QUANTUM FIELD THEORY

Fall 2021

Scalar contribution to the anomalous magnetic moment of the electron

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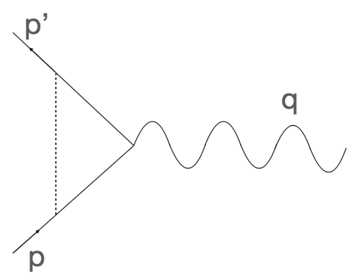
Consider a real scalar ϕ with physical mass M coupled to the electron field by $g\phi\bar{\Psi}\Psi$, with g a dimensionless coupling. We want to find the one-loop contribution of this scalar to the magnetic form factor $F_2(q^2)$ of the electron. To do so, we must compute this diagram



While the full diagram is UV divergent, the contribution to $F_2(q^2)$ is UV finite, so you can work in $d = 4$. Assume that the incoming and outgoing electrons are on-shell, so $p^2 = p'^2 = m^2$. Also, you can substitute $\not{p} \to m$ when $\not{p}$ is rightmost in the numerator, and $\not{p}' \to m$ when $\not{p}'$ is leftmost.

1. Following the steps sketched in class for the similar diagram with an internal photon (but with much less algebra!), arrive at an expression of the form

$$\int_0^1 dxdydz\, \delta(x+y+z-1) \int \frac{d^4\ell}{(2\pi)^4} \frac{N_1(\ell^2, q^2, m^2, x, y, z)\gamma^\mu + N_2(z, m) i\sigma^{\mu\nu} q_\nu}{[\ell^2 - \Delta]^3}$$

   Make sure that the N_1, N_2 that you find depend only on the Lorentz scalars specified above **(7 points)**.

2. From the previous expression, write the one-loop contribution to $F_2(q^2 = 0)$ as an integral over a single Feynman parameter **(2 points)**

$$\Delta F_2(q^2 = 0) = \int_0^1 dz\, f(z, m^2/M^2)$$

3. The previous integral can be carried out exactly, but the result is rather complicated. Assume that $M \gg m$, and expand the result to leading order in m^2/M^2. You should find a result of the form

$$\Delta F_2(q^2 = 0) = \frac{g^2 m^2}{M^2} \left(a + b \log \frac{m^2}{M^2} + \dots \right)$$

with a, b numbers that you have to determine **(1 point)**.