

# Learning Relational Rules from Rewards

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## ABSTRACT

Humans perceive the world in terms of objects and relations between them. In fact, for any given pair of objects, there is a myriad of relations that apply to them. How does the cognitive system learn which relations are useful to characterize the task at hand? And how can it use these representations to build a relational policy to interact effectively with the environment? In this paper we propose that this problem can be understood through the lens of a sub-field of symbolic machine learning called relational reinforcement learning (RRL). To demonstrate the potential of our approach, we build a simple model of relational policy learning based on a function approximator developed in RRL. We trained and tested our model in three Atari games that required to consider an increasingly number of potential relations: Breakout, Pong and Demon Attack. In each game, our model was able to select adequate relational representations and build a relational policy incrementally. We discuss the relationship between our model with models of relational and analogical reasoning, as well as its limitations and future directions of research.

## 1. Introduction

Humans can generalise what they learn about specific instances of a problem to a potentially unbound set of new instances of the same problem (Pouncy, Tsividis, & Gershman, 2021). For example, after learning to play checkers on a black and white  $8 \times 8$  board, a person can easily play the game on a yellow and green board or on a  $10 \times 10$  board. However, we also regularly go beyond task-specific generalizations and use what we have learned about one problem in service of new problems. For example, one can use what they know about checkers when learning to play chess (“learning to learn”; Lake, Ullman, Tenenbaum, & Gershman, 2017). Generalization of what one has learned about one domain to a seemingly unrelated domain on the basis of deep structural similarity is known as cross-domain generalization (e.g., Gick & Holyoak, 1983; Halford, Bain, Maybery, & Andrews, 1998). A potential mechanism for this kind of generalization is role-based relational reasoning. In this type of reasoning, inferences about objects are based on the commonalities or differences in the roles they play on a situation instead of the literal features of those objects (Holyoak, 2012). Analogy is a prime example of role-based relational reasoning in which inferences about a target domain are generated on the basis of patterns of relational roles in a source domain.


Doumas, Puebla, Martin, and Hummel (2022) developed a computational model of human cross-domain generalization based on the idea that it is special a case of analogical inference over explicitly relational representations. The model consisted of a version of the DORA model for learning structured relational representations from non-structured inputs (Doumas, Hummel, & Sandhofer, 2008),


augmented with the capacity to discover relational invariants from absolute magnitude representations and to integrate the learned representations with the capacity for simple reinforcement learning (RL, Sutton & Barto, 2018). Doumas et al. showed that after experience with a source domain (e.g., images from the CLVR dataset, Johnson et al., 2017), DORA was able to transfer its learned representations to a novel target domain (e.g., the video game Breakout) without any additional experience. Moreover, using RL DORA could learn strategies (or policies) for performing one task (e.g., playing Breakout), and transfer these policies to a new task (e.g., playing Pong) with no additional training.

One limitation of that work, however, was that standard RL algorithms do not scale well with relational representations (Otterlo, 2012). Consequently, during RL, the model learned using only a small set of pre-selected relations between objects (from the much larger set of relations it had previously learned) to learn policies. In contrast, throughout their lives humans acquire a vast vocabulary of relations that might apply freely to any given situation (Gentner & Hoyos, 2017). This disparity presents a key computational problem for a system using representations of relations to learn to behave effectively in the environment: How does the system choose which of all available relations to use to represent the task at hand? The present work aims to bridge the theoretical gap between psychological theories of role-based relational reasoning and RL by presenting a model that is capable of selecting relevant relations from a larger representational vocabulary in order to learn a relational policy.

This problem is closely related to the representational learning problem studied in feature-based sequential decision making research (e.g., Jones & Canas, 2010; Niv, 2019). Similarly to that line of work, we propose that RL itself can be responsible for selecting the relevant relations (in their case feature dimensions) over which the model learns a policy. However, because relational representations pose distinct problems for standard RL algorithms, we adapt

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methods developed in the sub-field of relational RL (RRL; for a review see Otterlo, 2012) to build a simple model that learns to select adequate relational representations when learning a policy for a particular task. We test our model on versions of three Atari games: Breakout, Pong and Demon Attack. In the following, we give a brief overview of RL, RRL and then describe our model.

### 1.1. Reinforcement Learning

In RL an agent interacts with the environment taking actions in order to maximise rewards and avoid punishment. The environment is conceptualised as a set of states with transitions between them probabilistically determined by the agent’s actions. RL algorithms are designed to learn an optimal policy (i.e., a mapping between states and actions) through this interaction (Sutton & Barto, 2018). In general terms, RL algorithms can be classified into *model-based* and *model-free* methods. In model-based methods the agent learns a transition function from states and actions to new states and a reward function from states to rewards. These two functions correspond to the *model* of the environment. The agent can use the model to plan the best course of action at a given state through simulation. In contrast, in model-free methods the agent uses prediction errors to learn the *value* (i.e., the expected cumulative reward) of taking each action in each state directly, without learning a model of the environment. These values can then be used to build the policy by greedy selection of the best action in each state. In the present paper we are mostly concerned with model-free learning (we elaborate on the relation of our approach to model-based learning and planning in the Discussion). The most influential model-free algorithm is q-learning (Watkins, 1989). This method approximates the action value function according to the following update rule:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[ R_t + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right] \quad (1)$$

where  $Q(S_t, A_t)$  is the current value of taking action  $A$  on state  $S$ ,  $\alpha$  is the learning rate,  $\gamma$  is a discount factor that has the effect of weighting more rewards closer in time to rewards farther away in the future, and the subtraction term is the prediction error. Note that q-learning updates its value estimates based on the best next action,  $\max_a Q(S_{t+1}, a)$ , even though the agent’s current policy might choose a different next action. This is important because it allows q-learning to approximate the optimal policy while following a different sub-optimal policy. As is usually done in q-learning, in the present paper we use a suboptimal policy known as  *$\epsilon$ -greedy*, where all the non-optimal actions are given a selection probability of  $\frac{\epsilon}{|\mathcal{A}|}$ , where  $|\mathcal{A}|$  is the size of the action space, and the action with the highest value is given a selection probability of  $1 - \epsilon + \frac{\epsilon}{|\mathcal{A}|}$ . During training, the value of  $\epsilon$  is slowly decreased, so as to allow the agent to explore progressively less as it improves its value estimate.

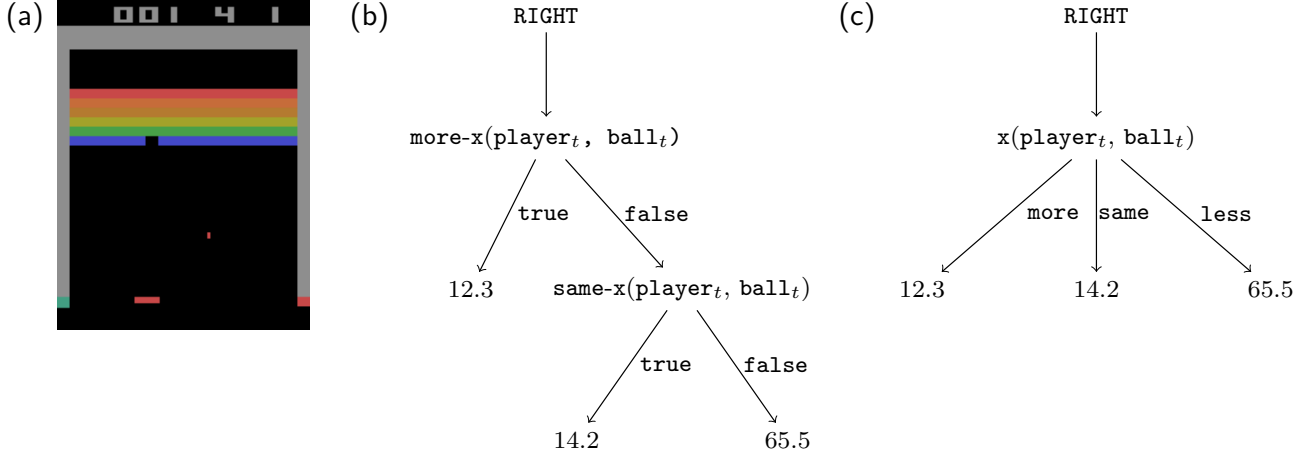
While q-learning is guaranteed to converge to the optimal policy as long as all state-action pairs are updated during learning (Watkins & Dayan, 1992), it becomes prohibitively for large state spaces. This is especially problematic in relational settings, where the size of the state space grows combinatorially on the number of relations and objects. As explained below, RRL algorithms use specialized function approximators to handle this problem.

### 1.2. Relational Reinforcement Learning

The goal of RRL is to learn an optimal policy in an environment described as a set of objects and relations between them (Džeroski, De Raedt, & Driessens, 2001). Notably, the policy is represented as a set of *variabilized* rules in (a subset of) first order logic. For example, in a version of the classic problem known as *blocks world* (Slaney & Thiébaux, 2001), where the agent is rewarded for unstacking a group of blocks, one such rule could be:  $\text{move}(X, \text{floor}) \leftarrow \text{on}(X, Y) \wedge \text{top}(X)$  (i.e., “if block  $X$  is on any other block  $Y$ , and block  $X$  is on top of a pile, then move block  $X$  to the floor”). In contrast this approach, the present paper concentrates in the learning of *ground* rules, i.e., rules that apply to specific objects in a specific task, such as:  $\text{LEFT} \leftarrow \text{more-x}(\text{player}, \text{ball})$  (i.e., “if the player is to the right of the ball, then move left”). This is because we think that is more likely that people learn relational rules that apply to specific situations and, later on, those rules are generalized through the process of schema induction (e.g., Chen & Mo, 2004; Gick & Holyoak, 1983). From a cognitive point of view, an interesting attribute of classic RRL algorithms is that, in general, they build policies *incrementally*, gradually adding rules to the policy that improve its the overall quality (Driessens, Ramon, & Blockeel, 2001, see next section). This is in stark contrast to a theory-based Bayesian approach to reinforcement learning (e.g., Tsividis et al., 2021), where the complete space of programs (which includes relational policies) is defined *a priori* and learning equates to infer the best-fitting program from all possible programs through probabilistic inference (we elaborate on the relation between top-down and bottom-up approaches in section 5). Another important attribute of RRL algorithms is that they have to deal with the discrete nature of relational representations. This is because describing the state of the environment in terms of relations imposes sharp partitions of the state space. For instance, on the aforementioned  $\text{LEFT}$  rule, the relation  $\text{more-x}(\text{player}, \text{ball})$  partitions the state space into states where the player is to the right of the ball, and states where it is not. This fact requires the use of an specialized function approximator that can make use of relational representations to *abstract away* irrelevant aspects of the state space. Our model makes use of a specialized function approximator developed for RRL.

### 1.3. Relational Regression Tree Learner

Our model, which we term relational regression tree learner (RRTL), is based on the function approximator for RRL proposed by (Driessens et al., 2001, see also Driessens, 2004). This algorithm uses regression trees to represent the state-action value function. For each action there is a



**Figure 1:** Two different ways of representing state-action values. (a) Typical state of the Breakout environment where the player is to the left of the ball. The fact that the value of the action RIGHT is higher when the player is to the left of the ball, lower when the player and the ball are at the same x-coordinate, and lowest when the the player is to the right of the ball, can be represented as a logical relational regression tree (b) or as a comparative relational regression tree (c). See text for details.

tree where each node represents a conjunction of ground relations and each leaf is a q-value. In the original model of Driessens et al. (2001) these trees were based on logical splits of the state-action space. To illustrate this, Fig. 1a shows a typical state of the game Breakout, where the player gains points by making the ball bounce against the wall of bricks at the top of the screen. In general, the action RIGHT has the highest value when the player is to the left of the ball, has a lower value when the ball and the player are at the same position on the x-axis, and has the lowest value when the player is to the right of the ball. To represent this ranking of values a logical regression tree needs to make two splits (true and false for more-x and true and false for same-x), as shown in Fig. 1b. An alternative way of representing the same ranking is to make splits based on the comparative values more, same and less of the x relation between the player and the ball. In this case the tree needs to make only a single split, as depicted in Fig. 1c.

To make a prediction the agent traverses the tree corresponding to each action according to the relations present in the state until it reaches a leaf. The predicted q-value can then be used to select an action and can be updated according to Eq. (1). At the beginning of the learning process, all the trees have a single leaf. At this stage, the q-value represents the overall value of the action in the environment. All state-action trees consider the same initial set of candidate relations to grow new leaves. As the agent interacts with the environment, each state-action tree keeps track of the current number of visits to the candidate relation,  $n$ , the mean,  $\mu_n$ , and the scaled variance,  $J_n = \sigma_n^2 \cdot n$ , of the q-values produced at each time step, as well as the same statistics for all potential partitions induced by the candidate (i.e., true and false for logical partitions and more, same and less for comparative ones). These statistics are calculated

incrementally according to Eqs. (2) and (3) (for derivations see Finch, 2009)<sup>1</sup>:

$$\mu_n = \mu_{n-1} + \frac{x_n - \mu_{n-1}}{n} \quad (2)$$

$$J_n = J_{n-1} + (x_n - \mu_{n-1})(x_n - \mu_n) \quad (3)$$

were  $\sigma_n^2 = J_n/n$ .

After a *minimal sample size* (a free parameter of the model) has been reached, these statistics can be used to compute, for each candidate, the  $F$ -ratio between the variance of the q-values if the leaf was split according to the candidate and the variance of the q-values of the unsplit leaf. Eqs. (4) and (5) show the  $F$ -ratio for logical and comparative splits, respectively:

$$F = \frac{\frac{n_T}{n_O} \sigma_T^2 + \frac{n_F}{n_O} \sigma_F^2}{\sigma_O^2} = \frac{J_T/n_O + J_F/n_O}{J_O/n_O} \quad (4)$$

$$F = \frac{\frac{n_M}{n_O} \sigma_M^2 + \frac{n_S}{n_O} \sigma_S^2 + \frac{n_L}{n_O} \sigma_L^2}{\sigma_O^2} = \frac{J_M/n_O + J_S/n_O + J_L/n_O}{J_O/n_O} \quad (5)$$

were  $\sigma^2$  is the variance,  $n$  is the total number of visits to the partition, and the subscript now indicates the partition ( $T$  = true,  $F$  = false,  $M$  = more,  $S$  = same,  $L$  = less and  $O$  = overall). With this ratio, the tree calculates the  $p$ -values

<sup>1</sup>The implementation described by Driessens (2004) uses the sum of squared q-values to calculate the variance, however, this can be numerically unstable.

**Table 1**  
Breakout State Representation

Dimension	Object-1	Object-2	Relation	
			Logical	Comparative
x	player <sub>t</sub>	ball <sub>t</sub>	more-x(player <sub>t</sub> , ball <sub>t</sub> ) same-x(player <sub>t</sub> , ball <sub>t</sub> ) less-x(player <sub>t</sub> , ball <sub>t</sub> )	x(player <sub>t</sub> , ball <sub>t</sub> )
y	player <sub>t</sub>	ball <sub>t</sub>	more-y(player <sub>t</sub> , ball <sub>t</sub> ) same-y(player <sub>t</sub> , ball <sub>t</sub> ) less-y(player <sub>t</sub> , ball <sub>t</sub> )	y(player <sub>t</sub> , ball <sub>t</sub> )
x	ball <sub>t</sub>	ball <sub>t-1</sub>	more-x(ball <sub>t</sub> , ball <sub>t-1</sub> ) same-x(ball <sub>t</sub> , ball <sub>t-1</sub> ) less-x(ball <sub>t</sub> , ball <sub>t-1</sub> )	x(ball <sub>t</sub> , ball <sub>t-1</sub> )
y	ball <sub>t</sub>	ball <sub>t-1</sub>	more-y(ball <sub>t</sub> , ball <sub>t-1</sub> ) same-y(ball <sub>t</sub> , ball <sub>t-1</sub> ) less-y(ball <sub>t</sub> , ball <sub>t-1</sub> )	y(ball <sub>t</sub> , ball <sub>t-1</sub> )

of a standard one-tailed  $F$ -tests for all candidates. If the smallest  $p$ -value is smaller than the *significance level* the leaf is split according to the candidate and the process continues until the tree cannot find new splits or reaches a *maximum tree depth*. In all our simulations we set the maximum tree depth to 10, the minimal sample size to 100,000, and the significance level to 0.001.

## 2. Simulation 1: Breakout

In Simulation 1, we tested our model on the game Breakout (see Fig. 1a). This is the simplest environment that we used. As mentioned above, in Breakout the player controls a paddle and receives points when the ball bounces the wall on the top of the screen. The player loses points if the ball passes the paddle when it is going down (the player  $y$ -position is fixed at bottom of the screen). The actions available to the agent are: NOOP, FIRE, RIGHT, LEFT. Note that the actions NOOP and FIRE have the same effect during the game (i.e., the paddle does not move), however, at the beginning of the game the agent has to execute the FIRE action to start. To succeed on this game the agent needs to learn to follow the ball, which requires to pay more attention to the relations across the  $x$ -dimension than to the relations across the  $y$ -dimension<sup>2</sup>.

To represent the state of the environment, we used the  $x$  and  $y$  relations between the player and the ball and (henceforth, object relations) and the  $x$  and  $y$  relations between the ball at the current time step and the ball at the previous time step (henceforth, trajectory relations)<sup>3</sup>. In the object

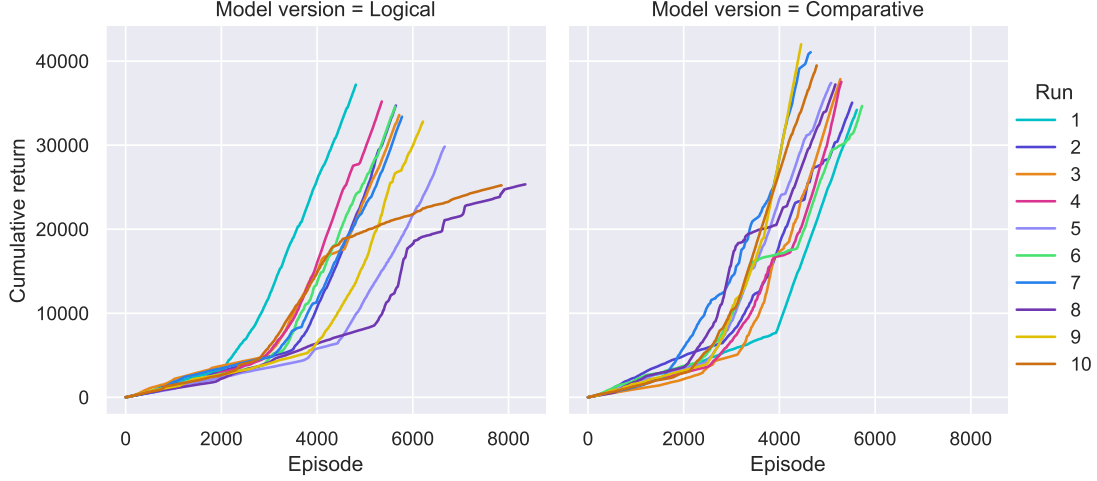
relations the first object was always the player and the second object was always the ball. In the trajectory relations the first object was always the object at the current time step and the second was the object at the previous time step. We created two versions of the state, a logical version and a comparative version. Table 1 presents all relations considered in this simulation. Note that even in this simple environment a tabular representation of the same state-action space would need  $3^4 = 81$  rows  $\times$  4 columns. During the construction of the state, we filtered out all states where the ball was not present (i.e., those states were treated as empty). This was done because of the frequentist statistics approach used to determine the state-action tree splits: to compete in equal grounds, all candidates should have the same number of visits.

We trained and tested our model in the environment “BreakoutDeterministic-v4” of OpenAI’s gym toolkit (Brockman et al., 2016). To obtain a relational state from the gym environment we created a visual pre-processor, which used the color and shape of the objects to calculate the  $x$  and  $y$  positions of the centers of the ball and the paddle. To obtain the relations shown on Table 1, we set the roles “Object-1” and “Object-2” according to a hierarchy of objects were “Object-1” was always the player and “Object-2” was always the ball. Furthermore, to calculate the comparative value associated with each relation we subtracted the  $x$  and  $y$  positions according to the same hierarchy and categorized the difference with a tolerance level of six pixels (e.g., if the  $x$ -center of the player was above 6 pixels to the right of the ball the comparative value was more, if the difference was below six pixels the comparative value was same, and else the comparative value was less). We calculate the trajectory relations in the same way, except that we used a tolerance level of zero. All RL agents in the present paper included an action buffer that store the last 10 actions and checked whether the agent had taken the same action 10 times in a row, in which case the current action was uniformly sampled from the action space. For each of the two versions of the

<sup>2</sup>There are certainly more complex and effective policies like digging a tunnel through the wall to allow the ball to bounce above the blocks (e.g., Mnih et al., 2015). However, as we are not representing the wall, following the ball is the optimal policy in the environment represented at this level of abstraction.

<sup>3</sup>We did not represent the trajectory relations for the player because, as noted above, the  $y$ -trajectory of the player is a constant and, for any consistent policy in Breakout, there is a correlation between the  $x$ -trajectory of the player  $x(\text{player}_t, \text{player}_{t-1})$  and the action taken  $x(\text{player}_t, \text{player}_{t+1})$ .





**Figure 2:** Cumulative return by model version and run on Breakout.

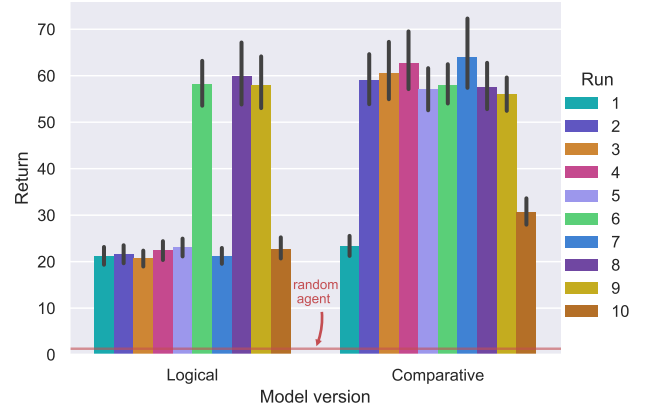
model, we trained 10 runs on 2,000,000 iterations with RL parameters  $\alpha = 0.1$ ,  $\gamma = 0.99$  and  $\epsilon$  was decayed from 1.0 to 0.1 according to the formula:  $\epsilon_t \leftarrow d\epsilon_{t-1}$  with  $d = 0.9999995$ . We transformed the reward using the sign function. The code for all our simulations is available at <https://github.com/GuillermoPuebla/rri1>.

## 2.1. Results and Discussion

Fig. 2 shows the cumulative return for the logical and comparative versions of the model for each one of the 10 runs. Overall, the runs of the comparative version received more rewards during training. Because in Breakout episodes with higher returns are necessarily longer, the runs in this version reached the 2,000,000 iterations in fewer episodes. Furthermore, the runs on the comparative version were also more consistent, namely, there was less variability on total return and number of episodes across runs than in the logical version. During training, we saved the agent’s set of state-action trees every 200,000 iterations. After the training session finished, we tested each set of trees on 10 games and picked the best one, which became the agent test trees. For testing we set  $\epsilon$  to 0.05.

Fig. 3 shows the average return on 100 test episodes. As can be seen, all runs in both models achieved higher than chance returns (defined as the average returns of an agent taking random actions on 100 episodes). Furthermore, the maximum average return across runs was similar in both versions. However, the results were much more consistent in the comparative version, with eight out of 10 runs achieving average returns above 50 in comparison to only three in the logical condition.

Fig. 4 shows the state-action trees of the best run of the logical (a) and comparative (b) versions of RRTL. As can be seen, the best run of the logical version of RRTL achieved to discriminate between the different comparative values of the object  $\times$  relation only regarding the LEFT action. However, this LEFT tree also introduced a test involving the trajectory of the ball in the  $y$  dimension, which increases the

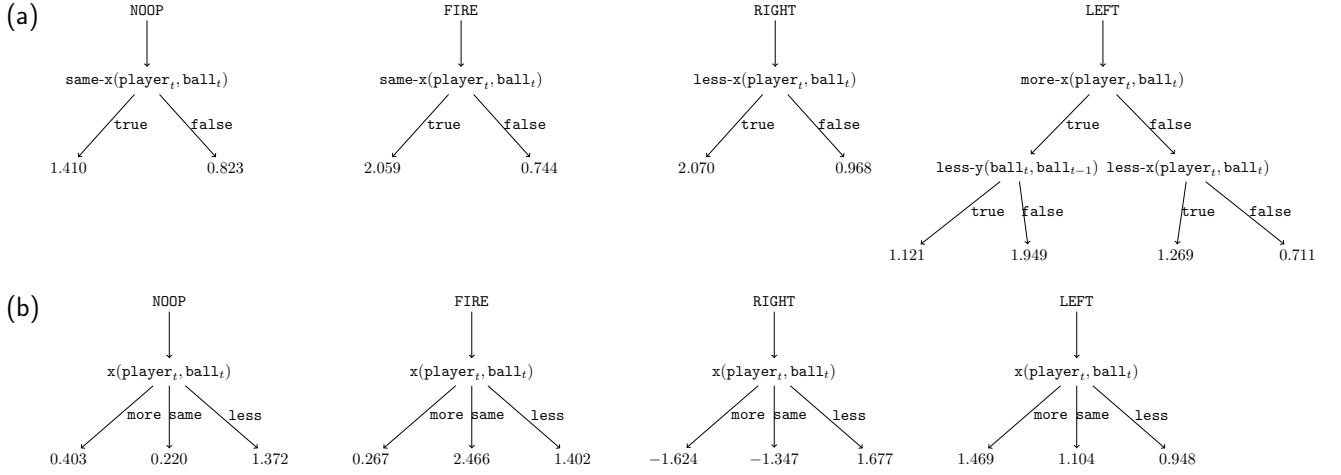


**Figure 3:** Mean returns by model version and run on 100 test episodes of Breakout. Error bars are 95% confidence intervals. The horizontal shadowed line represents the mean return and 95% confidence intervals for a random agent on 100 test episodes.

overall complexity of the policy. All other state-action trees made splits based on the on the object  $\times$  relation but did not discriminate between all its comparative values. In contrast, the best run of the comparative version of RRTL achieved to discriminate between all the comparative values of the object  $\times$  relation for all actions. To appreciate the relational policy learned by the agent in this run, the state-action trees can be translated into the following relational rule:

```

if more-x(playert, ballt) in state
|   LEFT
else if same-x(playert, ballt) in state
|   FIRE
else if less-x(playert, ballt) in state
|   RIGHT
else
|   sample_action_space()
    
```



**Figure 4:** Best state-action trees on Breakout for the logical (a) and comparative (b) versions of RRTL.

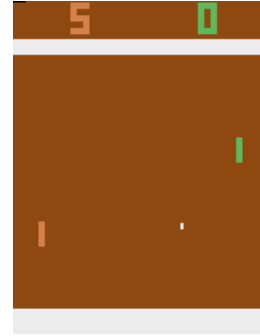
Clearly, the agent learned to follow the ball, going left if the player was to the right of the ball, going right if the player to the left of the ball and not moving if the player and the ball were at the same x-position. Notably, the agent preferred the FIRE action to the NOOP action for not moving, since the game would not start unless the FIRE action was executed and therefore the agent would not receive any rewards.

The results of Simulation 1 showed that the RRTL model is capable to learn a relational policy for a simple game — that nonetheless implies a large state-action space unmanageable for a straightforward tabular approach. Furthermore, we found a clear advantage for using comparative splits vs. the original logical splits. In Simulations 2 and 3 we tested our model in two environments that included increasing numbers of potentially relevant relations.

### 3. Simulation 2: Pong

In the Pong environment (see Fig. 5) two paddles, one corresponding to the player and the other to the enemy, hit a ball in order to make the ball pass the opponent. The player receives positive rewards when the ball passes over the enemy, and negative rewards when the ball passes over itself. The episode ends then either the player or the enemy make the ball pass the other 21 times. Both the player and the enemy can move only on the y-axis, while the ball can move on the x- and y-axis. The actions available to the player in this environment are: NOOP, FIRE, RIGHT, LEFT, RIGHTFIRE, LEFTFIRE. However, as the last two actions had the same effect as RIGHT and LEFT, we omitted them for the sake of simplicity. Because in Pong there are three objects instead of two, the number of potential object and trajectory relations increases accordingly. Furthermore, besides the object and trajectory relations used in the previous simulation, we added two contact relations between the player and the ball, and the ball and the enemy (these relations are necessarily logical). Table 2 on Appendix A presents all the relations used to

represent the state of the Pong environment. Note that to represent the state of Pong in a tabular fashion using the relations of Table 2 one would need to build a  $3^9 \cdot 2^2 = 78,732$  rows  $\times$  4 columns table.



**Figure 5:** A typical state of the Pong environment.

We trained and tested our model in the environment “PongDeterministic-v4” of OpenAI’s gym toolkit. As in Simulation 1 we created a visual pre-processor that calculated the x- and y-positions of the center of the objects. We used a tolerance level of 4 pixels to calculate the comparative value associated with each object relation. For the trajectory relations we used a tolerance level of zero. The following hierarchy of objects was used to represent the state: player > ball > enemy (see Table 2). For each of the two versions of the model, we trained 10 runs on 2,000,000 iterations with the same parameters as Simulation 1, except for two modifications. First, the  $\epsilon$  decay parameter,  $d$ , was set to 0.9999977. This allowed for a longer period of initial exploration. The second modification is related to the fact that the enemy in Pong follows the ball by default ( $r_{\text{enemy,ball}} = 0.84$  in 10 random games). This has the effect of making the reward signal very sparse for an agent that follows a random policy (as is any RL agent following a  $\epsilon$ -greedy policy at the beginning of learning). To address this issue, we added 0.1

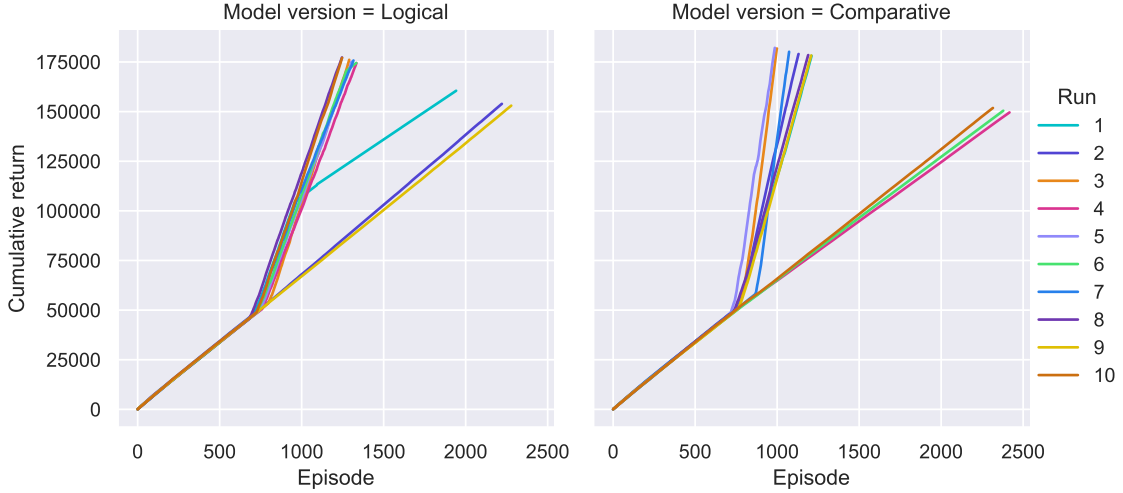


Figure 6: Cumulative return by model version and run on Pong.

to the reward at each time step to encourage the agent to play for as long as possible.

### 3.1. Results and Discussion

Fig. 6 shows the cumulative return for the logical and comparative versions of RRTL for each one of the 10 runs. The cumulative returns on Pong are smoother in comparison to the ones on Breakout because of the extra reward term. Overall, the runs of the comparative version received slightly more rewards during training. However, it is clear from the training data that the three runs of the comparative version received only the rewards corresponding to the extra term. this also happened for two runs of the logical version.

We tested the model runs in the same way as Simulation 1. We rescaled the returns to the  $[0, 21]$  interval by adding 21. This can be interpreted as the number of times the player scored a point against the enemy. Fig. 7 shows the average return on 100 test episodes. As can be seen, eight out of 10 runs achieved performance above chance in the logical version and seven out of 10 in the comparative version. However, three runs of the comparative version score around 14 points against the enemy in average, clearly surpassing the best logical run, which scored around nine points. Fig. 8 shows the state-action trees of the best (a) and worst (b) run on on Pong. Both are runs of the comparative version of RRTL. As can be seen, the best run selected the  $y$  relation between the player and the ball and achieved to discriminate between all its comparative values for all actions. This state-action tree can be translated in to the following relational rule:

```

if more-y(playert, ballt) in state
|   RIGHT
else if same-y(playert, ballt) in state
|   FIRE
else if less-y(playert, ballt) in state
|   LEFT
else
|   sample_action_space()
    
```

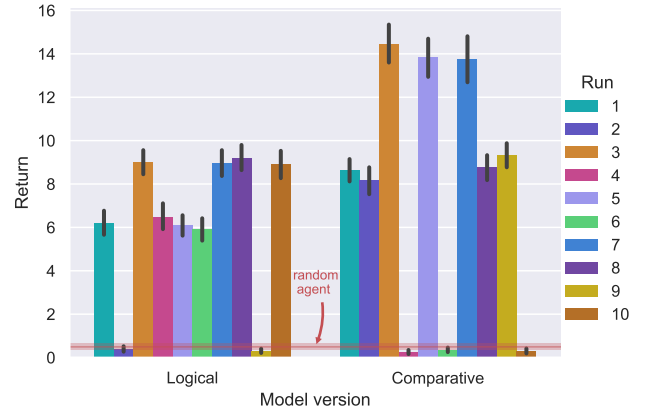
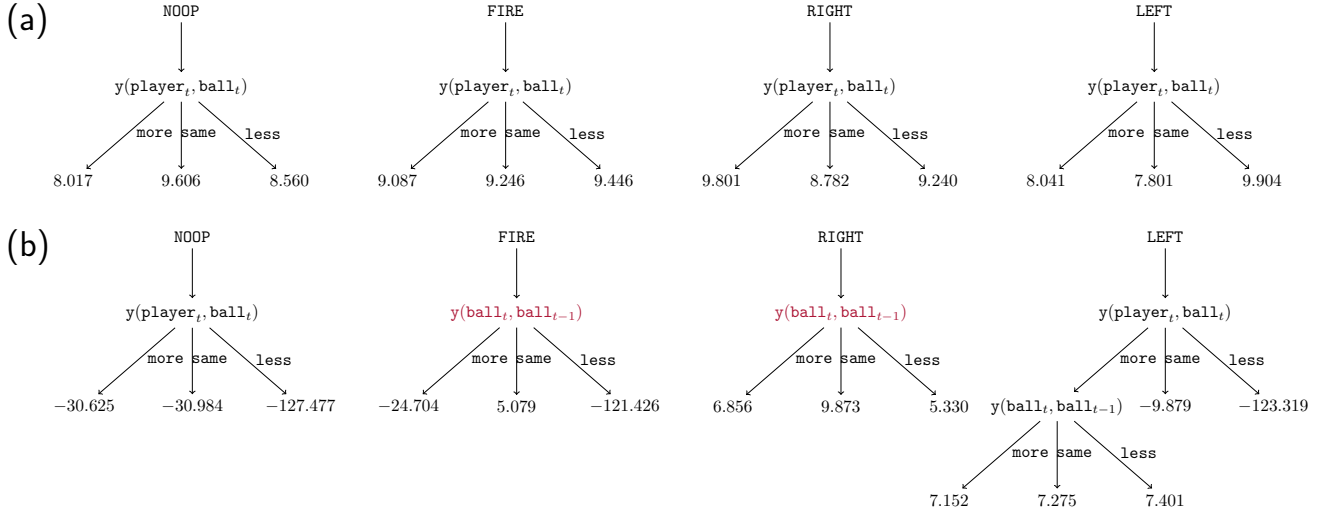


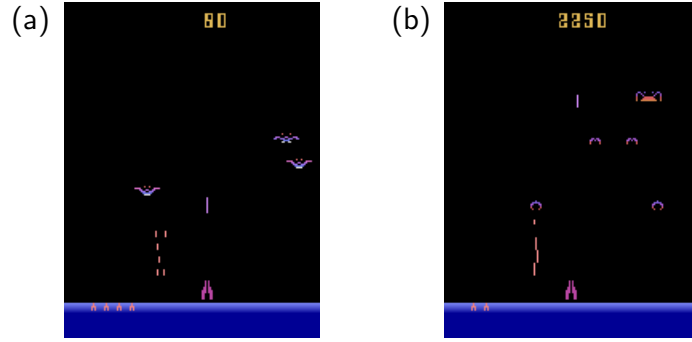
Figure 7: Mean returns by model version and run on 100 test episodes of Pong. Error bars are 95% confidence intervals. The horizontal shadowed line represents the mean return and 95% confidence intervals for a random agent on 100 test episodes.

As in Breakout, in this environment RRTL also learned to follow the ball, but this time in the  $y$  dimension, going down if the player was above the ball, going up if the player was below the ball and not moving if the player and the ball were at the same  $y$ -position. The agent preferred the FIRE action to the NOOP action for not moving, although a close inspection of Fig. 8 (a) shows that the predicted  $q$ -values are similar for both actions. To evaluate the learning process of the worst run, Fig. 8 (b) shows the state-action trees at the last training iteration. As can be seen, in this run the FIRE and RIGHT actions based their first split on the  $y$  trajectory relation of the ball, which lead to a set of trees that was not able to compensate for these early splits. This highlights one of the limitations of the current version of RRTL model: it cannot unsplit a node that led to bad performance. We elaborate on this and other limitations of the RRT model in the section 5.

Simulation 2 showed that the RRTL model is capable to learn a relational policy for a game larger than the one used



**Figure 8:** State-action trees on Pong for the best run (a) and the worst run (b). Both are runs from the comparative version of RRTL. The state-action trees of the worst run in (b) are based on the last training iteration.



**Figure 9:** Two typical states of the Demon Attack environment. On state (A) there are big enemies that can shoot missiles. On state (B) besides big enemies there are small enemies. The player loses a life if it is touched by a small enemy.

in Simulation 1. Additionally, we found further evidence in favor of using comparative splits vs. logical splits in the regression trees. In simulation 3 we tested the RRTL model in an even larger environment that included many more objects than Pong, providing the most stringent test of the capabilities of the RRTL model to select relevant relations to learn a policy.

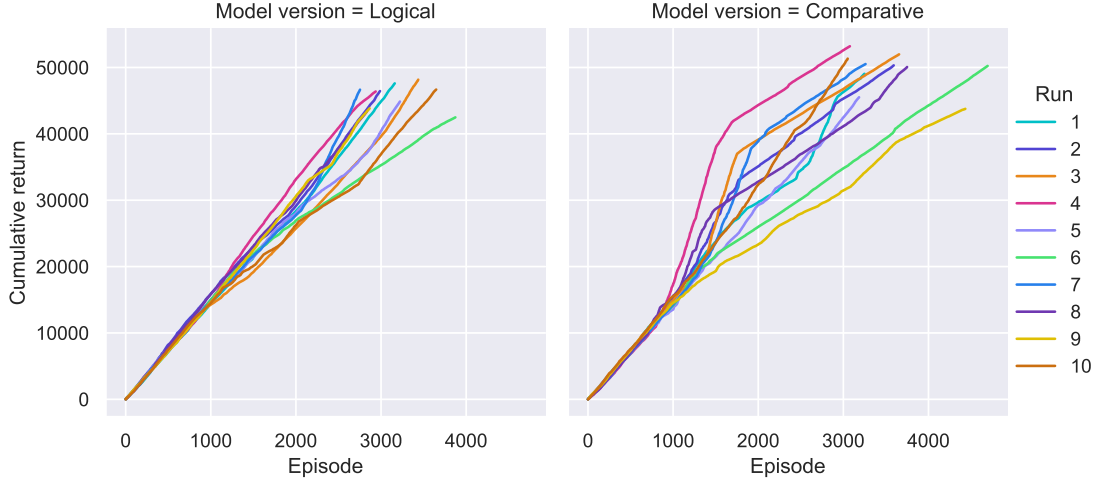
#### 4. Simulation 3: Demon Attack

In Demon Attack (see Fig. 9) the player controls a spaceship at the bottom of the screen that can only move on the x-dimension. In the initial levels, *big enemies* (or demons) appear in waves of three in the upper part of the screen. The bottom-most enemy shoots projectiles that make the player to lose a life and receive a negative reward. The player can shoot missiles that destroy the enemies at contact, in which case the player receives a positive reward. When an enemy is destroyed a new one appears to take its place until the current level is completed. Once all the enemies on a particular level

are destroyed, the player moves on to the next, more difficult wave. On advanced levels the big enemies will split into two bird-like *small enemies* the first time they are shot. The small enemies will eventually attempt descent onto the spaceship, which will also cause the player to lose one life and receive a negative reward. The actions available to the player in this environment are: NOOP, FIRE, RIGHT, LEFT, RIGHTFIRE, LEFTFIRE. We used all the available actions in this simulation. Because in Demon Attack there can be up to three big enemies and up to six small enemies at any given time<sup>4</sup>, the number of potential object relations is quite large. In this simulation we only used objects relations between the player and the other objects in the screen, with the exception of the player missile, which we treated as part of the player’s action. Furthermore, we treated the enemy’s projectiles as a single object, which we termed e-missile. We did not use any trajectory relations

<sup>4</sup>With the constrain that for each pair of small enemies there is one less possible big enemy.





**Figure 10:** Cumulative return by model version and run on Demon Attack.

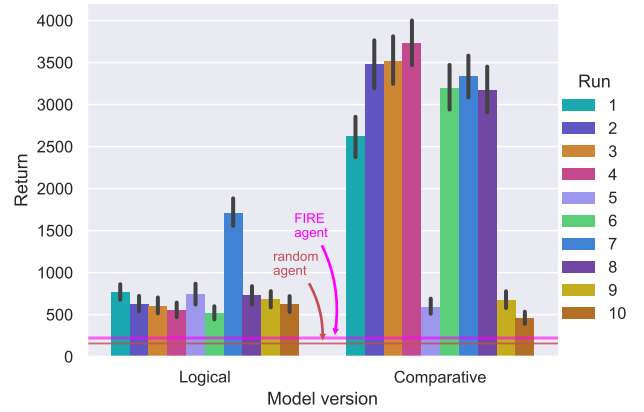
in this simulation<sup>5</sup>. When building the relational state we only considered states where there was an enemy missile. Tables 3 and 4 on Appendix A present all the relations used to represent the state of this environment. Note that to represent the state of Demon Attack in a tabular fashion using the relations of Tables 3 and 4 one would need to build a  $4^{19} \cdot 3 = 824,633,720,832$  rows  $\times$  6 columns table.

We trained and tested our model in the environment of “DemonAttackDeterministic-v4” OpenAI’s gym toolkit. As in Simulations 1 and 2 we created a visual pre-processor that calculated the x- and y-positions of the center of the objects. We used a tolerance level of 3 pixels to calculate the comparative value associated with each object relation. For each of the two versions of the model, we trained 10 runs on 3,000,000 iterations with the same parameters as Simulation 1<sup>6</sup>. In addition to the action buffer described earlier, RL agents in this simulation included a reward buffer that stored the last 300 rewards and checked whether the last 300 iterations yielded zero rewards, in which case the current episode was terminated and a new one was started.

#### 4.1. Results and Discussion

Fig. 10 shows the cumulative return for the logical and comparative versions of RRTL for each one of the 10 runs. Overall, the runs of the comparative version received more rewards during training. Interestingly, the episodes tended to be shorter in the comparative version, which resulted in more training episodes in comparison to the logical version.

We tested the model runs in the same way as Simulation 1. Fig. 11 shows the average return on 100 test episodes. We considered two baselines, a first random agent that sampled actions uniformly from the full set of actions, and a second random agent that sampled actions only from the “fire”



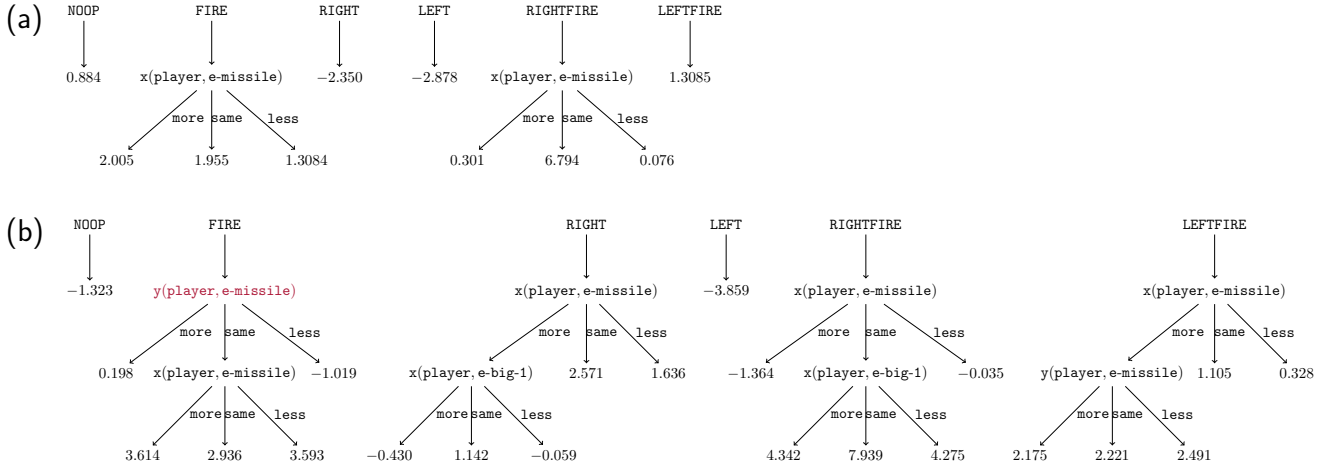
**Figure 11:** Mean returns by model version and run on 100 test episodes of Demon Attack. Error bars are 95% confidence intervals. The horizontal shadowed red line represents the mean return and 95% confidence intervals for a random agent on 100 test episodes. The horizontal shadowed pink line shows the same information for a random agent that only has access to the “fire” actions.

actions (i.e., FIRE, RIGHTFIRE, and LEFTFIRE), and therefore was always firing missiles. As can be seen, all runs in all conditions achieved performance above our two baselines. However, seven runs of the comparative version score above 2500 points in average, clearly surpassing the best logical run, which scored around 1700 points in average.

Fig. 12 shows the state-action trees of the best (a) and worst (b) run on Demon Attack. Both are runs of the comparative version of RRTL. As shown, the best run selected the x relation between the player and the enemy missile, although it only made splits for the FIRE and RIGHTFIRE actions. This can be translated into the following relational rule:

<sup>5</sup>This was due to the fact the demons follow a non-linear trajectory in the x- and y-dimensions even when stationary, circling around a fixed point.

<sup>6</sup>Note that all runs on the comparative version of RRTL achieved their best performance before two million iterations, so the last million of training iterations was, in fact, unnecessary.



**Figure 12:** State-action trees on Demon Attack for the best run (a) and the worst run (b). Both are runs from the comparative version of RRTL. The state-action trees of the worst run in (b) are based on the last training iteration.

```

if more-x(player, e-missile)
|   FIRE
else if same-x(player, e-missile)
|   RIGHTFIRE
else if less-x(player, e-missile)
|   LEFTFIRE
else
|   sample_action_space()
    
```

As can be seen, this model run learned a simple policy that essentially amounts to stay away from the enemy missile while shooting as much as possible. In particular, the agent will stay put and fire if the player was to the right of the enemy missile, will go right and fire if the player and the enemy missile are at the same x-position, and will go left and fire if the player is to the left of the enemy missile. As in Simulation 2, to evaluate the learning process of the worst run, Fig. 12 (b) shows the state-action trees of the worst run at the last training iteration. As can be seen, in this run the FIRE action based its first split on the y relation between the player and the enemy missile, which led to a set of trees that was not able to compensate for this split. This further highlights the incapability of RRTL of undo a bad split during learning.

## 5. General Discussion

Historically, theories of analogy making have been disconnected from notions of utility such as rewards or prediction errors (but see Foster & Jones, 2017, for an model of reward-driven schema induction). One reason for this disconnect is the emphasis of these theories on unsupervised discovery of relational representations and higher order abstractions, such as schemas or relational categories. However, this state of affairs is odd. After all, the capacity to form and manipulate relational representations provides humans with a clear evolutionary advantage over any other species (Penn, Holyoak, & Povinelli, 2008) and therefore allows

humans to act more effectively in their environment. To start to bridge this theoretical gap, this work investigated ways of integrating relational representations with the reinforcement learning framework. This is a hard problem because of the discrete nature of relational representations and the combinatorial explosion that results from applying them to describe the state of the environment.

In particular, we studied the problem of how humans select appropriate relational representations when learning policies to solve problems, when there is a large vocabulary of relations available to describe the state of the environment. Using a function approximator developed in RRL, we developed a model, RRTL, that learns grounded relational policies by making ternary splits based on the comparative values *more*, *same* and *less* that characterize comparative relations like “above” or “bigger-than”. We tested our model on three Atari games, Breakout, Pong and Demon Attack, that involved increasing numbers of potential relations. In each case, RRTL built simple relational policies based on a set of few relevant relations. RRTL can be considered a proof-of-principle of the idea of using RL to learn which relations between the objects in the environment are relevant to characterize the task at hand.

As previous models developed in RRL (e.g., Driessens et al., 2001), RRTL builds relational policies incrementally, adding relational representations gradually during learning. In contrast, the Bayesian model of theory-based RL of Tsividis et al. (2021), represents the full set of possible programs and selects, through Bayesian inference, the best possible program that explains the observed data and yields high rewards. We believe that, ultimately, top-down and bottom-up approaches are complementary. On the one hand, RRL can be seen as a largely associative process running “in the background”, that pre-selects relevant relational representations upon which Bayesian inference over potential programs could occur. On the other hand, theory-based

hypothesis generation could, occasionally, inform the generation of splits in RRTL. As incremental learning of programs is a hard problem, this might end up being necessary to learn relational policies in more complex environments than the ones studied in this work. In either case, we think that RRTL provides a starting point to model these interactions.

In its current form, RRTL has several limitations. The first one has to do with the frequentist approach used to select the relation candidates for splitting the state-action trees. Because frequentist tests are highly sensitive to the sample size, all candidates should have similar numbers of visits to be comparable. This is a problem for any realistic environment, since there can be relations that apply only rarely that, nonetheless, have a large impact of the optimal policy. For example, imagine a version of Demon Attack where the small enemies descend onto the spaceship too fast to try to shoot them from below, in which case the best option would be to avoid the small enemies and, consequently, the  $x$  relation between the player and the small enemy would be very important, but only so when there is a small enemy on the screen. We think that in this case taking a Bayesian approach to candidate selection would be useful to account for this kind of phenomenon. A related problem is the fact that RRTL currently does not have a way to undo a bad split made during learning, which can completely ruin the learning trajectory of the state-action trees (as seen in Simulations 2 and 3). Fortunately, Ramon, Driessens, and Croonenborghs (2007) developed a set of tree restructuring operations that they used for partial policy transfer on RRL. We think that combining these operations with a Bayesian approach to splitting candidate selection is a promising future direction for RRTL.

In contrast to most RRL systems developed in AI, RRTL learns policies composed of a set of ground rules. We proposed that instead of learning variabilized rules directly through interaction with the environment, humans “lift” initially ground policies to a representation more akin to a first-order relational policy through the process of schema induction. This process involves the extraction of the shared relational structure through analogical comparison and mapping of two different situations (Hummel & Holyoak, 2003). This same process could be used to compare different instances of the ground policy to isolate its relational content and abstract away the specific objects attached to it. Foster and Jones (2017) developed a computational model that uses RL to *guide* the induction of useful schemas to solve a task. Analogy-making also plays a mayor role in *transfer learning*, allowing humans to apply what they learned about one task to another that only shares abstract relational structure. As previously discussed, Dumas et al. (2022) showed that DORA, was able to perform relational policy transfer between Breakout and Pong through analogical inference. We think that RRTL could be naturally integrated with both approaches.

RRTL uses the relational regression tree developed by Driessens et al. (2001) to approximate the state-action value function. We chose this function approximator because it

integrates naturally with existing models of analogical reasoning based on relational representations (e.g., Dumas et al., 2008; Falkenhainer, Forbus, & Gentner, 1989). However, it is worth noting that recently several deep neural network models of RRL have been proposed (e.g., Dong et al., 2019; Jiang & Luo, 2019; Zimmer et al., 2021). In general, these models define a *soft logic* (i.e., one with truth values falling into the  $[0, 1]$  interval) and a series of differentiable gating operations that allow to approximate the behaviour of a logic program. While these models are an impressive demonstration of the possibility of combining symbolic and neural-based computation in AI, they tend to suffer from scalability (Jiang & Luo, 2019), or lack of interpretability (Dong et al., 2019). Both of these issues are the subject of current research in neuro-symbolic AI (e.g., Zimmer et al., 2021). In terms of biologically plausible mechanisms for RRL, our results show that prediction error driven learning does not need to be based on gradients. Certainly, it is possible that biological RRL is multi-faceted, exploiting both gradients and step functions. We think that Building an efficient and neurally plausible function approximator suited to work with relational representations is an important goal for RRL research.

To conclude, RRL provides a computational framework to understand how the cognitive system learns which relational representations are relevant for a given task, as well as how to build relational policies incrementally. We hope that the present paper helps to stimulate further theoretical and empirical research about how humans and other biological agents build relational policies to interact effectively with their environment.

## A. Appendix

State representations for Pong (Table 2) and Demon Attack (Tables 3 and 4).

## CRedit authorship contribution statement

**Guillermo Puebla:** Conceptualization of this study, Methodology, Software, Formal analysis, Writing - Original draft preparation. **Leonidas A. A. Dumas:** Supervision, Writing - Review & Editing .

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**Table 2**  
Pong State Representation.

Dim	Obj-1	Obj-2	Relation	
			Logical	Comparative
x	player <sub>t</sub>	ball <sub>t</sub>	more-x(player <sub>t</sub> , ball <sub>t</sub> ) same-x(player <sub>t</sub> , ball <sub>t</sub> ) less-x(player <sub>t</sub> , ball <sub>t</sub> )	x(player <sub>t</sub> , ball <sub>t</sub> )
x	player <sub>t</sub>	enemy <sub>t</sub>	more-x(player <sub>t</sub> , enemy <sub>t</sub> ) same-x(player <sub>t</sub> , enemy <sub>t</sub> ) less-x(player <sub>t</sub> , enemy <sub>t</sub> )	x(player <sub>t</sub> , enemy <sub>t</sub> )
x	ball <sub>t</sub>	enemy <sub>t</sub>	more-x(ball <sub>t</sub> , enemy <sub>t</sub> ) same-x(ball <sub>t</sub> , enemy <sub>t</sub> ) less-x(ball <sub>t</sub> , enemy <sub>t</sub> )	x(ball <sub>t</sub> , enemy <sub>t</sub> )
y	player <sub>t</sub>	ball <sub>t</sub>	more-y(player <sub>t</sub> , ball <sub>t</sub> ) same-y(player <sub>t</sub> , ball <sub>t</sub> ) less-y(player <sub>t</sub> , ball <sub>t</sub> )	y(player <sub>t</sub> , ball <sub>t</sub> )
y	player <sub>t</sub>	enemy <sub>t</sub>	more-y(player <sub>t</sub> , enemy <sub>t</sub> ) same-y(player <sub>t</sub> , enemy <sub>t</sub> ) less-y(player <sub>t</sub> , enemy <sub>t</sub> )	y(player <sub>t</sub> , enemy <sub>t</sub> )
y	ball <sub>t</sub>	enemy <sub>t</sub>	more-y(ball <sub>t</sub> , enemy <sub>t</sub> ) same-y(ball <sub>t</sub> , enemy <sub>t</sub> ) less-y(ball <sub>t</sub> , enemy <sub>t</sub> )	x(ball <sub>t</sub> , enemy <sub>t</sub> )
x	ball <sub>t</sub>	ball <sub>t-1</sub>	more-x(ball <sub>t</sub> , ball <sub>t-1</sub> ) same-x(ball <sub>t</sub> , ball <sub>t-1</sub> ) less-x(ball <sub>t</sub> , ball <sub>t-1</sub> )	x(ball <sub>t</sub> , ball <sub>t-1</sub> )
y	ball <sub>t</sub>	ball <sub>t-1</sub>	more-y(ball <sub>t</sub> , ball <sub>t-1</sub> ) same-y(ball <sub>t</sub> , ball <sub>t-1</sub> ) less-y(ball <sub>t</sub> , ball <sub>t-1</sub> )	y(ball <sub>t</sub> , ball <sub>t-1</sub> )
y	enemy <sub>t</sub>	enemy <sub>t-1</sub>	more-y(enemy <sub>t</sub> , enemy <sub>t-1</sub> ) same-y(enemy <sub>t</sub> , enemy <sub>t-1</sub> ) less-y(enemy <sub>t</sub> , enemy <sub>t-1</sub> )	y(enemy <sub>t</sub> , enemy <sub>t-1</sub> )
	player <sub>t</sub>	ball <sub>t</sub>	in-contact(player <sub>t</sub> , ball <sub>t</sub> )	
	ball <sub>t</sub>	enemy <sub>t</sub>	in-contact(ball <sub>t</sub> , enemy <sub>t</sub> )	



**Table 3**  
Demon Attack State Representation (continues next page).

Dim	Obj-1	Obj-2	Relation	
			Logical	Comparative
x	player	e-missile	more-x(player, e-missile) same-x(player, e-missile) less-x(player, e-missile)	x(player, e-missile)
x	player	e-big-1	more-x(player, e-big-1) same-x(player, e-big-1) less-x(player, e-big-1)	x(player, e-big-1)
x	player	e-big-2	more-x(player, e-big-2) same-x(player, e-big-2) less-x(player, e-big-2)	x(player, e-big-2)
x	player	e-big-3	more-x(player, e-big-3) same-x(player, e-big-3) less-x(player, e-big-3)	x(player, e-big-3)
x	player	e-small-1	more-x(player, e-small-1) same-x(player, e-small-1) less-x(player, e-small-1)	x(player, e-small-1)
x	player	e-small-2	more-x(player, e-small-2) same-x(player, e-small-2) less-x(player, e-small-2)	x(player, e-small-2)
x	player	e-small-3	more-x(player, e-small-3) same-x(player, e-small-3) less-x(player, e-small-3)	x(player, e-small-3)
x	player	e-small-4	more-x(player, e-small-4) same-x(player, e-small-4) less-x(player, e-small-5)	x(player, e-small-4)
x	player	e-small-5	more-x(player, e-small-5) same-x(player, e-small-5) less-x(player, e-small-5)	x(player, e-small-5)
x	player	e-small-6	more-x(player, e-small-6) same-x(player, e-small-6) less-x(player, e-small-6)	x(player, e-small-6)

**Table 4**  
Demon Attack State Representation (continued).

Dim	Obj-1	Obj-2	Relation	
			Logical	Comparative
y	player	e-missile	more-y(player, e-missile) same-y(player, e-missile) less-y(player, e-missile)	y(player, e-missile)
y	player	e-big-1	more-y(player, e-big-1) same-y(player, e-big-1) less-y(player, e-big-1)	y(player, e-big-1)
y	player	e-big-2	more-y(player, e-big-2) same-y(player, e-big-2) less-y(player, e-big-2)	y(player, e-big-2)
y	player	e-big-3	more-y(player, e-big-3) same-y(player, e-big-3) less-y(player, e-big-3)	y(player, e-big-3)
y	player	e-small-1	more-y(player, e-small-1) same-y(player, e-small-1) less-y(player, e-small-1)	y(player, e-small-1)
y	player	e-small-2	more-y(player, e-small-2) same-y(player, e-small-2) less-y(player, e-small-2)	y(player, e-small-2)
y	player	e-small-3	more-y(player, e-small-3) same-y(player, e-small-3) less-y(player, e-small-3)	y(player, e-small-3)
y	player	e-small-4	more-y(player, e-small-4) same-y(player, e-small-4) less-y(player, e-small-5)	y(player, e-small-4)
y	player	e-small-5	more-y(player, e-small-5) same-y(player, e-small-5) less-y(player, e-small-5)	y(player, e-small-5)
y	player	e-small-6	more-y(player, e-small-6) same-y(player, e-small-6) less-y(player, e-small-6)	y(player, e-small-6)