Qualifier Review Quantum Mechanics

Guillermo A. Fidalgo Rodríguez

University of Alabama

June 13, 2025



TOC

- Wave Functions; State Functions and operators; uncertainty principle
 - Wave Functions
 - Stats review

Wave Functions

The Schrödinger equation describes how the quantum state of a physical system changes over time. The wave function, denoted by Ψ , is a complex-valued function that contains all the information about the system.

Schrödinger Equation

$$i\hbar \frac{\partial \Psi(\mathbf{r},t)}{\partial t} = \hat{H}\Psi(\mathbf{r},t)$$

where \hat{H} is the Hamiltonian operator, which represents the total energy of the system. In one dimension, the time-dependent Schrödinger equation can be written as:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2\Psi(x,t)}{dx^2} + V(x)\Psi(x,t)$$

where V(x) is the potential energy and m is the mass of the particle.

Wave Functions (cont)

The wave function collapses when a measurement (i.e. observation) is made. In QM, we may only measure the probability of observing the position of a particle at specific time t by computing $|\Psi(x,t)|^2$ or more precisely

$$\int_{a}^{b} |\Psi(x,t)|^{2} dx = \int_{a}^{b} \Psi^{*}(x,t)\Psi(x,t) dx$$

Some important stats review

mean is the numerical average of multiple measurements at a time.

median the 50th percentile, second quantile, i.e. the value at which there is the same probability to measure any value below or after this value.

mpv the number that has the highest probability to be measured. Another name for the "mode".

expectation value this a little misnomer. Same as the mean value in our case.

We comput these as follows

mean/expectation value

The average is computed for a number of different moments of j as follows

$$\langle j \rangle = \sum_{j=0}^{\infty} j P(j) \quad ; \quad \langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j) \quad \dots \quad \langle j^n \rangle = \sum_{j=0}^{\infty} j^n P(j)$$
 (1)

$$\sqrt{\langle f(j)\rangle = \sum_{j=0}^{\infty} f(j)P(j)}$$
 (2)

Some important stats review

mean is the numerical average of multiple measurements at a time.

median the 50th percentile, second quantile, i.e. the value at which there is the same probability to measure any value below or after this value.

mpv the number that has the highest probability to be measured. Another name for the "mode".

expectation value this a little misnomer. Same as the mean value in our case.

We comput these as follows

median

The formula depends on the number of observations or data points. First we order the data list $\{X_1, X_2, \dots, X_n\}$ then

$$\mathsf{Median} = \begin{cases} \frac{X_{n/2} + X_{(n/2)+1}}{2} & \text{if } n \text{ is even} \\ X_{(n+1)/2} & \text{if } n \text{ is odd} \end{cases}$$

Variance and Standard Deviation

These measure the spread of a distribution (particularly the standard deviation). First we find the deviation of each value from the mean.

$$\Delta j = j - \langle j \rangle$$

then we find the average of the square of the deviations (why? Because the average deviation is **always** zero!). So now

$$\begin{split} \sigma^2 &= \left\langle (\Delta j)^2 \right\rangle = \sum (\Delta j)^2 P(j) = \sum (j - \langle j \rangle)^2 P(j) \\ &= \sum \left(j^2 - 2j \langle j \rangle + \langle j \rangle^2 \right) P(j) \\ &= \sum j^2 P(j) - 2 \langle j \rangle \sum j P(j) + \langle j \rangle^2 \sum P(j) \\ &= \left\langle j^2 \right\rangle - 2 \langle j \rangle \langle j \rangle + \left\langle j \right\rangle^2 = \left\langle j^2 \right\rangle - \left\langle j \right\rangle^2. \end{split}$$

Finally

$$\sigma = \sqrt{\langle j^2
angle - \langle j
angle^2}$$

Probability density and properties for continuous functions

$$P_{ab} = \int_{a}^{b} \rho(x) \, \mathrm{d}x$$

is the probability that x lies between a and b. The other properties are:

$$\int_{-\infty}^{+\infty} \rho(x)dx = 1 \tag{3}$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x \rho(x) dx \tag{4}$$

$$\langle f(x) \rangle = \int_{-\infty}^{+\infty} f(x) \rho(x) dx$$
 (5)

$$\sigma^2 \equiv \langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 \tag{6}$$