

Formalization of our placement problem

Components and Assembly

Let $\mathcal{C} = \{c\}$, the set of components.

Let $\mathcal{A} \subset \mathcal{C}$ be the “assembly”. All the components of the assembly **must** be deployed.

Ports

Components can have multiple ports. Each port can be either either type **CONTAINS** or **CONTAINED**. A connection between two ports requires that the types are different.

Let $P(c) = \{p \mid p \text{ is a port of } c\}$ and $C(p)$ the component containing the port p .

Let $P_{\text{CONTAINS}}(c) = \{p \mid p \text{ is a port } \text{CONTAINS} \text{ of } c\}$ and let $P_{\text{CONTAINED}}(c) = \{p \mid p \text{ is a port } \text{CONTAINED} \text{ of } c\}$.

Let $\mathcal{P}_{\text{CONTAINS}} = \bigcup_{c \in \mathcal{C}} P_{\text{CONTAINS}}(c)$, and $\mathcal{P}_{\text{CONTAINED}} = \bigcup_{c \in \mathcal{C}} P_{\text{CONTAINED}}(c)$.

Dimensions of the problem

Let $D \in \mathbb{N}^*$ be the number of dimensions of the problem: $\forall p, \forall k \in [1, D], p^k \in \mathbb{R}^+$

Examples: CPU, MEMORY, etc.

Tags of the Resource Ports

Let \mathcal{T} be the set of tags associated to the ports.

Let $T(p) = \{t \mid t \text{ tags of } p\} \subset \mathcal{T}$

Placement

If a **CONTAINED** port p is connected on a port **CONTAINS** q , then $d_q^p = 1$, else 0.

This relation is **not** symmetric!

Constraints for Incomplete Assemblies

Constraint I.1: Matching Tags

The tags of two ports must match for a connection between the ports.

$$\forall (p, q) \in \mathcal{P}_{\text{CONTAINED}} \times \mathcal{P}_{\text{CONTAINS}}, \text{ such that } d_q^p = 1 \Rightarrow \forall t \in T(p), t \in T(q)$$

Constraint I.2: Capacities Constraints

All the ports connected the same port must not require more than available.

$$\forall q \in \mathcal{P}_{\text{CONTAINS}}(c), \forall k \in [1, D], \sum_{p \in \mathcal{P}_{\text{CONTAINED}}} d_q^p \times p^k \leq q^k$$

Constraint I.3: Cannot Deploy on Itself

A **CONTAINED** port of a component cannot be connected to a **CONTAINS** port of that same component.

$$\forall p \in \mathcal{P}_{\text{CONTAINED}} \text{ and } \forall q \in \mathcal{P}_{\text{CONTAINS}}, d_q^p = 1 \Rightarrow C_p \neq C_q$$

Objective ?

We want to connect as many ports as possible:

$$\max \sum_{p, q \in \mathcal{P}_{\text{CONTAINS}} \times \mathcal{P}_{\text{CONTAINED}}} d_q^p$$

But we do not want to simply connect random useless resources between themselves...

Constraints for Complete Assemblies

Constraint C.1: Deploying the entire Assembly

For all the components in the assembly, all of their **CONTAINED** ports must be connected.

$$\forall c \in \mathcal{A}, \forall p \in P_{\text{CONTAINED}}(c), \exists q \in \mathcal{P}_{\text{CONTAINS}}, d_q^p = 1$$

Constraint C.2: Deploying the Containers

If a component is deployed onto another component, the latter must also be deployed.

$$\forall c \in \mathcal{C}, \text{if, } \exists p \in P_{\text{CONTAINED}}(c), \text{and } \exists q \in \mathcal{P}_{\text{CONTAINS}}, \text{such that } d_q^p = 1$$

$$\text{then } \forall p_j \in P_{\text{CONTAINED}}(C(q)), \exists p_k \in \mathcal{P}_{\text{CONTAINS}}, \text{such that } d_{p_k}^{p_j} = 1$$

Constraint C.3: Deploying the Entire Component

If a port **CONTAINED** of a component has been connected, all the **CONTAINED** ports of that component must be connected.

$$\forall c \in \mathcal{C}, \text{if } \exists p_i \in P_{\text{CONTAINED}}(c), \text{such that } \exists p_j \in \mathcal{P}_{\text{CONTAINS}}, d_{p_j}^{p_i} = 1$$

$$\text{then } \forall p_k \in P_{\text{CONTAINED}}(c), \exists p_l \in \mathcal{P}_{\text{CONTAINS}}, d_{p_l}^{p_k} = 1$$

Objective ?

We want to connect the full assembly with, for example, the least amount of components deployed

$$\min \sum_{p, q \in \mathcal{P}_{\text{CONTAINS}} \times \mathcal{P}_{\text{CONTAINED}}} d_q^p$$