

**Quantum Field Theory, 2021/2022**

**Exercise sheet 3 part a: Complex Klein-Gordon Field**

**Hand-in: October 13, 2021**

- 3.1. Consider the complex Klein-Gordon field  $\phi(x)$ , and define the charge-conjugation transformation such that:

$$\phi(x) \rightarrow \mathcal{C}\phi(x)\mathcal{C}^{-1} = \eta_c\phi^\dagger(x)$$

with  $\mathcal{C}$  a unitary operator which leaves the vacuum invariant  $\mathcal{C}|0\rangle = |0\rangle$ , and  $\eta_c$  is a phase factor  $|\eta_c| = 1$ .

- (a) Show that, under this transformation, the Lagrangian density of the complex Klein-Gordon field is invariant, and the (electromagnetic) charge-current density changes sign.
- (b) Show that:

$$\mathcal{C}a_{\mathbf{k}}\mathcal{C}^{-1} = \eta_cb_{\mathbf{k}} \quad ; \quad \mathcal{C}b_{\mathbf{k}}\mathcal{C}^{-1} = \eta_c^*a_{\mathbf{k}} \quad ;$$

and, therefore:

$$\mathcal{C}|a, \mathbf{k}\rangle = \eta_c^*|b, \mathbf{k}\rangle \quad ; \quad \mathcal{C}|b, \mathbf{k}\rangle = \eta_c|a, \mathbf{k}\rangle \quad ;$$

where  $|a, \mathbf{k}\rangle$  is the state with a single  $a$ -particle of momentum  $\mathbf{k}$  present, etc.

[3 points]