Quantum Field Theory: Introduction

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What is Quantum Field Theory?

- Field Theory:
  Theory of Fields (space-time functions), e.g.: Classical Electrodynamics: $\partial_\mu F^{\mu\nu} = 0$
- Quantum Field Theory (QFT):
  Application of Quantum Mechanics to Field Theory
- Historically, it appears in the context of:

  Quantum Mechanics + Relativity

  $\Rightarrow$ Relativistic QFT, as used in e.g. particle physics
- Non-relativistic QFT also exists:

  $\Rightarrow$ Condensed matter
- Present lectures:

Relativistic QFT, applications to particle physics

Conventions

Metric & vectors

$$
\begin{aligned}
g^{\mu\nu} &= (1, -1, -1, -1) \\
x^{\mu} &\equiv (x^0, \vec{x}) \equiv (x^0, x^i) \equiv (x^0, \boldsymbol{x}) \\
x^2 &\equiv x^{\mu} x_{\mu} = (x^0)^2 - \boldsymbol{x}^2 \\
p^2 &\equiv p^{\mu} p_{\mu} = \left(\frac{E}{c}\right)^2 - \boldsymbol{p}^2 = m^2 c^2 \\
xp &= p^{\mu} x_{\mu} = x^0 p^0 - \boldsymbol{x} \cdot \boldsymbol{p}
\end{aligned}
$$

Natural Units: $\hbar = c = 1$

$$
\begin{array}{rcl}
p^2 & \equiv & p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2 \quad , \\
e^{-iEt/\hbar} & = & e^{-iEt} \\
[E] & = & [x]^{-1} = [t]^{-1} \\
\text{Lagrangian} & & L = [E] \\
\text{Action} & & S = \displaystyle\int \mathrm{d}t\, L = \text{ no units} \\
\text{Lagrangian density: } \mathcal{L}: & & S = \displaystyle\int \mathrm{d}^4x\, \mathcal{L} \Rightarrow \mathcal{L} \equiv [E]^4 \\
\dfrac{\partial}{\partial x^\mu} & \sim & \dfrac{1}{x^\mu} \sim [E]
\end{array}
$$

Conversion constants

$$
\begin{aligned}
\hbar c &= 197.32\ \mathrm{MeV}\cdot\ \text{fm} \simeq 200\ \mathrm{MeV}\cdot\ \text{fm}\\
1\ \text{fm} &\simeq 10^{-15}\ \text{m} \equiv \text{ proton size}\\
1\ \text{barn} &= 100\ \text{fm}^2 = 10^{-28}\ \text{m}^2\\
(\hbar c)^2 &= 0.389\,\mathrm{GeV}^2\cdot\ \text{mbarn} \simeq 0.4\,\mathrm{GeV}^2\cdot\ \text{mbarn}\\
\text{mbarn} &\simeq \frac{1}{0.4}\mathrm{GeV}^{-2} = 2.5\,\mathrm{GeV}^{-2}\\
10\ \text{mbarn} &\simeq 25\,\mathrm{GeV}^{-2}
\end{aligned}
$$

Why QFT?

Relativistic QM:

- Non-relativistic free-particle:

$$\hat{E} = \frac{\hat{\boldsymbol{p}}^2}{2m} \Rightarrow i\frac{\partial \phi}{\partial t} = -\frac{\nabla^2}{2m}\phi \ , \ \ (\hat{E}, \hat{\boldsymbol{p}}) = i\left(\frac{\partial}{\partial t}, -\nabla\right)$$

- Relativistic free-particle:

$$\begin{aligned} \hat{E}^2 - \hat{\boldsymbol{p}}^2 &= m^2 \\ -\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi - m^2 \phi &= 0 \\ -\partial^\mu \partial_\mu \phi - m^2 \phi &= 0 \end{aligned}$$

Klein-Gordon equation

$$\partial^\mu \partial_\mu \phi + m^2 \phi \equiv \Box \phi + m^2 \phi = 0$$

2nd order in $t \Rightarrow$ 2 solution for each energy: $e^{-iEt} \ , \ \ e^{+iEt}$

$\Rightarrow$ Negative energy solutions!

Plane waves:

$$\phi_p = e^{\pm i(Et - \boldsymbol{p}\cdot\boldsymbol{x})} = e^{\pm ip^\mu x_\mu} = e^{\pm ipx}$$

We can write a **probability current**:

$$J^\mu = i\left(\phi^*\partial^\mu\phi - (\partial^\mu\phi^*)\phi\right)$$

The **probability density**:

$$J^0 = i\left(\phi^*\partial_t\phi - (\partial_t\phi^*)\phi\right)$$

not positive definite $\Rightarrow$ no probability interpretation!

- QM offers **no explanation of light "*quanta*"**
  particles are *quantized* into *waves* (fields), but **Electromagnetism** is treated as a **classical "field"**, with a quantum ad-hoc rule $E = h\nu$!
- QM offers **no explanation of anti-particles**
- QM offers **no explanation of particle creation**
- Levels of description of light/particles

Light	Particles	
Geometrical Optics Fermat principle	Classical Mechanics Action principle	linear trajectory
Maxwell eqs.	Schrödinger eq.	wave
Light Quanta $E = h\nu$ Creation	???	???

$\Rightarrow$ **Quantum Field Theory**

QFT setup

- The objects to quantize are the **Fields**: wave functions of QM.
- Need to study **Classical Field Theory**
- Study its symmetries, specifically:
  Lorentz & Poincaré symmetries of special relativity