Curso : Visión Computational 1
Año : 2024

Assignment 5. Variational models for image processing

Exercice 1 (2.5 pts). ***(Implementation of basic operators)***

Define functions :
*- **forward_derivative_x** which implements the forward partial derivative with respect to x of a color image.*
*- **forward_derivative_y** which implements the forward partial derivative with respect to y of a color image.*
*- **backward_derivative_x** which implements the backward partial derivative with respect to x of a color image.*
*- **backward_derivative_y** which implements the backward partial derivative with respect to y of a color image.*

Tip : Use matrix operations instead of convolutions with kernels to implement these methods. It makes their executions faster.

From these operators define :
*- a function **gradient** which constructs the gradient of a color image using forward derivatives.*
*- a function **divergence** which constructs the divergence operator using backward derivatives.*

Exercice 2 (2.5 pts). ***(Joint denoising and deblurring)***

***img1_degradation1.png** is the result of applying Gaussian blur of standard deviation 2 and additive white Gaussian noise of standard deviation 5 to **img1.png**.*

*1. Implement the Algorithm 3 seen in class by using the operators constructed in Exercise 1. Use **fast_gaussian_convolution** to implement the component-wise convolution with a Gaussian kernel.*

*2. Apply the algorithm to **img1_degradation1.png** with the following parameters :*
$\eta = 0.0001$, $\epsilon = 0.001$, $\alpha = 0.01$, $K = 15000$. *You will test different values of the regularization parameter γ and select the one providing the best PSNR with respect to **img1.png**.*

Exercice 3 (2.5 pts). ***(Impulse noise removal)***

We would like to test the following model

$$\arg\min_u \int_\Omega \|u(x) - u_0(x)\| + \gamma \|\nabla u(x)\| dx \tag{0.1}$$

to remove impulse noise in the color image $u_0 \colon \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$.

1. Write a differentiable approximation of the functional in (0.1) and determine its gradient.

2. Adapt the Algorithm 2 seen in class to the minimization problem (0.1).

*3. Apply the algorithm constructed in 2. to remove the impulse noise in **img1_degradation2.png** with the following parameters :* $\gamma = 1$, $\eta = 0.0001$, $\epsilon = 0.001$, $\alpha = 0.01$, $K = 15000$.

*4. What PSNR value do you obtain ? (with respect to **img1.png**).*

5. Test Algorithm 2 with the following parameters : $\gamma = 100, \eta = 0.0001, \epsilon = 0.001, \alpha = 0.0001, K = 15000$. *Which algorithm is the best for impulse noise removal ?*

Exercice 4 (2.5 pts). ***(Contrast processing)***

Let $u_0 \colon \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^n$ *be a continuous function, and the functional* E *given by*

$$E(u) = \frac{1}{2} \int_\Omega \|u(x) - u_0(x)\|^2 \, dx - \frac{\gamma}{4} \int_\Omega \int_\Omega w(x,y) \|u(x) - u(y)\|^2 \, dx \, dy, \tag{0.2}$$

where w *is a normalized Gaussian kernel of standard deviation* σ.

1. Show that the gradient $\nabla E(u)$ *of* E *at* u *is*

$$\nabla E(u) = u - u_0 - \gamma(u - w * u),$$

where $*$ *stands for the component-wise convolution.*

*2. Apply the gradient descent algorithm associated to the functional (0.2) in order to enhance simultaneously the local and the global contrasts of **img2.png**.*

Parameters of the models :
- Local contrast enhancement model : $\gamma_1 = 0.5$ *and* $\sigma_1 = 5$.
- Global contrast enhancement model : $\gamma_2 = 0.75$ *and* $\sigma_2 = 3000$.

Parameters of the algorithm :
- Stopping criteria : MSE between two consecutive images is less than $\epsilon = 0.001$
- Step size : $\alpha = 0.01$
- Maximum number of iterations : $K = 15000$

*Tip : Use the function **fast_gaussian_convolution** to implement the component-wise convolution with a Gaussian kernel.*