

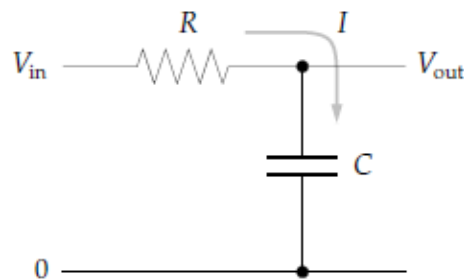
**Universidad de Puerto Rico
Recinto Universitario de Mayagüez
Departamento de Física
Asignación 6**

Instrucciones: Para entregar en o antes del jueves, 18 de marzo de 2021 a las 11:59 PM (20 puntos = 100%).

1.

Exercise 8.1: A low-pass filter

Here is a simple electronic circuit with one resistor and one capacitor:



This circuit acts as a low-pass filter: you send a signal in on the left and it comes out filtered on the right.

Using Ohm's law and the capacitor law and assuming that the output load has very high impedance, so that a negligible amount of current flows through it, we can write down the equations governing this circuit as follows. Let I be the current that flows through R and into the capacitor, and let Q be the charge on the capacitor. Then:

$$IR = V_{\text{in}} - V_{\text{out}}, \quad Q = CV_{\text{out}}, \quad I = \frac{dQ}{dt}.$$

Substituting the second equation into the third, then substituting the result into the first equation, we find that $V_{\text{in}} - V_{\text{out}} = RC (dV_{\text{out}}/dt)$, or equivalently

$$\frac{dV_{\text{out}}}{dt} = \frac{1}{RC}(V_{\text{in}} - V_{\text{out}}).$$

- a) Write a program (or modify a previous one) to solve this equation for $V_{\text{out}}(t)$ using the fourth-order Runge-Kutta method when in the input signal is a square-wave with frequency 1 and amplitude 1:

$$V_{\text{in}}(t) = \begin{cases} 1 & \text{if } \lfloor 2t \rfloor \text{ is even,} \\ -1 & \text{if } \lfloor 2t \rfloor \text{ is odd,} \end{cases} \quad (1)$$

where $\lfloor x \rfloor$ means x rounded down to the next lowest integer. Use the program to make plots of the output of the filter circuit from $t = 0$ to $t = 10$ when $RC = 0.01, 0.1$, and 1 , with initial condition $V_{\text{out}}(0) = 0$. You will have to make a decision about what value of h to use in your calculation. Small values give more accurate results, but the program will take longer to run. Try a variety of different values and choose one for your final calculations that seems sensible to you.

- b) Based on the graphs produced by your program, describe what you see and explain what the circuit is doing.

A program similar to the one you wrote is running inside most stereos and music players, to create the effect of the “bass” control. In the old days, the bass control on a stereo would have been connected to a real electronic low-pass filter in the amplifier circuitry, but these days there is just a computer processor that simulates the behavior of the filter in a manner similar to your program.

2.

Exercise 8.2: The Lotka–Volterra equations

The Lotka–Volterra equations are a mathematical model of predator–prey interactions between biological species. Let two variables x and y be proportional to the size of the populations of two species, traditionally called “rabbits” (the prey) and “foxes” (the predators). You could think of x and y as being the population in thousands, say, so that $x = 2$ means there are 2000 rabbits. Strictly the only allowed values of x and y would then be multiples of 0.001, since you can only have whole numbers of rabbits or foxes. But 0.001 is a pretty close spacing of values, so it’s a decent approximation to treat x and y as continuous real numbers so long as neither gets very close to zero.

In the Lotka–Volterra model the rabbits reproduce at a rate proportional to their population, but are eaten by the foxes at a rate proportional to both their own population and the population of foxes:

$$\frac{dx}{dt} = \alpha x - \beta xy,$$

where α and β are constants. At the same time the foxes reproduce at a rate proportional the rate at which they eat rabbits—because they need food to grow and reproduce—but also die of old age at a rate proportional to their own population:

$$\frac{dy}{dt} = \gamma xy - \delta y,$$

where γ and δ are also constants.

- a) Write a program to solve these equations using the fourth-order Runge–Kutta method for the case $\alpha = 1$, $\beta = \gamma = 0.5$, and $\delta = 2$, starting from the initial condition $x = y = 2$. Have the program make a graph showing both x and y as a function of time on the same axes from $t = 0$ to $t = 30$. (Hint: Notice that the differential equations in this case do not depend explicitly on time t —in vector notation, the right-hand side of each equation is a function $f(\mathbf{r})$ with no t dependence. You may nonetheless find it convenient to define a Python function $\mathbf{f}(\mathbf{r}, \tau)$ including the time variable, so that your program takes the same form as programs given earlier in this chapter. You don't have to do it that way, but it can avoid some confusion. Several of the following exercises have a similar lack of explicit time-dependence.)
- b) Describe in words what is going on in the system, in terms of rabbits and foxes.

3.

Exercise 8.4: Building on the results from Example 8.6 above, calculate the motion of a nonlinear pendulum as follows.

- a) Write a program to solve the two first-order equations, Eqs. (8.45) and (8.46), using the fourth-order Runge–Kutta method for a pendulum with a 10 cm arm. Use your program to calculate the angle θ of displacement for several periods of the pendulum when it is released from a standstill at $\theta = 179^\circ$ from the vertical. Make a graph of θ as a function of time.
- b) Extend your program to create an animation of the motion of the pendulum. Your animation should, at a minimum, include a representation of the moving pendulum bob and the pendulum arm. (Hint: You will probably find the function rate discussed in Section 3.5 useful for making your animation run at a sensible speed. Also, you may want to make the step size for your Runge–Kutta calculation smaller than the frame-rate of your animation, i.e., do several Runge–Kutta steps per frame on screen. This is certainly allowed and may help to make your calculation more accurate.)

For a bigger challenge, take a look at Exercise 8.15 on page 398, which invites you to write a program to calculate the chaotic motion of the double pendulum.

4.

Exercise 8.5: The driven pendulum

A pendulum like the one in Exercise 8.4 can be driven by, for example, exerting a small oscillating force horizontally on the mass. Then the equation of motion for the pendulum becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell} \sin \theta + C \cos \theta \sin \Omega t,$$

where C and Ω are constants.

- a) Write a program to solve this equation for θ as a function of time with $\ell = 10$ cm, $C = 2 \text{ s}^{-2}$ and $\Omega = 5 \text{ s}^{-1}$ and make a plot of θ as a function of time from $t = 0$ to $t = 100$ s. Start the pendulum at rest with $\theta = 0$ and $d\theta/dt = 0$.
- b) Now change the value of Ω , while keeping C the same, to find a value for which the pendulum resonates with the driving force and swings widely from side to side. Make a plot for this case also.

Instrucciones para Entregar sus Asignaciones

- 1) Prepare un archivo en pdf con la información que pide el ejercicio. Por ejemplo, si el ejercicio pide que escriba un programa, deberá mostrar su programa. Si el ejercicio pide output para un input dado, deberá mostrar el input y el output. Este archivo lo subirá a la plataforma Moodle del curso.
- 2) Suba también en archivos separados los programas usados para la hacer la asignación a la plataforma Moodle.