Curso : Vision Computacional 1
Año : 2024

Assignment 6. Applications of Fourier analysis

Exercice 1 (2 pts). ***(Amplitude VS Phase)***.

The aim of this exercise is to determine which component in the Fourier domain of an image contains more information about the image content : the amplitude or the phase.

*1. Construct two images **res1.png** and **res2.png** such that :*
*- The Fourier domain of **res1.png** contains the amplitude of **img1.png** and the phase of **img2.png**.*
*- The Fourier domain of **res2.png** contains the amplitude of **img2.png** and the phase of **img1.png**.*
2. Conclude

Exercice 2 (2 pts). ***(Periodic degradation removal)***

*The image **img3.png** present a periodic degradation. Construct an image **img3_proc.png** in which this degradation has been removed.*

Exercice 3 (3 pts). ***(Image enhancement).***

Let $u_0 \colon \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^n, n \geq 1$, *be a continuous function, and the functional* E *given by*

$$E(u) = \int_\Omega \frac{1}{2}\|u(x) - u_0(x)\|^2 \, dx - \frac{\gamma}{4}\int_\Omega \int_\Omega w(x-y)\|u(x) - u(y)\|^2 \, dx \, dy, \tag{0.1}$$

where w *is a normalized Gaussian kernel of standard deviation* σ. *It can be shown that* E *possesses an unique minimum if* γ *is not too high.*

1. Determine an expression of the minimum of the functional E.

*2. Implement the expression found in 1. and apply it to enhance simultaneously the details and the global contrast of **img4.png**.*

Parameters of the models :
- Details enhancement model : $\gamma_1 = 0.5$ *and* $\sigma_1 = 5$.
- Global contrast enhancement model : $\gamma_2 = 0.75$ *and* $\sigma_2 = 300$.

*Hint : You can use the implementation of the 2D Gaussian kernel in **gaussian_kernel.py**.*

Exercice 4 (3 pts)**. *(Shape classification).***

*The numpy array **shapes.npy**, of size* $25 \times 201 \times 2$*, contains a set of 25 closed curves in* $\mathbb{R}^2$*, each one of them being discretized with 201 points. Figure 0.1 shows 5 curves in the set. Each of the 20 other curves is obtained by applying a translation, a rotation and a scale transformation to one of these 5 curves. The aim of this exercise is to classify automatically the set of 25 curves into 5 classes, each class containing the curves which are deduced from each others by a translation, a rotation and a scale transformation. We know that the method of Fourier descriptors can be used to solve this classification problem.*

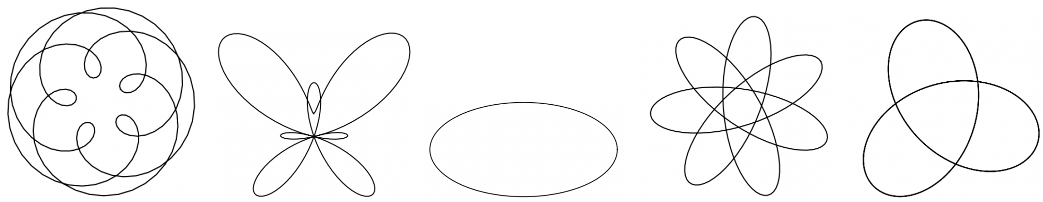

FIGURE 0.1: 5 curves of the dataset, each one being a represent of a class.

1. Show that the Fourier descriptors are invariant to translations, rotations and scales.
2. Implement a method which, given a curve, returns its Fourier descriptors.
*3 Apply the K-Means method to the Fourier descriptors to generate the 5 classes (`https://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans`). For each class, determine the curves which belong to the class (you can determine a curve by its index in **shapes.npy**).*
*4. Let us consider the 2D curve **new_shape.npy** which does not belong to the dataset. By using the function **predict** of **sklearn.cluster.KMeans**, determine the class the curve belongs to.*