Review of Quantum Teleportation landmark papers

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In this review, we start by introducing the historical development of quantum teleportation. In the next two sections, we discuss the theoretical concept of quantum teleportation, followed by a review of the first experimental proof. In the last section, we present simulations of quantum teleportation made on *qiskit* and an algorithm that automatically generates a circuit for securely sending a state, given a quantum network.

I. INTRODUCTION

When Anton Zeilinger, winner of last year's Nobel Prize in Physics, first read Bennet et al. (1993) [1] on the "Teleportation of unknown Quantum States", he thought to himself that it was a typical theoretical idea that wouldn't be realizable with an experimental setup.

At that time Quantum Information was still a very young field and the production and measurement of entanglement in labs still posed many challenges. For example, while it was possible to produce an entangled pair of photons, multipartite entanglement systems were not. But little did Zeilinger know, that by that time, they had already started to develop the needed tools to such achievement, with their GHZ experiment[2].

Four years later the experimental implementation was successfully developed [3], proving Quantum Teleportation. The experiment not only paved the way for many future applications but proved once again Quantum Mechanics' nonlocality and further established this new field of Quantum Information (...and won a Nobel Prize).

II. THE CONCEPT OF QUANTUM TELEPORT

Quantum teleportation states that it's theoretically possible to teleport a quantum state to another place instantaneously (without violating special relativity, as we will see later). Even more surprising, the sender who we will name Alice is able to send a quantum state to Bob, the receiver, without even knowing the state she is sending or where he is located.

But what do you need for this mechanism to work? Briefly said, for Alice to teleport a quantum state, she must share an entangled quantum state with Bob. One of these states is now with Alice in her lab while the other is with Bob in his lab (wherever it is). Of course, the state that Alice wants to send is not the same as the one entangled with Bob.

For the teleportation to happen, Alice needs to entangle her two systems and measure them. This measurement will make Bob's system collapse at the same time. Precisely speaking, it will collapse into the state Alice originally wanted to send, except for a unitary transformation that depends on what Alice has measured. So, if Alice sends such information through a classical channel, Bob will be able to apply the proper unitary transformation, obtaining the state Alice wanted to send him, which she no longer has. Quantum teleportation has happened.

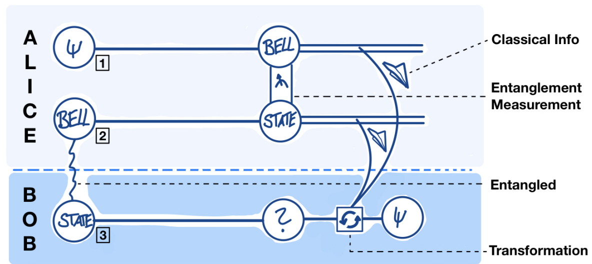

FIG. 1: Visualization of the Teleportation of Alice State ψ.

Let us explain an example in further detail. Alice wants to send an unknown state $|\psi\rangle_1$ of the qubit 1:

$$|\psi\rangle_1 = \alpha\,|0\rangle_1 + \beta\,|1\rangle_1 \tag{1}$$

To do so, Alice and Bob must have previously entangled two qubits, 2 and 3. In this case, for example, they have chosen to do so with the following Bell-state:

$$\left|\Psi^{(-)}\right\rangle_{23} = \frac{1}{\sqrt{2}}\left(|01\rangle_{23} - |10\rangle_{23}\right) \tag{2}$$

The complete system composed of Alice's and Bob's qubits can be described by the product state $|\Psi_{123}\rangle = |\psi\rangle_1 \otimes |\Psi^-\rangle_{23}$:

$$\begin{aligned} |\Psi\rangle_{123} = &\frac{\alpha}{\sqrt{2}}\left(|0\rangle_1|01\rangle_{23} - |0\rangle_1|10\rangle_{23}\right) + \\ &+ \frac{\beta}{\sqrt{2}}\left(|1\rangle_1|01\rangle_{23} - |1\rangle_1|10\rangle_{23}\right) \end{aligned} \tag{3}$$

Now Alice who wants to couple her first qubit with the other two, performs a complete measurement on her two qubits with the Bell operator basis consisting of:

$$\begin{aligned} \left|\Psi^{(\pm)}_{12}\right\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle_{12} \pm |10\rangle_{12}\right) \\ \left|\Phi^{(\pm)}_{12}\right\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle_{12} \pm |11\rangle_{12}\right) \end{aligned} \tag{4}$$

and since these states form a 2 qubit basis, the state of the complete system can be expressed in terms of these vectors, entangled states of qubits 1 and 2. The equation 3 expressed in this basis is as follows:

$$
\begin{aligned}
|\Psi\rangle_{123} = \frac{1}{2}\Big\{ &\left|\Psi_{12}^{(-)}\right\rangle (-\alpha\,|0\rangle_3 - \beta\,|1\rangle_3) \\
+ &\left|\Psi_{12}^{(+)}\right\rangle (-\alpha\,|0\rangle_3 + \beta\,|1\rangle_3) \\
+ &\left|\Phi_{12}^{(-)}\right\rangle (+\beta\,|0\rangle_3 + \alpha\,|1\rangle_3) \\
+ &\left|\Phi_{12}^{(+)}\right\rangle (-\beta\,|0\rangle_3 + \alpha\,|1\rangle_3)\Big\}
\end{aligned}
\quad (5)
$$

If Alice proceeds to do a Bell measurement, the system will collapse into one of these terms with a probability of 25% for each. Then the global state of the system will be a product state between the entangled state of qubits 1 and 2 and the new state of qubit 3, $|\Psi\rangle_{123} = |\Psi^{(\pm)}/\Phi^{(\pm)}\rangle_{12} \otimes (U|\psi\rangle_3)$. This change in the entanglement of the particles is known in the literature as entanglement swapping. For Bob to obtain the initial state of qubit 1, he only needs to do a unitary transformation. Then, qubit 3 will be in the state $|\psi_1\rangle$ and teleportation has taken place.

Now let us see why this does not conflict with special relativity. If the system collapses into the state $|\Psi_{12}^{(-)}\rangle(-\alpha|0\rangle_3 - \beta|1\rangle_3$, then qubit 3 is already in the state we wanted to teleport. Since this transformation of the qubit 3 happens at the same time as Alice's measurement, one could think that information traveled faster than the speed of light(in fact, instantaneously). However, it is essential to note that without the classical information of Alice, Bob does not know which of the four possible unitary transformations he has applied to his state. Thus he also doesn't know what Alice's original state was and the limit for information transfer is not violated.

On the other hand, we need to point out why quantum teleportation does not violate the no-cloning theorem. When Alice does the entanglement between 1 and 2, qubit 1 is no longer in its initial state, and immediately qubit 3 is projected into another state, qubits 1 and 3 are never in the same state so, the no-cloning theorem is never violated.

In summary, Quantum teleportation can send quantum states, but it still needs classical communication. This led us to the next question: what is the point to do quantum teleportation? Quantum teleportation gives the possibility to send a quantum state without knowing it, conserving its superposition's relative phases, and with a theoretical accuracy of 100%. Classically, to Alice send Bob a quantum state, she would need to have thousands of copies of the system and measure them to reproduce the statistics (probabilities) of each state. In quantum teleportation, Alice sends the exact state, meanwhile, in the classic situation, she would send an approximation of the system state.

III. EXPERIMENTAL PROOF

It is clear that quantum teleportation is based on entanglement. In the laboratory, different systems with this property can be reproduced, but photons and their polarization are particularly suitable to perform the teleportation of a quantum property. In the original experiment, this was done with type II parametric down-conversion[4]. A laser beam is directed onto a non-linear crystal, annihilating an entering photon and creating two photons in accordance with energy and momentum conservation laws. A property of this non-linear crystal is the changing index of diffraction concerning to polarization leading to two separate cones for horizontally and vertically polarized light. The points of intersection then correspond to an entangled pair which can be expressed by the following state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H_1, V_2\rangle + e^{i\alpha}\,|V_1, H_2\rangle) \quad (6)$$

with H representing horizontal and V vertical Polarization. By slightly rotating the crystal the relative phase α can be arbitrarily adjusted and a half-wave plates in one of the paths switches the polarization from horizontal to vertical and vice versa. This allows for the experimental implementation of any of the four Bell-states.

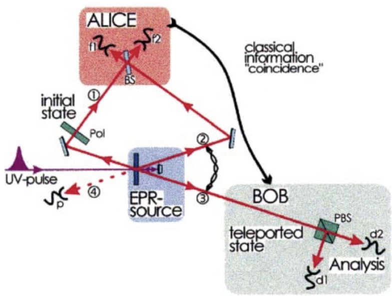

FIG. 2: Experiment setup of the first Quantum Teleportation.

To ensure the entanglement swapping from particles 2 and 3 to particles 1 and 2, allowing for the teleportation of Alice's original state $|\psi\rangle_1$, Alice has to perform a Bell-state measurement on 1 and 2. We want to point out that at the time of the original experiment, it wasn't possible to experimentally distinguish all four Bell-states from one another (and is still a challenge today). While it is fairly easy to determine the two states $|\Psi^{\pm}\rangle$ using linear optical elements, one can't differentiate between $|\Phi^{\pm}\rangle$[5]. On closer examination, one notices that $|\Psi^{-}\rangle$ is antisymmetric upon particle interchange as opposed to the other three Bell-states, leading to an indistinguishable behaviour when interfering with a beam splitter.

But how can this property be exploited? One can prove that a coincidence measurement (two detectors f1, f2 set up behind the beam splitter) of two indistinguishable particles whose wave functions overlap at the beam splitter inevitably correspond to a Bell-state measurement of $|\Psi^-\rangle$[6].

After Alice has measured a coincidence f1f2, she informs Bob. In the general case, he would now have to apply the unitary transformation corresponding to the classical information he has received from Alice. But if the experimentalist is clever and chooses Alice's state also to be $|\psi^-\rangle$(the same Bell-state photons 1 and 2 are projected on), Bob does not have to perform any further transformation.

He can now analyze the state in front of him with a polarizing beam splitter adjusted to the basis of the state that is to be teleported and another detector pair(one for each degree of freedom). Detector d1 should measure light polarized parallel and d2 orthogonal to $|\psi\rangle_1$ (or vice versa). The experimental proof of Quantum Teleportation is then given by the three-fold coincidence measurement d1f1f2 and a simultaneous absence of a d2f1f2 coincidence.

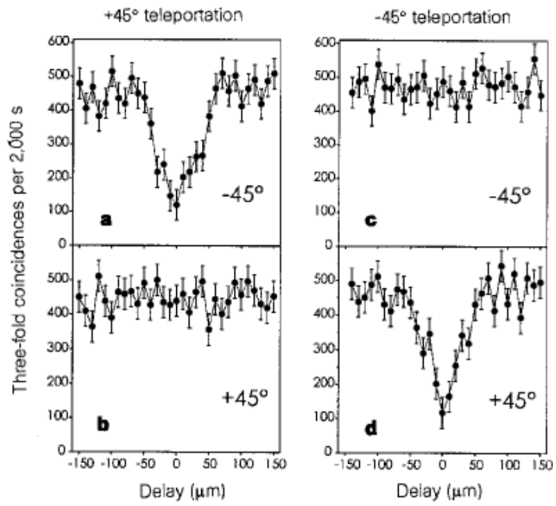

FIG. 3: Experimental results; outputs of d1f1f2 (upper half) and d1f1f2 (lower half) for photon 1 polarized in $\pm 45°$.

As already stated earlier it is essential for the Bell-state measurement for the two photons to be indistinguishable and their wavefunctions to overlap. What happens outside of the region of overlap? Classical statistics.

The probability of particles 1 and 2 exiting the beam splitter in different directions is 50%, and for photon 3 to be projected onto the original state of particle 1 at the second beam splitter is 50% as well. Without overlap, Bob's photon is still in a maximally entangled state with Alice's photon 2 and thus in a state of undefined polarization. We thereby derive a probability of 25% to measure a three-fold coincidence outside of the region of overlap.

Figure 3 shows the actual measured three-fold coincidences for a state to be teleported with polarization +45° and -45° from Zeilinger et al. The visible dip for the projection onto the orthogonal state regarding the original state of Alice corresponds to the expected dip to zero in the region of overlap. We further note that in the original paper, the experiment was repeated on another basis and with circularly polarized light, representing a superposition of states. They all showed a visibility between $57-66\%$, experimentally proving the Quantum Teleportation concept.

IV. SIMULATIONS AND Q. NETWORKS

If we wanted to implement quantum teleportation using more modern technologies, we could use IBM quantum computers and simulators, for example, by writing the following circuit in their high-level language, Qiskit:

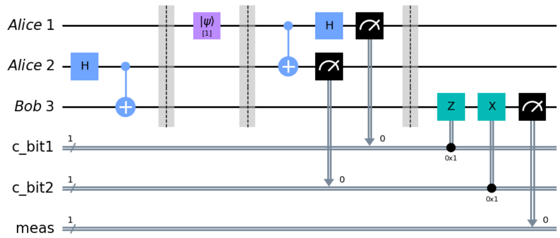

FIG. 4: Basic teleportation circuit of a random ψ state, with 3 "qubits", 2 classical registers and a measurement.

In such quantum computers/circuits, all channels start in the state $|0\rangle$. The first step is to entangle particle 2 and 3 of Alice and Bob using a Hadamard and a CNOT-gate:

$$|\Phi_{23}^{(+)}\rangle = CNOT(2,3)\ H(2)|00\rangle_{23} = \frac{1}{\sqrt{2}}|00\rangle_{23} + |11\rangle_{23}$$

Afterwards, Alice generates the state she wants to send with our quantum teleportation circuit. Now everything needed for quantum teleportation is set and the whole system is in the initial state:

$$|\Psi\rangle_{123} = (\alpha|0\rangle_1 + \beta|1\rangle_1)\left|\Phi_{23}^{(+)}\right\rangle$$

To proceed with the teleportation, we need to do a complete Bell operators basis measurement. Since in these quantum computers/circuits, we only measure individual qubits, and only in their Z axis, we need to translate such Bell basis measurement. In this case, such translation is given by an entanglement between both states and then measuring each one separately on their Z axis! Specifically, you need to apply a $CNOT_{1,2}$, as shown in the above circuit, followed by a H_1, which transforms your state into:

$$\begin{aligned}|\Psi_{123}\rangle =& \frac{1}{2}|00\rangle_{12}(\alpha|0\rangle_3 + \beta|1\rangle_3) + \frac{1}{2}|01\rangle_{12}(\alpha|1\rangle_3 + \beta|0\rangle_3) + \\ &+ \frac{1}{2}|10\rangle_{12}(\alpha|0\rangle_3 - \beta|1\rangle_3) + \frac{1}{2}|11\rangle_{12}(\alpha|1\rangle_3 - \beta|0\rangle_3)\end{aligned}$$

here we can see that Bob has these 4 possible states, which depend on what Alice measures in the Z axis now!

And since depending on the result of this measurement, you get one state or another, and each one of those corresponds to a different operation to go back to Alice's original send state, you can codify the Z measurement in classical bits of information that Alice sends to Bob!

Alice measure	Chance	Bob state	Gates to get ψ
00	1/4	$\alpha\|0\rangle + \beta\|1\rangle$	$\mathbb{I}$
01	1/4	$\alpha\|1\rangle + \beta\|0\rangle$	$\mathbb{X}$
10	1/4	$\alpha\|0\rangle - \beta\|1\rangle$	$\mathbb{Z}$
11	1/4	$\alpha\|1\rangle - \beta\|0\rangle$	$\mathbb{ZX}$

In this case, then, we can see that the first and second classic bits, exactly correspond to applying or not applying a Z and a X respectively! That is why in the above circuit, you can see that the measures get saved into classical channels, retrieved later to apply or not apply a Z and a X respectively, finishing our more modern teleportation implementation!

On a side note: If one looks closely, the operations previous to the measurements are an entanglement operation in reverse order. If we define an entangling operator $\hat{E}_{12} = CNOT(12)\ H(1)$ then we have the equivalent, but acting on the bras of the measurements projection operators. This corresponds to transforming our single-qubit measurements into a Bell-state measurement.

To expand on this, we can notice that by using previous output as input for a new teleportation, you can do teleportation between two people who have never shared an entangled state! Of course, passing through an intermediary who shares one state with the initial sender and another with the final receiver!

Even better, if you move all the classical information decoding to the end, these teleportations can even be secure between the sender and the receiver without any intermediary being able to get back to the original sent state:

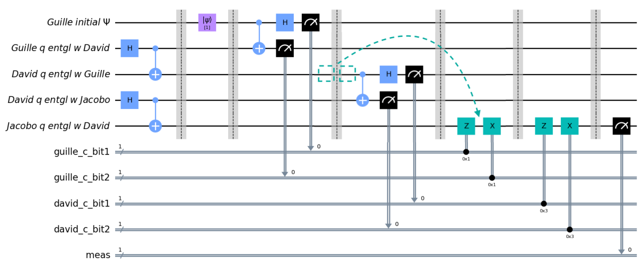

FIG. 5: Secure double quantum teleportation, where only the final receiver can decode the original state.

were you can notice, that we are doing all the decoding in that same original order, but together in the end! Of course, for this to work the intermediaries need to cooperate and classically send the results of their Bell measurements. Also, such receivers will always be able to destroy the communication, but never read it!

Finally, we can realize that with this, we could build a quantum teleportation network, a quantum internet!

This would be useful for preparation labs that could send their states to other labs that have more precise measurements for example, or many more reasons!

But since each teleportation consumes one entanglement, for this to work we would need a lot of entangled states. This could maybe be done by a company that basically constantly entangles states and then distributes them to different cities where the local laboratories/citizens can then entangle a state with them! Having a kind of entanglement highways!

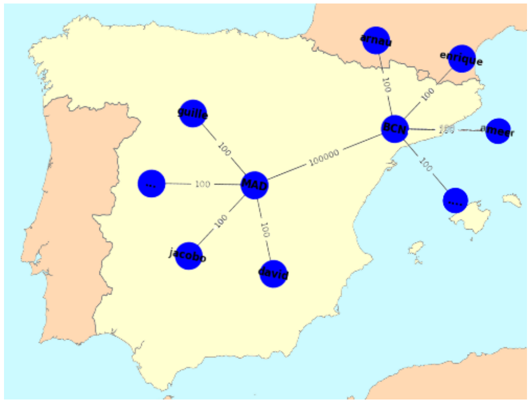

FIG. 6: Spain quantum teleportation network/internet.

Finally, we can also write an algorithm to which if you pass a concrete network, who you are, and who you want to teleport your state to, would automatically create the Qiskit circuit needed.

For example, in the case, that Guille in Madrid (MAD), wanted to send a state to his friend Ameer in Barcelona (BCN), for the above network, the algorithm correctly, automatically generates:

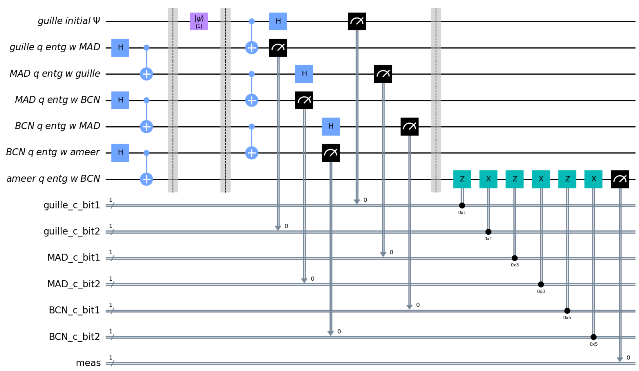

FIG. 7: Automatically generated Qiskit circuit, to send a state to Ameer in BCN, from MAD, given the network graph.

APPENDIX

All the code and its corresponding results (final algorithm included) are available in the following GitHub repository: `https://github.com/GuillermoAbadLopez/Quantum-Teleportation/blob/main/src/main.ipynb`

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[6] S. L. Braunstein and A. Mann. Measurement of the bell operator and quantum teleportation. *PHYSICAL REVIEW A*, 51:1727–1730, Mar 1995.