

# Formalization of our placement problem

## Components and Assembly

Let  $\mathcal{C} = \{c\}$ , the set of components.

Let  $\mathcal{A} \subset \mathcal{C}$  be the “assembly”. All the components of the assembly **must** be deployed.

## Ports

Components can have multiple ports. Each port can be either type CONTAINS or CONTAINED. A connection between two ports requires that the types are different.

Let  $P(c) = \{p \mid p \text{ is a port of } c\}$  and  $C(p)$  the component containing the port  $p$ .

Let  $P_{\text{CONTAINS}}(c) = \{p \mid p \text{ is a port CONTAINS of } c\}$  and let  $P_{\text{CONTAINED}}(c) = \{p \mid p \text{ is a port CONTAINED of } c\}$ .

Let  $\mathcal{P}_{\text{CONTAINS}} = \cup_{c \in \mathcal{C}} P_{\text{CONTAINS}}(c)$ , and  $\mathcal{P}_{\text{CONTAINED}} = \cup_{c \in \mathcal{C}} P_{\text{CONTAINED}}(c)$ .

## Dimensions of the problem

Let  $D \in \mathbb{N}^*$  be the number of dimensions of the problem:  $\forall p, \forall k \in \llbracket 1, D \rrbracket, p^k \in \mathbb{R}^+$

Examples: CPU, MEMORY, etc.

## Tags of the Resource Ports

Let  $\mathcal{T}$  be the set of tags associated to the ports.

Let  $T(p) = \{t \mid t \text{ tags of } p\} \subset \mathcal{T}$

## Placement

If a CONTAINED port  $p$  is connected on a port CONTAINS  $q$ , then  $d_q^p = 1$ , else 0.

This relation is **not** symmetric!

## Constraints for Incomplete Assemblies

### Constraint I.1: Matching Tags

The tags of two ports must match for a connection between the ports.

$$\forall (p, q) \in \mathcal{P}_{\text{CONTAINED}} \times \mathcal{P}_{\text{CONTAINS}}, \text{ such that } d_q^p = 1 \Rightarrow \forall t \in T(p), t \in T(q)$$

### Constraint I.2: Capacities Constraints

All the ports connected the same port must not require more than available.

$$\forall q \in \mathcal{P}_{\text{CONTAINS}}(c), \forall k \in \llbracket 1, D \rrbracket, \sum_{p \in \mathcal{P}_{\text{CONTAINED}}} d_q^p \times p^k \leq q^k$$

### Constraint I.3: Cannot Deploy on Itself

A CONTAINED port of a component cannot be connected to a CONTAINS port of that same component.

$$\forall p \in \mathcal{P}_{\text{CONTAINED}} \text{ and } \forall q \in \mathcal{P}_{\text{CONTAINS}}, d_q^p = 1 \Rightarrow C_p \neq C_q$$

## Objective ?

We want to connect as many ports as possible:

$$\max_{p, q \in \mathcal{P}_{\text{CONTAINS}} \times \mathcal{P}_{\text{CONTAINED}}} \sum d_q^p$$

But we do not want to simply connect random useless resources between themselves...

## Constraints for Complete Assemblies

### Constraint C.1: Deploying the entire Assembly

For all the components in the assembly, all of their CONTAINED ports must be connected.

$$\forall c \in \mathcal{A}, \forall p \in P_{\text{CONTAINED}}(c), \exists q \in \mathcal{P}_{\text{CONTAINS}}, d_q^p = 1$$

### Constraint C.2: Deploying the Containers

If a component is deployed onto another component, the latter must also be deployed.

$$\begin{aligned} &\forall c \in \mathcal{C}, \text{ if } \exists p \in P_{\text{CONTAINED}}(c), \text{ and } \exists q \in \mathcal{P}_{\text{CONTAINS}}, \text{ such that } d_q^p = 1 \\ &\text{ then } \forall p_j \in P_{\text{CONTAINED}}(C(q)), \exists p_k \in \mathcal{P}_{\text{CONTAINS}}, \text{ such that } d_{p_k}^{p_j} = 1 \end{aligned}$$

### Constraint C.3: Deploying the Entire Component

If a port CONTAINED of a component has been connected, all the CONTAINED ports of that component must be connected.

$$\begin{aligned} &\forall c \in \mathcal{C}, \text{ if } \exists p_i \in P_{\text{CONTAINED}}(c), \text{ such that } \exists p_j \in \mathcal{P}_{\text{CONTAINS}}, d_{p_j}^{p_i} = 1 \\ &\text{ then } \forall p_k \in P_{\text{CONTAINED}}(c), \exists p_l \in \mathcal{P}_{\text{CONTAINS}}, d_{p_l}^{p_k} = 1 \end{aligned}$$

### Objective ?

We want to connect the full assembly with, for example, the least amount of components deployed

$$\min \sum_{p, q \in \mathcal{P}_{\text{CONTAINS}} \times P_{\text{CONTAINED}}} d_q^p$$