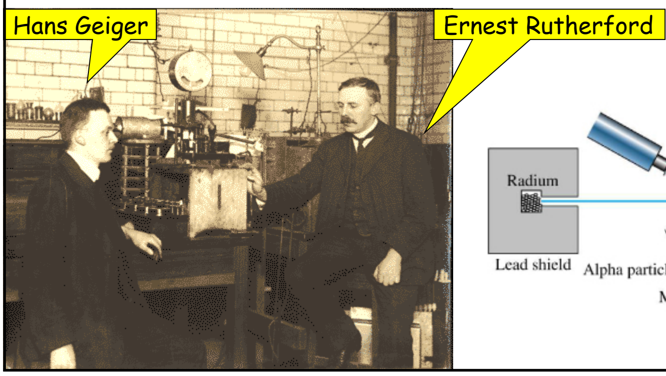
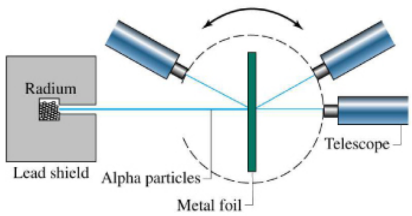
Intermezzo: Rutherford experiment

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of Charge Ze.



Classical: Rutherford experiment





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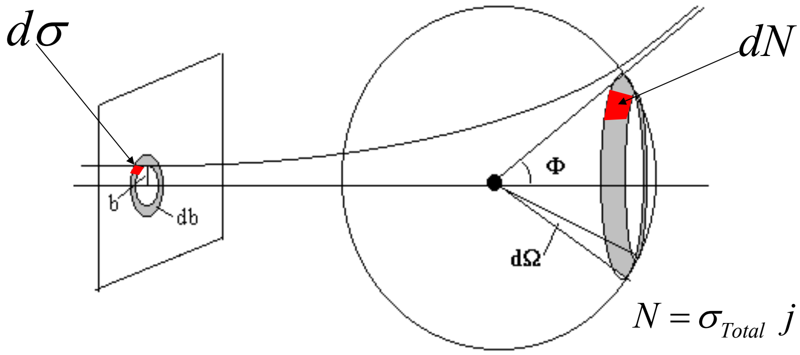
Differential cross section



$$dN = d\sigma\ j$$

flux

$$\boxed{\frac{dN}{d\Omega} = j\,\frac{d\sigma}{d\Omega}}$$

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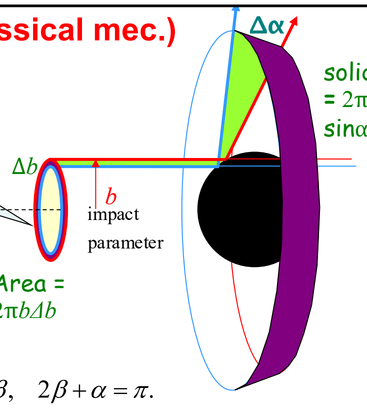
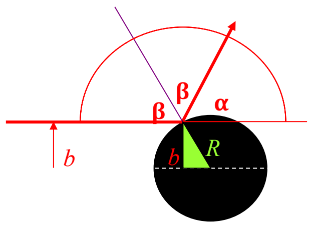
Example: rigid sphere (classical mec.)

All the balls that pass through this ring area $\Delta S = 2\pi b\Delta b$ are dispersed in the interval Angle $\Delta\alpha$, i.e. the solid angle $\Delta\Omega = 2\pi\sin\alpha\,\Delta\alpha$



Area = $2\pi b\Delta b$

$$\frac{dN/j}{d\Omega} = \frac{j(2\pi b\,db)/j}{2\pi\sin\alpha\,d\alpha} = \frac{d\sigma}{d\Omega}$$



$$\frac{b}{R} = \sin\beta, \quad 2\beta + \alpha = \pi.$$

$$b = R\sin\frac{\pi-\alpha}{2} = R\cos\frac{\alpha}{2}.$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\,db}{2\pi\sin\alpha\,d\alpha} = \frac{b}{\sin\alpha}\frac{db}{d\alpha} = \frac{R\cos\frac{\alpha}{2}}{\sin\alpha}R\sin\frac{\alpha}{2}\left(-\frac{1}{2}\right) = -\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\alpha}R^2 = -\frac{R^2}{4}.$$

$$\sigma = \int_0^\pi \frac{d\sigma}{d\Omega}2\pi\sin\alpha\,d\alpha = \int_0^\pi \frac{\pi R^2}{2}\sin\alpha\,d\alpha = \frac{\pi R^2}{2}\left[-\cos\alpha\right]_0^\pi = \pi R^2$$

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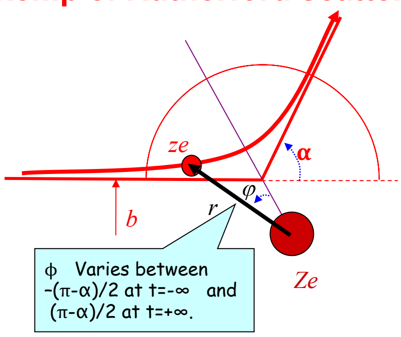
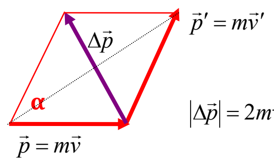
Exemple: Rutherford scattering (classical mec.)





$\vec{p}\,' = m\vec{v}\,'$

$|\Delta\vec{p}| = 2mv\sin\frac{\alpha}{2}$

$\vec{p} = m\vec{v}$

The change in P is due to the Coulomb force:

$$\Delta\vec{p} = \int_{-\infty}^{+\infty}\vec{F}(r)\,dt. \qquad |\Delta\vec{p}| = \int_{-\infty}^{+\infty}F(r)\cos\varphi\,dt.$$

$L = mr^2\omega$ is conserved for a particle in a central field. initial condition. Initial condition $L = mvb$

$$\boxed{|\Delta\vec{p}| = 2mv\sin\frac{\alpha}{2}} \quad |\Delta\vec{p}| = \int_{-\frac{\pi-\alpha}{2}}^{+\frac{\pi-\alpha}{2}} F(r(\cos(\varphi)))\cos\varphi\frac{d\varphi}{\left|\frac{d\varphi}{dt}\right|} \quad \boxed{\left|\frac{d\varphi}{dt}\right| = \omega = \frac{vb}{r^2}} \quad \boxed{F(r) = \frac{Zze^2}{r^2}}$$

$$2mv\sin\frac{\alpha}{2} = Zze^2\int_{-\frac{\pi-\alpha}{2}}^{+\frac{\pi-\alpha}{2}}\frac{1}{r^2}\cos\varphi\frac{d\varphi}{\frac{vb}{r^2}} \qquad b\sin\frac{\alpha}{2} = \frac{Zze^2}{2mv^2}\int_{-\frac{\pi-\alpha}{2}}^{+\frac{\pi-\alpha}{2}}\cos\varphi\,d\varphi = \frac{Zze^2}{2mv^2}\cos\frac{\alpha}{2}$$

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$b = \frac{Zze^2}{mv^2}\,\mathrm{cotan}\,\frac{\alpha}{2}.$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\alpha}\left|\frac{db}{d\alpha}\right| = \left(\frac{Zze^2}{mv^2}\right)^2 \frac{1}{\sin\alpha}\frac{1}{2\sin^2\frac{\alpha}{2}} = \left(\frac{Zze^2}{mv^2}\right)^2 \frac{1}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\frac{1}{2\sin^2\frac{\alpha}{2}}.$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{4(mv^2/2)}\right)^2 \frac{1}{\sin^4\frac{\alpha}{2}}}$$

What is the probability that the α particle is deflected more than 1 degree?

(typical kinetic energy of alpha particles1 MeV = 1,60 × 10^{-13} J)

$$\sigma^{a\geq 1^\circ} = (...)^2 \int_{1^\circ}^{180^\circ} \frac{2\pi\sin\alpha\, d\alpha}{\sin^4\frac{\alpha}{2}} = 2\pi(...)^2 \int_{1^\circ}^{180^\circ} \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\, d\alpha}{\sin^4\frac{\alpha}{2}} = 8\pi(...)^2 \int_{1^\circ}^{180^\circ} \frac{\cos\frac{\alpha}{2}\, d\frac{\alpha}{2}}{\sin^3\frac{\alpha}{2}} =$$

substituting $\sin\frac{\alpha}{2} = x$

$$= 8\pi(...)^2 \int_{\sin 0.5^\circ}^{\sin 90^\circ} \frac{dx}{x^3} = 8\pi(...)^2 \left[\frac{-1}{2x^2}\right]_{\sin 0.5^\circ}^{\sin 90^\circ} = 5.33\times 10^{-22}\,\mathrm{m}^2 = 5.33\times 10^{6}\,\mathrm{barns}$$

$(1 barn = 1b = 10^{-28} m^2)$

What is the impact parameter that corresponds to an angle of 1° ?

$$b = \frac{Zze^2}{mv^2}\,\mathrm{cotan}\,\frac{\alpha}{2} = 1.14\times 10^{-13}\,\mathrm{cotan}\,0.5^\circ\mathrm{m} = 1.30\times 10^{-11}\ \mathrm{m}$$

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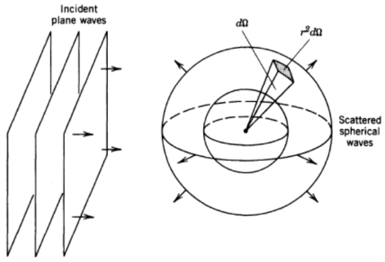
Differential cross section (QM, H not depending on t)

A plane wave is scattered by the object and a spherical wave emerges



$$\psi = \psi_{inn} + \psi_{scattered}$$

$$\psi_{inn} = Ce^{i\vec{k}\cdot\vec{r}}$$

$$\psi_{scattered} = Cf(\theta,\phi)\frac{e^{ikr}}{r}$$

Procedure:

- Solve the time-independent Schrödinger equation
- Find a valid approach for the solution far from the center of dispersion.
- Write this solution as a sum of the initial incoming plane wave and an outgoing spherical wave

$$\psi \cong C\left(e^{i\vec{k}\cdot\vec{r}} + f(\theta,\phi)\frac{e^{ikr}}{r}\right)$$

First we determine the relationship between the "f" and the differential cross section

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relationship between the function "f (θ, φ)" and the differential cross section

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r}) \qquad with \quad E = \frac{\hbar^2 k^2}{2m} \quad \text{i} \quad U(\vec{r}) = \frac{2m}{\hbar^2}V(\vec{r}) \quad \text{we have:}$$

$$\boxed{\left(\nabla^2 + k^2\right)\psi(\vec{r}) = U(\vec{r})\psi(\vec{r})}$$

Current density: $$\vec{j} = -\frac{i\hbar}{2m}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) = \mathrm{Re}\left(\psi^*\frac{\hbar}{im}\vec{\nabla}\psi\right)$$

Incoming current density:

$$\psi_{in} = Ce^{i\vec{k}\cdot\vec{r}} = Ce^{ikz}$$

$$\boxed{\vec{j}_{in} = \frac{\hbar k}{m}|C|^2\hat{k}}$$

Outgoing current density:

$$\psi_{sc} = Cf(\theta,\phi)\frac{e^{ikr}}{r}$$

$$\frac{\partial}{\partial r}Cf(\theta,\phi)\frac{e^{ikr}}{r} = ikCf(\theta,\phi)\frac{e^{ikr}}{r} + O\left(r^{-2}\right)$$

$$\boxed{\vec{j}_{sc} = |C|^2|f(\theta,\phi)|^2\frac{\hbar k}{mr^2}\vec{e}_r}$$

$$\vec{\nabla}f(r) = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z = \frac{\partial f}{\partial r}\vec{e}_r + \ldots\vec{e}_\theta + \ldots\vec{e}_\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{j_{sc}r^2 d\Omega}{d\Omega\, j_{in}} = \frac{|C|^2\frac{\hbar k}{mr^2}|f(\theta,\phi)|^2 r^2 d\Omega}{d\Omega|C|^2\frac{\hbar k}{m}} = |f(\theta,\phi)|^2 \Rightarrow \boxed{\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2}$$

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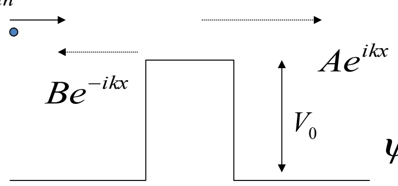
before a 1D example:

$$\psi_{inn} = Ce^{ikx}$$



$$\psi = \psi_{inn} + \psi_{scattered}$$

$$\psi_{scattered} = C\left(Ae^{ikx} + Be^{-ikx}\right)$$

$$f(\theta,\phi) = \begin{cases} A & \text{Forward scattering} \\ B & \text{Reflection} \end{cases}$$

In this case we can find exactly "f" by the usual procedure of continuity

$$\psi \quad and \quad d\psi/dx$$

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Determination of the function "f (θ, φ)"

Using the method of Green's functions.
A formal solution of the Schrodinger equation

$$\left(\nabla^2 + k^2\right)\psi(\vec{r}) = U(\vec{r})\psi(\vec{r})$$

is :

$$\boxed{\psi(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi(\vec{r}\,')\, d^3r'}$$

if:

$$\left(\nabla^2 + k^2\right)\psi_0(\vec{r}) = 0 \quad \Rightarrow \quad \psi_0(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$$

$$\left(\nabla^2 + k^2\right)G(\vec{r}-\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

effectively ....

$$(\nabla^2 + k^2)\psi(\vec{r}) = \underbrace{(\nabla^2 + k^2)\psi_0(\vec{r})}_{\text{This term is 0}} + \int \underbrace{(\nabla^2 + k^2)G(\vec{r}-\vec{r}\,')}_{\text{This is } \delta(\vec{r}-\vec{r}\,')}U(\vec{r}\,')\psi(\vec{r}\,')\, d^3r'$$

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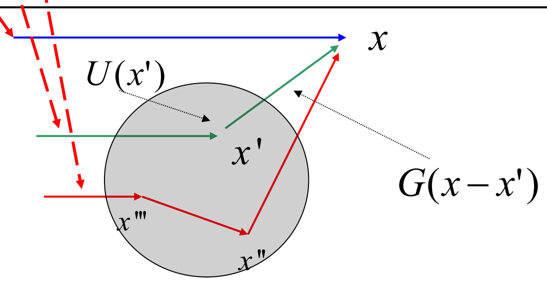
BORN series: Feynman diagrams

$$\boxed{\psi(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi(\vec{r}\,')\, d^3r'}$$

As U is small we can obtain it iterating:

$$\psi(\vec{r}) = \psi_0(\vec{r}) + \int d^3r' G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi_0(\vec{r}\,') +$$
$$+ \int d^3r'' \int d^3r''' \, G(\vec{r}-\vec{r}\,'')U(\vec{r}\,'')G(\vec{r}\,''-\vec{r}\,''')U(\vec{r}\,''')\psi_0(\vec{r}\,''') + \cdots$$

interpretation:
(Feynman diag.)



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What is the Green's function associated with the Schrodinger equation?

$$\left(\nabla^2 + k^2\right)G(\vec{r}-\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

Notice: $\delta(\vec{r}) = \frac{1}{(2\pi)^3}\int e^{i\vec{s}\cdot\vec{r}} d^3s$, **and:** $\nabla^2 e^{i\vec{s}\cdot\vec{r}} = -s^2 e^{i\vec{s}\cdot\vec{r}}$

with $\int (\nabla^2 + k^2) e^{i\vec{s}\cdot\vec{r}} d^3s = \int (-s^2 + k^2) e^{i\vec{s}\cdot\vec{r}} d^3s$

The function is: $G(\vec{r}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} \frac{e^{i\vec{s}\cdot\vec{r}}}{-s^2 + k^2} d^3s \Rightarrow \boxed{G(\vec{r}) = -\frac{e^{ikr}}{4\pi r}}$

Approximations for large r…….

If the potential is short-range, i.e. active only for small r' :

$$|\vec{r}-\vec{r}\,'| = \sqrt{r^2 + r'^2 - 2\vec{r}\cdot\vec{r}\,'} = r\left(1 - 2\frac{\vec{r}\,'\cdot\vec{r}}{r^2} + \frac{r'^2}{r^2}\right)^{1/2} \approx r\left(1 - \frac{\vec{r}\,'\cdot\vec{r}}{r^2}\right)$$

1) $e^{ik|\vec{r}-\vec{r}\,'|} \approx e^{ikr} e^{-ik\frac{\vec{r}}{r}\cdot\vec{r}\,'} = e^{ikr} e^{-i\vec{k}_f\cdot\vec{r}\,'}$ $\quad k\frac{\vec{r}}{r} = \frac{\vec{p}_f}{\hbar} \equiv \vec{k}_f$ $\quad$ **2)** $\frac{1}{|\vec{r}-\vec{r}\,'|} \approx \frac{1}{r}$

$$\boxed{G(\vec{r}-\vec{r}\,') = -\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|} \approx \frac{e^{ikr}}{r} e^{-i\vec{k}_f\cdot\vec{r}\,'}}$$

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Therefore the exact formal solution for large r (where we have the detectors), which allows us to obtain the function f (θ, φ), is :

$$\psi(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi(\vec{r}\,')d^3r' = e^{i\vec{k}\cdot\vec{r}} - \frac{e^{ikr}}{r}\underbrace{\frac{1}{4\pi}\int e^{-i\vec{k}_f\cdot\vec{r}\,'}U(\vec{r}\,')\psi(\vec{r}\,')d^3r'}_{f(\theta,\phi)}$$

$$= \;\rightarrow x' \rightarrow x \;+\; \rightarrow x''' \rightarrow x'' \rightarrow x \;+\; \ldots.$$

$$\psi_0(x)\,U(x')\,G(x-x')$$

We can iterate to obtain it. These iterations (Born series) we have seen that we can be expressed in a diagrammatic form: the Feynman diagrams

QM differential cross section. Born approximation

If we keep only the first iteration (first term): $\psi_0(\vec{r}) = e^{i\vec{k}?\vec{r}}$ i $U(\vec{r}) = \frac{2m}{\hbar^2}V(\vec{r})$

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{e^{ikr}}{r}\frac{2m}{4\pi\hbar^2}\int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}\,'}V(\vec{r}\,')d^3r' \qquad \psi \cong C\left(e^{i\vec{k}\cdot\vec{r}} + f(\theta,\phi)\frac{e^{ikr}}{r}\right)$$

and therefore f (θ, φ) is (Born approximation):

$$\boxed{f^B(\theta,\phi) = \frac{m}{2\pi\hbar^2}\int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}}V(\vec{r})d^3r} \qquad \boxed{\frac{d\sigma}{d\Omega} \approx \left|f^B(\theta,\phi)\right|^2}$$

= the Fourier transform of the potential !!!!

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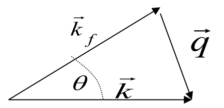
Differential cross section for potentials with spherical symmetry

$$V(\vec{r}) = V(r)$$



$$q^2 = k_f^2 + k^2 - 2k_f k\cos(\theta) = 2k^2(1-\cos(\theta))$$

$$q = 2k\sin\left(\frac{\theta}{2}\right)$$

$$f^B(\theta,\phi) = \frac{m}{2\pi\hbar^2}\int e^{i\overbrace{(\vec{k}-\vec{k}_f)}^{\vec{q}}\cdot\vec{r}}V(r)d^3r$$

$$= \frac{2m}{4\pi\hbar^2}\int_0^\infty V(r)r^2dr\int_0^\pi e^{iqr\cos(\eta)}\sin(\eta)d\eta\int_0^{2\pi}d\vartheta$$

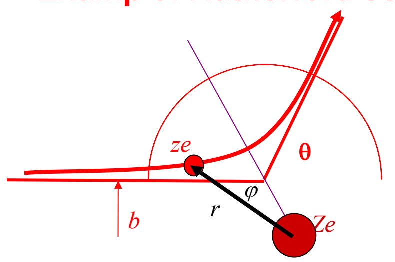
$$= \frac{2m}{q\hbar^2}\int_0^\infty rV(r)\sin(qr)dr$$

The TOTAL cros section: $\sigma = \int_\Omega \left|f^B(\theta,\phi)\right|^2 d\Omega = \int_\Omega \left|f^B(\theta,\phi)\right|^2 \sin(\theta)d\theta d\varphi$

$$\sin(\theta) = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \qquad dq = k\cos\left(\frac{\theta}{2}\right)d\theta$$

$$\sigma = 2\pi\int_0^\pi \left|f^B(\theta)\right|^2\sin(\theta)d\theta = \frac{2\pi}{k^2}\int_0^{2k}\left|f^B(q)\right|^2 q\,dq$$

Example: Rutherford scattering (QM)



$$V(\vec{r}) = \frac{zZe^2}{|\vec{r}\,|}$$

$$q = 2k\sin\left(\frac{\theta}{2}\right)$$

$$f^B(\theta,\phi) = \frac{m}{2\pi\hbar^2}\int e^{i\vec{q}\cdot\vec{r}}V(\vec{r})d^3r = \frac{zZe^2m}{2\pi\hbar^2}\int d^3r e^{i\vec{q}\cdot\vec{r}}\frac{1}{|\vec{r}\,|}$$

This integral diverges. Can be found by applying a convergence factor

$$\lim_{\mu\to 0}\int d^3x\frac{e^{i\vec{q}\cdot\vec{x}}}{x}e^{-\mu x} = \lim_{\mu\to 0}\frac{4\pi}{q^2+\mu^2} = \frac{4\pi}{q^2} \qquad (T = mv^2/2)$$

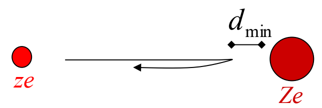
$$\frac{d\sigma}{d\Omega} = \left|f(\theta,\phi)\right|^2 = \frac{(zZe^2)^2m^2}{(2\pi)^2\hbar^4}\frac{(4\pi)^2}{q^4} = \frac{(zZe^2)^2m^2}{4p^4\sin^4(\theta/2)} = \left(\frac{zZe^2}{4T}\right)^2\frac{1}{\sin^4(\theta/2)}$$

(coincides with that obtained classically!)

Rutherford scattering: experimental results

Using alpha particles with kinetic energy of about 7.6 MeV over Au, no differences were observed with the prediction.

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{4T}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}}$$

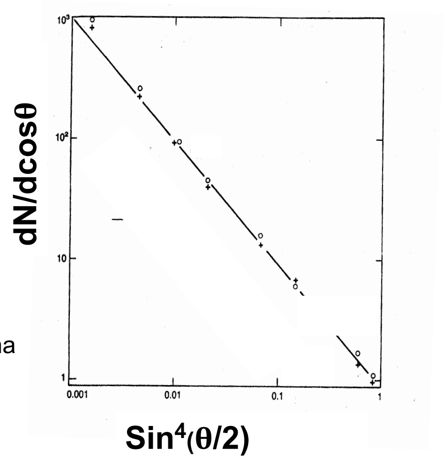


$$T = \frac{1}{2}mv^2 = \frac{zZe^2}{d_{\min}} \Rightarrow d_{\min} = \frac{zZe^2}{T}$$

For Au (A = 197, Z = 79) and 7.6 MeV alpha

$$d_{\min} \approx 30\,fm$$

Not entering the nucleus (10 fm): "point like”



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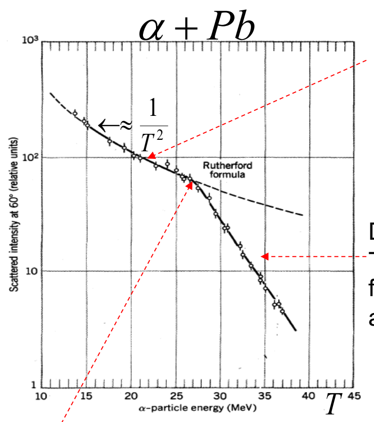
Deviation from Rutherford scattering at high energy -> determines the charge distribution of the nucleus



$$\leftarrow \approx \frac{1}{T^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{4T}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}}$$

Decreases !!!! The nuclear force is attractive

$$d_{\min} = \frac{zZe^2}{27\,MeV} = 8.7\,fm$$

From 27 MeV "see" the nucleus of Pb:
- effects of the strong interaction
- Effects of no point-like charge distribution

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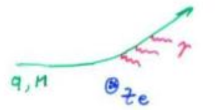
Bremsstrahlung, Classical



$q = Z_1 \cdot e$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 Z_1 Z_2 e^2}{4\pi\varepsilon_0\, p \cdot v}\right)^2 \frac{1}{(2\sin\frac{\theta}{2})^4} \qquad p = M\gamma v$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer $Q^2 = 2p^2(1-\cos\theta)$

$$\frac{d\sigma}{dQ} = 8\pi\left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \beta c}\right)^2 \cdot \frac{1}{Q^3}$$

$Q = |\vec{p} - \vec{p}\,'|$

→ From Maxwell's Eq (Jackson)

$$\lim_{\omega\to 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\varepsilon_0} Q^2$$ Radiated Energy between $\omega, \omega+d\omega$

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of Charge Ze.

Because of the acceleration the particle radiated EM waves → energy loss.

Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

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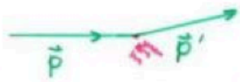
Bremsstrahlung, Classical

$$\frac{d\sigma}{dQ} = 8\pi\left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \beta c}\right)^2 \cdot \frac{1}{Q^3}$$



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$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \int_0^{\omega_{max}} d\omega \int_{Q_{min}}^{Q_{max}} dQ\, \frac{dI}{d\omega} \cdot \frac{d\sigma}{dQ}, \quad \omega_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \frac{16}{3}\alpha \cdot Z^2 \cdot \left(\frac{Z_1^2 e^2}{4\pi\varepsilon_0 M c^2}\right)^2 \cdot E \cdot \ln\frac{Q_{max}}{Q_{min}}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \sim \frac{1}{137}$$

$$\frac{dE}{dx} \propto \frac{E}{M^2}$$

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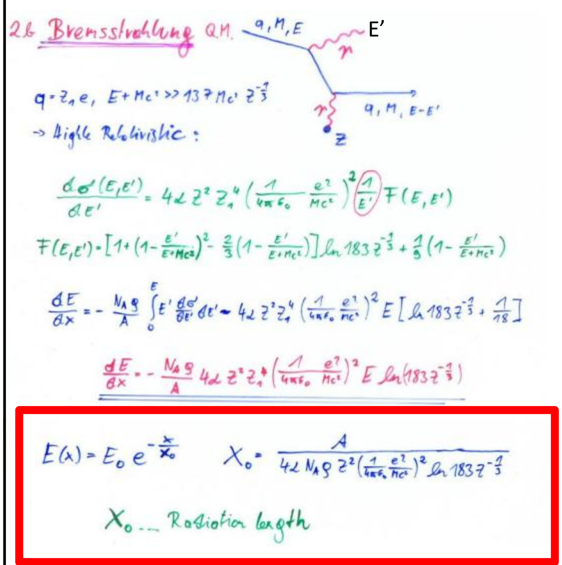
Bremsstrahlung, QED



Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional to ρ of the material.

Proportional $1/M^2$ of the incoming particle.

Proportional to the Energy of the Incoming particle →

$E(x)=\mathrm{Exp}(-x/X_0)$ – 'Radiation Length'

$X_0 \propto M^2A/(\rho Z_1^4 Z^2)$

X_0: Distance where the Energy E_0 of the incoming particle decreases $E_0\mathrm{Exp}(-1)=0.37E_0$.

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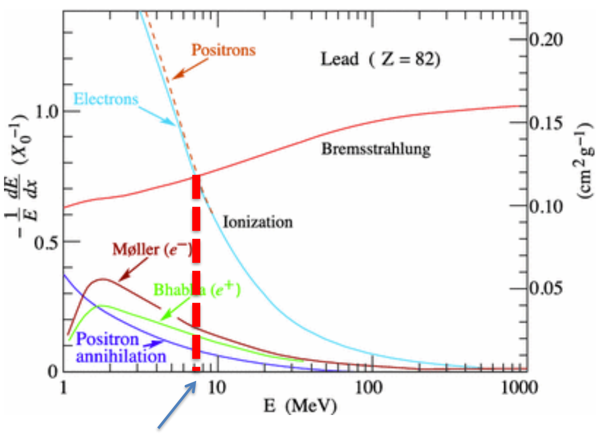
Critical Energy



$$for \quad E >> 0$$

$$\left(\frac{dE}{dx}\right)_{ion} \propto const$$

$$\left(\frac{dE}{dx}\right)_{bre} \propto \frac{E}{M^2}$$

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

Electron in Copper: p ≈ 20MeV

Muon in Copper: p ≈ 400GeV

Above Critical Energy Bremsstrahlung dominates, below Ionization

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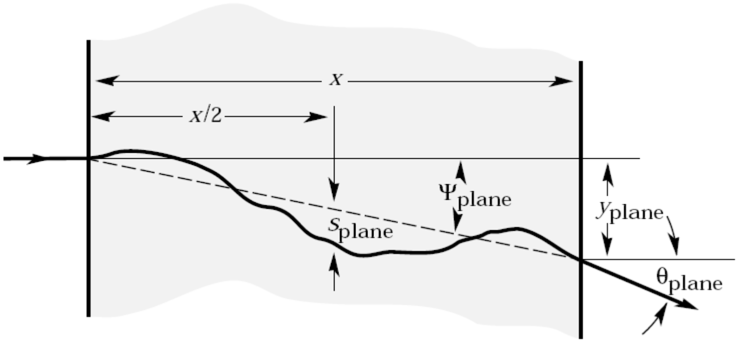
Effect of Bremsstrahlung: Multiple Scattering

- Particles don't only loose energy …



… they also change direction

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Multiple Scattering

Statistical (quite complex) analysis of multiple collisions gives:

Probability that a particle is defected by an angle θ after travelling a distance x in the material is given by a Gaussian distribution with sigma of:

$$\Theta_0 = \frac{0.0136}{\beta cp[\mathrm{GeV/c}]} Z_1 \sqrt{\frac{x}{X_0}}$$

X_0 … Radiation length of the material
Z_1 … Charge of the particle
p … Momentum of the particle

Angular distributions are given:

$$\frac{dN}{d\Omega} \propto \frac{1}{2\pi\theta_0^2} \exp\left(-\frac{\theta_{space}^2}{2\theta_0^2}\right)$$

$$\frac{dN}{d\theta_{plane}} \propto \frac{1}{\sqrt{2\pi}\theta_0} \exp\left(-\frac{\theta_{plane}^2}{2\theta_0^2}\right)$$

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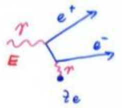
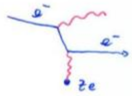
Interaction of Particles with Matter (photons: pair production)



$\gamma + Nucl. \rightarrow e^+ + e^- + Nucl$

The Diagram is very similar to Bremsstrahlung



$e^- + Nucl \rightarrow \gamma + e^- + Nucl$

"Crossing Symmetry: bring particle to the other side and make it the antiparticle" → 'same' crosssection ...

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 r_e^2 \frac{1}{E}\cdot G(E,E') \qquad E \gg 137 m_e c^2 Z^{-\frac{1}{3}}$$

$$G(E,E') = \left[\left(\frac{E'+m_ec^2}{E}\right)^2 + \left(1-\frac{E'+m_ec^2}{E}\right)^2 + \frac{2}{3}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\ln Z^{-\frac{1}{3}} - \frac{1}{9}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\right]$$

$$\sigma = \int_0^{E-2m_ec^2} \frac{d\sigma}{dE'}dE' = 4\alpha Z^2 r_e^2 \cdot \frac{7}{9}\ln 183 Z^{-\frac{1}{3}}$$

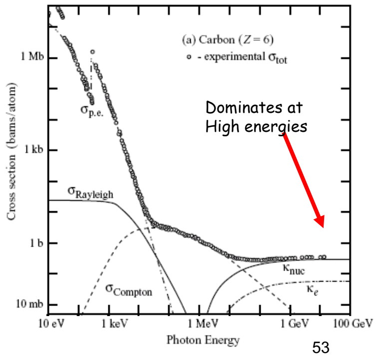
$$P(x) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}} \qquad \lambda = \frac{A}{\rho N_A \sigma} = \frac{9}{7}X_0$$

↳ Probability that Photon converts to e^+e^- after a distance x.

(exponential distribution)

For $E\gamma \gg m_e c^2 = 0.5$ MeV : $\lambda = 9/7 X_0$

Average distance a high energy photon has to travel before it converts into an $e^+ e^-$ pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing it's energy from E_0 to E_0*Exp(-1) by photon radiation.



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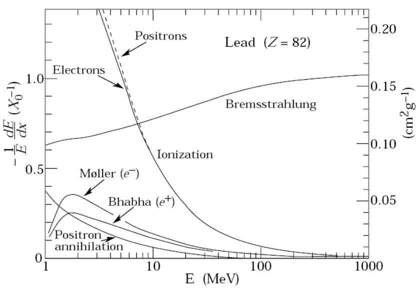
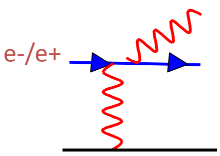
Summary electron Energy Loss

electron energy loss primarily
- bremsstrahlung (e -> γ + e) at E > 20 MeV
- ionization below 20 MeV

Crossover depends on Z (example is for cooper).



For E> 20 MeV:



Nucleus (quarks)

(Xo is the Average distance a high energy E electron emits a photon of 0.37E)

$$\frac{dE}{dx_{Brem}} = -\frac{E}{X_0}$$

radiation length (for electrons)

$$X_0 = \frac{180A}{Z^2}\ g\ cm^{-2}$$

divide by ρ to get in *cm*

Radiation probability depends on radiation length X_0

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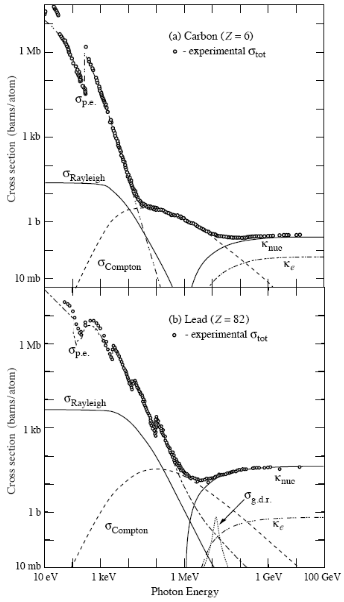
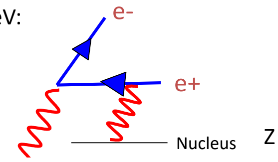
Summary *Photon Energy Loss*

Photon energy loss primarily
- pair production at E > 20 MeV
- Compton scattering ($\gamma + e \rightarrow \gamma + e$) below 20 MeV.

Crossover depends on Z (example is for cooper).
- P.e at very low E



For E> 20 MeV:



$\gamma + N \rightarrow e^+ + e^- + N$

(9/7 Xo is the Average distance a high energy photon has to travel before it converts into an $e^+ e^-$ pair)

Pair Production, also known as photon conversion or Bethe-Heitler process

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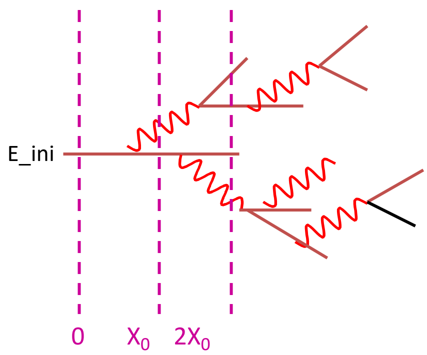
electron/photon interaction with matter

Bremsstrahlung + Pair Production → EM Shower



Assume (E_ini > E_c)

1. Each electron E > E_c travels 1 X_0 and gives up 50% E (37% E) to photon
2. Each photon travels 1 X_0 (9/7 X_0) and pair produces with 50% E (E(e+)=E(e-)) to each
3. Electrons with E< E_c stop (ionization)

E_c = 1 – 20 MeV

Electrons/positrons or photons will form a cascade by the combination of Bremsstrahlung and pair production

(Note: Electrons are losing energy constantly by ionization the media)

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3) Cherenkov radiation

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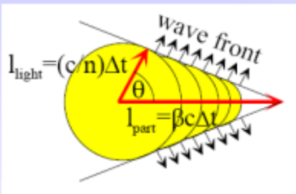
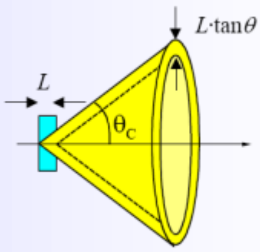
Cherenkov Radiation

with velocity $\beta \geq \beta_{thr} = \frac{1}{n}$ n: refractive index



$$\boxed{\begin{array}{l}\cos\theta_C = \frac{1}{n\beta} \\ \text{with } \; n = n(\lambda) \geq 1\end{array}}$$



- $\beta_{thr} = \frac{1}{n} \rightarrow \theta_C \approx 0$ Cherenkov threshold
- $\theta_{max} = \arccos\frac{1}{n}$ 'saturated' angle (β=1)

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Different Cherenkov Detectors

- Threshold Detectors
  - Yes/No on whether the speed is $\beta>1/n$
- Differential Detectors
  - $\beta_{max} > \beta > \beta_{min}$
- Ring-Imaging Detectors
  - Measure β

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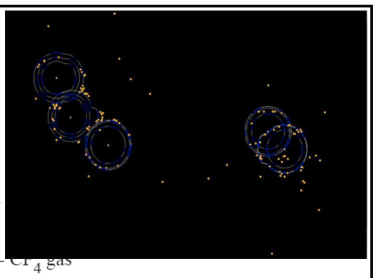
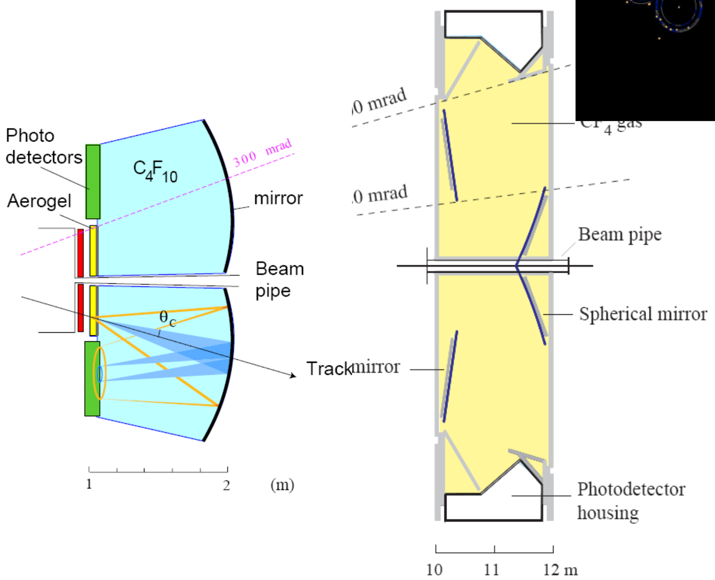
LHCb RICH





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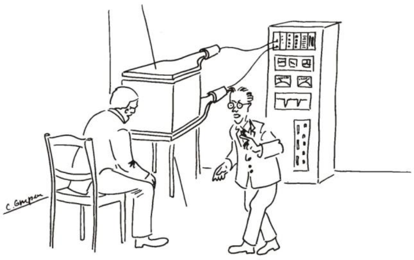
Interaction of Particles with Matter (neutrinos)

Any device that is to detect a particle must interact with it in some way
→ almost …

In many experiments neutrinos are measured by missing transverse momentum.

E.g. e^+e^- collider. $P_{tot}=0$,
If the $\Sigma\, p_i$ of all collision products is $\neq 0$ → neutrino escaped.



"Did you see it?"
"No nothing."
"Then it was a neutrino!"

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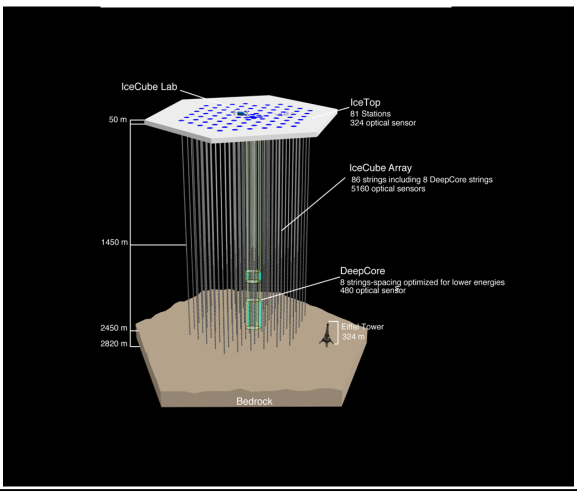
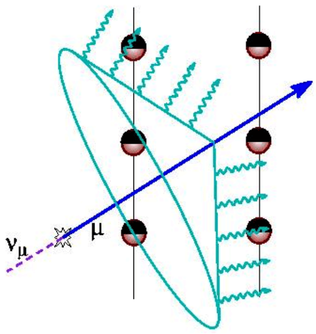
Neutrino detector

Neutrinos hardly interact: a neutrino that crosses one light year of lead has only a 50% probability of having an interaction





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Electromagnetic Interaction of Particles with Matter

Ionization and Excitation:

Charged particles traversing material are exciting and ionizing the atoms.

The average energy loss of the incoming particle by this process is to a good approximation described by the Bethe Bloch formula.

The energy loss fluctuation is well approximated by the Landau distribution.

Multiple Scattering and Bremsstrahlung:

The incoming particles are scattering off the atomic nuclei which are partially shielded by the atomic electrons.

The deflection of the particle on the nucleus results in an acceleration that causes emission of Bremsstrahlungs-Photons.

These photons in turn produced e+e- pairs in the vicinity of the nucleus, which causes an EM cascade. This effect depends on the 2^{nd} power of the particle mass, so it is only relevant for electrons.

Cherenkov Radiation:

If a particle propagates in a material with a velocity larger than the speed of light in this material, Cherenkov radiation is emitted at a characteristic angle that depends on the particle velocity and the refractive index of the material.