Review of Quantum Teleportation landmark papers

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Outline

1. Introduction
2. The concept of quantum teleportation
3. Experimental proof
4. Simulations and Quantum Networks

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Historical introduction

QED lagrangian:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi \tag{1.1}$$

In terms of the chiral states:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R - m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R) \tag{1.2}$$

The two currents:

$$\begin{cases} j_V^\mu = j_R^\mu + j_L^\mu = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L = \bar{\psi}\gamma^\mu\psi \ \ \text{(always conserved)} \\ j_A^\mu = j_R^\mu - j_L^\mu = \bar{\psi}_R\gamma^\mu\psi_R - \bar{\psi}_L\gamma^\mu\psi_L = \bar{\psi}\gamma^\mu\gamma^5\psi \ \ \text{(conserved only if m=0)} \end{cases} \tag{1.3}$$

The Schwinger model Lagrangian (m=0):

$$\mathcal{L}_{QED_{m=0}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{\partial}\psi - ej_V^\mu A_\mu \tag{1.4}$$

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Theoretical introduction to Quantum teleportation

Quantum teleportation states that it's possible to teleport a quantum state to another place instantaneously.

It does not violate special relativity

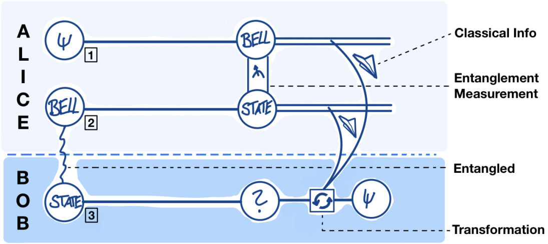
It isn't necessary to know which state is sent or where the receiver is placed!

The state is sent with a theoretical accuracy of 100% and with the exact relative phases.

Theoretical introduction to Quantum teleportation

The sender *Alice* must have two systems: the one that is in the state she wants to teleport, and another one in an entagled state with a system the receiver *Bob* has.

Alice has to entangle her two systems and measure them, then Bob's system will **inmediatily** collapse in another state equal to the one Alice wanted to teleport, except for an unitary transformation.



Alice has to send Bob which unitary transformation he has to do **through classical communication**.

Example of quantum teleportation

Alice has a qubit in the state $|\psi\rangle_1$

$$|\psi\rangle_1 = \alpha\,|0\rangle_1 + \beta\,|1\rangle_1 \tag{2.1}$$

The entangled state Alice and Bob share is the following Bell-state

$$\left|\Psi^{(-)}\right\rangle_{23} = \frac{1}{\sqrt{2}}\left(|01\rangle_{23} - |10\rangle_{23}\right) \tag{2.2}$$

The system is described by the product state $|\Psi_{123}\rangle = |\psi\rangle_1 \otimes |\Psi^-\rangle_{23}$

$$\begin{aligned}|\Psi\rangle_{123} = {} & \frac{\alpha}{\sqrt{2}}\left(|0\rangle_1\,|01\rangle_{23} - |0\rangle_1\,|10\rangle_{23}\right) + \\ & + \frac{\beta}{\sqrt{2}}\left(|1\rangle_1\,|01\rangle_{23} - |1\rangle_1\,|10\rangle_{23}\right)\end{aligned} \tag{2.3}$$

Example of quantum teleportation

Alice has to entangle her two qubits, so she performs a measurement in the Bell operator basis:

$$\begin{aligned} \left|\Psi_{12}^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle_{12} \pm |10\rangle_{12}\right) \\ \left|\Phi_{12}^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle_{12} \pm |11\rangle_{12}\right) \end{aligned} \tag{2.4}$$

The system's state can be expressed in terms of this basis

$$\begin{aligned} |\Psi\rangle_{123} = \frac{1}{2}\Big\{ &\left|\Psi_{12}^{(-)}\right\rangle\left(-\alpha\,|0\rangle_3 - \beta\,|1\rangle_3\right) \\ &+ \left|\Psi_{12}^{(+)}\right\rangle\left(-\alpha\,|0\rangle_3 + \beta\,|1\rangle_3\right) \\ &+ \left|\Phi_{12}^{(-)}\right\rangle\left(+\beta\,|0\rangle_3 + \alpha\,|1\rangle_3\right) \\ &+ \left|\Phi_{12}^{(+)}\right\rangle\left(-\beta\,|0\rangle_3 + \alpha\,|1\rangle_3\right)\Big\} \end{aligned} \tag{2.5}$$

Example of quantum teleportation

After the Bell measurement, qubit 3 is projected in an unitary transformation of $|\psi\rangle_1$, that depends on the Bell state measured.

$$|\Psi\rangle_{123} = |\Psi^{(\pm)}/\Phi^{(\pm)}\rangle_{12} \otimes (U|\psi\rangle_3) \tag{2.6}$$

Without the classical sent of information the teleport **cannot** happen, so special relativity is not violated.

When Alice does the Bell measurement qubit 3 is projected in its new state, but qubit 1 is no longer in its first state, so they are never in the same state and the no-cloning theorem is not violated.

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Experimental quantum teleportation

The quickest difference we can see, is the crossed term that appears in the eq. of motion:

$$\begin{cases} \left(i\partial_0 + i\partial_1 - e(A^0 + A^1)\right)\psi_R = m\psi_L \\ \left(i\partial_0 - i\partial_1 - e(A^0 - A^1)\right)\psi_L = m\psi_R \end{cases} \quad \begin{cases} \left(i\partial_0 + i\partial_1 + e(A^0 + A^1)\right)\psi_R^\dagger = -m\psi_L^\dagger \\ \left(i\partial_0 - i\partial_1 + e(A^0 - A^1)\right)\psi_L^\dagger = m\psi_R^\dagger \end{cases}$$

the classical solutions would look as:

$$\begin{cases} \psi(t,x)_R \;\; \alpha \;\; e^{-i(eA^0+eA^1)t}\; e^{ip_k^R(x-t)} - imt\psi_L \\ \psi(t,x)_L \;\; \alpha \;\; e^{-i(eA^0-eA^1)t}\; e^{ip_k^L(x+t)} - imt\psi_R \end{cases} \tag{3.1}$$

where their movement doesn't depend only on $x \pm t$ anymore, they don't move at the speed of light anymore!

- Change reference frame, changes right and left moving
- Screening is harder, flux tubes more resilient (minimum energy required to create particle-antiparticle): $L > 4m/e^2$
- Spectrum of the theory: tower of neutral particle-antiparticle meson-like states
- Our bosons are no longer free, they interact.
- Seem we won't be able to reabsorb θ

The two regimes of the massive Schwinger model

The strong coupling regime ($\frac{m}{e} << 1$ or $m << e$):

- In this regime we will have massive highly energetic states bounded through the gauge field, which we can describe pretty well through bosonization.
- We can make perturbation expansion with the (m/e) to the bosonization an see how it changes.

The weak coupling regime ($\frac{e}{m} << 1$ or $m >> e$):

- In the weak coupling regime, we will have almost free massive fermions interacting, like in all the weak coupling theories with the typical perturbative expansion of the propagators in the coupling (e/m).
- Also because the mass is much bigger than the coupling we can work on the non-relativistic limit with a Schrödinger equation.

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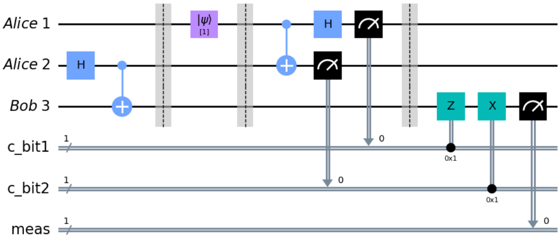
Basic quantum teleportation circuit



For teleportation to be setup, Alice and Bob need to have an entangled state:

$$|\Phi_{23}^{(+)}\rangle = CNOT(2,3)\ H(2)|00\rangle_{23} = \frac{1}{\sqrt{2}}|00\rangle_{23} + |11\rangle_{23}$$

Now we add to the total state, that which Alice wants to teleport:

$$|\Psi\rangle_{123} = |\psi\rangle_1|\Phi_{23}^{(+)}\rangle = (\alpha|0\rangle_1 + \beta|1\rangle_1)\ \frac{1}{\sqrt{2}}|00\rangle_{23} + |11\rangle_{23}$$

Basic quantum teleportation

Since in these quantum computers/circuits, we only measure individual qubits, and only in their Z axis, we need to translate such Bell basis measurement. We need to apply a $CNOT_{1,2}$, followed by a H_1, which transforms your state into:

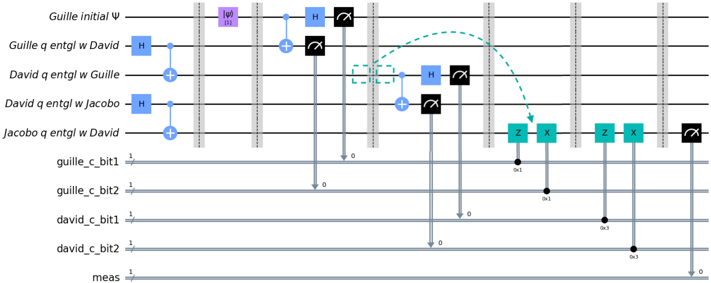
$$\begin{aligned}|\Psi_{123}\rangle =& \frac{1}{2}|00\rangle_{12}(\alpha|0\rangle_3 + \beta|1\rangle_3) + \frac{1}{2}|01\rangle_{12}(\alpha|1\rangle_3 + \beta|0\rangle_3) + \\ &+ \frac{1}{2}|10\rangle_{12}(\alpha|0\rangle_3 - \beta|1\rangle_3) + \frac{1}{2}|11\rangle_{12}(\alpha|1\rangle_3 - \beta|0\rangle_3)\end{aligned}$$

here we can see that Bob has these 4 possible states, which depend on what Alice measures in the Z axis now! And you can codify the Z measurements in classical bits of information that Alice sends to Bob!

Alice measure	Chance	Bob state	Gates to get ψ
00	1/4	$\alpha\|0\rangle + \beta\|1\rangle$	$\mathbb{I}$
01	1/4	$\alpha\|1\rangle + \beta\|0\rangle$	$\mathbb{X}$
10	1/4	$\alpha\|0\rangle - \beta\|1\rangle$	$\mathbb{Z}$
11	1/4	$\alpha\|1\rangle - \beta\|0\rangle$	$\mathbb{Z}\mathbb{X}$

Secure double quantum teleportation

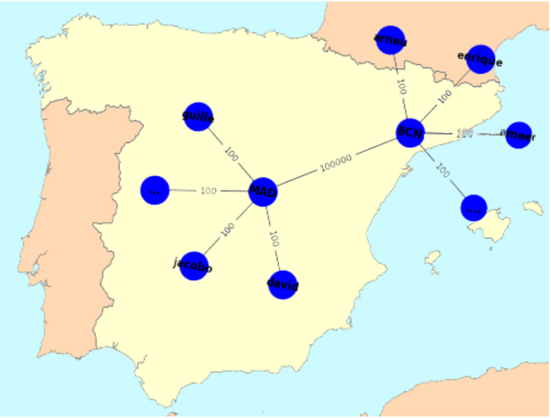
Using previous output as input for a new teleportation, you can do teleportation between two people who have never shared an entangled state!



where we have moved all the classical information decoding to the end, so these teleportations are secure between the sender and the receiver!

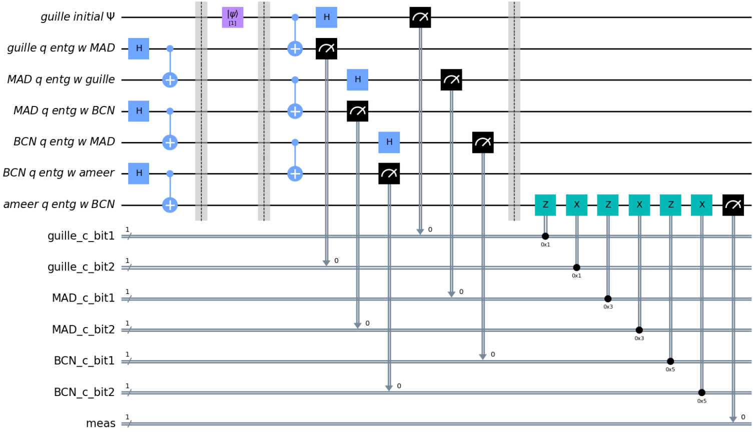
Quantum teleportation networks and algorithm

Quantum teleportation networks, with entanglement highways:



Quantum teleportation network algorithm circuit

In the case, Guille in MAD, wanted to send a state to his friend Ameer in BCN, for the above network, the algorithm automatically generates:



Thank you for your time