Quantum Teleportation

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Outline

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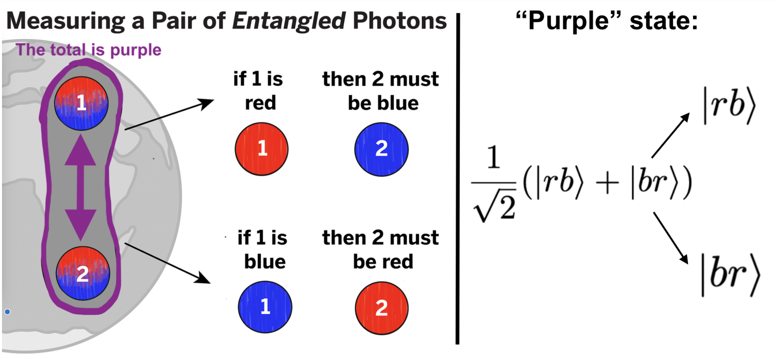
Quantum entanglement



So, if we have the following state and we measure the 1 in the first qubit:

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \xrightarrow{\text{q1} = 1} |q2 = 0\rangle$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{\text{q1} = 1} |q2 = ?\rangle$$

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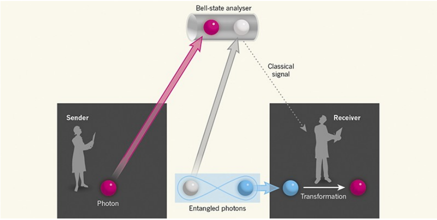
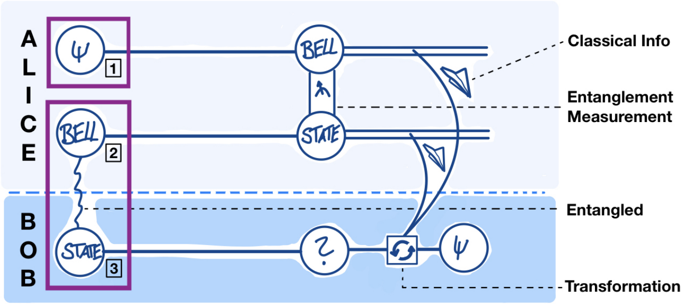
Introduction to Quantum Teleportation

Quantum teleportation:

- **IS NOT:** Instantaneous travel like depicted in films.
- **IS:** A way to transfer actual quantum states.



I. Initial set up for Quantum Teleportation



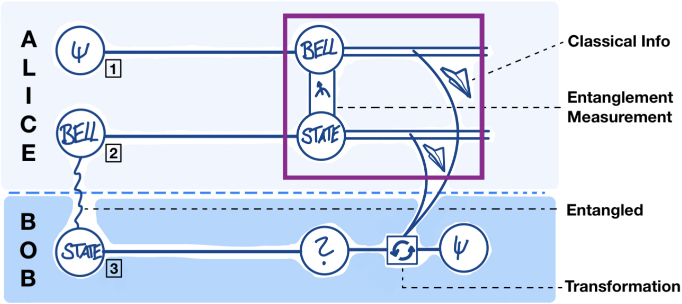
Alice has a **qubit 1** in an **UNKNOWN** state $|\psi\rangle_1$:

$$|\psi\rangle_1 = \alpha\,|0\rangle + \beta\,|1\rangle$$

Qubits **2 (Alice)** and **3 (Bob)** are **entangled**:

$$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

II. Bob states, depending on Alice's result



$$\left.\begin{array}{lcl} |A\rangle_{12} & \longrightarrow & |\psi\rangle_3 = \alpha\,|0\rangle_3 + \beta\,|1\rangle_3 \\ |B\rangle_{12} & \longrightarrow & |\psi\rangle_3 = \alpha\,|0\rangle_3 - \beta\,|1\rangle_3 \\ |C\rangle_{12} & \longrightarrow & |\psi\rangle_3 = \alpha\,|1\rangle_3 + \beta\,|0\rangle_3 \\ |D\rangle_{12} & \longrightarrow & |\psi\rangle_3 = \alpha\,|1\rangle_3 - \beta\,|0\rangle_3 \end{array}\right\} \xrightarrow{\hat{U}} \underbrace{|\psi\rangle_3 = \alpha\,|0\rangle + \beta\,|1\rangle}$$

qubit 3 **transforms** into the original state of qubit 1!!!

Teleportation happened!

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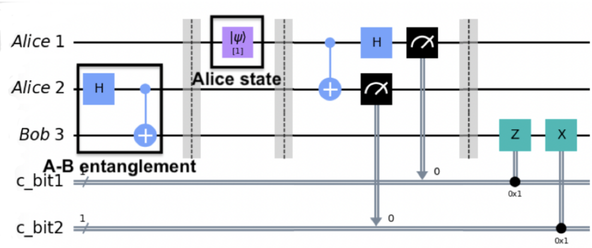
I. Set up, for basic Quantum Teleportation circuit




```python
# Define teleport circuit
c = Circuit(3)

# Initial Alice random state
c.add(gates.U1q(q=0, theta=theta, phi=phi))

# Initial Alice-Bob entangled state
c.add(gates.H(1))
c.add(gates.CNOT(1,2))
```

```
Circuit:
q0: -U1q──o─H─M─o─────
q1: -H────o─X─M──|─o──
q2: ──────X──────Z─X─M─
```

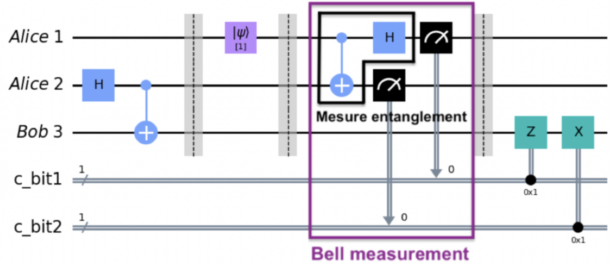
Alice's state, to teleport:

$$|\psi\rangle_1 = \alpha\,|0\rangle + \beta\,|1\rangle$$

And the initial Alice-Bob entangled state:

$$|\psi\rangle_{23} = CNOT(2,3)\ H(2)\ |00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

II. Bell measurement in a Quantum Teleportation circuit



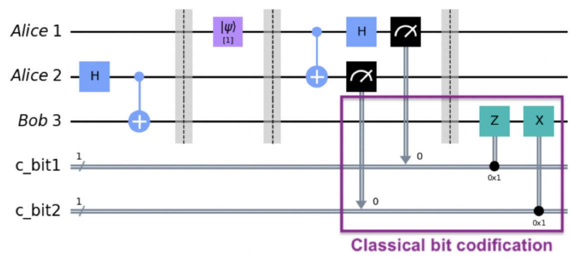

```
# Entanglement measure of Alice
c.add(gates.CNOT(0,1))
c.add(gates.H(0))
c.add(gates.M(0))
c.add(gates.M(1))
```

```
Circuit:
q0: ─U1q──o─H─M─o──────
q1: ─H───o─X─M───|─o───
q2: ─────X─────────Z─X─M─
```

Since we can only measure individual qubits in their Z axis, we translate Alice's double measurement with this "reverse" entanglement measure.

$$|\Psi_{123}\rangle = \frac{1}{2}\{\,|00\rangle_{12}\,(\alpha\,|0\rangle_3 + \beta\,|1\rangle_3) + |01\rangle_{12}\,(\alpha\,|1\rangle_3 + \beta\,|0\rangle_3) + \\ + |10\rangle_{12}\,(\alpha\,|0\rangle_3 - \beta\,|1\rangle_3) + |11\rangle_{12}\,(\alpha\,|1\rangle_3 - \beta\,|0\rangle_3)\,\}$$

III. C-bits encoding in a Quantum Teleportation circuit




```
# From Alice's results, Bob does control Z's & X's:
c.add(gates.CZ(0,2))
c.add(gates.CNOT(1,2))
c.add(gates.M(2, register_name="measure"))

# Plot the circuit:
print(f"Circuit:\n{c.draw()}\n")
```

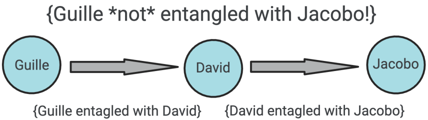
```
Circuit:
q0: ─U1q──o─H─M─o─────
q1: ─H──o─X─M───|─o───
q2: ────X───────Z─X─M─
```

Depending on what Alice measures, we have these 4 possibilities:

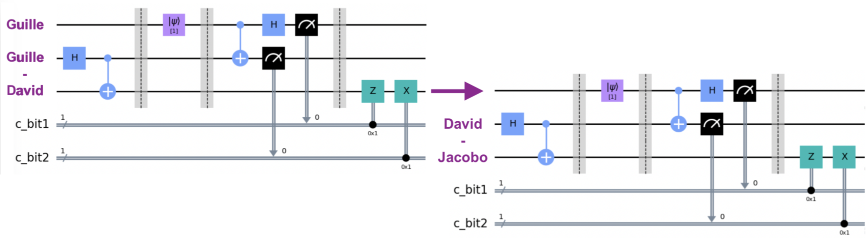
Alice measure	Bob state (ψ_3)	Gates to get ψ_1 on ψ_3
00	$\alpha\|0\rangle + \beta\|1\rangle$	$\mathbb{I}$
01	$\alpha\|1\rangle + \beta\|0\rangle$	$\mathbb{X}$
10	$\alpha\|0\rangle - \beta\|1\rangle$	$\mathbb{Z}$
11	$\alpha\|1\rangle - \beta\|0\rangle$	$\mathbb{ZX}$

IV. Consecutives Quantum Teleportations

Using previous output as input for a new teleportation:

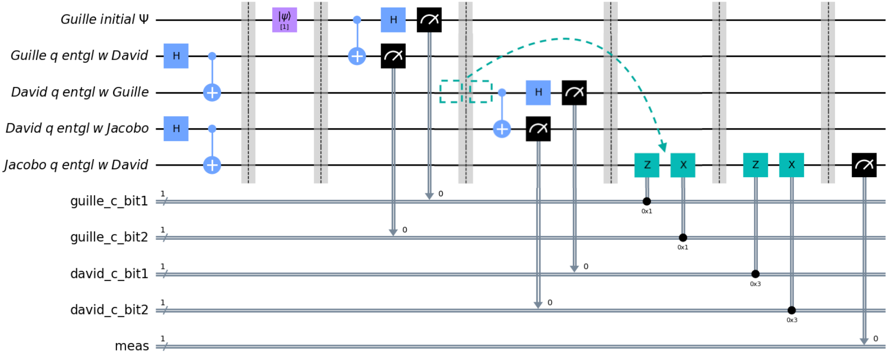


Notice that this will "consume" two entanglements!



V. Secure consecutive Quantum Teleportations

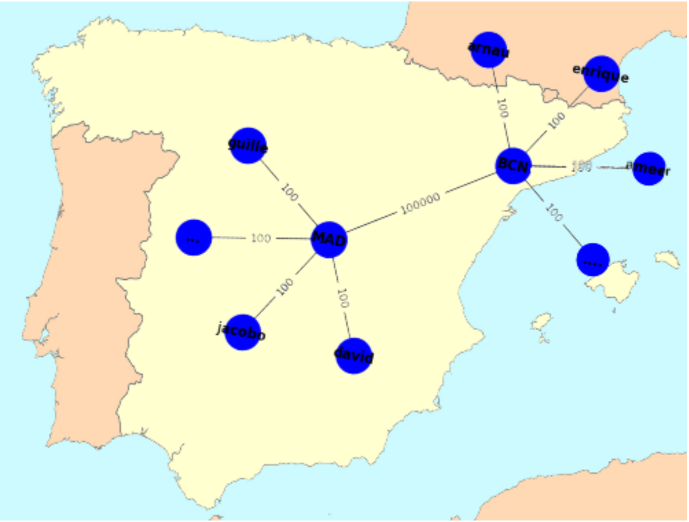
Consecutive teleportations can be secure, if we move all the decoding to the end:



the intermediaries can interrupt such communication, but never obtain it!

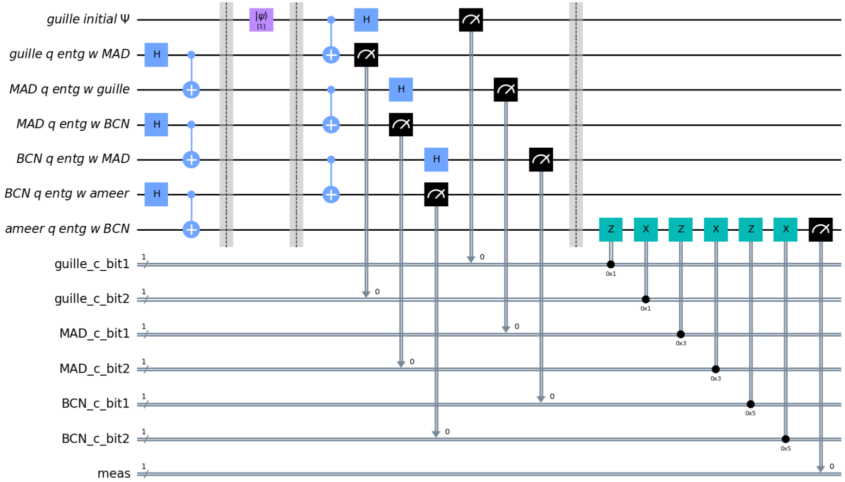
V. Quantum teleportation networks

Quantum teleportation networks, with entanglement highways:



VI. Quantum teleportation network algorithm

In the case, Guille in MAD, wanted to send a state to his friend Ameer in BCN, for the above network, the algorithm automatically generates:



Thank you for your time!