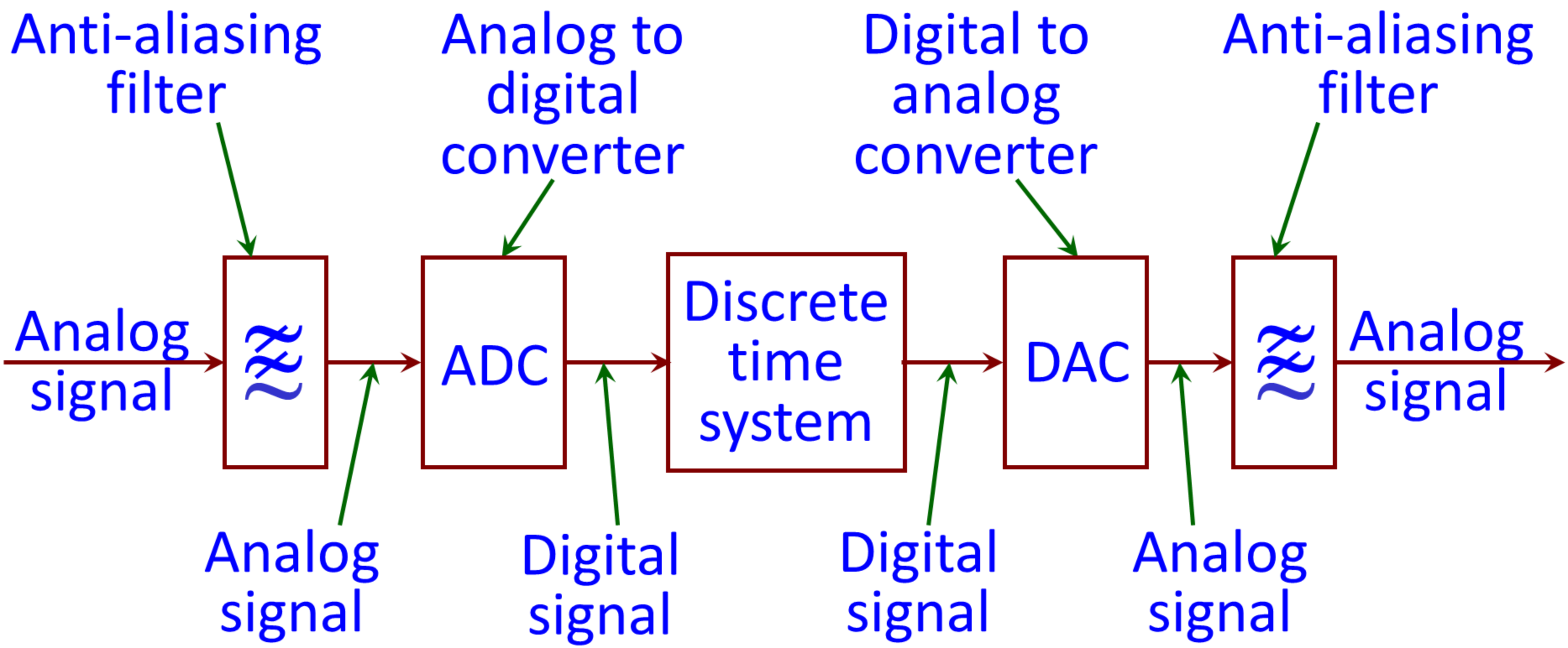
Week 4
Signal Processing Systems

Typical DSP Systems



Where Does the Signal Come From?

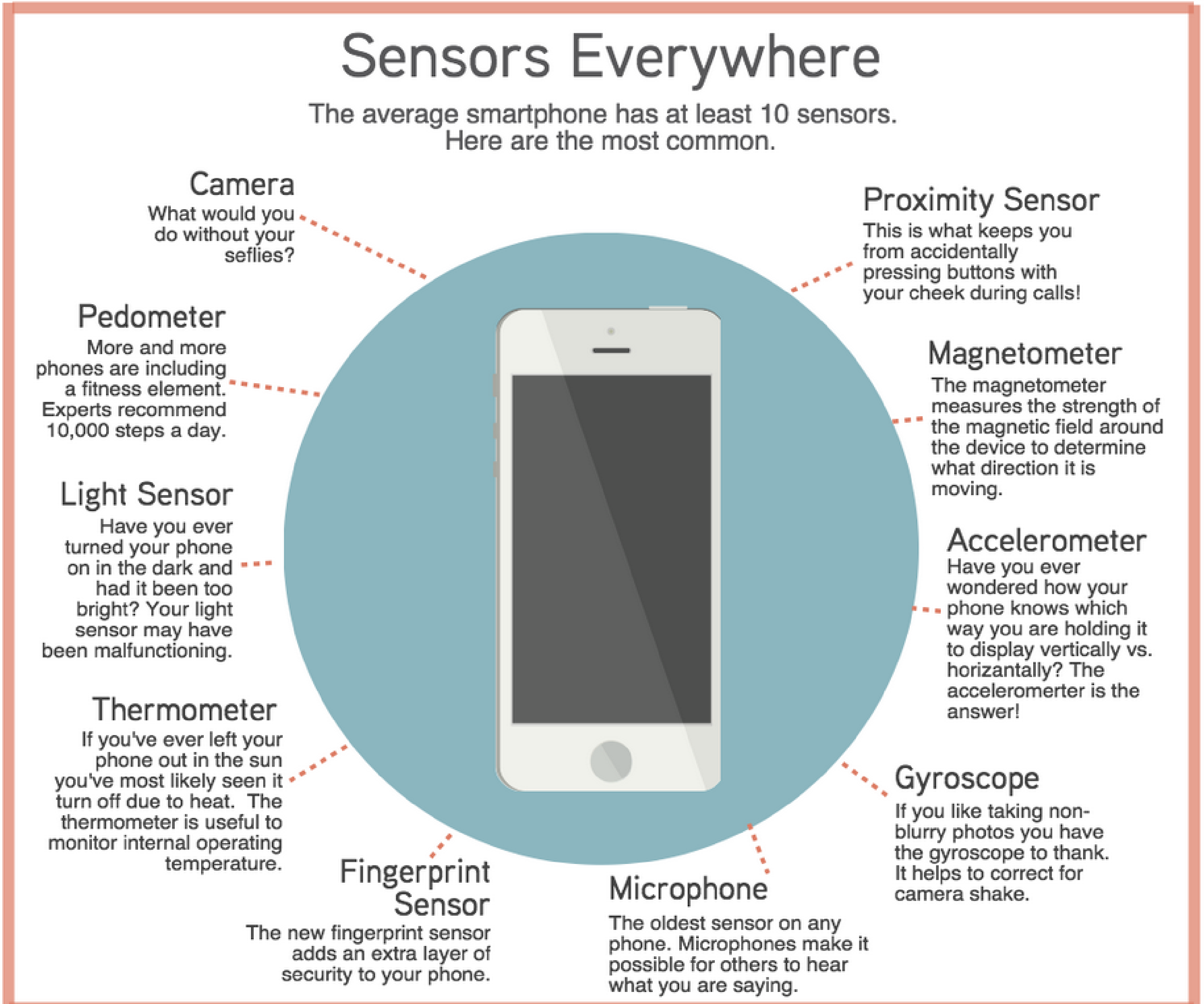
Where Does the Signal Come From?

- A sensor acquires a physical parameter and converts it into a signal suitable for processing

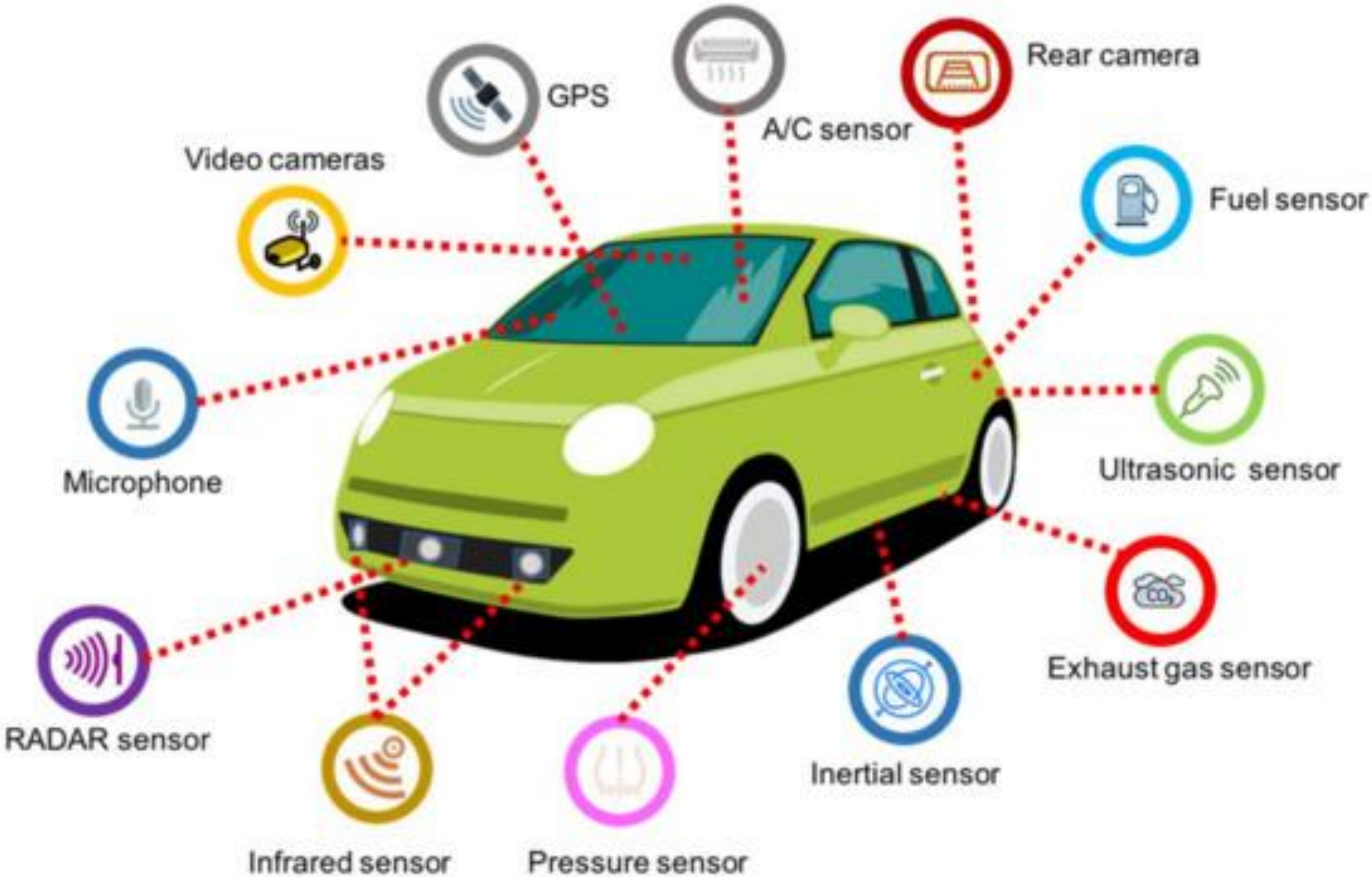
Where Does the Signal Come From?

- Sensors
  - Temperature Sensor
  - Light Sensor
  - Accelerometer
  - Magnetic Field Sensor
  - Ultrasonic Sensor
  - Photogate
  - CO_2 Gas Sensor

Sensors In a Smart Phone



Sensors In a Car



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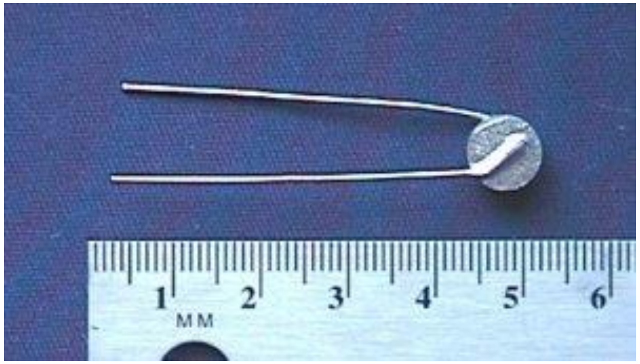
Temperature Sensor

thermal resistor
"thermistor"



resistance changes with temperature

Light Sensor

photo-resistor

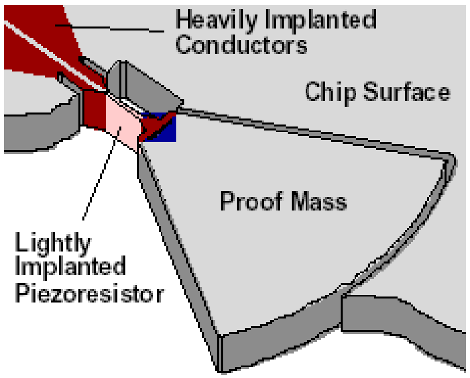
resistance changes with light intensity

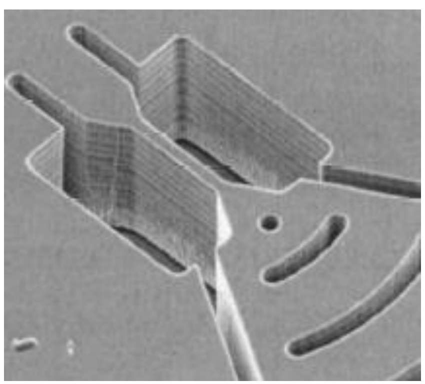
Physical Principles

- Ampere's Law
  - A current carrying conductor in a magnetic field experiences a force (e.g. galvanometer)
- Faraday's Law of Induction
  - A coil resist a change in magnetic field by generating an opposing voltage/current (e.g. transformer)
- Photoconductive Effect
  - When light strikes certain semiconductor materials, the resistance of the material decreases (e.g. photoresistor)

Different Sensing Techniques

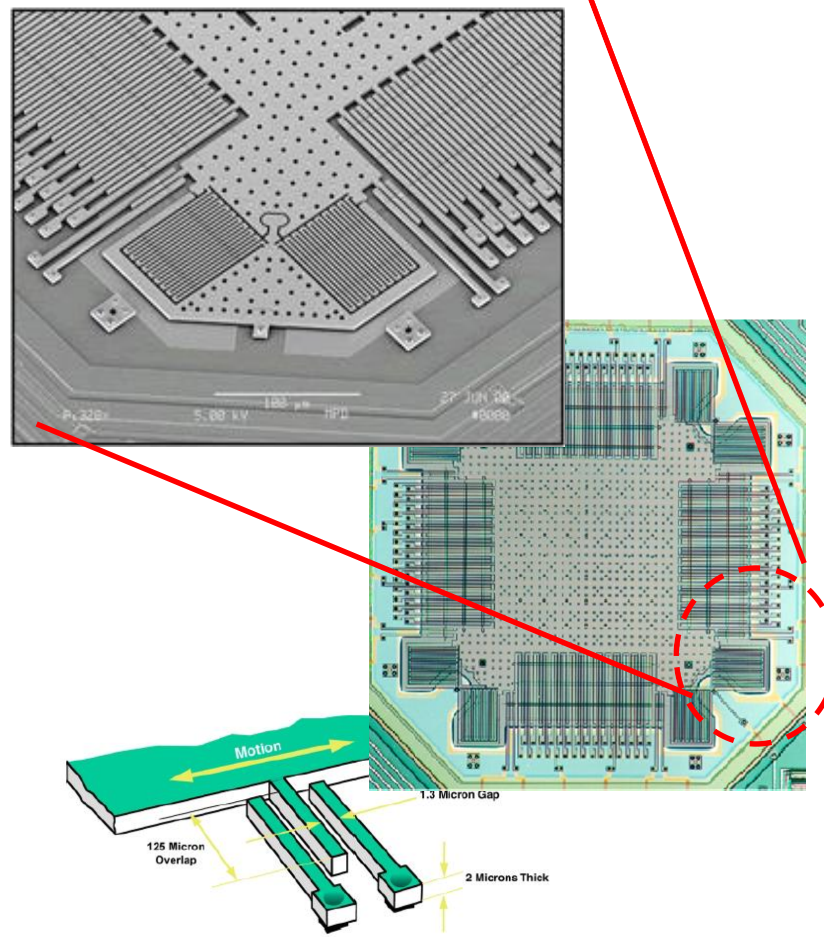
Piezoresistive MEMS accelerometer





Courtesy of JP Lynch, U Mich.

Capacitive MEMS accelerometer



Courtesy of Analog Devices, Inc.

Sensor Signal Conditioning

- Manipulation of an analog signal in such a way that it meets the requirements of the next stage for further processing
  - Amplification
  - Limiting
  - Linearization
  - Anti-aliasing filtering
  - …

EE111 Electric Circuits

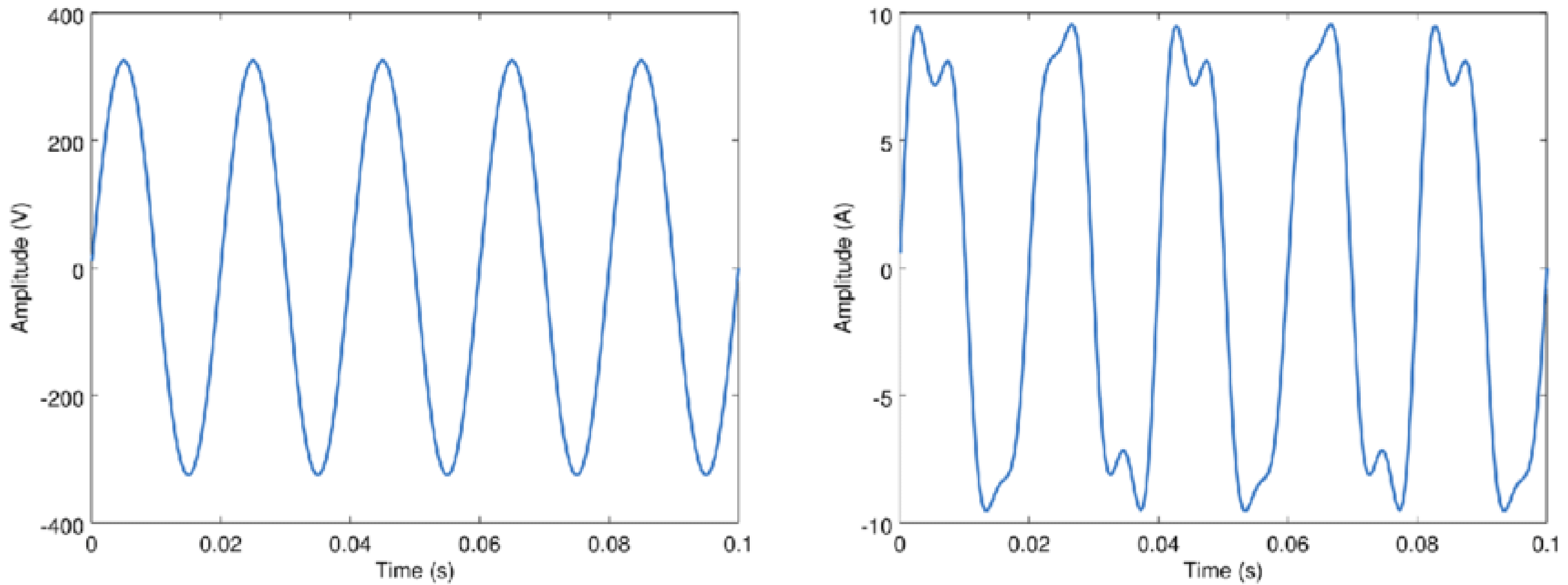
From Analog to Digital

The World is Analog

- All the sensed signal is analog

The World is Analog

- Common sensor output: voltage and current



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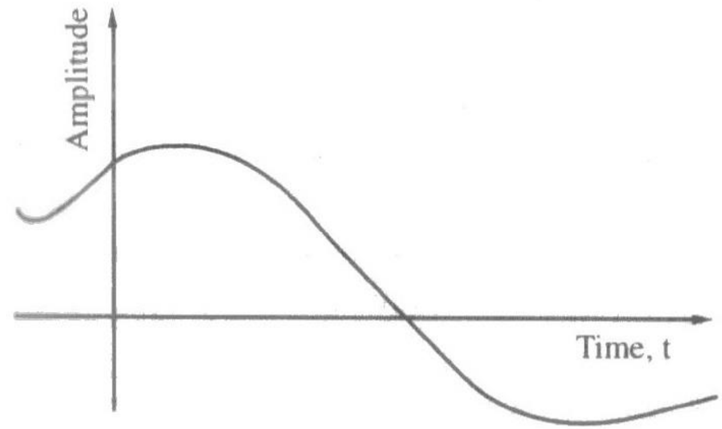
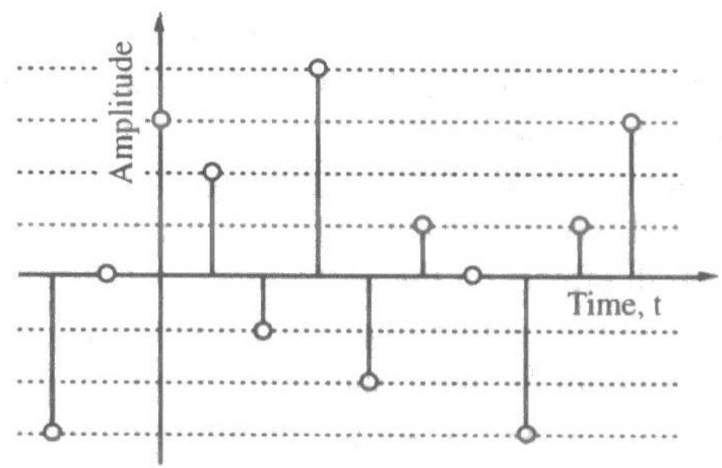
Analog & Digital Signal

- Analog signal
  - Continuous-time signal with continuous-valued amplitude
  - Most of the natural signals are analog



- Digital signal
  - Discrete-time signal with discrete-valued amplitude
  - A digital signal is a quantized sampled-data signal



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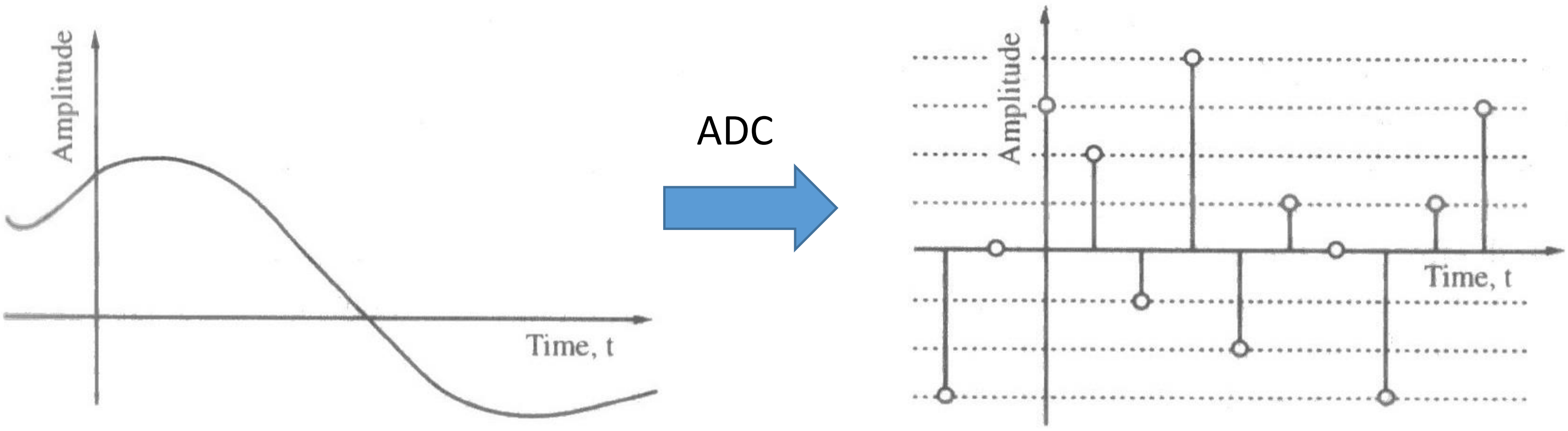
Digital Processing Has Many Advantages

- Digital processing has many advantages
  - Refer to slides of week 1

The Bridge Between Analog and Digital



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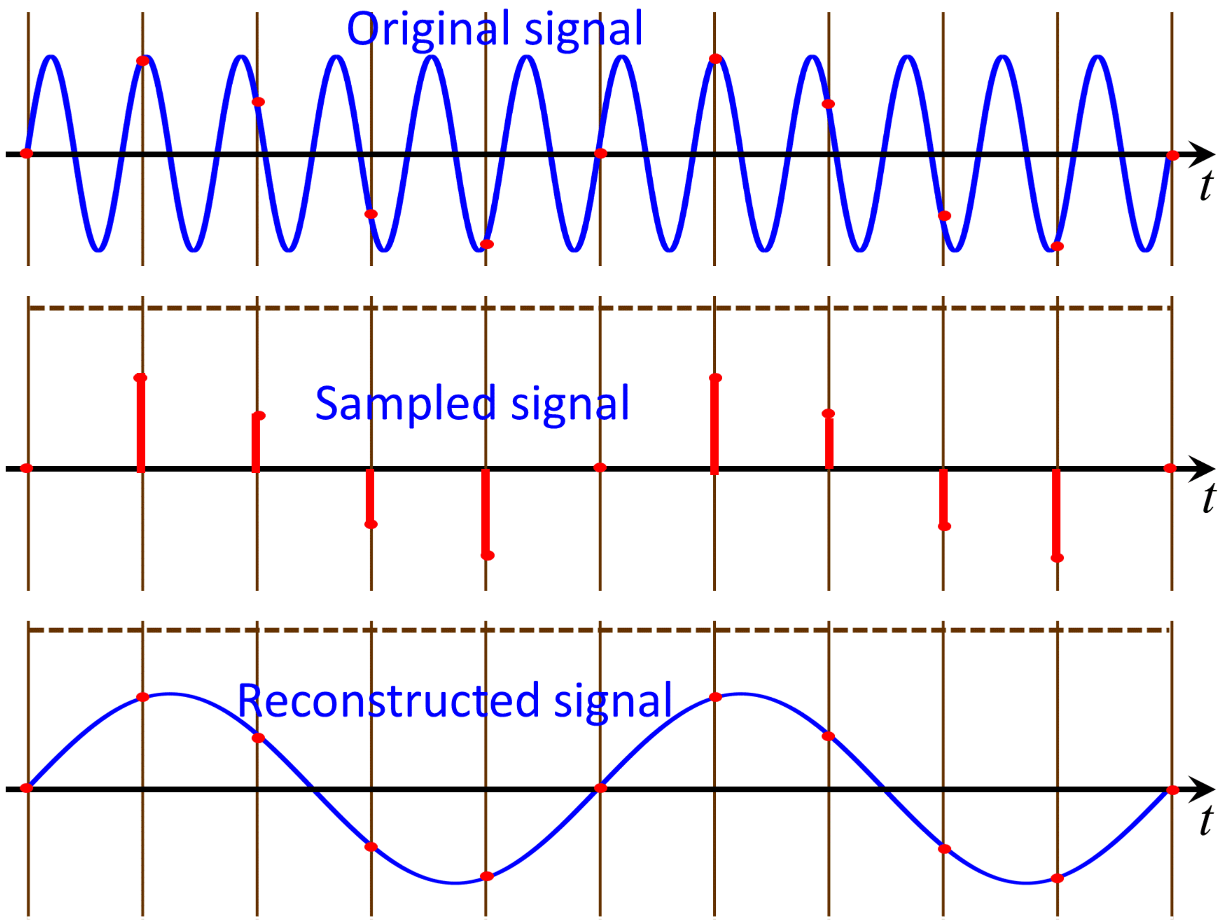
Analog-to-Digital Conversion

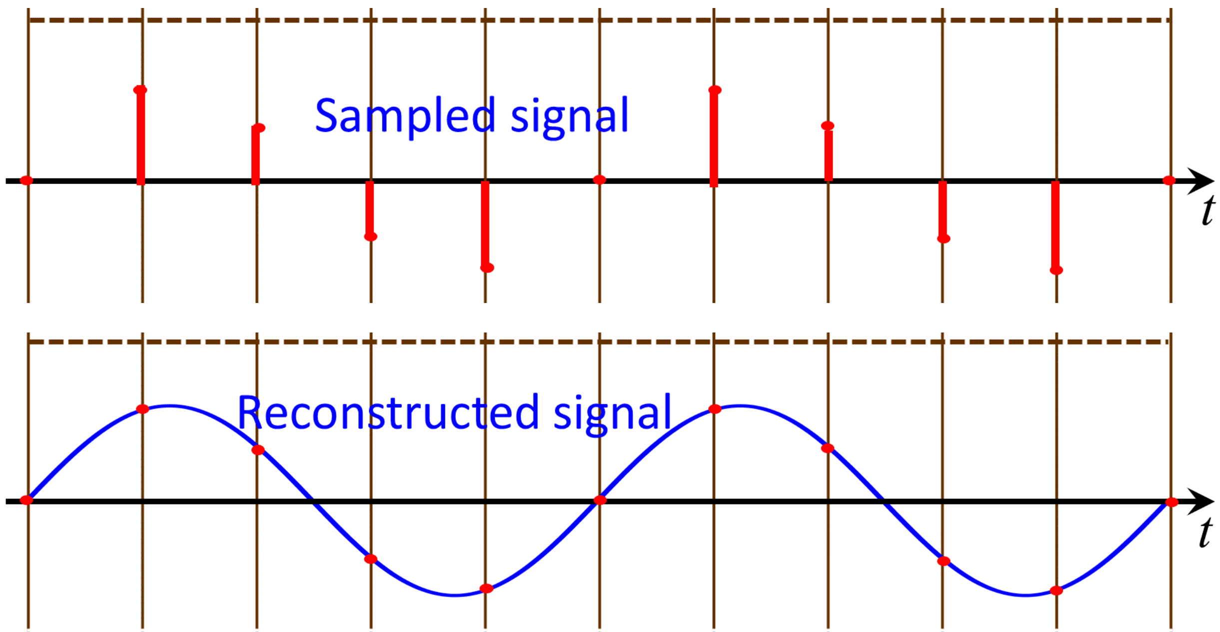
- Q1: can we recover the original continuous signal?



If the sampling rate is not sufficiently high, the reconstructed signal is different from the original signal.

Analog-to-Digital Conversion

- Q1: can we recover the original continuous signal?



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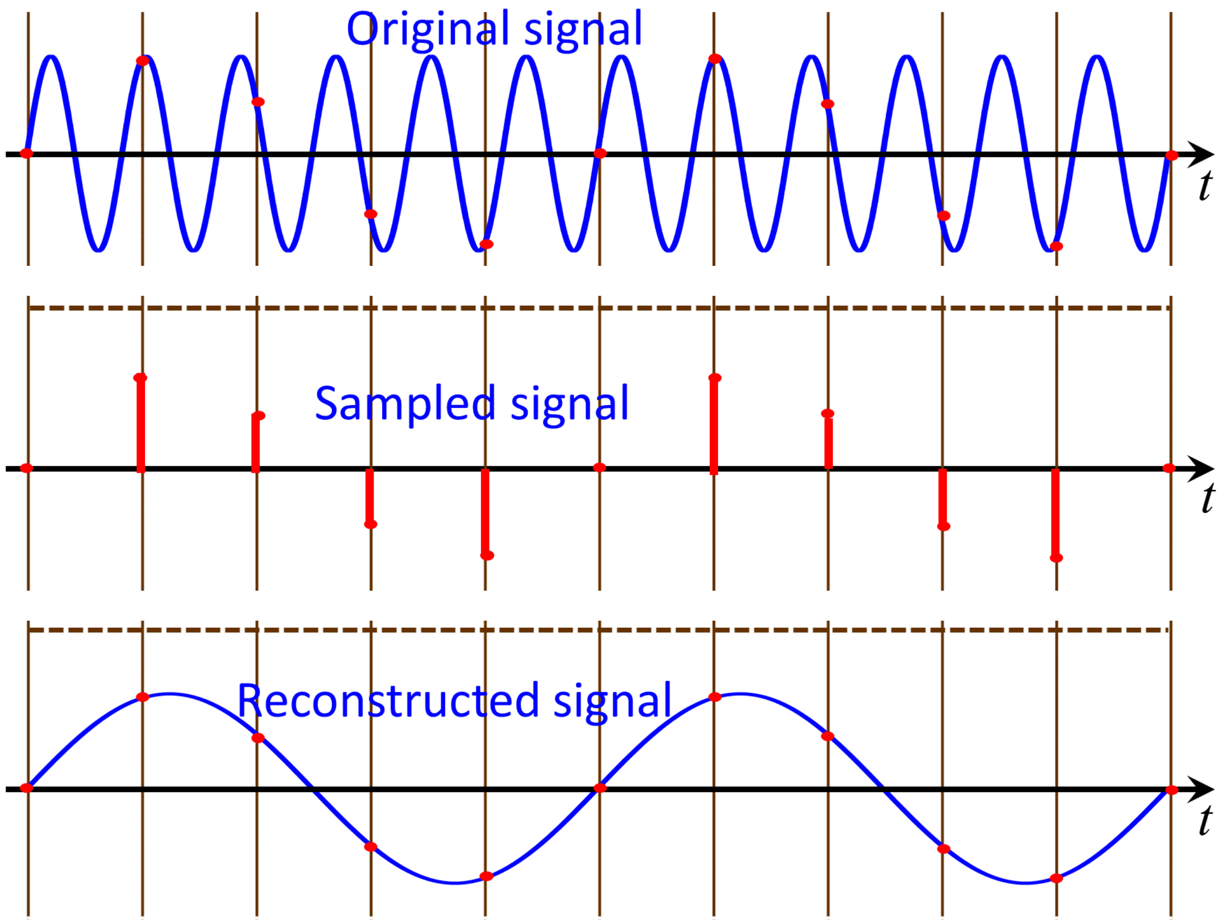

If the sampling rate is not sufficiently high, the reconstructed signal is different from the original signal.

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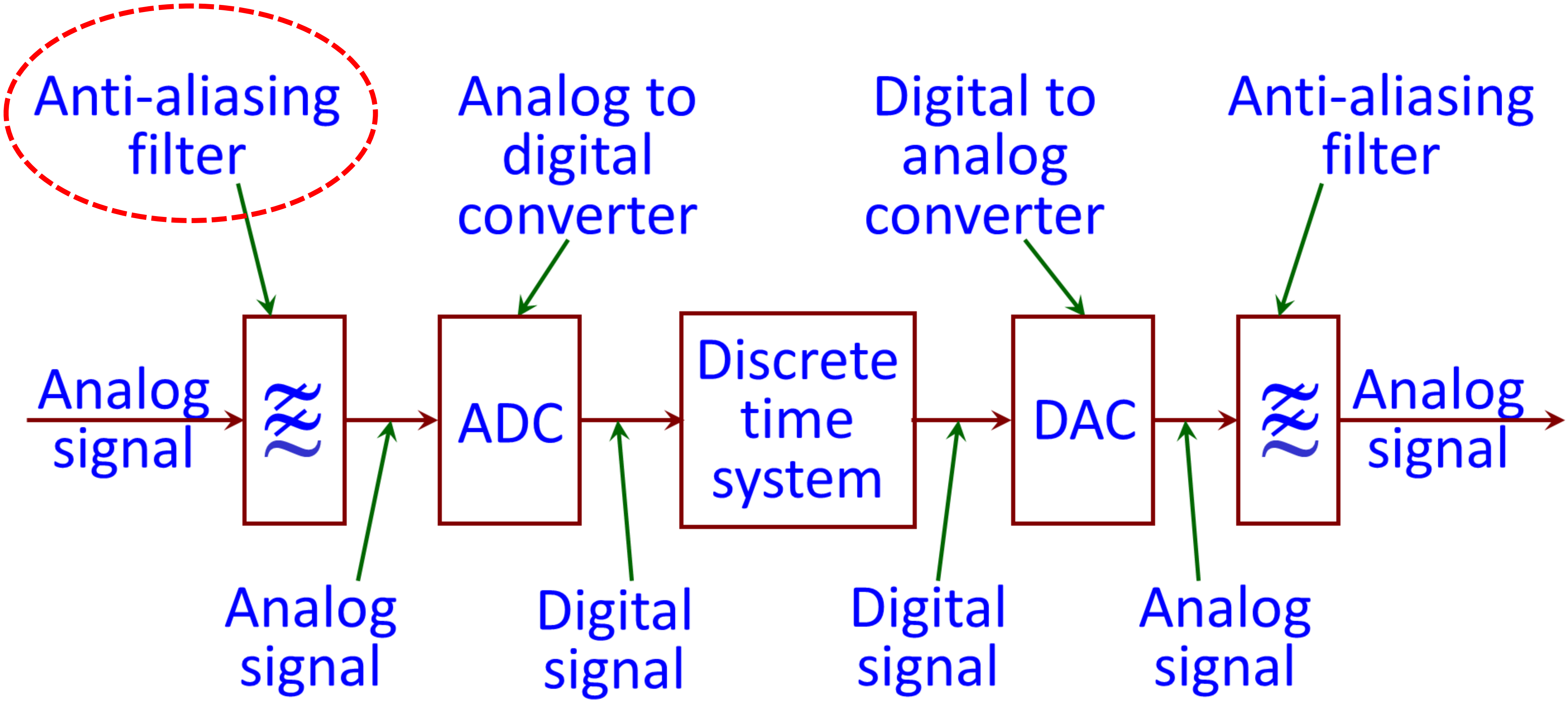
The Famous Nyquist Theorem

Birthdate
1889/02/07
Birthplace
Nilsby, Sweden
Death date
1976/04/04
Associated organizations
Bell Labs
Fields of study
Signal processing
Awards
IRE Medal of Honor, Stuart Ballantine Medal of the Franklin Institute, Mervin J. Kelly award

- The Nyquist Theorem states that in order to adequately reproduce the original signal it should be periodically sampled at a rate that is 2X the highest frequency you wish to record

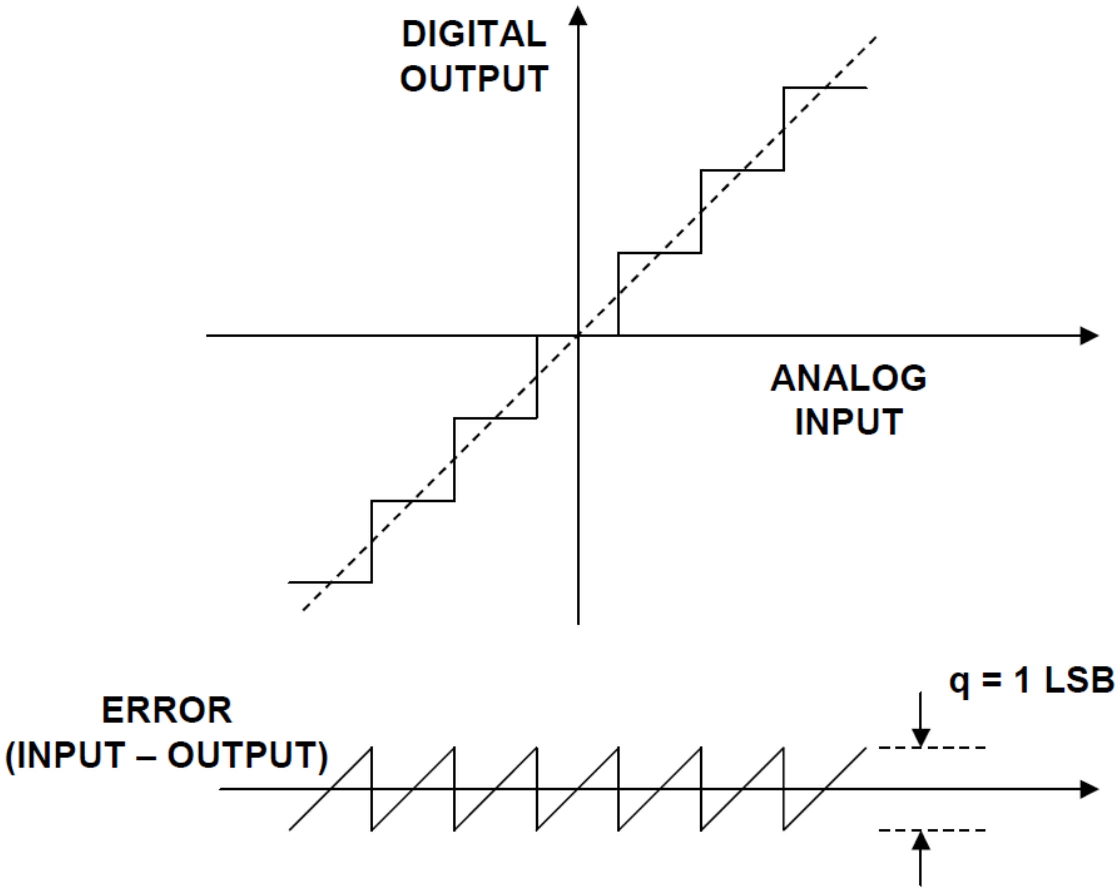
Typical DSP Systems

- ❑ Analog filter or digital filter?
- ❑ High-pass? Low-pass? Band-pass?



Analog-to-Digital Conversion

- Q2: how many bits we need to represent a sample?



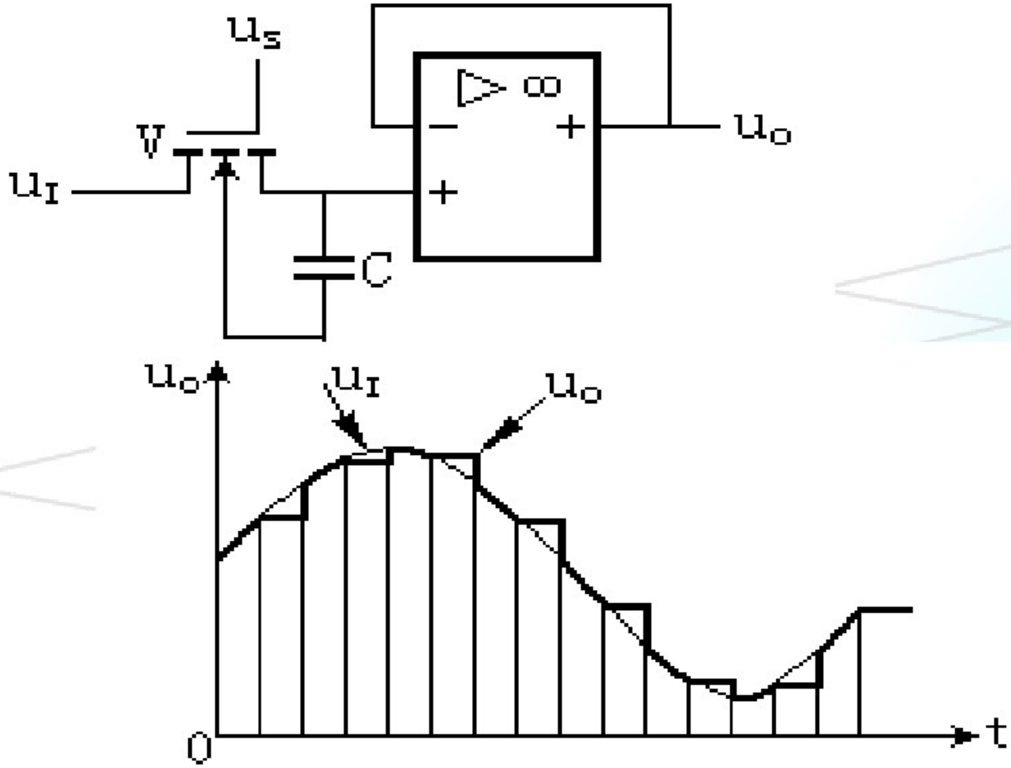
Analog-to-Digital Conversion

- Commonly used ADC
  - 8-bit, 10-bit, 12-bit, 14-bit, 16-bit, 24-bit

模拟电压U_i	量化结构	二进制码
0~1/8V	0V	0 0 0
1/8~2/8V	1/8V= △	0 0 1
2/8~3/8V	2/8V=2△	0 1 0
3/8~4/8V	3/8V=3△	0 1 1
4/8~5/8V	4/8V=4△	1 0 0
5/8~6/8V	5/8V=5△	1 0 1
6/8~7/8V	6/8V=6△	1 1 0
7/8~8/8V	7/8V=7△	1 1 1

How Does an ADC Work?

- Sample & hold



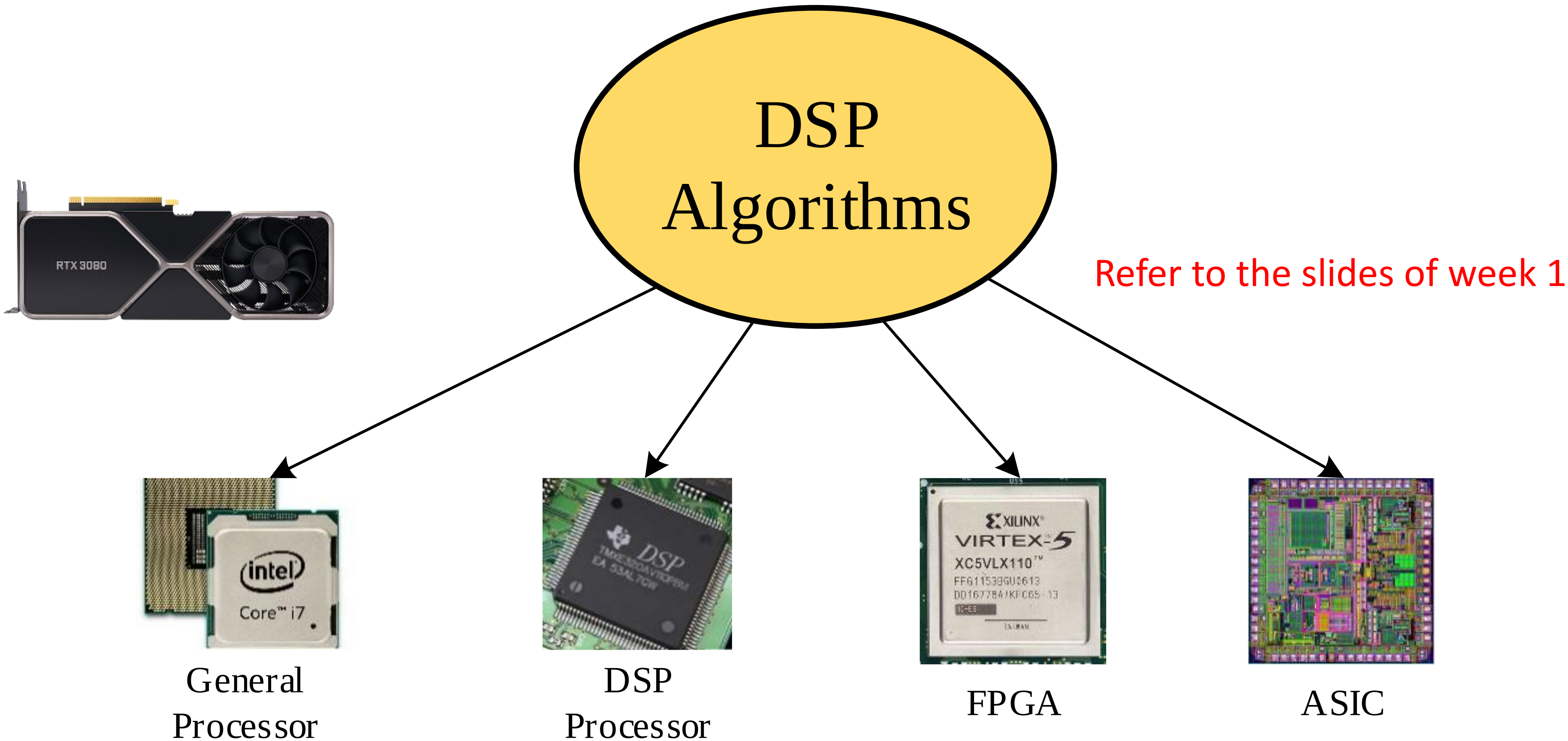
How Does an ADC Work?

- Quantize & coding

模拟电压U_i	量化结构	二进制码
0~1/8V	0V	0 0 0
1/8~2/8V	1/8V= Δ	0 0 1
2/8~3/8V	2/8V=2Δ	0 1 0
3/8~4/8V	3/8V=3Δ	0 1 1
4/8~5/8V	4/8V=4Δ	1 0 0
5/8~6/8V	5/8V=5Δ	1 0 1
6/8~7/8V	6/8V=6Δ	1 1 0
7/8~8/8V	7/8V=7Δ	1 1 1

The Discrete-time System

- A given DSP algorithm can be implemented in various ways



The Discrete-time System

- Fixed point VS Floating point

Fixed Point Number

- Fixed-point arithmetic
  - high speed
  - Low complexity
- Represented by an integer with a scaling factor

$$X = x_{W-1}x_{W-2}...x_M \ . \ x_{M-1}...x_0 = x_{W-1}x_{W-2}...x_0 \times r^{-M}$$

Decimal Number System

- Decimal number system uses the 10 symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to represent a number

- Example:

$$(456)_{10} = 4\times10^{2} + 5\times10^{1} + 6\times10^{0}$$

$$(3705.86)_{10} = 3\times10^{3} + 7\times10^{2} + 0\times10^{1} + 5\times10^{0} + 8\times10^{-1} + 6\times10^{-2}$$

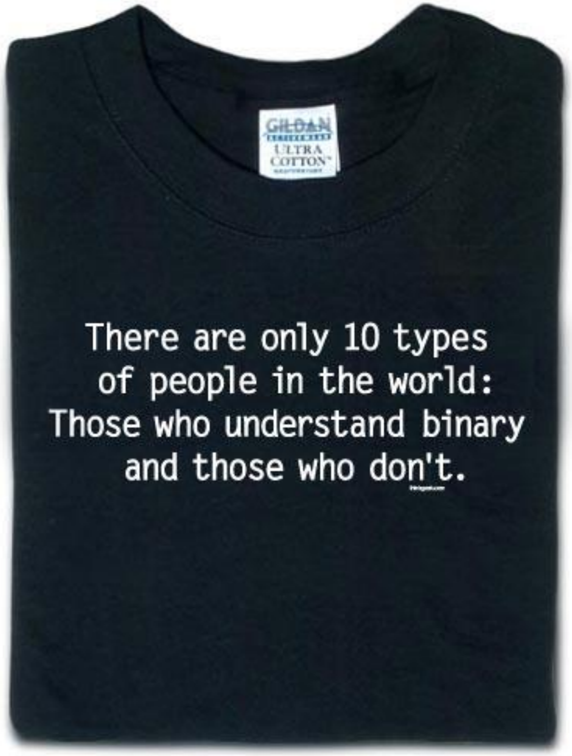
Binary Number System

- In binary number system, 2 symbols (0 and 1) are used to represent a number
- Example:

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$(11001)_2 = (2^4)_{10} + (2^3)_{10} + (2^0)_{10}$$
$$= (16)_{10} + (8)_{10} + (1)_{10}$$
$$= (25)_{10}$$

$$(101.01)_2 = (2^2)_{10} + (2^0)_{10} + (2^{-2})_{10}$$
$$= (4)_{10} + (1)_{10} + (0.25)_{10}$$
$$= (5.25)_{10}$$



Binary Number System (cont'd)

- Unsigned binary

$$X = x_{W-1}x_{W-2}\ldots x_0 = \sum_{k=0}^{W-1} x_k \cdot 2^k,\ \ x_k \in \{0,1\}$$

- The range of an *N*-bit unsigned binary number is $[0, 2^N\text{-}1]$
  - The largest 4-bit number is $(1111)_2 = 16$
- Negative number is not represented. To represent negative numbers, an extra bit, called sign bit is needed

Negative Numbers

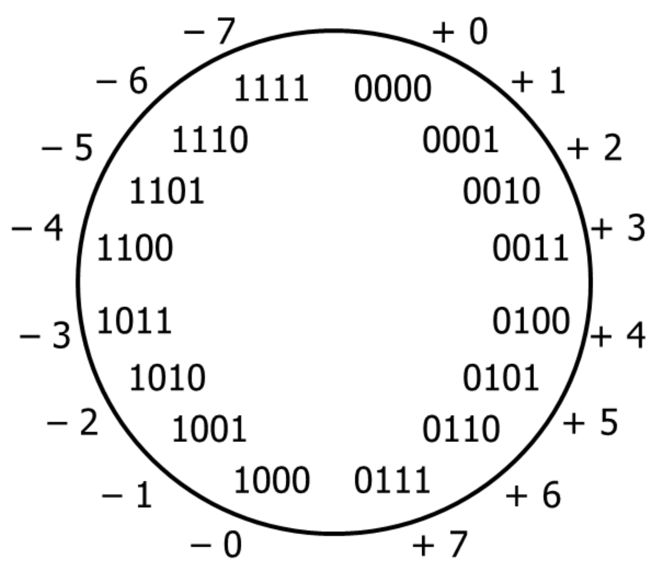
- Three approaches to represent negative numbers
  - Sign and magnitude
  - Two's-complement

- The two approaches represent positive numbers in the same way

Signed-Magnitude

- The most significant bit (MSB) is the sign bit
- Remaining bits are the number's magnitude

$$X = x_{W-1}x_{W-2}\ldots x_0 = (-1)^{x_{W-1}} \sum_{k=0}^{W-2} x_k \cdot 2^k,\ \ x_k \in \{0,1\}.$$



Sign and Magnitude (cont'd)

- Problem 1: Two representations of for zero
  - +0 = 0000 and –0 = 1000
- Problem 2: Arithmetic is cumbersome
  - $4 - 3 \neq 4 + (-3)$

Add		Subtract			Compare and subtract		
4	0100	4	0100	0100	– 4	1100	1100
+ 3	+ 0011	– 3	+ 1011	– 0011	+ 3	+ 0011	– 0011
= 7	= 0111	= 1	≠ 1111	= 0001	– 1	≠ 1111	= 1001

Two's complement

- Negative number
  - $0111 \equiv 7_{10}$
  - $100\mathbf{1} \equiv -7_{10}$
- The value of a two's complement number is

$$X = x_{W-1}x_{W-2}...x_0 = -x_{W-1} \cdot 2^{W-1} + \sum_{k=0}^{W-2} x_k \cdot 2^k,\ x_k \in \{0,1\}.$$

- The MSB carries a negative weight

$$(1101)_{2's} = -2^3 + 2^2 + 2^0 = -8 + 4 + 1 = -3$$

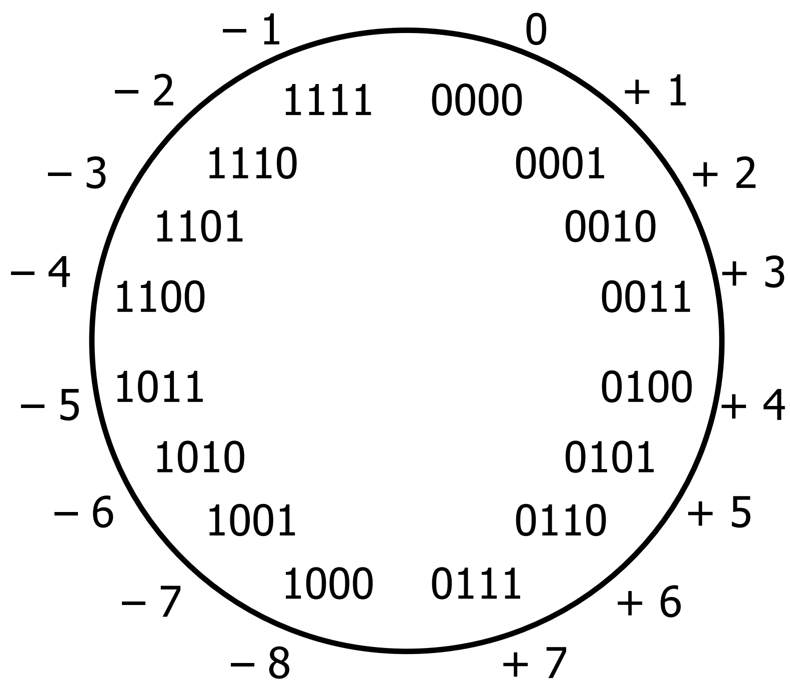
$$(1001)_{2's} = -2^3 + 2^0 = -8 + 1 = -7$$

$$(0110)_{2's} = 2^2 + 2^1 = 4 + 2 = 6$$

$$(110)_{2's} = -2^2 + 2^1 = -4 + 2 = -2$$

Two's complement (cont'd)

- The range of an N-bit two's complement number is $[-2^{N-1}, 2^{N-1}-1]$
- For a 4-bit two's complement number



Two's complement (cont'd)

- Benefits:
  - Simplified arithmetic
  - Only one zero!

EE115 Analog and Digital Circuits

Add		Invert and add		Invert and add	
4 + 3	0100 + 0011	4 – 3	0100 + 1101	– 4 + 3	1100 + 0011
= 7	= 0111	= 1 drop carry	1 0001 = 0001	– 1	1111

- As long as the results can be represented (no overflow)!

Floating Point Number

exponent;
characteristic

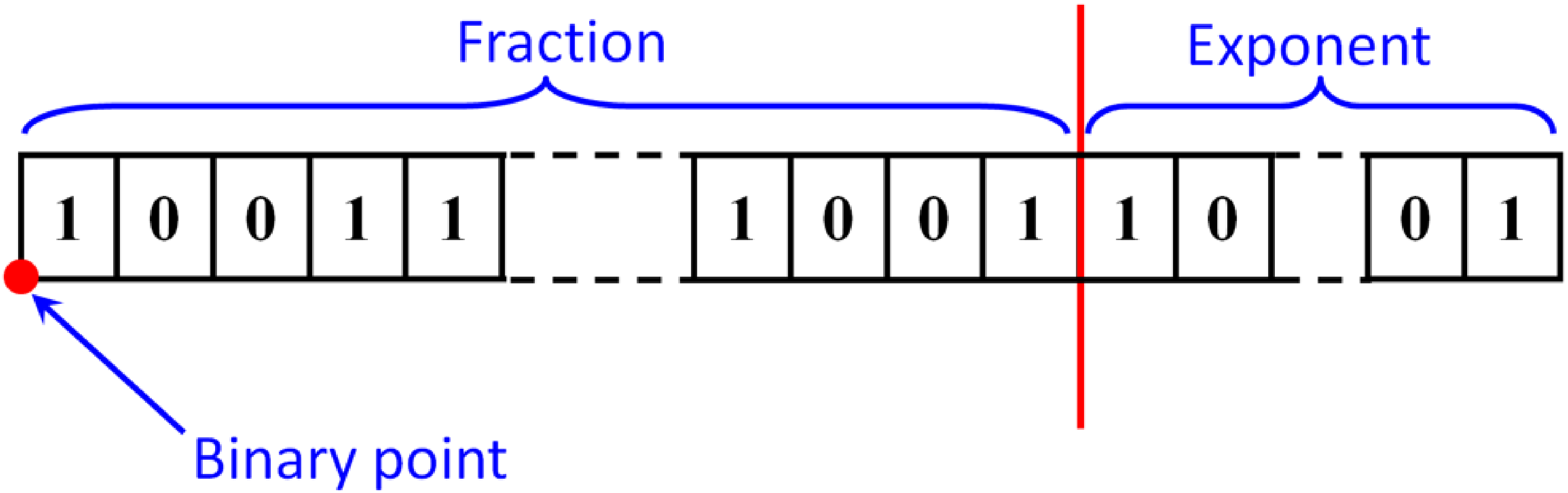
$$A = P \times Q^D$$

base; radix

fraction;
mantissa

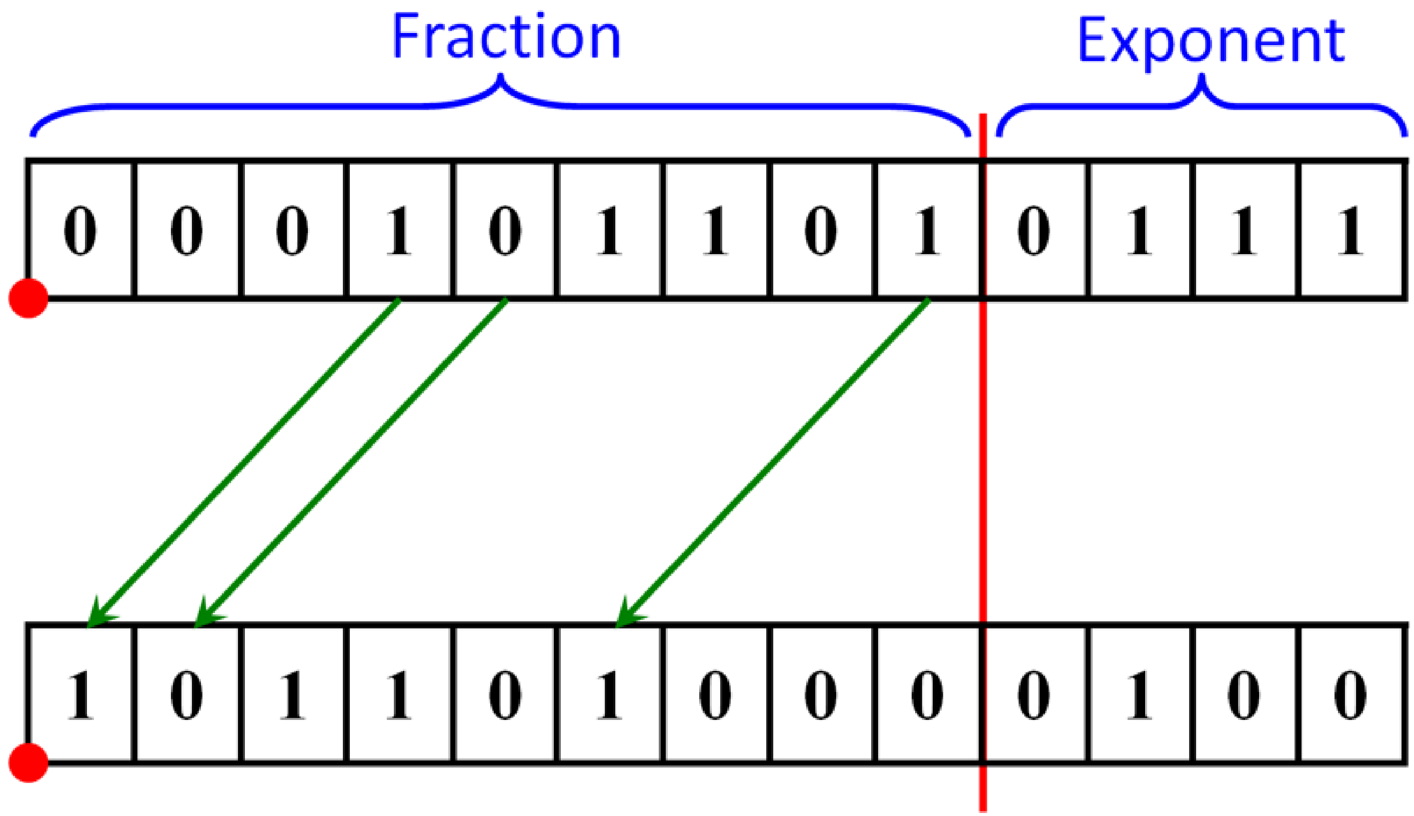
Example: 8934 can be written as 0.8934×10^4

Binary Representation of Floating Point Number



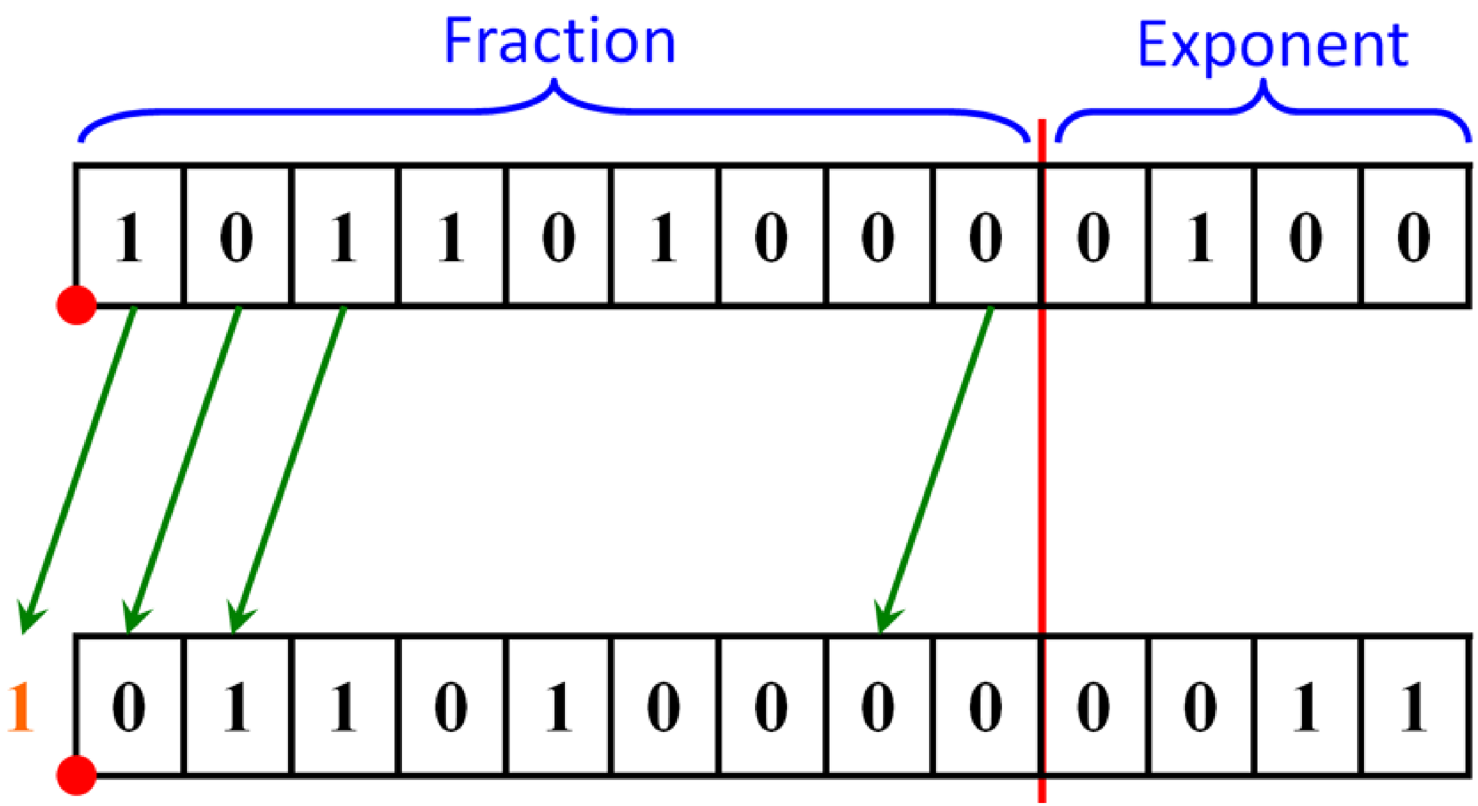
Binary Representation of Floating Point Number (cont'd)

- For maximum precision, the number can be normalized until the first digit is "1"



Binary Representation of Floating Point Number (cont'd)

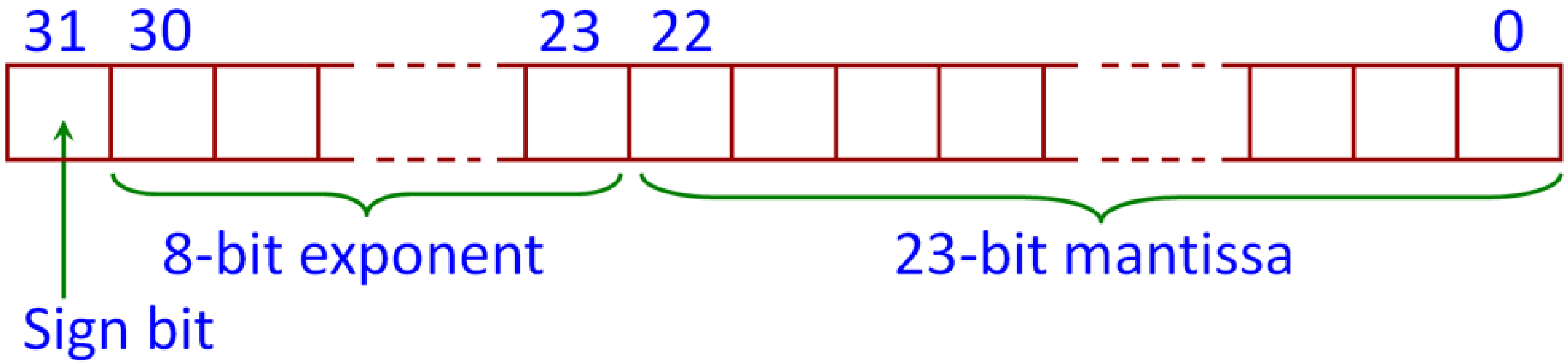
- Since the first digit is a "1", it is not necessary to record it. Thus it is discarded



IEEE 754

- IEEE standard for binary floating-point arithmetic
- IEEE 754 specifies 4 formats
  - Single-precision (32-bit)
  - Double-precision (64-bit)
  - Signal-extended precision (≥43-bit, seldom used)
  - Double-extended precision (≥79-bit, usually 80-bit)

32-bit Single Precision Format



Value = $(-1)^{S} \times 2^{Exp-127} \times M$

Where

Exp = Recorded exponent.

M = 1.(value represented by fractional bit).

The Mantissa Value

Mantissa value = 1.(value represented by fractional bit).

Example

	22																						0
	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

23-bit mantissa