# Qualifier Review
# Quantum Mechanics

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# TOC

1. Wave Functions; State Functions and operators; uncertainty principle
   - Wave Functions
   - Stats review

# Wave Functions

The Schrödinger equation describes how the quantum state of a physical system changes over time. The wave function, denoted by $\Psi$, is a complex-valued function that contains all the information about the system.

## Schrödinger Equation

$$i\hbar \frac{\partial \Psi(\mathbf{r},t)}{\partial t} = \hat{H}\Psi(\mathbf{r},t)$$

where $\hat{H}$ is the Hamiltonian operator, which represents the total energy of the system. In one dimension, the time-dependent Schrödinger equation can be written as:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2\Psi(x,t)}{dx^2} + V(x)\Psi(x,t)$$

where $V(x)$ is the potential energy and $m$ is the mass of the particle.

# Wave Functions (cont)

The wave function collapses when a measurement (i.e. observation) is made. In QM, we may only measure the probability of observing the position of a particle at specific time $t$ by computing $|\Psi(x,t)|^2$ or more precisely

$$\int_a^b |\Psi(x,t)|^2 \, \mathrm{d}x = \int_a^b \Psi^*(x,t)\Psi(x,t) \, \mathrm{d}x$$

# Some important stats review

**mean** is the numerical average of multiple measurements at a time.

**median** the 50th percentile, second quantile, i.e. the value at which there is the same probability to measure any value below or after this value.

**mpv** the number that has the highest probability to be measured. Another name for the "mode".

**expectation value** this a little misnomer. Same as the mean value in our case.

We comput these as follows

## mean/expectation value

The average is computed for a number of different moments of j as follows

$$\langle j \rangle = \sum_{j=0}^{\infty} jP(j) \quad ; \quad \langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j) \quad \ldots \quad \langle j^n \rangle = \sum_{j=0}^{\infty} j^n P(j) \tag{1}$$

In General

$$\boxed{\langle f(j) \rangle = \sum_{j=0}^{\infty} f(j) P(j)} \tag{2}$$

# Some important stats review

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## median

The formula depends on the number of observations or data points. First we order the data list $\{X_1, X_2, \ldots, X_n\}$ then

$$\text{Median} = \begin{cases} \frac{X_{n/2}+X_{(n/2)+1}}{2} & \text{if } n \text{ is even} \\ X_{(n+1)/2} & \text{if } n \text{ is odd} \end{cases}$$

# Variance and Standard Deviation

These measure the spread of a distribution (particularly the standard deviation). First we find the deviation of each value from the mean.

$$\Delta j = j - \langle j \rangle$$

then we find the average of the *square* of the deviations (why? Because the average deviation is **always** zero!). So now

$$\begin{aligned}
\sigma^2 &= \left\langle (\Delta j)^2 \right\rangle = \sum (\Delta j)^2 P(j) = \sum (j - \langle j \rangle)^2 P(j) \\
&= \sum \left( j^2 - 2j \langle j \rangle + \langle j \rangle^2 \right) P(j) \\
&= \sum j^2 P(j) - 2 \langle j \rangle \sum j P(j) + \langle j \rangle^2 \sum P(j) \\
&= \left\langle j^2 \right\rangle - 2 \langle j \rangle \langle j \rangle + \langle j \rangle^2 = \left\langle j^2 \right\rangle - \langle j \rangle^2 .
\end{aligned}$$

Finally

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

# Probability density and properties for continuous functions

$$P_{ab} = \int_a^b \rho(x)\,\mathrm{d}x$$

is the probability that $x$ lies between $a$ and $b$. The other properties are:

$$\int_{-\infty}^{+\infty} \rho(x)dx = 1 \tag{3}$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x\rho(x)dx \tag{4}$$

$$\langle f(x) \rangle = \int_{-\infty}^{+\infty} f(x)\rho(x)dx \tag{5}$$

$$\sigma^2 \equiv \left\langle (\Delta x)^2 \right\rangle = \left\langle x^2 \right\rangle - \langle x \rangle^2 \tag{6}$$