## Problem I

Consider a one-dimensional crystal, with lattice constant $a$ and monatomic base, formed by atoms of mass $M$ that via the oscillations parallel to the crystal interact harmonically in first and second neighbors, with constants with constants $C_1$ and $C_2 \equiv zC_1$, respectively, with $-1 \leq z \leq 1$.

1. Determine, based on the parameters $M$, $a$, $C_1$ i $z$:
   (a) The phonon dispersion relation.
   (b) The speed of sound.
   (c) Check that when $z = 0$ the corresponding results are retrieved in the case with interactions only up to first neighbors.

2. In materials that experience phase transitions or ferroelectricity, the so-called soft phonons are relevant. The aforementioned materials are characterized, among other particularities, by having a Debye temperature much lower than the expected value. Here, we study a material with a Debye temperature that is a quarter of the value that would be expected if only first-neighbor interactions were considered. Use the results from the previous section to determine the value of the constant $z$ of this material.

## Problem II

Assuming a two-dimensional crystal, with a square lattice of constant $a$ and basis of two atoms, of masses $m$ and $M$ and located at $(0,0)$ and at $a(1/2,1/2)$, respectively. For vibrations perpendicular to the plane, the atoms only interact in first neighbors, harmonically, with a coupling constant $C$.

1. Write its equations of motion.

2. Show that the dispersion relations of the normal modes of these vibrations can be expressed through the equality:

$$\omega_{\pm}^2(\vec{q}) = 2C\,\frac{m+M}{mM}\left\{1 \pm \sqrt{1 - \frac{4mM}{(m+M)^2}\,(1-A^2)}\right\}$$

   where

$$|A| = \cos\frac{q_x a}{2}\cos\frac{q_y a}{2}.$$

3. Prove that for $\vec{q} \to 0$ the relation of dispersion of the acoustic branch depends on $|\vec{q}|$ and no of its direction.

4. Determine the speed of sound and check that in this case becomes isotropic.