Review of Quantum Teleportation landmark papers:
- Charles H. Bennett et al. 1993
- Dik Bouwmeester et al. 1997 (Zeilinger's group)

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Universitat de Barcelona

Facultat de Física

Outline

1. Introduction
2. The concept of quantum teleportation
3. Experimental proof
4. Simulations and Quantum Networks

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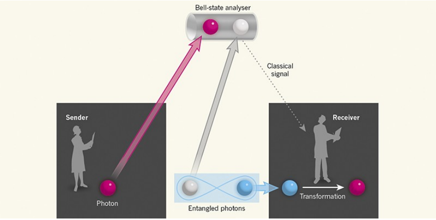
Introduction to Quantum Teleportation

Quantum teleportation:

- **IS NOT:** Instantaneous travel like depicted in films.
- **IS:** A way to transfer actual quantum states.



Does not violate causality of special relativity!

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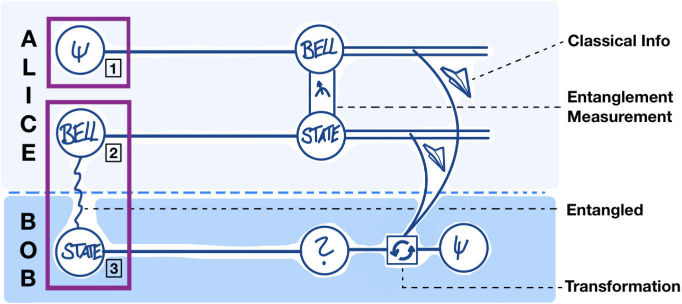
I. Initial set up for Quantum Teleportation



Alice has a **qubit 1** in an **UNKNOWN** state $|\psi\rangle_1$:

$$|\psi_1\rangle = \alpha\,|0\rangle_1 + \beta\,|1\rangle_1$$

Qubits **2 (Alice)** and **3 (Bob)** are **entangled**:

$$\left|\Psi_{23}^{(-)}\right\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle_{23} - |10\rangle_{23}\right)$$

II. Global initial state for Quantum Teleportation

The full system is described by the product:

$$|\psi_{123}\rangle = |\psi_1\rangle \otimes \left|\Psi_{23}^{(-)}\right\rangle$$

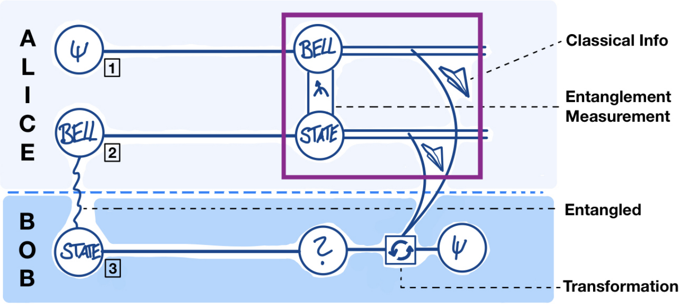
Which can be written in terms of the **Bell states** of **Alice's qubits**:

$$\left|\Psi_{12}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle_{12} \pm |10\rangle_{12}\right), \quad \left|\Phi_{12}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{12} \pm |11\rangle_{12}\right)$$

Obtaining the next expression to the **system's state**:

$$\boxed{\begin{aligned} |\Psi_{123}\rangle = \mathbf{\frac{1}{2}}\left|\boldsymbol{\Psi_{12}^{(-)}}\right\rangle\left(-\alpha\,|0\rangle_3 - \beta\,|1\rangle_3\right) \;+\; \mathbf{\frac{1}{2}}\left|\boldsymbol{\Psi_{12}^{(+)}}\right\rangle\left(-\alpha\,|0\rangle_3 + \beta\,|1\rangle_3\right) \;+ \\ \mathbf{\frac{1}{2}}\left|\boldsymbol{\Phi_{12}^{(-)}}\right\rangle\left(+\beta\,|0\rangle_3 + \alpha\,|1\rangle_3\right) \;+\; \mathbf{\frac{1}{2}}\left|\boldsymbol{\Phi_{12}^{(+)}}\right\rangle\left(-\beta\,|0\rangle_3 + \alpha\,|1\rangle_3\right) \end{aligned}}$$

III. Bob states, depending on Alice's result



$$\left.\begin{array}{lcl}
\left|\Psi_{12}^{(-)}\right\rangle & \longrightarrow & |\psi_3\rangle = -\alpha\,|0\rangle_3 - \beta\,|1\rangle_3 \\
\left|\Psi_{12}^{(+)}\right\rangle & \longrightarrow & |\psi_3\rangle = -\alpha\,|0\rangle_3 + \beta\,|1\rangle_3 \\
\left|\Phi_{12}^{(-)}\right\rangle & \longrightarrow & |\psi_3\rangle = +\beta\,|0\rangle_3 + \alpha\,|1\rangle_3 \\
\left|\Phi_{12}^{(+)}\right\rangle & \longrightarrow & |\psi_3\rangle = -\beta\,|0\rangle_3 + \alpha\,|1\rangle_3
\end{array}\right\} \xrightarrow{\hat{U}} \underbrace{|\psi_3\rangle = \alpha\,|0\rangle_3 + \beta\,|1\rangle_3}_{\text{qubit 3 transforms into the original state of qubit 1!!!}}$$

Teleportation happened!

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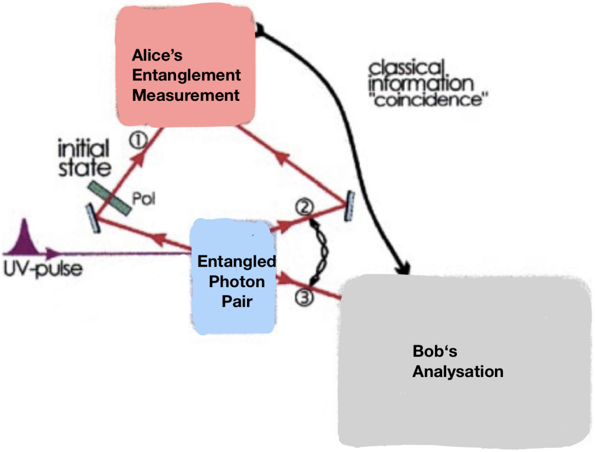
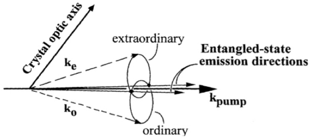
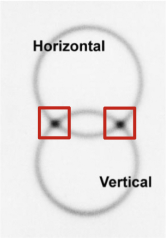
Experimental Setup

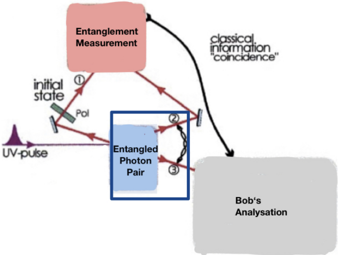


I. Type-II Parametric Down-Conversion

- Entanglement is the resource!
- Polarization of photons: very suitable for teleportation of a quantum property







$$\boxed{|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow_1, \updownarrow_2\rangle + e^{i\alpha}\,|\updownarrow_1, \leftrightarrow_2\rangle\right)}$$

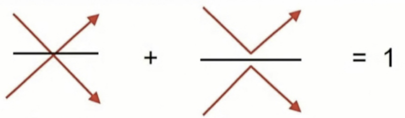
- Any Bell-state possible with Rotation of crystal and half-wave plates
- We choose $|\psi^-\rangle$

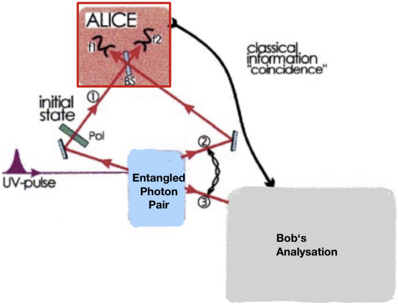
II. Bell-State Measurement - But How?

Property of the Beam Splitter

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\leftrightarrow_1, \updownarrow_2\rangle - |\updownarrow_1, \leftrightarrow_2\rangle)$$

$$\downarrow$$

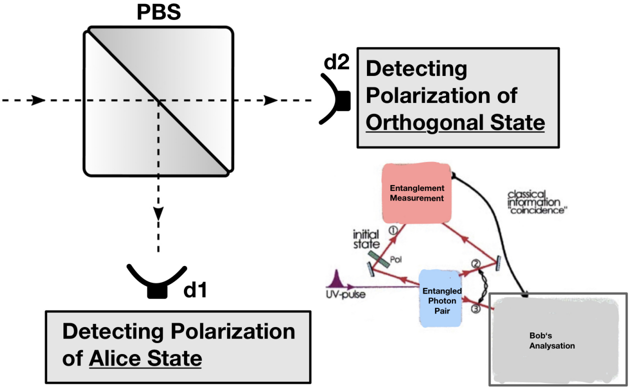




"2 clicks" $\Longrightarrow$ Projected onto antisymmetric Bell-state!

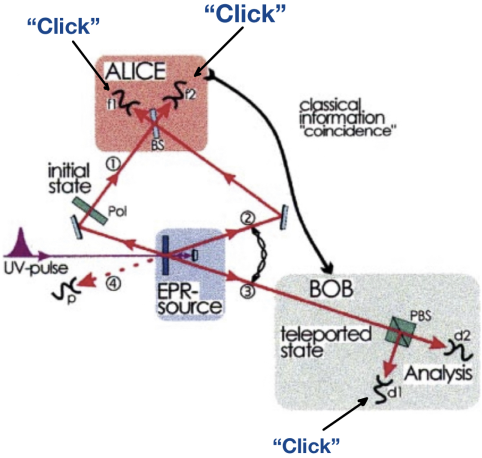
$$|\Psi_{123}\rangle \rightarrow \left|\Psi_{12}^{(-)}\right\rangle (-\alpha\,|0_3\rangle - \beta\,|1_3\rangle)$$

III. Bob's Analysation - The Confirmation



$$\left|\Psi_{12}^{(-)}\right\rangle \longrightarrow \left.|\psi_3\rangle = -\alpha\,|0\rangle_3 - \beta\,|1\rangle_3 \;\right\} \xrightarrow{\hat{U}=-\mathbb{I}} \underbrace{|\psi_3\rangle = \alpha\,|0\rangle_3 + \beta\,|1\rangle_3}_{\text{Alice's original state!}}$$

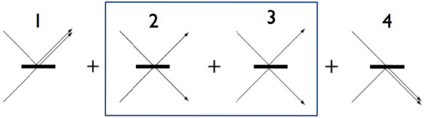
IV. Theoretical Data - What we would expect!

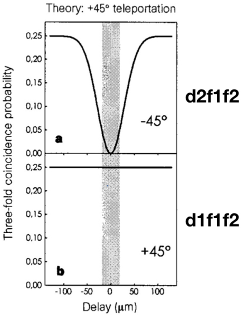


- Measured variable: Three-fold coincidence, **d1f1f2 and d2f1f2** ($\hat{=}$ "Click" at all 3 detectors)

IV. Theoretical Data - What we would expect!

Delay between photons:	Coincidence **f1f2**	Measure **d2(⊥)**		
• Outside region of overlap:	$P =$ **50%**	$P =$ **50%**	$\rightarrow$	**25%**
• Inside region of overlap:	$P =$ **25%**	$P =$ **0%**	$\rightarrow$	**0%**





V. Experimental Data

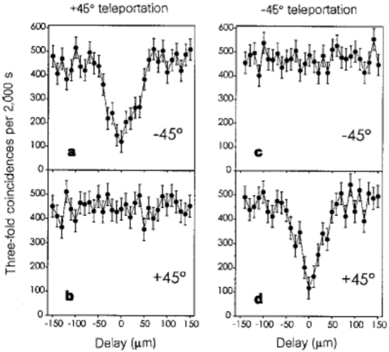


Table 1 Visibility of teleportation in three fold coincidences

Polarization	Visibility
+45°	0.63 ± 0.02
–45°	0.64 ± 0.02
0°	0.66 ± 0.02
90°	0.61 ± 0.02
Circular	0.57 ± 0.02

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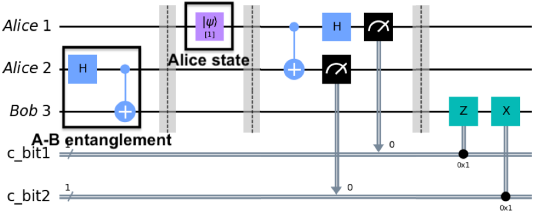
I. Set up, for basic Quantum Teleportation circuit



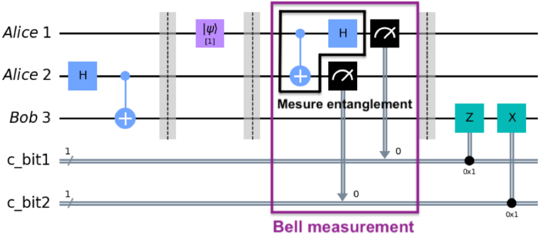
Alice's state, to teleport:

$$|\psi\rangle_1 = \alpha\,|0\rangle_1 + \beta\,|1\rangle_1$$

And the initial Alice-Bob entangled state:

$$\left|\Phi_{23}^{(+)}\right\rangle = CNOT(2,3)\ H(2)\ |00\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle_{23} + |11\rangle_{23})$$

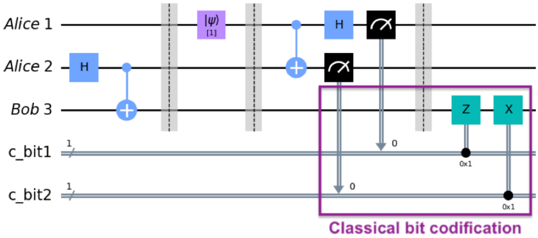
II. Bell measurement in a Quantum Teleportation circuit



Since we can only measure individual qubits in their Z axis, to translate the Bell measurement, we do this reverse/measure entanglement, getting:

$$|\Psi_{123}\rangle = \frac{1}{2}\{ |00\rangle_{12} (\alpha |0\rangle_3 + \beta |1\rangle_3) + |01\rangle_{12} (\alpha |1\rangle_3 + \beta |0\rangle_3) + \\ + |10\rangle_{12} (\alpha |0\rangle_3 - \beta |1\rangle_3) + |11\rangle_{12} (\alpha |1\rangle_3 - \beta |0\rangle_3) \}$$

III. C-bits encoding in a Quantum Teleportation circuit

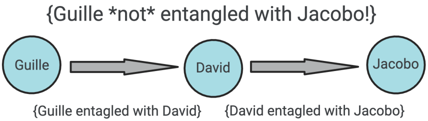


Depending on what Alice measures, we have these 4 possibilities:

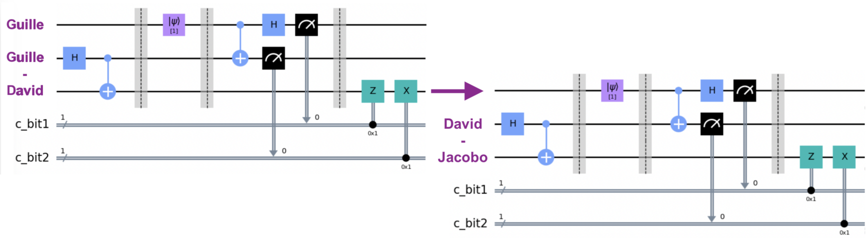
Alice measure	Chance	Bob state	Gates to get ψ
00	1/4	$\alpha\|0\rangle + \beta\|1\rangle$	$\mathbb{I}$
01	1/4	$\alpha\|1\rangle + \beta\|0\rangle$	$\mathbb{X}$
10	1/4	$\alpha\|0\rangle - \beta\|1\rangle$	$\mathbb{Z}$
11	1/4	$\alpha\|1\rangle - \beta\|0\rangle$	$\mathbb{ZX}$

IV. Consecutives Quantum Teleportations

Using previous output as input for a new teleportation:

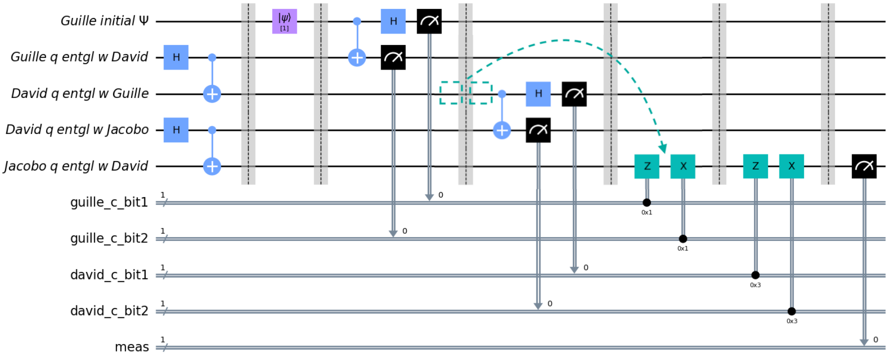


Notice that this will "consume" two entanglements!



V. Secure consecutive Quantum Teleportations

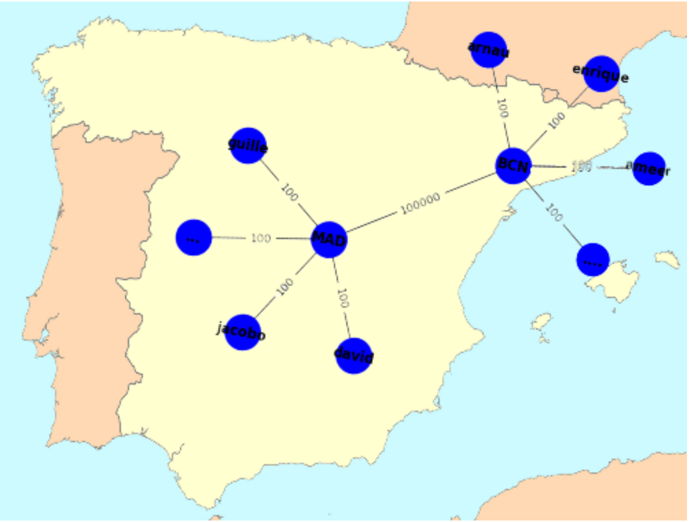
Consecutive teleportations can be secure, if we move all the decoding to the end:



the intermediaries can interrupt such communication, but never obtain it!

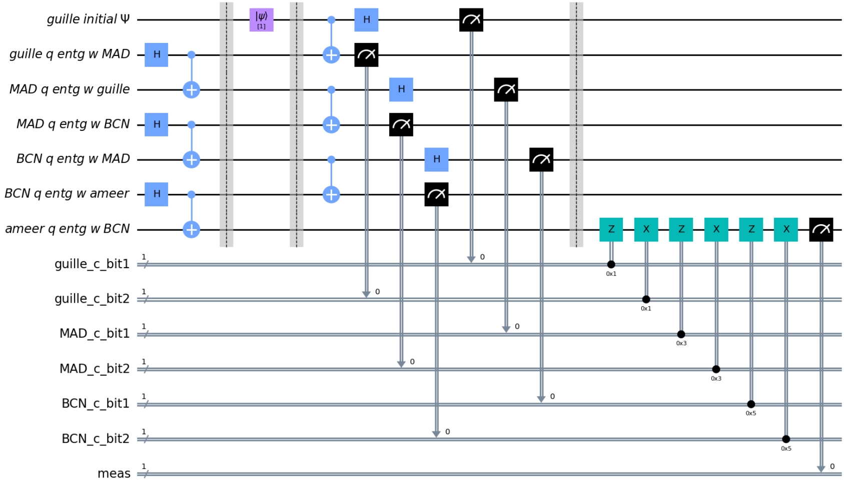
V. Quantum teleportation networks

Quantum teleportation networks, with entanglement highways:



VI. Quantum teleportation network algorithm

In the case, Guille in MAD, wanted to send a state to his friend Ameer in BCN, for the above network, the algorithm automatically generates:



Thank you for your time!

[1] Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, and Anton Zeilinger. Experimental quantum teleportation. *Nature*, 390(6660):575–579, Dec 1997.

[2] A. Shimony D. M. Greenberger, M. A. Horne and A. Zeilinger. Bell's theorem without inequalities. *Nature*, 58:1131–1143, Dec 1990.

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[5] Paul G. Kwiat Klaus Mattle, Harald Weinfurter and Anton Zeilinger. Dense coding in experimental quantum communication. *Phys. Rev. Lett.*, 76:4656–4659, Jun 1996.

[6] S. L. Braunstein and A. Mann. Measurement of the bell operator and quantum teleportation. *PHYSICAL REVIEW A*, 51:1727–1730, Mar 1995.