PH - 541
Quantum Mechanics
Lecture Notes

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TOC

Eigenkets of an observation

Theorem 1

The eigenvalues of a Hermitian operator A are real; the eigenkets of A corresponding to different eigenvalues are orthogonal.

Proof:

$$A\left|a'\right\rangle = a'\left|a'\right\rangle \tag{1}$$

$$\left\langle a''\right| A = a''^{*}\left\langle a''\right| \tag{2}$$

$$\left\langle a''\right| \cdot \color{blue}{1} - \color{blue}{2} \cdot \left|a'\right\rangle \to (a' - a'')\left\langle a''\middle|a'\right\rangle = 0 \tag{3}$$

If $a' = a''$, then $a' = a''^{*} = a'^{*}$ since $\langle a'|a'\rangle = 0$
$\therefore a'$ is real. If $a' \neq a'', a' - a'' \neq 0 \implies \langle a''|a'\rangle = 0$
i.e. $|a''\rangle$ & $|a'\rangle$ are orthogonal.

Can normalize $|a'\rangle$ so that $\{|a'\rangle\}$ forms an orthogonal set with $\langle a''|a'\rangle = \delta_{a'a''}$