Demystifying the Fourier Transform: The Intuition

Valerio Velardo

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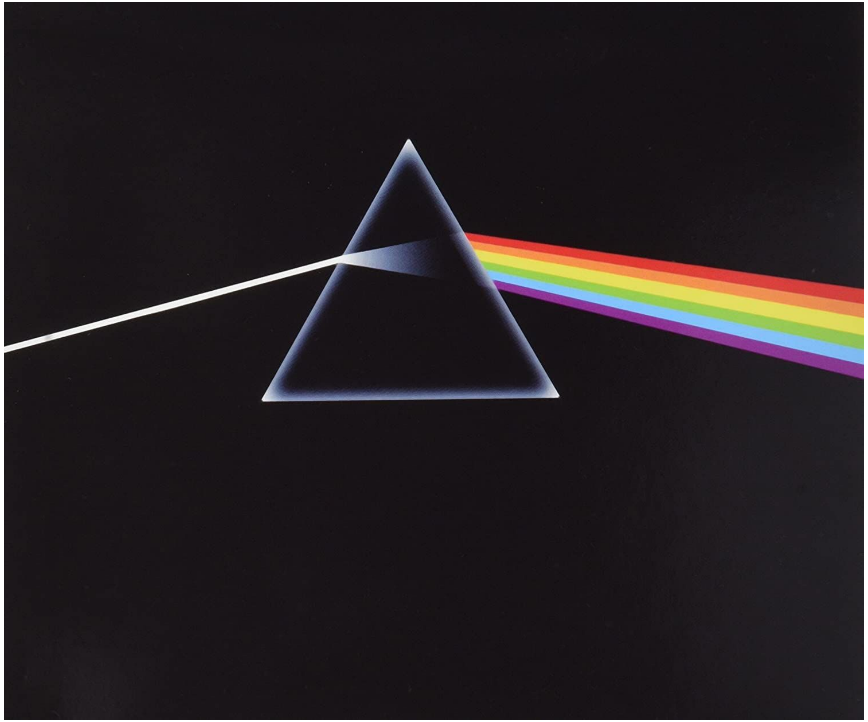
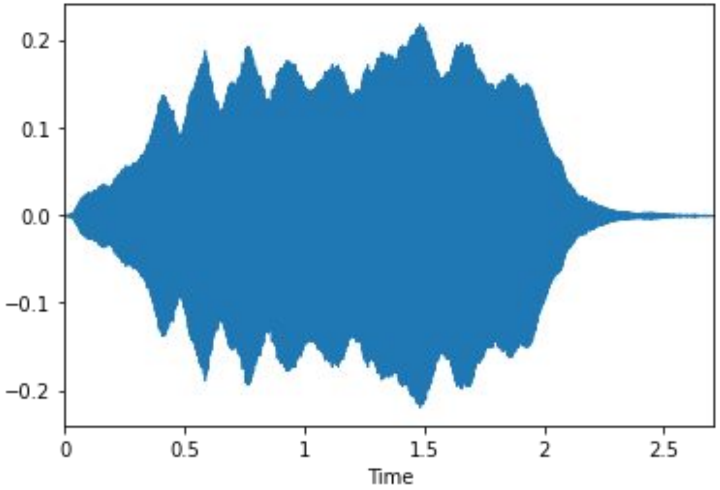


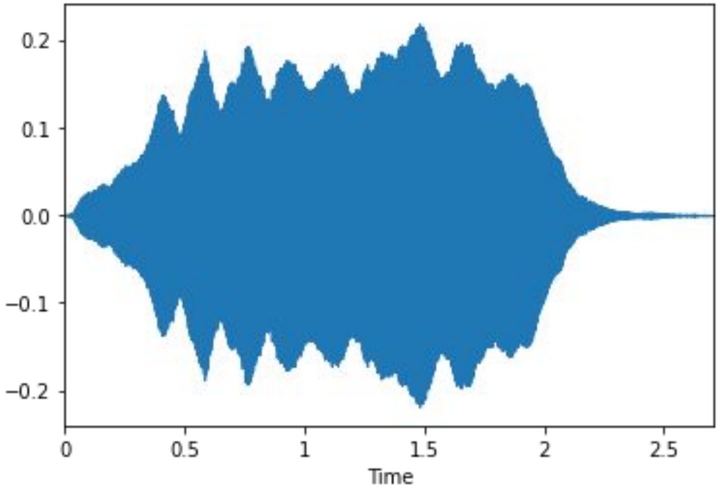
Intuition

- Decompose a complex sound into its frequency components

From time to frequency domain

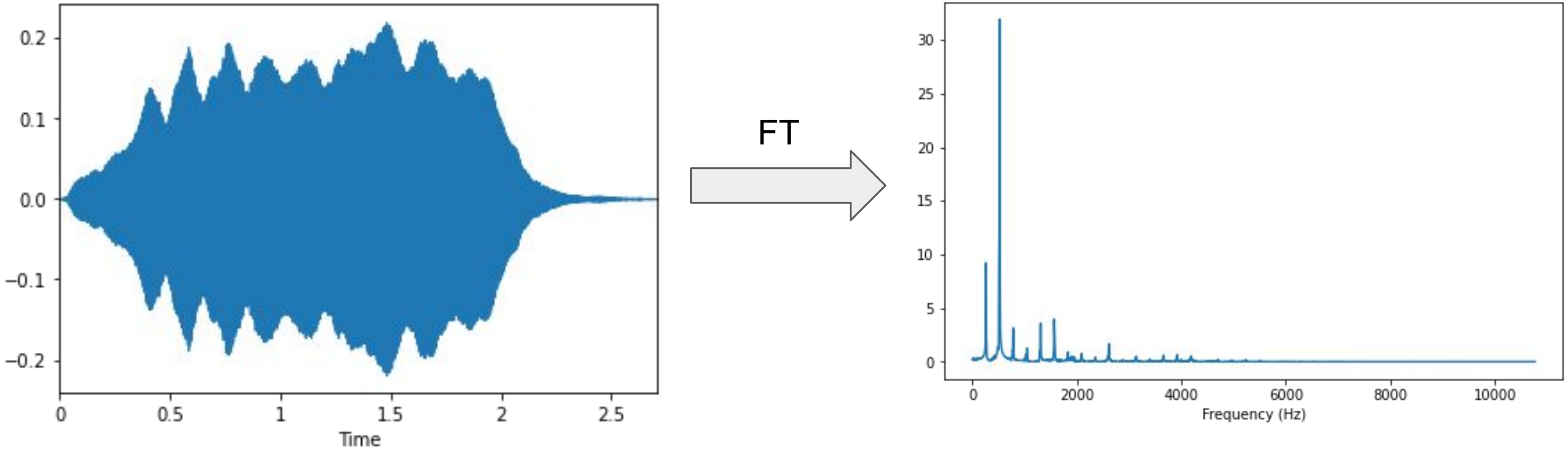


From time to frequency domain



FT

From time to frequency domain



Deeper intuition

- Compare signal with sinusoids of various frequencies

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- For each frequency we get a magnitude and a phase

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- High magnitude indicates high similarity between the signal and a sinusoid

Sine wave

$$sin(2\pi \cdot (ft - \varphi))$$

Deeper intuition

- Compare signal with sinusoids of various frequencies
- For each frequency we get a magnitude and a phase
- High magnitude indicates high similarity between the signal and a sinusoid

Fourier transform: Step by step

Choose a frequency

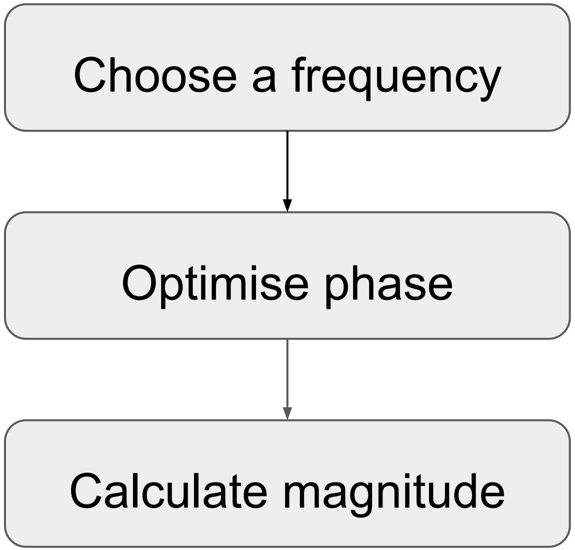
Fourier transform: Step by step

Choose a frequency

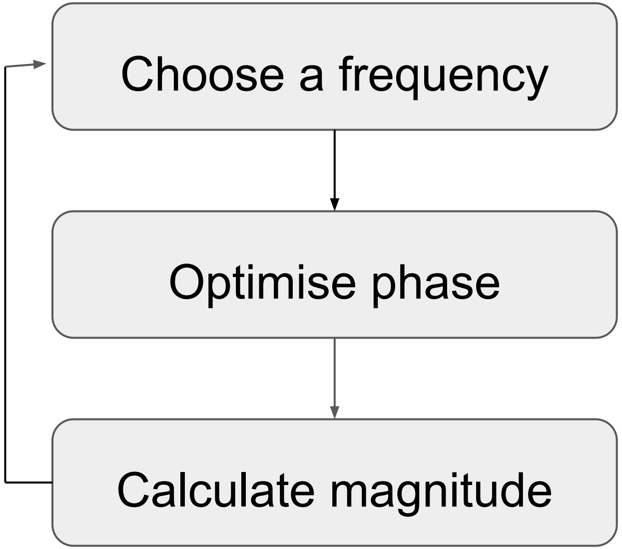
↓

Optimise phase

Fourier transform: Step by step



Fourier transform: Step by step



Fourier transform

$$\varphi_f = argmax_{\varphi \in [0,1)} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

Fourier transform

$$\varphi_f = argmax_{\varphi \in [0,1)} \left(\int \boxed{s(t) \cdot sin(2\pi \cdot (ft - \varphi))} \cdot dt \right)$$

Multiply signal and sinusoid

Fourier transform

$$\varphi_f = argmax_{\varphi \in [0,1)} \left(\boxed{\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt} \right)$$

Calculate area

Fourier transform

$$\varphi_f = \boxed{argmax_{\varphi \in [0,1)}} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

Select phase in [0, 1) that
maximises the area

Fourier transform

$$\varphi_f = argmax_{\varphi \in [0,1)} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

$$d_f = \max_{\varphi \in [0,1)} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

Fourier transform

$$\varphi_f = argmax_{\varphi \in [0,1)} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

$$d_f = \boxed{\max_{\varphi \in [0,1)}} \left(\int s(t) \cdot sin(2\pi \cdot (ft - \varphi)) \cdot dt \right)$$

Select max area

Fourier transform

$$\varphi_f = argmax_{\varphi\in[0,1)}\left(\boxed{\int} s(t)\cdot sin(2\pi\cdot(ft-\varphi))\cdot\boxed{dt}\right)$$

t ∈ R

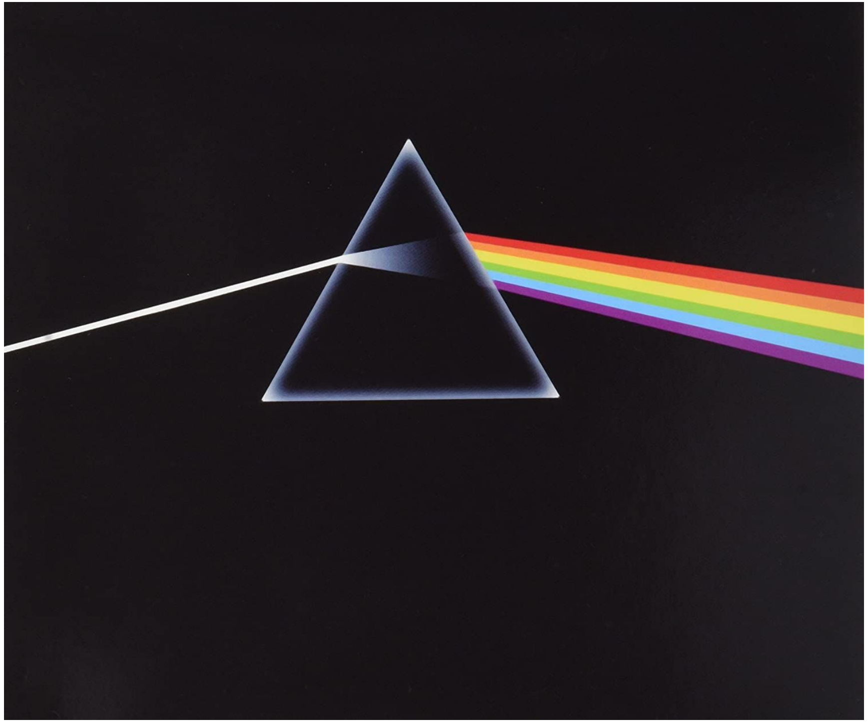
$$d_f = \max_{\varphi\in[0,1)}\left(\boxed{\int} s(t)\cdot sin(2\pi\cdot(ft-\varphi))\cdot\boxed{dt}\right)$$

Fourier transform

$$\varphi_f = argmax_{\varphi\in[0,1)}\left(\int s(t)\cdot sin(2\pi\cdot(\boxed{f}t-\varphi))\cdot dt\right)$$

f ∈ **R**

$$d_f = \max_{\varphi\in[0,1)}\left(\int s(t)\cdot sin(2\pi\cdot(\boxed{f}t-\varphi))\cdot dt\right)$$



Reconstructing a signal

- Superimpose sinusoids

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- Weight them by the relative magnitude

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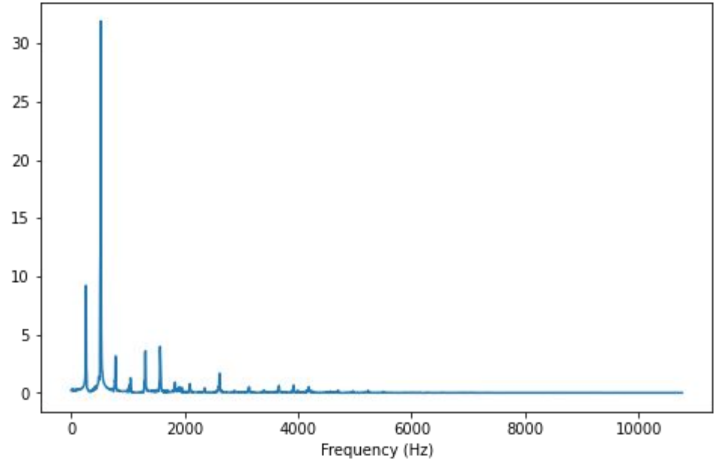
Reconstructing a signal

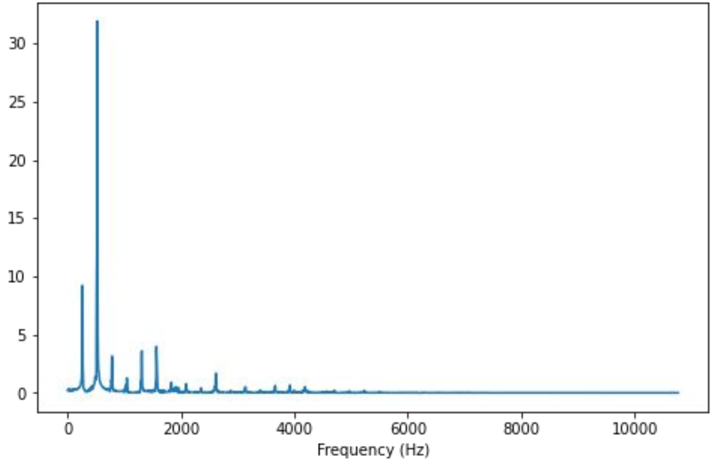
- Superimpose sinusoids
- Weight them by the relative magnitude
- Use relative phase
- Original signal and FT have same information

Inverse Fourier transform

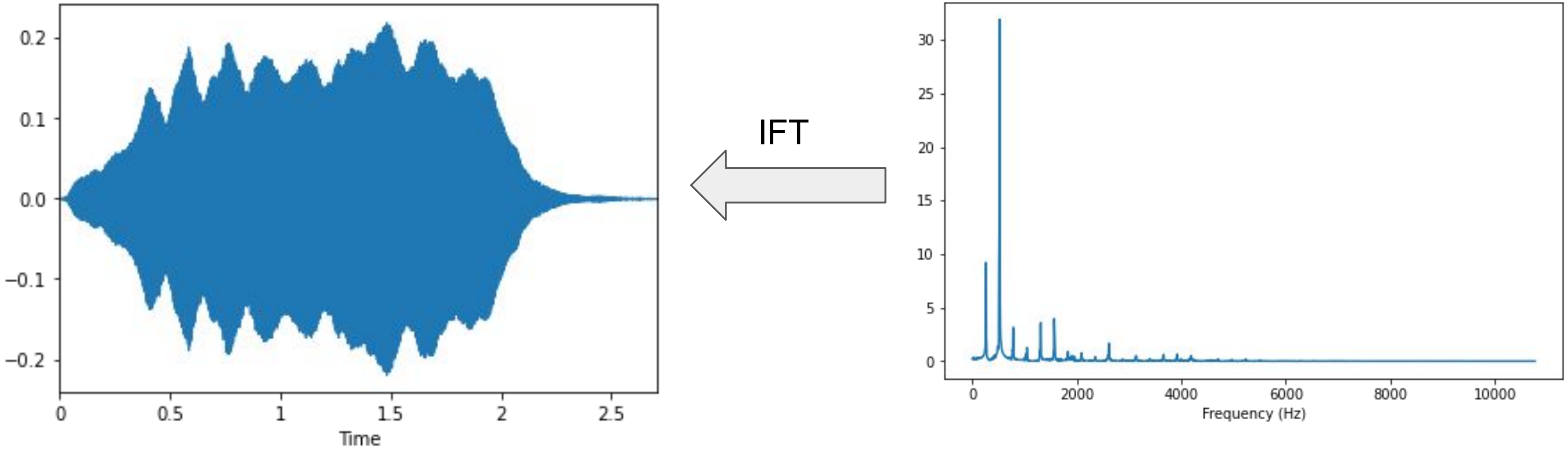


Inverse Fourier transform

IFT



Inverse Fourier transform



Additive synthesis

What’s up next?

- Complex numbers