Lecture 6:
The Cosmic Microwave Background and the Epoch of Recombination.

L: In 1964, the Cosmic Microwave Background was discovered by Penzias and Wilson. The blackbody spectrum at $T_0 = 2.725\,\mathrm{K}$ was consistent with the first observations from the ground, and was confirmed spectacularly with the COBE mission, where the FIRAS experiment measured the absolute intensity at many frequencies and found that it was precisely a blackbody.

L: Review Planck blackbody spectrum:

$$\epsilon_\nu(T)\, d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{e^{h\nu/kT} - 1} \ . \tag{1}$$

When we integrate the energy density over all frequencies we obtain $\epsilon = \alpha T_0^4 = 4.18 \times 10^{-13}\,\mathrm{erg\,cm^{-3}}$, where I write the Boltzmann constant as $\alpha = 7.56 \times 10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}$ to avoid confusion with the scale factor. You should show as an exercise that $\Omega_{CMB,0} = 5.05 \times 10^{-5}$ for $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Q: Why is there this blackbody radiation in the Universe?

L: In the Big Bang model, the Universe in the past was much denser and hotter. Matter then was in equilibrium, because a lot of interactions occurred and brought everything to equilibrium. So a remnant thermal radiation should be present from the early Universe, that has simply been redshifted from the early epoch.

L: There is a correction for neutrinos. The radiation in equilibrium also contained neutrinos at very high temperatures, which stopped interacting with the matter much earlier than photons. In detail, $\Omega_\nu = 3(7/8)(4/11)^{4/3}\,\Omega_{CMB} = 0.681\Omega_{CMB}$.

L: The temperature of the neutrinos is $(4/11)^{1/3}$ times that of photons, because when neutrinos decoupled, the temperature was high enough for electron-positron pairs to be also in equilibrium and nearly as abundant as photons. Later when these electron-positron pairs annihilated, the CMB temperature was increased relative to the expected decline as $T_{\mathrm{CMB}} \propto 1 + z$, because of the energy from these pairs, so the neutrinos were left cooler. In addition, fermions have a density 7/8 that of bosons when they are at the same temperature; and there are three independent neutrino families. In a more detailed calculation, there is an "effective" number of neutrino families, $N_{\mathrm{eff}} = 3.046$, arising from the fact that the neutrinos that were left over in the cosmic background were not exactly in thermodynamic equilibrium, because of the slow reaction rate during the epoch of their decoupling. This leads to this energy density slighly larger than assuming equilibrium before electron-positron annihilation. If there are other unknown particles (such as axions), which can contribute to the energy density of the Universe at early epochs, this can affect N_{eff}, which can be measured in the present Universe in various ways. This can also be affected by the mass of neutrinos, in particular if we want to compute the radiation contribution at late times (close to our epoch), but in practice N_{eff} has been measured from the CMB fluctuations, at the time of recombination. This will be treated in more detail later in the course.

L: So the total radiation density, with $N_{\mathrm{eff}} = 3.046$ is $\Omega_{r,0} = 1.692\Omega_{\mathrm{CMB},0} = 8.5 \times 10^{-5}$.

Q: How about the contribution to Ω_r from other types of radiation? How about light from stars?

L: The emissivity from all known galaxies at present has been measured from cosmological surveys, by counting how many galaxies of various luminosities exist, averaged over large volumes. This emissivity is $\rho_L \sim 2 \times 10^8\, L_\odot\,\mathrm{Mpc^{-3}}$, and in cgs units is $2.6 \times 10^{-32}\,\mathrm{erg\,s^{-1}\,cm^{-3}}$. If we assume the comoving emissivity is about constant for the age of the Universe of $4.4 \times 10^{17}\,\mathrm{s}$, we obtain about $10^{-14}\,\mathrm{erg\,cm^{-3}}$. More detail modeling of the galaxy evolution, and taking into account the redshifting of this light, increases this a bit, but not much: estimates of all radiation from galaxies and quasars are $\sim 10\%$ of the CMB energy density. It is smaller than the energy density in the Cosmic Microwave Background, but not by a big factor... There are also neutrinos from stars and supernovae, and gravitational waves, but this does not add much to the total radiation in the form of photons from stars.

Q: The radiation is clearly unimportant today. But was it important in the past? When were the energy densities of matter and radiation equal?

L: Radiation contribution in the past, neglecting dark energy at $z \gg 1$:

$$\Omega_r = \frac{\Omega_{r0}(1+z)}{\Omega_{m0} + \Omega_{r0}(1+z)} \ . \tag{2}$$

At $1+z = \Omega_{m0}/\Omega_{r0} = 3500 = 1/a_{rm}$. This is called the *equalization* epoch. The Big Bang Universe was radiation dominated before the equalization epoch.

Q: Was the vacuum energy important at that high redshift? $\Omega_\Lambda = \frac{\Omega_{\Lambda 0}}{\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{\Lambda 0}} \ll 1$.

Q: Let us consider the Universe when it was radiation dominated. Can we calculate its age, without any other information?

L: If the Universe is radiation dominated, $H(z) = H_0\sqrt{\Omega_{r0}}(1+z)^2$, so

$$t = \int_z^\infty \frac{dz}{H(z)(1+z)} = \frac{1}{2H_0\sqrt{\Omega_{r0}}}\,\frac{1}{(1+z)^2} = \ . \tag{3}$$

In terms of the radiation energy density ϵ_r, this is:

$$t = \frac{1}{2H} = \frac{1}{2}\sqrt{\frac{3c^2}{8\pi G \epsilon_r}} \ . \tag{4}$$

Q: What was the temperature of the universe at matter-radiation equality?

Summary: The universe was dominated by radiation before the equalization epoch at $z_{\rm eq} \simeq 3500$, then it became matter dominated, and very recently it has become dark energy dominated.

The epoch of recombination

Q: We imagine observing the CMB today. What was the last time these photons interacted with matter? What was the physical process determining this interaction?

Q: First of all, let us imagine all the matter was ionized. What do you thing is the physical process that causes the dominant interaction with matter?

L: Compton scattering, or at the low frequencies we are interested in, Thompson scattering as described by classical electromagnetism.

Q: What are the particles that dominate the interaction? Electrons.

L: Thompson cross section:

$$\sigma_e = \frac{8\pi}{3} r_e^2 = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\,\mathrm{cm}^2 \ . \tag{5}$$

Q: What is the column density of matter necessary for ionized matter to start being opaque?

$$N_e = \sigma_e^{-1} = 1.5 \times 10^{24}\,\mathrm{cm}^{-2} \ . \tag{6}$$

Compare that to two things: a piece of paper, of width ~ 0.1 mm. Typical organic matter has a density of $1\,\mathrm{g\,cm}^{-3}$, with $\sim 3\times10^{23}$ electrons per gram. This implies an electron column density of $\sim 3\times10^{21}\,\mathrm{cm}^{-2}$. If the matter were ionized the optical depth would be only 0.002, but it is much greater for visual photons! Not so for radio or X-ray. Molecules in the solid greatly increase the optical depth in optical bands.

Q: What is the column density of the Earth atmosphere? Density is $\sim 10^{-3}\,\mathrm{g\,cm}^{-3}$, and height is $\sim 10^6$ cm, this gives $\sim 3\times10^{26}\,\mathrm{cm}^{-2}$, if it were ionized the optical depth would be ~ 200. But we can see the stars!

L: Atoms and molecules are neutral, and in a gas this greatly reduces the cross section, except for a few resonance lines of the atoms and molecules.

Q: So what is the optical depth due to ionized baryonic matter?

$$d\tau = n_e \sigma_e\, c\, dt \ ; \qquad \tau = c\int_{t_e}^{t_0} n_e\sigma_e\, dt = n_{e0}\sigma_e c \int_0^{z_e} \frac{(1+z)^2\, dz}{H(z)} \ . \tag{7}$$

If we integrate this optical depth to high redshift ($z_e \gg 1$), but still low compared to the equalization epoch, and we neglect dark energy and radiation in the epoch where matter dominates, and neglect the small contribution from very low redshift at which dark energy dominates, then we use $H(z) = H_0\Omega_{m0}^{1/2}(1+z)^{3/2}$, and

$$\tau = \frac{2n_{e0}\sigma_e c}{3H_0\Omega_{m,0}^{1/2}}(1+z)^{3/2} \ . \tag{8}$$

If we put numbers (we can assume for fully ionized matter $n_{e0} \simeq \rho_{c0}\Omega_{b0}/m_p(1-Y/2)$, where $Y \simeq 0.25$ is the helium abundance by mass and the rest $1-Y$ is hydrogen), we find that τ reaches unity at a redshift $z_e \simeq 62$, when the temperature was $T_{CMB} \simeq 170$ K. So, the CMB photons would come from roughly redshift 60, and no photons of any kind would be able to reach us from higher redshift without having been scattered in between by the intergalactic medium.

L: But in fact, the Universe was not always ionized! So the Universe is transparent up to a higher redshift.

Q: Why did the Universe stop being ionized? At some stage, atoms formed. The ionization potential of hydrogen is: $Q = 1\,\mathrm{Rydberg} = 13.6\,\mathrm{eV}$. If we are very naive, we would think atoms will be favored starting at a temperature $T \simeq Q/k \simeq 1.6\times10^5$ K, which was reached at redhift $z \simeq 60000$.

Q: Why is this estimate off by a large factor?

L: Comoving number density of CMB photons: $n_\gamma = 411\,\mathrm{cm}^{-3}$. Comoving number density of baryons: $n_b \simeq 2.5\times10^{-7}\,\mathrm{cm}^{-3}$. A free electron has a very large number of quantum states available, compared to few ones in an atom, so it may prefer to be in one ionized state even if some atomic states are at much lower energy compared to kT.

L: Statistical mechanics: a non-relativistic species x will have number density in thermodynamic equilibrium of

$$n_x = g_x \left(\frac{2\pi m_x kT}{h^2}\right)^{3/2} e^{-m_x c^2/(kT)}\, e^{\mu_x/(kT)} \ . \tag{9}$$

For simplicity we ignore the presence of helium. The particle mass is m_x, g_x is the degeneracy factor and μ_x is the chemical potential. The factor $(2\pi m_x kT)^{3/2}/h^3$ represents the density of kinematic quantum states with energies that are not too high compared to kT. Remember that the quantum state density in phase space is h^{-3}, and the momentum of a non-relativistic particle with energy kT is $(2m_x kT)^{1/2}$. This applies to electrons, protons and hydrogen atoms.

Saha equation: in equilibrium of the reaction

$$H + \gamma \to p + e \tag{10}$$

(meaning that atoms are in equilibrium with ions and electrons), the chemical potentials are related by $\mu_H = \mu_e + \mu_p$ (the chemical potential of photons in a blackbody spectrum is zero). This leads to the Saha equation:

$$\frac{n_H}{n_e n_p} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_e m_p}\right)^{3/2} \left(\frac{2\pi kT}{h^2}\right)^{-3/2} e^{(m_p+m_e-m_H)c^2/(kT)} \simeq \left(\frac{2\pi m_e kT}{h^2}\right)^{-3/2} e^{Q/(kT)} \ . \tag{11}$$

Q: Solve as exercise: when was the neutral fraction equal to 1/2?

If $n_p = n_e = xn$, $n_H = (1-x)n$, then

$$\frac{1-x}{x^2} = n \left(\frac{2\pi m_e kT}{h^2}\right)^{-3/2} e^{Q/(kT)} \ . \tag{12}$$

L: Example: at $T = 3000$ K, the quantity $(2\pi m_e kT/h^2)^{3/2} \simeq 4 \times 10^{20}\,\mathrm{cm}^{-3}$. This quantity represents the density of kinematic quantum states accessible to electrons with energies comparable to kT.

L: The redshift of last-scattering is that when the optical depth is one:

$$\tau = n_{e0}\sigma_{e0}c \int_0^z \frac{(1+z)^3 x(z)\,dz}{H(z)(1+z)} = 1 \tag{13}$$

The detailed computation, including details like helium and departures from thermal equilibrium due to scattering of Lyman alpha photons and slow rate of recombination of protons and electrons to hydrogen compared to the Hubble expansion rate, is that the last scattering surface is at $z = 1089$, at $T \simeq 3000K$.

L: Recombination is rather abrupt: because of the exponential factor $e^{Q/(kT)}$, hydrogen recombines suddenly when T decreases. This means the last-scattering surface is rather narrow (with $\Delta z \ll 1 + z$). However, when the ionization becomes rather small, recombinations become slow compared to the Hubble rate of expansion. This means there is a residual ionization left over from the early Universe at the level $x \sim 10^{-4}$.

Q: Is the Universe still atomic at present?

L: We know the Universe is ionized at present because we do not see much Lyα absorption from the intergalactic medium. The Universe was reionized when galaxies and quasars started emitting ultraviolet light from their hot, young stars and accreting black holes. This new radiation resulting from gravitational collapse of galaxies and formation of stars and black holes reionized the Universe. We have measured an optical depth to the CMB of $\sim 6\%$ from the latest Planck mission measurements.

Q: What does that imply about the redshift when reionization occurred?