Temas selectos de física

Tarea 1

① → Muestre

$$S_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}\right]$$

$$= \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n(n-1)}$$

Desarrollamos $S_{xy} = \frac{1}{n-1}\sum_{i=1}^{n} x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x}\bar{y}$

$$\Rightarrow \frac{1}{n-1}\left[\sum x_i y_i - \sum \bar{x} y_i - \sum \bar{y} x_i + \bar{x}\bar{y}\right]$$

$$\frac{1}{n-1}\left[\sum x_i y_i - \frac{\sum y_i \sum x_i}{n} - \frac{\sum y_i \sum x_i}{n} + \frac{\sum x_i \sum y_i}{n}\right]$$

$$\boxed{S_{xy} = \frac{1}{n-1}\left[\sum x_i y_i - n\bar{y}\bar{x}\right]}$$

Se sigue $S_{xy} = \frac{1}{n-1}\sum x_i y_i - \frac{\cancel{n}\sum x_i \sum y_i}{n^{\cancel{2}}}$

$$\Rightarrow \boxed{S_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n(n-1)}}$$

② → Continue con el procedimiento Para $\ell_1 = 1.4465$

$$[S - \ell_1 I]\, t_1 = \begin{bmatrix} 0.7486 - 1.4465 & 0.6793 \\ 0.6793 & 0.7343 - 1.4465 \end{bmatrix}\begin{bmatrix} t_{11} \\ t_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.6979 & 0.6793 \\ 0.6793 & -0.7122 \end{bmatrix}\begin{bmatrix} t_{11} \\ t_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Haciendo el producto matricial tenemos:

$-0.6479\, t_{11} + 0.6793\, t_{21} = 0$ Elegimos $t_{11} = 1$

$\Rightarrow -0.6479 + 0.6793\, t_{21} = 0 \quad \therefore \boxed{t_{21} = \dfrac{0.6479}{0.6793} = 0.9537}$

Tenemos $t_1 = (1, 0.9537) \rightarrow$ Normalizando

$$\boxed{t_1 = \frac{t_1}{\| t_1 \|} = \frac{t_1}{1.3793} = \begin{bmatrix} 0.7236 \\ 0.6902 \end{bmatrix}}$$

Para $l_2 = 0.0864$

$$\Rightarrow [S - l_2 I]\, t_2 = \begin{bmatrix} 0.7122 & 0.6793 \\ 0.6793 & 0.6479 \end{bmatrix} \begin{bmatrix} t_{12} \\ t_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 0.7122\, t_{12} + 0.6793 = 0$ con $t_{22} = 1$

$\Rightarrow t_{12} = -0.9538$

$$\therefore \boxed{t_2 = \frac{(-0.9538, 1)}{\sqrt{(-0.9538)^2 + 1^2}} = \begin{bmatrix} -0.6902 \\ 0.7236 \end{bmatrix}}$$