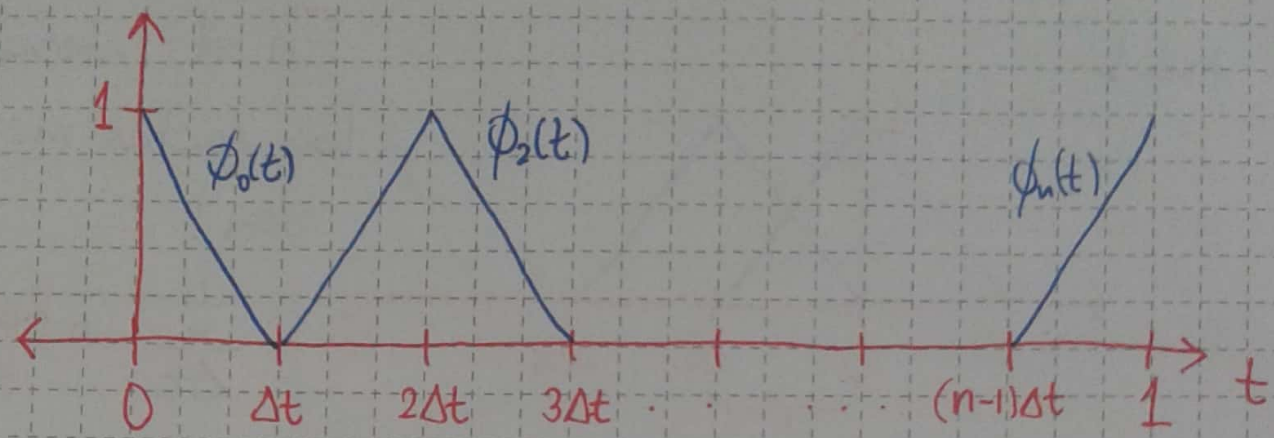
1.- a)



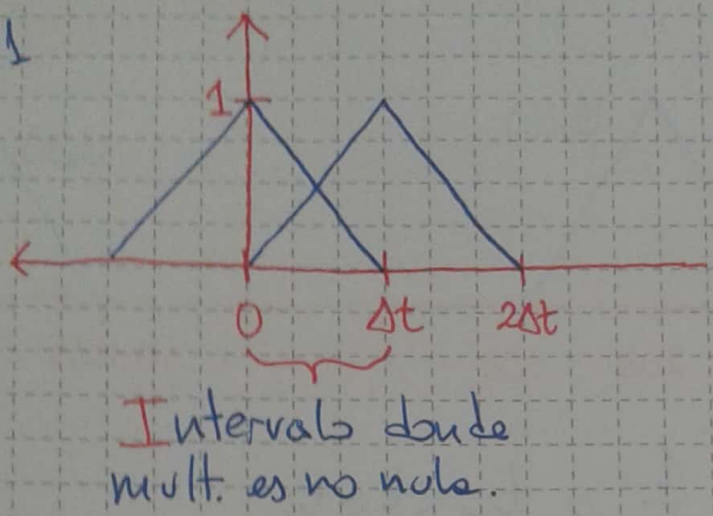
b) $$\phi_i'(t) = \begin{cases} \dfrac{1}{\Delta t}, & t_{i-1} < t \leq t_i \\ -\dfrac{1}{\Delta t}, & t_i < t \leq t_{i+1} \\ 0, & \text{en otro caso} \end{cases}$$

Derivadas corresponden a pendientes de rectas.

c) $$\int_0^1 \phi_i(t) \cdot \phi_{i+1}(t)\, dt$$ ~~Se tiene que dos funciones hat contiguas se intersectan~~

Se tiene que la multiplicación de dos funciones hat contiguas es <u>no</u> nula en un solo intervalo: $t \in [t_i, t_{i+1}]$. Además, se sabe que todas las funciones hat son idénticas. Por estas razones calcularemos la integral de las siguientes funciones:

graf. 1



En el intervalo $[0, \Delta t]$ las rectas multiplicandose son:

$\frac{t}{\Delta t}$ → Recta que sube

$-\frac{t}{\Delta t}+1$ → Recta que baja.

Así la integral:

$$\int_0^1 \phi_i(t)\cdot\phi_{i+1}(t)\,dt = \int_0^{\Delta t}\left(-\frac{t}{\Delta t}+1\right)\cdot\frac{t}{\Delta t}\,dt =$$

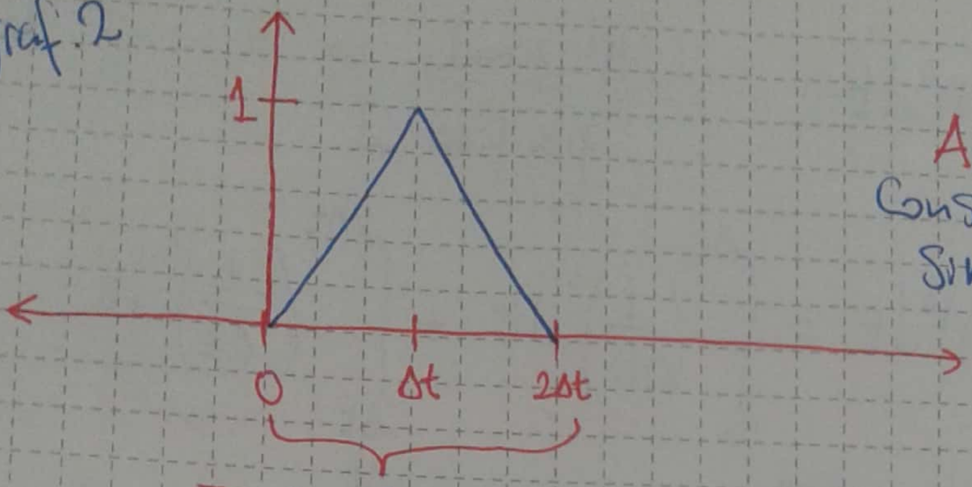
$$\int_0^{\Delta t}\left(-\frac{t^2}{\Delta t^2}+\frac{t}{\Delta t}\right)dt = \left.\frac{-t^3}{3\Delta t^2}\right|_0^{\Delta t} + \left.\frac{t^2}{2\Delta t}\right|_0^{\Delta t} = \frac{-\Delta t^3}{3\Delta t^2}+\frac{\Delta t^2}{2\Delta t}$$

$$\frac{-\Delta t}{3}+\frac{\Delta t}{2} = \boxed{\frac{\Delta t}{6}}$$

d) $\int_0^1 (\phi_i(t))^2\,dt =$ Esta vez, la función a integrar tiene valores no nulos en un intervalo $t \in [t_{i-1}, t_{i+1}]$.

#FUNGINEER

graf. 2



Además, si consideramos la simetría de la función hat respecto a t;

$$\int_0^1 (\phi_i(t))^2\,dt = \int_0^{2\Delta t} (\phi_i(t))^2\,dt \overset{\text{simetría}}{=} 2\int_0^{\Delta t} (\phi_i(t))^2\,dt$$

$$2\int_0^{\Delta t}\left(\frac{t}{\Delta t}\right)^2 dt = \frac{2}{\Delta t^2}\left(\frac{t^3}{3}\right)\Bigg|_0^{\Delta t} = \frac{2\,\Delta t^3}{\Delta t^2\cdot 3} = \boxed{\frac{2\,\Delta t}{3}}$$

e) $\int_0^1 \phi_i'(t)\,\phi_{i+1}'(t)\,dt$ Similar al item c). Integral es válida entre $t \in [t_i, t_{i+1}]$.

Además, si utilizamos las funciones del gráfico 1:

$$\int_0^1 \phi_i'(t)\,\phi_{i+1}'(t)\,dt = \int_0^{\Delta t}\left(\frac{1}{\Delta t}\right)\cdot\left(-\frac{1}{\Delta t}\right)dt = -\frac{1}{\Delta t^2}(t)\Big|_0^{\Delta t}$$

$$-\frac{\Delta t}{\Delta t^2} = \boxed{-\frac{1}{\Delta t}}$$

f) $\int_0^1 (\phi_i'(t))^2 dt$ Utilizando la función del gráfico 2.

$$\int_0^{2\Delta t} (\phi_i'(t))^2 dt = \int_0^{\Delta t} \left(\frac{1}{\Delta t}\right)^2 dt + \int_{\Delta t}^{2\Delta t} \left(-\frac{1}{\Delta t}\right)^2 dt$$

$$= \int_0^{\Delta t} \left(\frac{1}{\Delta t^2}\right) dt + \int_{\Delta t}^{2\Delta t} \left(\frac{1}{\Delta t^2}\right) dt = 2\int_0^{\Delta t} \frac{1}{\Delta t^2} dt = \frac{2}{\Delta t^2}(t)\Big|_0^{\Delta t}$$

$$= \frac{2}{\Delta t^2} \cdot \Delta t = \boxed{\frac{2}{\Delta t}}$$

2.- a) $\delta y''(t) = \gamma y(t) + f(t) \quad / \cdot v(t)$

$$\delta y''(t) \cdot v(t) = (\gamma y(t) + f(t)) v(t) \quad / \int_0^1$$

$$\int_0^1 \delta y''(t) \cdot v(t)\, dt = \int_0^1 \gamma y(t) v(t)\, dt + \int_0^1 f(t) v(t)\, dt$$ weak form

b) $$\delta y'(t) v(t)\Big|_0^1 - \delta\int_0^1 y'(t) \cdot v'(t)\, dt = \int_0^1 \gamma y(t) v(t)\, dt + \int_0^1 f(t) v(t)\, dt$$

Eligiendo $v(t)$ como $\phi_K(t)$, con $K \in \{1, 2, \ldots, n-1\}$

$$\rightarrow \delta\, y'(t)\,\phi_k(t)\Big|_0^1 - \delta\int_0^1 y'(t)\cdot\phi_k'(t)\,dt = \eta\int_0^1 y(t)\cdot\phi_k(t)\,dt + \int_0^1 f(t)\cdot\phi_k(t)\,dt$$

$$\phi_k(1) = \phi_k(0) = 0$$

(Funciones hat valen 0 en los extremos)

$$\rightarrow\ -\delta\int_0^1 y'(t)\,\phi_k'(t)\,dt - \eta\int_0^1 y(t)\,\phi_k(t)\,dt = \int_0^1 f(t)\,\phi_k(t)\,dt$$

Con $y(t) = \sum_{i=0}^{n} C_i\,\phi_i(t)$, $y'(t) = \sum_{i=0}^{n} C_i\,\phi_i'(t)$

En los extremos se tiene:

$$y(0) = \sum_{i=0}^{n} C_i\,\phi_i(0) = C_0\,\phi_0(0) = \boxed{C_0 = y_0}$$

$$y(1) = \sum_{i=0}^{n} C_i\,\phi_i(1) = C_n\,\phi_n(1) = \boxed{C_4 = y_4} \quad n=4$$

Para C_1, C_2 y C_3 se plantea:

$$-\delta\int_0^1\left(\sum_{i=0}^{n} C_i\,\phi_i'(t)\right)\cdot\phi_k'(t)\,dt - \eta\int_0^1\left(\sum_{i=0}^{n} C_i\,\phi_i(t)\right)\phi_k(t)\,dt = \int_0^1 f(t)\,\phi_k(t)\,dt$$

$$\sum_{i=0}^{n} C_i\left[-\delta\int_0^1 \phi_i'(t)\cdot\phi_k'(t)\,dt - \eta\int_0^1 \phi_i(t)\,\phi_k(t)\,dt\right] = g_k$$

Para $K=1$ (C_1)

$$C_0\left[-\delta\underbrace{\int_0^1 \phi_0'(t)\,\phi_1'(t)\,dt}_{e)} - \gamma\underbrace{\int_0^1 \phi_0(t)\,\phi_1(t)\,dt}_{c)}\right] +$$

$$C_1\left[-\delta\underbrace{\int_0^1 \phi_1'(t)\,\phi_1'(t)\,dt}_{f)} - \gamma\underbrace{\int_0^1 \phi_1(t)\,\phi_1(t)\,dt}_{d)}\right] +$$

$$C_2\left[-\delta\underbrace{\int_0^1 \phi_2'(t)\,\phi_1'(t)\,dt}_{e)} - \gamma\underbrace{\int_0^1 \phi_2(t)\,\phi_1(t)\,dt}_{c)}\right] = g_1$$

Las integrales de C_1 con C_3 y C_4 son nulos.

De la expresión anterior conocemos el valor de las integrales por el item 1.

$$C_0\left[-\delta\left(-\frac{1}{\Delta t}\right) - \gamma\left(\frac{\Delta t}{6}\right)\right] + C_1\left[-\delta\left(\frac{2}{\Delta t}\right) - \gamma\left(\frac{2\Delta t}{3}\right)\right] + C_2\left[-\delta\left(-\frac{1}{\Delta t}\right) - \gamma\left(\frac{\Delta t}{6}\right)\right]$$
$$= g_1$$

Para $K=2$ (C_2) Las integrales de C_2 con C_0 y C_4 son nulos

$$C_1\left[-\delta\left(-\frac{1}{\Delta t}\right) - \gamma\left(\frac{\Delta t}{6}\right)\right] + C_2\left[-\delta\left(\frac{2}{\Delta t}\right) - \gamma\left(\frac{2\Delta t}{3}\right)\right] + C_3\left[-\delta\left(-\frac{1}{\Delta t}\right) - \gamma\left(\frac{\Delta t}{6}\right)\right]$$
$$= g_2$$

El patrón se repite para $K=3$.

$$\alpha = \left[-\delta\left(-\frac{1}{\Delta t}\right) - \gamma\left(\frac{\Delta t}{6}\right)\right]$$

$$\beta = \left[-\delta\left(\frac{2}{\Delta t}\right) - \gamma\left(\frac{2\Delta t}{3}\right)\right]$$

El sistema será:

$$C_0\alpha + C_1\beta + C_2\alpha = g_1$$
$$C_1\alpha + C_2\beta + C_3\alpha = g_2$$
$$C_2\alpha + C_3\beta + C_4\alpha = g_3$$

Conocidos.

$$C_1\beta + C_2\alpha = g_1 - C_0\alpha$$
$$C_1\alpha + C_2\beta + C_3\alpha = g_2$$
$$C_2\alpha + C_3\beta = g_3 - C_4\alpha$$

$$\longrightarrow \begin{bmatrix} \beta & \alpha & 0 \\ \alpha & \beta & \alpha \\ 0 & \alpha & \beta \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} g_1 - g_0\alpha \\ g_2 \\ g_3 - y_4\alpha \end{bmatrix}$$

Luego, se pueden encontrar los valores de C_1, C_2 y C_3 mediante un solver lineal. Donde:

$$C_1 \approx y(1/4)$$
$$C_2 \approx y(1/2)$$
$$C_3 \approx y(3/4)$$