Problem 1: Homogeneous Fermi gas

Consider an ideal Fermi gas with a single spin component and dispersion relation $\epsilon_k = \hbar^2 k^2/2m$.

1. The ground state is a Fermi sea filled up to the Fermi wavevector k_F and Fermi energy $\epsilon_k = \epsilon(k_F)$. Compute the density n and total energy E in $d = 1, 2, 3$ dimensions. Give the polytropic index γ in the equation of state $\mu(T = 0) = \epsilon_k \propto n^\gamma$ in dimension d.

2. Compute the density of states

$$\rho(\epsilon) = \frac{1}{V} \sum \delta(\epsilon - \epsilon_k) \tag{1}$$

in d = 1, 2, 3 dimensions. Give $\rho(\epsilon_F)$ both in terms of k_F, and in terms of n and ϵ_F.

Problem 2: Phonons

A monoatomic linear chain consisting of atoms of mass $m = 6.81\text{x}10^{-26} kg$, with an equilibrium separation of 4.85 Å, has a propagation speed for sound waves of $1.08\text{x}10^4 m/s$. Assuming a classical model with nearest neighbor interaction, determine the value of the elastic constant and the maximum frequency of the modes.