7. Weak Interactions

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7.1 Weak interactions and parity violation
Weak interactions

- Typical decay width of hadron resonances (ρ,ω,Δ) $\sim$ 1-100 MeV
- But the hadrons in the lightest isospin multiplets ($\pi^{\pm}$,π^0,p,n) have much smaller decay widths
  - Isospin is (approximately) conserved in the strong interactions
  - The hadrons in the lightest isospin multiplets must decay through a different interaction
    - ★ Electromagnetic (π^0). Decay width $\sim 10^{-5}$ MeV
    - ★ Weak ($\pi^{\pm}$,n). Decay widths $\sim 10^{-14}$-10^{-24} MeV

Weak interactions are also necessary to explain:

- Muon decay $\Gamma_{\mu^- \to e^- \bar{\nu}_e \nu_\mu} = 2.6\, 10^{-16}$ MeV
- Decays of the lightest strange hadrons: $\Gamma_{K^\pm} = 4.6\, 10^{-16}$ MeV, $\Gamma_{K^0_S} = 6.3\, 10^{-12}$ MeV, $\Gamma_{K^0_L} = 1.1\, 10^{-14}$ MeV, $\Gamma_{\Lambda^0} = 2.1\, 10^{-12}$ MeV

- Historically, they were discovered in nuclear beta decay

$$
\begin{array}{cclcl}
{}^{A}_{Z}X & \to & {}_{Z+1}{}^{A}X + e^{-} & & \beta^{-} \text{ decay} \\
{}^{A}_{Z}X & \to & {}_{Z-1}{}^{A}X + e^{+} & & \beta^{+} \text{ decay}
\end{array}
$$

- Pauli postulated the existence of the neutrino ν, a massless spin $1/2$ particle, to explain the missing energy, momentum and angular momentum
- Then nuclear beta decays can be understood in terms of

$$
n \to p\, e^{-}\, \bar{\nu}_e \qquad , \qquad p \to n\, e^{+}\, \nu_e
$$

- The first interaction proposed by Fermi was

$$
\mathcal{L}_{int} = G\bar{\psi}_p\gamma^{\mu}\psi_n\bar{\psi}_e\gamma_{\mu}\psi_{\nu_e} + \text{H.c.} \equiv G\bar{p}\,\gamma^{\mu}\,n\,\bar{e}\,\gamma_{\mu}\nu_e + \text{H.c.}
$$

- This interaction Lagrangian is parity invariant
- With a parity invariant theory the K^{+} decays are difficult to understand

$$
\begin{array}{rcl}
K^{+} \to \pi^{+}\pi^{0} & \Longrightarrow & P(K^{+}) = + \\
K^{+} \to \pi^{+}\pi^{0}\pi^{0} & \Longrightarrow & P(K^{+}) = -
\end{array}
$$

Parity violation

- Lee & Yang proposal (56) to measure parity violation: polarized ${}^{60}Co \rightarrow {}^{60}Ni^* \, e^- \, \bar{\nu}_e$, look at e^- in the polarization direction

$$
\begin{array}{cccccc}
 & {}^{60}Co & \rightarrow & {}^{60}Ni^* & e^- & \bar{\nu}_e \\
J & 5 & & 4 & \frac{1}{2} & \frac{1}{2} \\
 & & & & \Uparrow \uparrow \vec{p} \;\; e_R & \Uparrow \downarrow -\vec{p} \;\; \bar{\nu}_R \\
 & \Uparrow & & \Uparrow & & \\
 & & & & \Uparrow \downarrow -\vec{p} \;\; e_L & \Uparrow \uparrow \vec{p} \;\; \bar{\nu}_L
\end{array}
$$

- The magnetic field used to polarize the nucleus is invariant under parity:
  - If parity is respected, one should observe the same number of electrons in the upward and downward directions
  - The experiment was carried out by Wu on the same year, and she observed a clear tendency for electrons to go downwards
- This suggests that only the left handed component of the leptons is sensitive to the weak interactions

- Fermi's original proposal was modified to

$$\mathcal{L}_{int} = 2\sqrt{2} G \bar{p} \gamma^{\mu} P_L \, n \, \bar{e} \gamma_{\mu} P_L \nu_e + \text{H.c.} \quad , \quad P_L = \frac{1-\gamma^5}{2}$$

G is the Fermi constant, $[G] = -2$. This is called $V - A$ theory

- If the mass of the outgoing charged lepton is neglegible $\Longrightarrow$ fully polarized
- Since all fields are at the same space-time point $\Longrightarrow$ zero range interaction
- A coupling constant with negative dimensions suggests that this is an effective interaction of a more fundamental theory
- A zero range interaction may be obtained as the low energy limit of a finite range interaction
- For a massive vector boson as the mediator

$$\langle 0 | T\{W_{\mu}(x) W^{\dagger}_{\nu}(y)\} | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik(x-y)}}{k^2 - m_W^2} \left(-g_{\mu\nu} + \frac{k_{\mu} k_{\nu}}{m_W^2} \right)$$

$$\underset{k^{\mu} \ll m_W}{\simeq} \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik(x-y)}}{-m_W^2} (-g_{\mu\nu}) = \frac{i g_{\mu\nu}}{m_W^2} \delta(x-y)$$

$\Longrightarrow \quad G \sim 1/m_W^2$

- Since $n = (udd)$ and $p = (uud)$, it is natural to interpret beta-decay as a $d \to u\, e^- \,\bar{\nu}_e$ process
- In terms of quark and lepton fields the Fermi theory was finally written as

$$\mathcal{L}_{int} = 2\sqrt{2} G J^\mu J_\mu^\dagger \quad , \quad J^\mu = \bar{u}\,\gamma^\mu P_L\, d + \bar{\nu}_e\,\gamma^\mu P_L\, e + \bar{\nu}_\mu\,\gamma^\mu P_L\,\mu + \dots$$

  - If a right-handed neutrino exists $\Longrightarrow$ it does not interact (sterile neutrino)
  - $\mathcal{L}_{int}$ is invariant under $l_L \to e^{i\theta_l} l_L$, $\nu_{lL} \to e^{i\theta_l}\nu_{lL}$, $l = e,\ \mu$
  - $\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$ is also invariant for $\theta_l \neq \theta_l(x)$ if $l_R \to e^{i\theta_l} l_R$ $\Longrightarrow$ individual leptonic numbers (electronic, muonic, ...) are conserved
  - $\mathcal{L}$ is also invariant under $q \to e^{i\theta_B} q$, $\theta_B \neq \theta_B(x)$, $q = u, d, \dots$ $\Longrightarrow$ baryon number is conserved
- If the week interaction reads as above at quark level, how does it read at hadronic level?
  - We know from the electromagnetic case that we need form factors
  - Is this compatible with the Lagrangian for beta-decay at nucleon level we wrote before?
- We will see that chiral symmetry constraints a lot the structure of week interactions at hadronic level

5.2 Low energy tests
Charged pion decay

$$\pi^+ \to l^+ \nu_l \quad , \quad l = \mu , e$$

- The relevant piece of the Fermi Lagrangian is

$$\mathcal{L}_{int} = 2\sqrt{2} G J_{q\,\mu}^{\dagger} J_l^{\mu} \quad , \quad J_{q\,\mu}^{\dagger} = \bar{d}\, \gamma_\mu P_L\, u \quad , \quad J_l^{\mu} = \bar{\nu}_l \gamma^\mu P_L l$$

- At first order, we have

$$\mathcal{M} = \langle f | \, \mathcal{L}_{int}(0) \, | i \rangle = 2\sqrt{2} G \; {}_q\langle f | \, J_{q\,\mu}^{\dagger}(0) \, | i \rangle_q \; {}_l\langle f | \, J_l^{\mu}(0) \, | i \rangle_l$$

$$|i\rangle_l = |0\rangle_{cl} |0\rangle_\nu \quad , \quad |i\rangle_q = |\vec{p}_A\rangle_\pi \quad , \quad |f\rangle_l = |; \vec{p}_1 \lambda_1\rangle_{cl} , |\vec{p}_2 \lambda_2 ;\rangle_\nu \quad , \quad |f\rangle_q = |0\rangle_\pi$$

  - The subscript $_{cl}$ stands for charged lepton
  - Note that since the outgoing neutrino is left handded, the helicity is fixed to $\lambda_2 = -1$
  - If the mass of the outgoing positively charged lepton is neglegible $\Longrightarrow$ $\lambda_1 = 1$ $\Longrightarrow$ angular momentum is not conserved $\Longrightarrow$ the amplitude must be proportional to the charged lepton mass

- For the leptonic part, we have

$$ {}_l\langle f|\, J_l^{\mu}(0)\,|i\rangle_l = {}_{cl}\langle ;\vec{p}_1\lambda_1|\ {}_{\nu}\langle \vec{p}_2\lambda_2\,;|\ \bar{\nu}_l\,\gamma^{\mu}P_L l\ |0\rangle_{cl}\,|0\rangle_{\nu} = \bar{u}(2)\gamma^{\mu}P_L v(1) $$

- For the hadronic part, we have

$$ {}_q\langle f|\, J_{q\,\mu}^{\dagger}(0)\,|i\rangle_q = {}_{\pi}\langle 0|\ \bar{d}\,\gamma_{\mu}P_L\, u\ |\vec{p}_A\rangle_{\pi} \equiv \frac{i}{\sqrt{2}}F_{\pi}p_{A\,\mu} = -\frac{1}{2}\,{}_{\pi}\langle 0|\ \bar{d}\,\gamma_{\mu}\gamma^5\, u\ |\vec{p}_A\rangle_{\pi} $$

  - The form factor F_{π} is just a constant since $p_A^2 = m_{\pi}^2$
  - Since the pion has $J^P = 0^-$, parity implies that only the part of the current proportional to γ^5 contributes
  - $F_{\pi} \equiv$ pion (weak) decay constant

- Then

$$ \mathcal{M} = 2iGF_{\pi}p_{A\,\mu}\bar{u}(2)\gamma^{\mu}P_L v(1) = -2iGF_{\pi}m_l\bar{u}(2)P_R v(1) $$

where we have used

$$ \bar{u}(2)\not{p}_A P_L v(1) = \bar{u}(2)\left(\not{p}_1 + \not{p}_2\right)P_L v(1) = -m_l\bar{u}(2)P_R v(1) $$

$$ \bar{u}(2)\not{p}_2 = 0\ ,\ \left(\not{p}_1 + m_l\right)v(1) = 0 $$

- Let us calculate $\bar{u}(2)P_R v(1)$

$$v_R(1) = P_R v(1) = \sqrt{E_1+m}\begin{pmatrix} \frac{1}{2}\left(1+\frac{\vec{p}_1\cdot\vec{\sigma}}{E_1+m}\right)\tilde{\chi}_\lambda(\hat{p}_1) \\ \frac{1}{2}\left(1+\frac{\vec{p}_1\cdot\vec{\sigma}}{E_1+m}\right)\tilde{\chi}_\lambda(\hat{p}_1) \end{pmatrix}$$

$$\bar{u}_L(2) = \bar{u}(2)P_R = \sqrt{E_2}\begin{pmatrix} \chi_-^\dagger(\hat{p}_2) & \chi_-^\dagger(\hat{p}_2) \end{pmatrix}$$

  - We take the z-axis in the direction of $\vec{p}_1 = -\vec{p}_2 = (0,0,p)$

$$\Longrightarrow \quad \vec{p}_1\cdot\vec{\sigma} = p\sigma^3 \quad , \quad \chi_-(\hat{p}_2) = \chi_+(\hat{p}_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \chi_-(\hat{p}_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\tilde{\chi}_\lambda(\hat{p}_1) = i\sigma^2\chi_\lambda^*(\hat{p}_1) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\chi_\lambda(\hat{p}_1) = -\lambda\chi_{-\lambda}(\hat{p}_1)$$

  - Then

$$\begin{aligned} \bar{u}(2)P_R v(1) &= \sqrt{E_2}\sqrt{E_1+m}\left(1+\frac{p}{E_1+m}\right)\chi_+^\dagger(\hat{p}_1)\tilde{\chi}_\lambda(\hat{p}_1) \\ &= \sqrt{E_2}\sqrt{E_1+m}\left(1+\frac{p}{E_1+m}\right)\delta_{-\,\lambda} \end{aligned}$$

- The charged lepton is fully polarized

$$\mathcal{M}\left(\underset{\vec{p}_A=0}{\pi^+} \to \underset{\vec{p}_1,+}{l^+}\ \underset{\vec{p}_2,-}{\nu_l}\right) = 0$$

$$\mathcal{M}\left(\underset{\vec{p}_A=0}{\pi^+} \to \underset{\vec{p}_1,-}{l^+}\ \underset{\vec{p}_2,-}{\nu_l}\right) = -2iGF_\pi m_l \sqrt{E_2}\sqrt{E_1+m}\left(1+\frac{p}{E_1+m}\right)$$

- Then, using $p = E_2 = \sqrt{E_1^2 - m_l^2}$ and $E_1 + E_2 = m_\pi$

$$\sum_{\lambda_1=+,-} |\mathcal{M}|^2 = 8G^2F_\pi^2 m_l^2 E_2 m_\pi$$

and taking into account that $p = (m_\pi^2 - m_l^2)/2m_\pi$

$$\Gamma = \frac{|\mathcal{M}|^2 p}{8\pi m_A^2} = \frac{G^2F_\pi^2 m_l^2 m_\pi}{4\pi}\left(1-\frac{m_l^2}{m_\pi^2}\right)^2$$

- If G is measured elsewhere (beta-decay o muon decay) we can obtain F_π
- A prediction independent of G and F_π is the following

$$1.2\,10^{-4} \underset{Exp}{\simeq} \frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+\to\mu^+\nu_\mu)} \underset{Th}{=} \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 \simeq 1.2\,10^{-4}$$

Non-linear sigma model

- Pion decay and beta-decay are low energy processes that should be describable within the non-linear sigma-model at hadronic level
- Knowing $\mathcal{L}_{int}$ at quark level, how do we find it at hadronic level?
- Implement the symmetry breaking pattern

$$\mathcal{L}_{int} = 2\sqrt{2} G J_{q\,\mu}^{\dagger} J_l^{\mu} + \text{H. c.} \quad , \quad J_{q\,\mu}^{\dagger} = \bar{d}\,\gamma_\mu P_L\, u \quad , \quad J_l^{\mu} = \bar{\nu}_l\,\gamma^\mu P_L l$$

- Introducing the isospin doublet field q and the isospin matrices $\tau^{\pm}$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad , \quad \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$J_{q\,\mu}^{\dagger} = \bar{q}\,\gamma_\mu P_L \tau^- \, q \quad , \quad J_{q\,\mu} = \bar{q}\,\gamma_\mu P_L \tau^+ \, q$$

$$\mathcal{L}_{int} = \bar{q}\,\gamma_\mu P_L a_L^{\mu}\, q \quad , \quad a_L^{\mu} \equiv 2\sqrt{2} G \begin{pmatrix} 0 & \bar{l}\,\gamma^\mu P_L \nu_l \\ \bar{\nu}_l\,\gamma^\mu P_L l & 0 \end{pmatrix}$$

- If we take the massless limit, $m_u = m_d = 0$, $\mathcal{L}_{int}$ breaks the chiral symmetry since $[g_L, a_L^{\mu}] \neq 0$ for $g_L \in SU(2)$, $g_L \neq g_L(x)$
- However, the chiral symmetry can be restored if

$$g_L = g_L(x) \quad , \quad a_L^{\mu}(x) \to g_L(x) a_L^{\mu}(x) g_L^{\dagger}(x) + i g_L(x) \partial^\mu g_L^{\dagger}(x) \quad , \quad g_R \neq g_R(x)$$

$$\mathcal{L} = \bar{q}_L (i\partial\!\!\!/ + a\!\!\!/_L) q_L + \bar{q}_R i\partial\!\!\!/ q_R$$

- For QCD, $\not\partial \to \not D$, where D_μ contains the gluon fields, the chiral symmetry still holds
- This symmetry should also hold in the non-linear sigma-model if we introduce $a_L^\mu(x)$ in it
- Now $U(x) \to g_L(x)U(x)g_R^\dagger$ rather than $U(x) \to g_L U(x) g_R^\dagger$
  - Then $\partial_\mu U(x) \not\to g_L(x)\partial_\mu U(x) g_R^\dagger$ anymore
  - However, if we replace $\partial_\mu U(x) \to D_\mu U(x) \equiv (\partial_\mu - i a_{L\,\mu})U(x)$ then

$$D_\mu U(x) \to g_L(x) D_\mu U(x) g_R^\dagger$$

- Hence

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) \to \mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\left((D_\mu U)^\dagger D^\mu U\right)$$

$$\begin{aligned}
\mathcal{L}_{int}^{weak} &= \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U^\dagger(-i a_L^\mu)U + U^\dagger i a_L^\mu(x)\partial_\mu U\right) + \mathcal{O}(G^2) \\
&\simeq i2\sqrt{2}G\frac{f_\pi^2}{4}\mathrm{tr}\left(\begin{pmatrix} 0 & \bar{l}\gamma^\mu P_L \nu_l \\ \bar{\nu}_l\gamma^\mu P_L l & 0 \end{pmatrix}\left(\partial_\mu U U^\dagger - U\partial_\mu U^\dagger\right)\right) \\
&\simeq -2Gf_\pi\left(\partial_\mu \pi^+ \bar{\nu}_l\gamma^\mu P_L l + \partial_\mu \pi^- \bar{l}\gamma^\mu P_L \nu_l\right)
\end{aligned}$$

- If we compare with the interaction at quark level we see that

$$J^{\dagger}_{q\,\mu} \to \frac{if_\pi^2}{4}\mathrm{tr}\left(\tau^- \left(\partial_\mu U U^\dagger - U\partial_\mu U^\dagger\right)\right) \simeq -\frac{f_\pi}{\sqrt{2}}\partial_\mu \pi^+ + \dots$$

- We achieved a representation of the current at hadronic level
- Then

$$\begin{array}{rcl} {}_\pi\langle 0|\ \bar{d}\,\gamma_\mu P_L\, u\ |\vec{p}_A\rangle_\pi & = & \dfrac{i}{\sqrt{2}}F_\pi p_{A\,\mu} \\ \| & & \\ {}_\pi\langle 0| -\dfrac{f_\pi}{\sqrt{2}}\partial_\mu \pi^+\ |\vec{p}_A\rangle_\pi & = & \dfrac{if_\pi}{\sqrt{2}}p_{A_\mu} \end{array}$$

$$\Longrightarrow \quad F_\pi = f_\pi$$

- Recall that f_π controls the size of the strong interactions at low energy among pions, and between pions and nucleons
- The non-linear sigma model not only provides an alternative way of calculating the pion decay, but also tells us that any other weak process at low energy can be calculated without introducing any additional form factor or parameter

- For instance $\pi^+ \to \pi^0\, e^+ \nu_e$ can be calculated by expanding $J^{\dagger}_{q\,\mu}$ at second order in the pion fields and sandwitching it between $|\pi^+\rangle_\pi$ and ${}_\pi\langle\pi^0|$

$$J^{\dagger}_{q\,\mu} = \frac{if_\pi^2}{4}\mathrm{tr}\left(\tau^-\left(\partial_\mu U U^\dagger - U\partial_\mu U^\dagger\right)\right) = -\frac{f_\pi}{\sqrt{2}}\partial_\mu\pi^+ + \frac{i}{\sqrt{2}}\left(\partial_\mu\pi^+\pi^0 - \partial_\mu\pi^0\pi^+\right) + \dots$$

- For nucleons we had

$$\bar{N}\left(i\left(\not\partial + \not v\right) + ig_A \not a\gamma^5 - m_N\right)N$$

$$v_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger + u^\dagger\partial_\mu u\right) \quad , \quad a_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u\right) \quad , \quad U = u^2$$

  - The transformation properties under global $SU_L(2)\otimes SU_R(2)$ were

$$u \to g_L u h^\dagger(u) = h(u) u g_R^\dagger \quad , \quad u^\dagger \to g_R u^\dagger h^\dagger(u) = h(u) u^\dagger g_L^\dagger \quad , \quad N \to h(u)N$$

$$\bar{N} \to \bar{N}h^\dagger(u)\ ,\ v_\mu \to h(u)v_\mu h^\dagger(u) + h(u)\partial_\mu h^\dagger(u)\ ,\ a_\mu \to h(u)a_\mu h^\dagger(u)$$

  - Now we have to generalize them to local $SU_L(2)$, $g_L \to g_L(x)$
  - This is achieved by replacing $\partial_\mu u \to D_\mu u = (\partial_\mu - ia_{L\,\mu})u$

$$v_\mu \to v'_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger + u^\dagger D_\mu u\right) \quad , \quad a_\mu \to a'_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger - u^\dagger D_\mu u\right)$$

- This leads to

$$\mathcal{L}_{int}^{weak} = \frac{1}{2}\bar{N}\left(u^\dagger \not{a}_L u - g_A u^\dagger \not{a}_L u\gamma^5\right) N$$

- If we compare with the interaction at quark level we see that

$$J^\dagger_{q\,\mu} \to \frac{1}{2}\bar{N}\left(u^\dagger\tau^- u - g_A u^\dagger \tau^- u\gamma^5\right) N \simeq \bar{n}\,\gamma^\mu \frac{1-g_A\gamma^5}{2} p + \dots$$

- If we are interested in a process with no pions, like beta-decay, we may set $u=1$. Then we finally get,

$$\mathcal{L}_{int}^{weak} = 2\sqrt{2}G\,\bar{n}\,\gamma^\mu \frac{1-g_A\gamma^5}{2} p\,\bar{\nu}_l\,\gamma_\mu P_L l + \text{ H.c.}$$

- Recall that $g_A \simeq 1.27 \implies$ at nucleon level the weak interaction is not purely left $\implies$ the original $V-A$ proposal at nucleon level is not totally correct

$$\frac{1-g_A\gamma^5}{2} = \underbrace{\frac{1+g_A}{2}}_{\sim 1.13} P_L + \underbrace{\frac{1-g_A}{2}}_{\sim 0.13} P_R$$

Nuclear beta-decay

- Nuclei are bound states of non-relativistic nucleons $\Longrightarrow$ nuclear beta-decay is a low energy process $\Longrightarrow$ we can use the non-linear sigma model as a starting point
- Since nucleons are non-relativistic we can further simplify $\mathcal{L}_{int}^{weak}$ using Schrödinger fields

$$u_\lambda(\vec{p}) = \sqrt{E+m}\begin{pmatrix} \chi_\lambda \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_\lambda \end{pmatrix} \underset{|\vec{p}|\ll m}{\simeq} \sqrt{2m}\begin{pmatrix} \chi_\lambda \\ 0 \end{pmatrix}$$

$$N(x) = e^{-imx^0}P_+\varphi_N(x) \quad , \quad P_+ = \frac{1+\gamma^0}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$\varphi_N(x)$ is a Schrödinger spin 1/2 field

- Then

$$J^{\dagger}_{q\,\mu} \simeq \bar{n}\gamma^\mu \frac{1-g_A\gamma^5}{2}p \simeq \varphi_n^\dagger P_+\gamma^\mu \frac{1-g_A\gamma^5}{2} P_+\varphi_p e^{-i(m_p-m_n)x^0}$$

$$P_+\gamma^\mu P_+ = \left(1, \vec{0}\right) \quad , \quad P_+\gamma^\mu\gamma^5 P_+ = (0, \vec{\sigma})$$

which leads to

$$\mathcal{L}_{int}^{weak} = \sqrt{2}G\left(\varphi_n^\dagger\varphi_p\,\bar{\nu}_l\,\gamma_0 P_L l - g_A\,\varphi_n^\dagger\sigma^i\varphi_p\,\bar{\nu}_l\,\gamma_i P_L l\right)e^{-i(m_p-m_n)x^0} \;+\; \text{H.c.}$$

- The first term $(\varphi_n^\dagger \varphi_p)$, which does not depend on the spin, induces the Fermi transitions
- The second term $(\varphi_n^\dagger \sigma^i \varphi_p)$, which depends on the spin, induces the Gamow-Teller transitions
- If the initial and final nuclear states have $J^P = 0^+$ the second term does not contribute $\Longrightarrow$ the decay width does not depend on g_A
- If, in addition, the initial and final nuclear state belong to the same isospin multiplet, the form factor

$$\langle {}_{Z-1}^{A}X | \, \varphi_n^\dagger(0)\varphi_p(0) \, | {}_{Z}^{A}X \rangle$$

can be evaluated exactly. This is because $\varphi_n^\dagger \varphi_p$ replaces a proton by a neutron, and hence plays the role of a τ^- operator
- For instance ${}^{14}_{6}C$, ${}^{14}_{7}N^*$ and ${}^{14}_{8}O$ have $J^P = 0^+$ and form and isospin triplet
- In the decay ${}^{14}_{8}O \rightarrow {}^{14}_{7}N^* \, e^+ \, \nu_e$

$$\langle {}^{14}_{7}N^* | \, \varphi_n^\dagger(0)\varphi_p(0) \, | {}^{14}_{8}O \rangle \rightarrow \langle \underset{I\, I_3}{10} | \, \tau^- \, | \underset{I\, I_3}{11} \rangle = \sqrt{2}$$

- Then, in these cases, the Fermi constant can be measured accurately

$$G = 1.136(3)\, 10^{-5} \;\; \text{GeV}^{-2}$$

$$ {}^{14}_{\ 8}\underset{A}{O} \to {}^{14}_{\ 7}\underset{1}{N^*}\ \underset{2}{e^+}\ \underset{3}{\nu_e} $$

- We have

$$
\begin{aligned}
\mathcal{M} &= \sqrt{2}G \left\langle 1\right| \varphi_n^\dagger \varphi_p \left| A \right\rangle \left\langle 2,\, 3\right| \bar{\nu}_l\, \gamma_0 P_L l \left| 0 \right\rangle \\
&= \sqrt{2}G \left(\underbrace{\sqrt{2}}_{\text{Isospin}}\ \underbrace{\sqrt{2m_A}\sqrt{2m_1}}_{\text{rel.normalization}} \right) \bar{u}(3)\gamma^0 P_L v(2)
\end{aligned}
$$

- If the positron polarization is not measured

$$
\begin{aligned}
\left|\overline{\mathcal{M}}\right|^2 &= 16G^2 m_A m_1 \sum_{\lambda_2,\lambda_3=+,-} \operatorname{tr}\left(u(3)\bar{u}(3)\gamma^0 P_L v(p_2)\bar{v}(p_2)\gamma^0 P_L\right) \\
&= 16G^2 m_A m_1 \operatorname{tr}\left(\not{p}_3\gamma^0 P_L \left(\not{p}_2 - m_2\right)\gamma^0 P_L\right) \\
&= 8G^2 m_A m_1 \operatorname{tr}\left(\not{p}_3\gamma^0 \not{p}_2\gamma^0\right) = 32G^2 m_A m_1 (2p_2^0 p_3^0 - p_2 p_3) \\
&= 32G^2 m_A m_1 (E_2 E_3 + |\vec{p}_2| E_3 \cos\theta)
\end{aligned}
$$

$\hat{p}_2\hat{p}_3 \equiv \cos\theta$

- The decay width reads

$$\Gamma = \frac{1}{2m_A}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}|\overline{\mathcal{M}}|^2(2\pi)^4\delta(m_A-E_1-E_2-E_3)\delta(\vec{p}_1+\vec{p}_2+\vec{p}_3)$$

- The integral over $\vec{p}_1$ can be carried out, $E_1 \simeq m_1$

$$\Gamma = \frac{1}{2m_A\, 2m_1}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}|\overline{\mathcal{M}}|^2(2\pi)\delta(m_A-m_1-E_2-E_3)$$

- The integral over $\vec{p}_3$ can also be carried out, $E_3 = |\vec{p}_3|$

$$\Gamma = \frac{1}{2m_A\, 2m_1}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\theta(m_A-m_1-E_2)\,\frac{E_3}{2\pi}32G^2 m_A m_1 E_2 E_3\Big|_{E_3=m_A-m_1-E_2\equiv E_0-E_2}$$

- In the region $E_2 \gg m_2$, $E_2 \simeq |\vec{p}_2|$, the positron energy spectrum reads

$$\frac{d\Gamma}{dE_2} = \frac{G^2}{\pi^3}|\vec{p}_2|E_2(E_0-E_2)^2 \simeq \frac{G^2}{\pi^3}E_2^2(E_0-E_2)^2$$

- The total decay width reads

$$\Gamma = \int_{m_2}^{E_0} dE_2\frac{d\Gamma}{dE_2} \simeq \int_0^{E_0} dE_2\frac{G^2}{\pi^3}E_2^2(E_0-E_2)^2 = \frac{G^2E_0^5}{30\pi^3}$$

- Remarks:
  - Γ is very sensitive to the mass difference between nuclei ($\Gamma \sim E_0^5 = (m_A - m_1)^5$)
  - The energy spectrum has a maximum at $E_2 \sim E_0/2$ and it is symmetric around this point
  - The positron (electron) mass is not always neglegible (for instance at the lower end of the energy spectrum, or even in the total decay width when $E_0 \gtrsim m_e$
  - The Coulomb interaction of the positron (electron) with the nucleus must be taken into account for the fine details of the energy spectrum and for an acurate decay width

Neutron decay

$$n \to p\, e^-\, \bar{\nu}_e$$

- This is going to be the exercise for this week
- The non-relativistic approximation holds
- The term proportional to g_A contributes $\Longrightarrow$ we can measure g_A from neutron decay width

Muon decay

$$\underset{A}{\mu^-} \rightarrow \underset{1}{e^-}\ \underset{2}{\bar{\nu}_e}\ \underset{3}{\nu_\mu}$$

- Since $m_\mu \sim 106$ MeV $\gg m_e \sim 0.5$ MeV, we shall neglect m_e
- The relevant piece of the interaction Lagrangian reads

$$\mathcal{L}_{int} = 2\sqrt{2}G J_{e\,\mu}^{\dagger} J_m{}^{\mu} + \text{H. c.} \quad , \quad J_{e\,\mu}^{\dagger} = \bar{e}\,\gamma_\mu P_L\,\nu_e \quad , \quad J_m{}^{\mu} = \bar{\nu}_m\,\gamma^\mu P_L m$$

  m stands for the muon field

- We obtain

$$\mathcal{M} = 2\sqrt{2}G\bar{u}(3)\gamma^\mu P_L u(A)\bar{u}(1)\gamma_\mu P_L v(2)$$

- If the muon is not polarized and we do not measure the polarizations of the final particles

$$\left|\overline{\mathcal{M}}\right|^2 = \frac{1}{2}\sum_{\lambda_A,\lambda_1,\lambda_2,\lambda_3=+,-} 8G^2\left|\bar{u}(3)\gamma^\mu P_L u(A)\bar{u}(1)\gamma_\mu P_L v(2)\right|^2$$

$$= 4G^2\sum_{\lambda_A,\lambda_3=+,-}\text{tr}\left(u(3)\bar{u}(3)\gamma^\mu P_L u(A)\bar{u}(A)\gamma^\nu P_L\right)\sum_{\lambda_1,\lambda_2=+,-}\text{tr}\left(u(1)\bar{u}(1)\gamma_\mu P_L v(2)\bar{v}(2)\gamma_\nu P_L\right)$$

$$= 4G^2\text{tr}\left(\not{p}_3\gamma^\mu P_L\left(\not{p}_A + m_A\right)\gamma^\nu P_L\right)\text{tr}\left(\not{p}_1\gamma_\mu P_L\not{p}_2\gamma_\nu P_L\right)$$

$$\left|\overline{\mathcal{M}}\right|^2 = 4G^2 \mathrm{tr}\left(\not{p}_3\gamma^\mu \not{p}_A\gamma^\nu P_L\right) \mathrm{tr}\left(\not{p}_1\gamma_\mu \not{p}_2\gamma_\nu P_L\right) \equiv 4G^2 L^{\mu\nu}(p_3,p_A)L_{\mu\nu}(p_1,p_2)$$

$$L^{\mu\nu}(p_3,p_A) = 2\left(p_3^\mu p_A^\nu - g^{\mu\nu}p_3.p_A + p_3^\nu p_A^\mu\right) + 2i\epsilon^{\alpha\mu\beta\nu}p_{3\,\alpha}p_{A\,\beta}$$

$$L^{\mu\nu}(p_3,p_A)L_{\mu\nu}(p_1,p_2) = 16(p_Ap_2)(p_3p_1) \Longrightarrow \left|\overline{\mathcal{M}}\right|^2 = 64G^2(p_Ap_2)(p_3p_1)$$

Recall that $\epsilon^{\alpha\mu\beta\nu}\epsilon_{\alpha'\mu\beta'\nu} = -2(g^\alpha_{\alpha'}g^\beta_{\beta'} - g^\alpha_{\beta'}g^\beta_{\alpha'})$

- Notice that the amplitude may be written so that it does not depend on any angle:

$$(p_Ap_2) = m_AE_2 \quad , \quad (p_3p_1) = \frac{1}{2}(p_3+p_1)^2 = \frac{1}{2}(p_A-p_2)^2 = \frac{m_A^2}{2} - m_AE_2$$

- The decay width reads

$$\begin{aligned}
\Gamma &= \frac{1}{2m_A}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\int\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\int\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}|\overline{\mathcal{M}}|^2(2\pi)^4\delta(p_A-p_1-p_2-p_3)\\
&= \frac{1}{2m_A}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\int\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\int\frac{d^4p_3}{(2\pi)^3}\theta(p_3^0)\delta(p_3^2)|\overline{\mathcal{M}}|^2(2\pi)^4\delta(p_A-p_1-p_2-p_3)\\
&= \frac{1}{2m_A}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\int\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\theta(m_A-E_1-E_2)\delta((p_A-p_1-p_2)^2)|\overline{\mathcal{M}}|^2(2\pi)
\end{aligned}$$

- Consider

$$0 = (p_A - p_1 - p_2)^2 = m_A^2 - 2m_A(E_1 + E_2) + 2E_1E_2(1 - \cos\theta) \quad \Longrightarrow$$

$$\cos\theta = 1 + \frac{m_A\left(m_A - 2(E_1 + E_2)\right)}{2E_1E_2}$$

$$\cos\theta \leq 1 \Longrightarrow E_1 + E_2 \geq \frac{m_A}{2} \quad , \quad \cos\theta \geq -1 \Longrightarrow E_1 \leq \frac{m_A}{2} \, , \; E_2 \leq \frac{m_A}{2}$$

- Then

$$\begin{aligned}
\Gamma &= \frac{1}{2m_A}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\theta(\frac{m_A}{2} - E_1)\int_{\frac{m_A}{2}-E_1}^{\frac{m_A}{2}} \frac{dE_2 E_2}{(2\pi)^2 2}\frac{1}{2E_1E_2}|\overline{\mathcal{M}}|^2(2\pi) \\
&= \frac{1}{2m_A}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1^2}\theta(\frac{m_A}{2} - E_1)\int_{\frac{m_A}{2}-E_1}^{\frac{m_A}{2}} \frac{dE_2}{(8\pi)} 64G^2 m_A E_2\left(\frac{m_A^2}{2} - m_A E_2\right) \\
&= \frac{G^2}{\pi^3}\int_0^{\frac{m_A}{2}} dE_1\left(\frac{m_A E_1^2}{4} - \frac{E_1^3}{3}\right)
\end{aligned}$$

- The electron energy spectrum reads ($m_A = m_\mu$)

$$\frac{d\Gamma}{dE_1} = \frac{G^2 m_\mu^2 E_1^2}{4\pi^3}\left(1 - \frac{4E_1}{3m_\mu}\right)$$

  - Increases till the maximum energy kinematically allowed $E_1 = \frac{m_\mu}{2}$
  - It reaches the maximum energy with a zero slope
  - At small E_1 distorsions due to finite m_e are expected

- The decay width reads

$$\Gamma = \frac{G^2 m_\mu^5}{192\pi^3}$$

  - It allows to measure G independently of beta-decay measurements

$$G|_{\mu-\mathrm{decay}} = 1.16632(2)\, 10^{-5}\,\mathrm{GeV}^{-2}$$

  - Notice that there is a small but significant discrepancy with the value from beta decay measurements:

$$G|_{\beta-\mathrm{decay}} = 1.136(3)\, 10^{-5}\,\mathrm{GeV}^{-2}$$

The decay of the strange quark

- Let us consider $K^+ \to e^+\nu_e$
  - $\Gamma(K^+ \to e^+\nu_e) = 1.28\, 10^3 \text{ s}^{-1}$
  - Since K^+ has $J^P = 0^-$ like the π^+, if the s quark couples to the u quark in the same way as the d quark does in the Fermi Lagrangian, then, neglecting the electron mass,
  $$\frac{\Gamma(K^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to e^+\nu_e)} \simeq \frac{m_K f_K^2}{m_\pi f_\pi^2}$$
  f_K is the analogous form factor to f_π for the kaon
  - Approximate chiral $SU_L(3) \times SU_R(3)$ $\Longrightarrow$ $f_K \simeq f_\pi$
  - Then
  $$\frac{\Gamma(K^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to e^+\nu_e)} \simeq \frac{m_K}{m_\pi} \simeq 3.5$$
  - However
  $$\left.\frac{\Gamma(K^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to e^+\nu_e)}\right|_{\text{Exp}} \simeq 0.27$$
  $\Longrightarrow$ the s quark does not couple with the same strength to the u quark as the d quark does

- Let us then introduce new parameters, V_{us} and V_{ud}, to account for the different strenghts that the d and s quarks couple to u

$$J^\mu \to J^\mu = \bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu + V_{ud} \bar{u} \gamma^\mu P_L d + V_{us} \bar{u} \gamma^\mu P_L s + \dots$$

  - If we assume $V_{ud} \simeq 1$, we can estimate $|V_{us}|^2$ from the ratio $\Gamma(K^+ \to e^+ \nu_e)/\Gamma(\pi^+ \to e^+ \nu_e)$

$$|V_{us}| \sim 0.27$$

  - A more accurate estimate can be obtained from $K^0 \to \pi^- e^+ \bar{\nu}_e$, which does not depend on f_π or f_K. The current experimental value is

$$|V_{us}| \simeq 0.2249(10)$$

- Historically, V_{us} and V_{ud}, were introduced by Cabibbo (63) as $\sin\theta_c = V_{us}$ and $\cos\theta_c = V_{ud}$, with the following rational: the quark fields in J^μ are unitary transformations of the quark fields that have well defined mass

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J^\mu = \bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu + \bar{u} \gamma^\mu P_L d' + \dots$$

More quarks

- No quark field couples to s'? It seems natural that a new quark should exists that couples to s', this is the charm quark c

$$J^\mu = \bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu + \bar{u} \gamma^\mu P_L d' + \bar{c} \gamma^\mu P_L s' + \dots$$

- We shall discuss later on the rational for the unitary transformation
- With Cabibbo's modification we have

$$\begin{array}{rcl} G|_{\beta\,\text{decay}} & = & 1.136(3)\, 10^{-5}\,\text{GeV}^{-2} \\ \| & & \\ G|_{\mu\,\text{decay}} \cos\theta_c & = & 1.16632(2)\, 10^{-5}\,\text{GeV}^{-2} \times 0.9744 = 1,13644\, 10^{-5}\,\text{GeV}^{-2} \end{array}$$

  which solves the discrepancy

- The Cabibbo matrix is the most general 2×2 matrix that can be build once as many phases as possible have been absorbed by redefinitions of the quark fields
- It can be easily generalized to N d-type quarks (with $Q = -1/3$)
  - Consider V a $N \times N$ unitary matrix $\Longrightarrow$ $V_{ij} V^*_{ik} = \delta_{jk}$
  - For $j = k$, we have N constraints on the moduli
  - For $j \neq k$ we have $N(N-1)/2$ constraints on the modulus and the phases

- Then from the N^2 moduli and N^2 phases of an arbitrary $N \times N$ remain

$$\begin{aligned} \text{Moduli}: \quad N^2 - N - \tfrac{N(N-1)}{2} &= \frac{N(N-1)}{2} \\ \text{Phases}: \quad N^2 - \tfrac{N(N-1)}{2} &= \frac{N(N+1)}{2} \end{aligned}$$

- In addition, $2N-1$ phases can be absorbed in redefinitions of the quark fields q and q'
- The final count is then

$$\begin{aligned} \text{Moduli}: \quad & & \frac{N(N-1)}{2} \\ \text{Phases}: \quad & \tfrac{N(N+1)}{2} - (2N-1) = & \frac{(N-2)(N-1)}{2} \end{aligned}$$

- Note that for $N=2$, we have one modulus and no phase, in agreement with Cabibbo's parameterization
- For $N=3$, we have three moduli and one phase. This phase has important consequences: it leads to CP violation
- This analysis was first carried out by Kobayashi and Maskawa (73). The corresponding matrix for $N=3$ is called CKM matrix

CP violation

- For $N = 3$, three quark families, there is a phase in the CKM matrix $\Longrightarrow$ there is a complex term in the Fermi Lagrangian
- Suppose V_{ub} complex, there will be a term

$$V_{ub} V_{cs}^* \bar{u} \gamma^\mu P_L\, b\, \bar{s} \gamma_\mu P_L\, c + V_{ub}^* V_{cs} \bar{b} \gamma^\mu P_L\, u\, \bar{c} \gamma_\mu P_L\, s$$

- The CP transformation of a piece of current is, for instance,

$$(\bar{s} \gamma_\mu P_L\, c)(x) \underset{P}{\rightarrow} (\bar{s} \gamma^\mu P_R\, c)(\tilde{x}) \underset{C}{\rightarrow} -(\bar{c} \gamma_\mu P_L\, s)(\tilde{x})$$

- Then, upon changing $\tilde{x} \to x$ in the Lagrangian, the currents on the left transform into the currents on the right and viceversa
- However, that part of the Fermi Lagrangian is not invariant unless $V_{ub} V_{cs}^* = V_{ub}^* V_{cs}$, which is not the case if V_{ub} is complex
- The possibility that the weak interactions violate CP if a third family of quarks existed was realized before the b quark was discovered
- Its discovery (77) opened up the possibility of explaning the misterious CP violation observed in neutral kaon decays since the 60's in the electroweak theory

Neutral kaon decays

$$K^0 = (\bar{s}d) \qquad \bar{K}^0 = (\bar{d}s) \qquad J^P = 0^-$$

$$|\bar{K}^0\rangle \equiv C\,|K^0\rangle \quad \Longrightarrow \quad |K^0\rangle = C\,|\bar{K}^0\rangle$$

$$|K_S\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad |K_L\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP\,|K_S\rangle = |K_S\rangle \qquad CP\,|K_L\rangle = -\,|K_L\rangle$$

- K^0 and $\bar{K}^0$ may decay to $\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$
- In the two pion decay case the orbital angular momentum must be zero $\Longrightarrow$ $C\,|\pi\pi\rangle = |\pi\pi\rangle \;\Longrightarrow\; CP\,|\pi\pi\rangle = |\pi\pi\rangle$
- The three pion decay is dominated by the $L = 0$ orbital momenta $\Longrightarrow$ $C\,|\pi\pi\pi\rangle = |\pi\pi\pi\rangle \;\Longrightarrow\; CP\,|\pi\pi\pi\rangle = -\,|\pi\pi\pi\rangle$
- Then, if CP is conserved,

$$|K_S\rangle \to |\pi\pi\rangle \qquad |K_L\rangle \to |\pi\pi\pi\rangle$$

- Since $\tau(K_S) \sim 8.9\,10^{-11}$ s and $\tau(K_L) \sim 5.1\,10^{-8}$ s, if we wait long enough in a mixed sample of K^0 and $\bar{K}^0$ produced in a strong interaction process only K_L remain
- It was observed that K_L also decay to $\pi\pi$ with branching fractions (64)

$$\mathrm{BF}(K_L \to \pi^+\pi^-) \sim 2\,10^{-3} \qquad \mathrm{BF}(K_L \to \pi^0\pi^0) \sim 8\,10^{-4}$$

$\Longrightarrow$ *CP* is violated

- *CP* violation was not observed in other systems until the new milenium:
  - 2001: the so called *B*-factories (Belle and Babar) observed it in *B*-meson decay
  - 2019: LHCb observed it in *D*-meson decay
- *CP* violation $\Longrightarrow$ particles and antiparticles do not behave in the same way, for instance

$$\Gamma(K_L \to \pi^+\, e^-\, \bar{\nu}_e) \neq \Gamma(K_L \to \pi^-\, e^+\, \nu_e)$$

- *CP* violation is one of the three Sakharov conditions to explain the asymmetry betweeen matter and antimatter in the universe

T violation

- The same terms that violate CP in the Fermi Lagrangian also violate T (check it!)
- Time reversal violation has also been directly observed:
  - 1998: in K^0 decays (CPLEAR)
  - 2012: in B-meson decays (Babar)
- CPT is always respected

The CKM matrix

- The CKM matrix is nowadays known rather accurately
- Its structure is often summarize as follws:

$$V_{ij} = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{pmatrix} \qquad \lambda \sim 0.22$$

$$j = d, s, b \qquad i = u, c, t$$

- If kinematically allowed, quarks like first to decay within the same family and next to the closest family
- The size of CP violation can be estimated from Jarlskog's determinant: $\sim 2.96\,10^{-5}$ (note that $|\det V| \sim 1$)

Neutrino masses and oscillations

- The introduction of a right handed neutrino seems unnecessary since it does not interact, and hence it was not introduced neither in the Fermi theory nor in the electroweak theory
- However, it allows to write down a Dirac mass term for the neutrino
- Then we have a case similar to the quarks one: it may well be that the neutrino fields to put in the weak currents are not the ones with well defined mass but a unitary transformation of those
- This automatically leads to the analogous of the CKM matrix for the leptons, the Pontecorvo-Maki-Nakagawa-Sakata matrix, and hence to neutrino oscillations
- Nowadays neutrino oscillations have been observed by several kinds of experiments
  - Solar neutrinos: electron neutrinos emited in the Sun nuclear reactions. A deficit observed by Davis and others since the late 60's, and definitively confirmed by SNO in 2001
  - Atmospheric neutrinos: from muon decays induced by cosmic rays. The inbalance between muon to electron ratio confirmed by Kamiokande in 1998
  - Reactor neutrinos: neutrinos emitted in nuclear power plants
  - Beam neutrinos: neutrino beams produced in particle accelerators

- The current status of the PMNS matrix may be summarized as follows

$$U_{\mathrm{PMNS}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \mathcal{O}(\lambda) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad , \quad \lambda \simeq 0.22$$

- Note that the pattern is very different from the one in the quark sector
- This is an extra source of *CP*-violation
- If the neutrino masses are of Majorana type:
    - Two more phases appear $\Longrightarrow$ more sources of *CP*-violation
    - One does not need a right handed neutrino anymore
    - Lepton number violation should be observed, for instance by detecting a neutrinoless double beta decay (e.g. ${}^{128}_{52}\mathrm{Te} \rightarrow {}^{128}_{54}\mathrm{Xe}\ e^-\ e^-$)

5.3 Neutral currents

- The Fermi Lagrangian we have used so far contains a current J^μ which changes the electric charge by one ($J^\dagger_\mu$ by minus one). They are called charged currents.
- We may write

$$F = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \dots$$

$$J^\mu = \sum_F \bar{F}\tau^+\gamma^\mu P_L F' \quad , \quad J^\dagger_\mu = \sum_F \bar{F}'\tau^-\gamma_\mu P_L F$$

$F' = F$ for leptons, $F' = VF$ for quarks, $V =$ CKM-matrix

- Recall that $\tau^\pm$ are combinations of the τ^1 and τ^2 generators of $SU(2)$. If a $SU(2)$ group structure underlies the week interactions, we need the τ^3 generator $\Longrightarrow$ neutral currents
- Experimentally it was known that neutral currents that change flavor (FCNC) are very suppressed, e.g.

$$\frac{\Gamma(K^0_L \to \mu^+\mu^-)}{\Gamma(K^0_L \to \mathrm{all})} \simeq 9\ 10^{-9}$$

- We only need to include neutral currents that do not change the flavor. This is easy if the CKM matrix is unitary, since , for instance

$$J^\mu_{nc} \sim \sum_{F'} \bar{F}'\tau^3\gamma^\mu P_L F' = \sum_F \bar{F}\tau^3\gamma^\mu P_L F$$

- We will see that the structure of neutral currents is actually more complicated, but the key feature that they are invariant under unitary transformations of the fields remains
- The existence of neutral currents was suggested in 1958 by Bludman, and incorporated in the early version of the electroweak theory by Glashow (61) (with massless $W^{\pm}$ and Z^0 !)
- Since they do not change flavor $\Longrightarrow$ compete with the e.m. interactions $\Longrightarrow$ difficult to observe
- They were first detected at CERN in 1973 by Gargamelle, a bubble chamber experiment
  - A neutrino (antineutrino) beam was produced from decaying positively (negatively) charged pions and kaons $\pi^+, K^+ \to \mu^+ \nu_\mu$ ($\pi^-, K^- \to \mu^- \bar{\nu}_\mu$)
  - Boosted pions and kaons had been produced by colliding protons to a fixed target (Be)
  - The following processes were observed:

$$\bar{\nu}_\mu\, e^- \to \bar{\nu}_\mu\, e^-$$

$$\bar{\nu}_\mu\, N \to \bar{\nu}_\mu\, X \quad , \quad N = p\,,n \quad , \quad X = \text{hadrons with no muon}$$

$$\nu_\mu\, N \to \nu_\mu\, X \quad , \quad N = p\,,n \quad , \quad X = \text{hadrons with no muon}$$

- The above processes are second order in the Fermi Lagrangian, and hence very suppressed

- However, the experiment found

$$R_\nu = \frac{\sigma^{nc}(\nu_\mu N \to \nu_\mu X)}{\sigma^{cc}(\nu_\mu N \to \mu^- X)} = 0.31 \pm 0.01$$

$$R_{\bar\nu} = \frac{\sigma^{nc}(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma^{cc}(\bar\nu_\mu N \to \mu^+ X)} = 0.38 \pm 0.02$$

- $\Longrightarrow$ charged and neutral currents have roughly the same size
- Neutral currents were parameterized as follows

$$J^\mu_{nc} = \sum_f \bar f \gamma^\mu \frac{c_V^f - c_A^f\gamma^5}{2} f$$

$$\mathcal{L} \to \mathcal{L} = \mathcal{L}_{cc} + \mathcal{L}_{nc}$$

$$\mathcal{L}_{nc} = 4\sqrt{2} G \rho J^{\dagger\,\mu}_{nc} J_{nc\,\mu}$$

  f stands for fermion fields, either quarks or leptons. From now on the previous current $J^\mu \equiv J^\mu_{cc}$, and $\mathcal{L}_{cc}$ stands for the Fermi Lagrangian made with these charged currents
- The parameters ρ, c_V^f and c_A^f can be measured in high energy neutrino DIS

Neutrino deep inelastic scattering (DIS)

$$\nu_l \, N \to \nu_l \, X \quad , \quad \nu_l \, N \to l^- \, X$$
$$\bar{\nu}_l \, N \to \bar{\nu}_l \, X \quad , \quad \bar{\nu}_l \, N \to l^+ \, X$$

$l = e \, , \mu$

- For e^- DIS, the parton model provides a formula ($s \gg m_N$)

$$\frac{d\sigma}{dxdy} \simeq F_2(x) \frac{2\pi\alpha^2 s}{q^4} \left(1 + (1-y)^2\right) \quad , \quad F_2(x) = \sum_i f_i(x) Q_i^2 x$$

$$x = -\frac{q^2}{2m_N \nu} \quad , \quad \nu = \frac{p_N q}{m_N} \quad , \quad y = \frac{p_N q}{p_N p_A} \quad , \quad q = p_A - p_1$$

  p_A = momentum of the incoming e^-, p_1 = momentum of the outgoing e^-

- This formula allows to extract from data $F_2(x)$, which is a combination of $xf_{u_v}(x)$, $xf_{d_v}(x)$ and $xf_s(x)$
- Analogous formulas can be worked out for neutrino DIS:

$$\frac{d\sigma(\nu_l \, N \to l^- \, X)}{dxdy} = \frac{G^2 s}{\pi} \left(xf_d(x) + xf_{\bar{u}}(x)(1-y)^2\right)$$

$$\frac{d\sigma(\bar{\nu}_l \, N \to l^+ \, X)}{dxdy} = \frac{G^2 s}{\pi} \left(xf_{\bar{d}}(x) + xf_u(x)(1-y)^2\right)$$

- From these formulas $xf_{u_v}(x)$, $xf_{d_v}(x)$ and $xf_s(x)$ can be extracted from data $\Longrightarrow$ we know now how the nucleons (proton and neutron) are made out of partons (quarks)
- Similar formulas for $\nu_l\, N \to \nu_l\, X$ and $\bar{\nu}_l\, N \to \bar{\nu}_l\, X$ allow to extract ρ, c_V^f, c_A^f, and the parton charges Q^f, which we have assumed to coincide with the quark charges so far
- $\rho \simeq 1$ and the following results are obtained

	C_V^f	C_A^f	Q^f	
ν_l	$1/2$	$1/2$	0	(input)
e^-	-0.03	$-1/2$	-1	
u	0.19	$1/2$	$2/3$	
d	-0.34	$-1/2$	$-1/3$	

- The remarkable relation below holds

$$C_V^f = C_A^f - 2Q^f x \quad , \quad x \simeq 0.23 \quad \Longrightarrow \quad J^\mu_{nc} = \sum_F \bar{F}\frac{\tau^3}{2}\gamma^\mu P_L F - x\,\bar{F} Q_F \gamma^\mu F$$

$$Q_F = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \text{ for leptons} \quad , \quad Q_F = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \text{ for quarks}$$

- FCNC very suppressed $\Longrightarrow$ C_V^f and C_A^f do not depend on the family F