Designing roller coasters

A manufacturer of roller coasters wants to improve their designs and innovate with proposals that surprise visitors to amusement parks. For this reason, it is needed to develop analytical and numerical tools that allow to determine the motion features of a car on the roller coaster.

In Fig. 1 the reader will find a three-dimensional curve intended to represent the path followed by the roller coaster train.

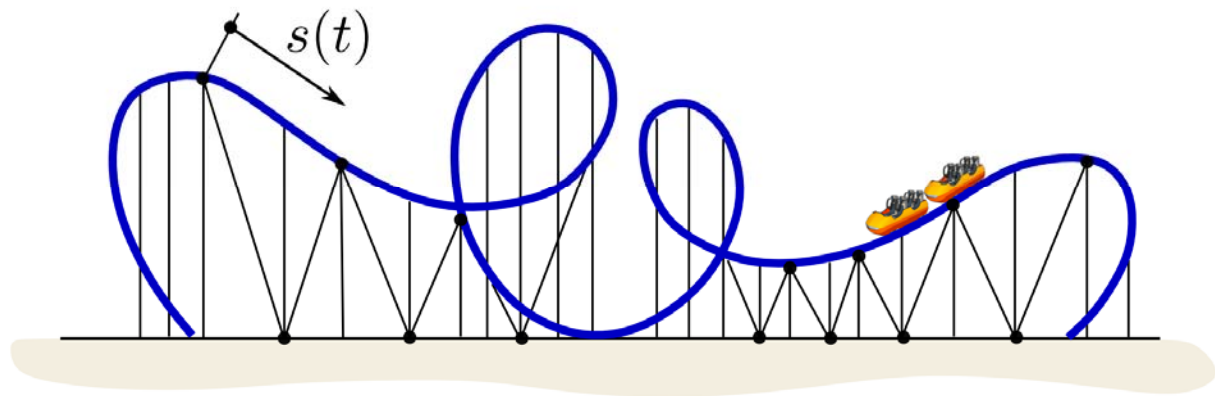$$\mathbf{r} = x_1(u)\hat{\mathbf{n}}_1 + x_2(u)\hat{\mathbf{n}}_2 + x_3(u)\hat{\mathbf{n}}_3$$

Fig. 1 A roller coaster path.

The manufacturer wants to analyse a possible path given by the following parameterization,

$$\mathbf{r} = (40\cos u - 10)\hat{\mathbf{n}}_1 + 20\sin u\,\hat{\mathbf{n}}_2 + (60\cos^2 u - 40\cos u + 25)\hat{\mathbf{n}}_3$$

On this basis, it is required to characterize the kinematics and forces that a roller coaster car will be subjected to. In addition, the manufacturer requires to determine what the stable and unstable equilibrium points of the path are.

After analysing the curve that the manufacturer is interested in, do you dare to design a section of roller coaster that is irresistible for park visitors?