SI 100B 图像处理应用

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2020/10/24

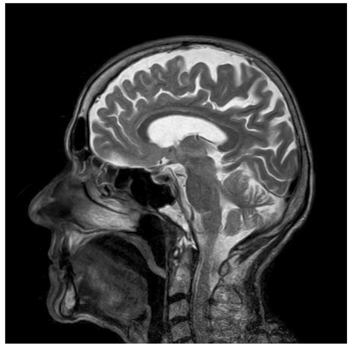
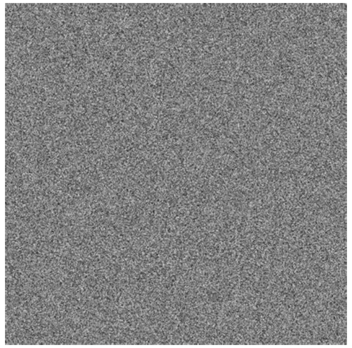
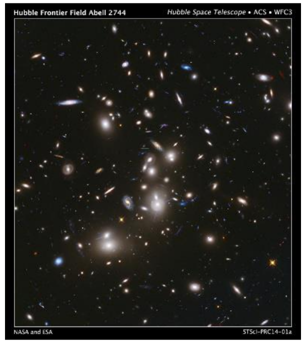
Outline

- Understand digital image
- Principle of Image Processing
- Image Blur
- Histogram Equalization

What is Digital Image?

- What is an image?



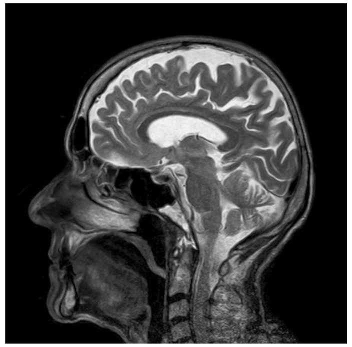
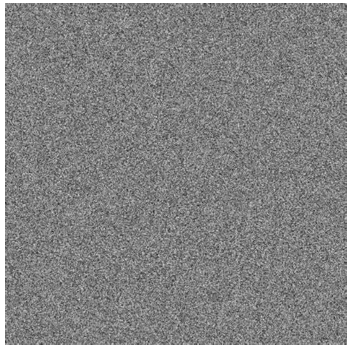
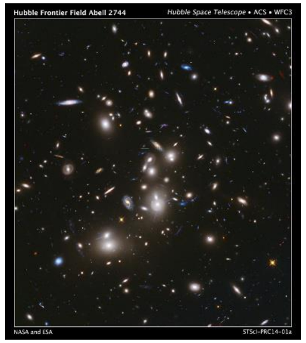




- **___?___ image.**
- **Digital image.**

What is Digital Image?

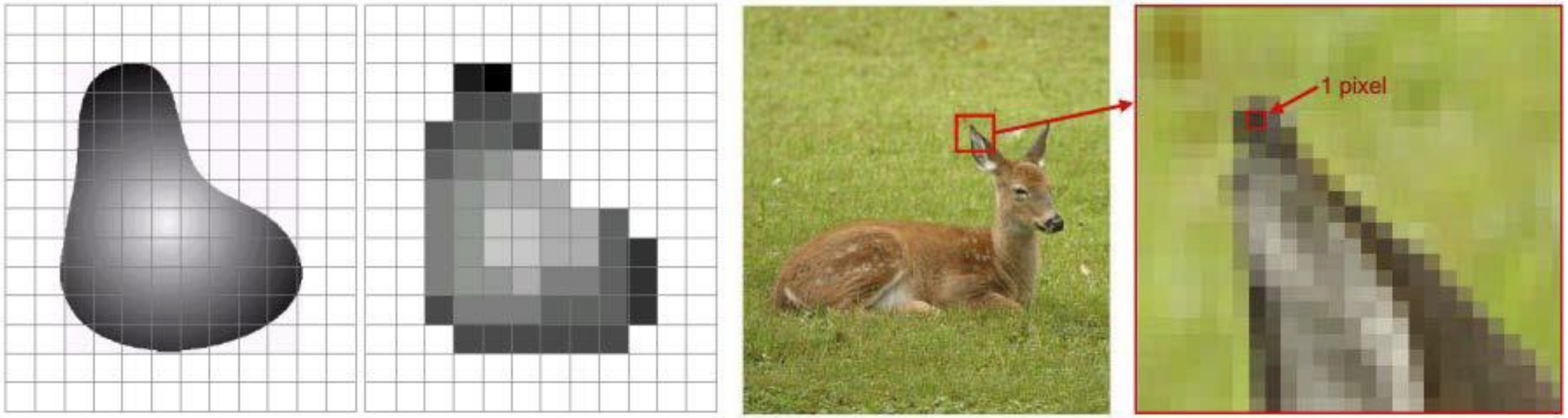
- What is an image?



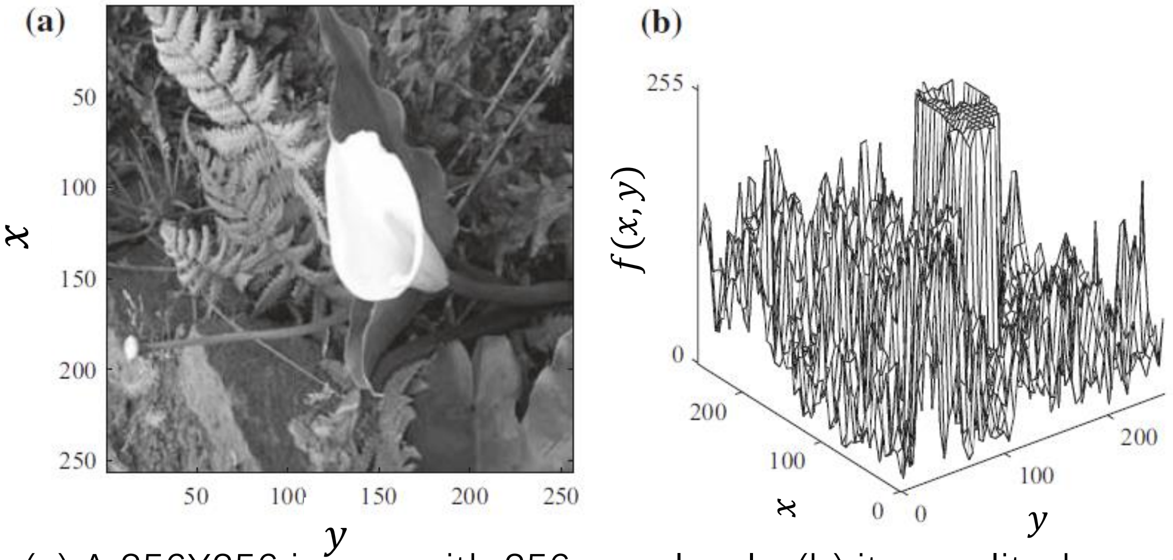
- **Analog image.**
- **Digital image.**


Analog vs. Digital Image



Pixel / Voxel

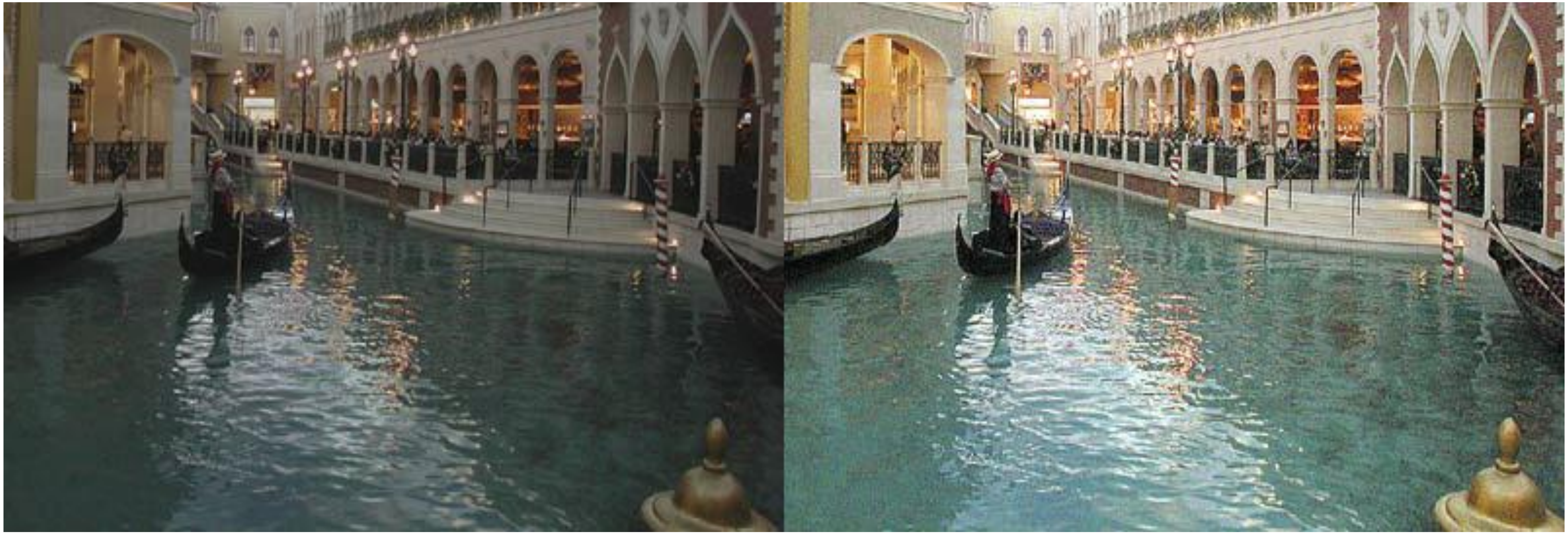
- **Digitization implies that a digital image is an approximation of a real scene.**
- **Digital image compose of a finite number of elements – Pixel.**
- **A pixel has a location and intensity information typically represent gray levels, colors, heights, opacities, etc.**
- **A visual representation in form of a function *f(x,y)*, where**
- *f* is related to the intensity or brightness (color) at point
- *(x, y)* are spatial coordinates
- *x*, *y*, and the amplitude of *f* are finite and discrete quantities



(a) A 256X256 image with 256 gray levels; (b) its amplitude profile

Digital image processing

- **Definition: Processing digital images by means of a digital computer.**



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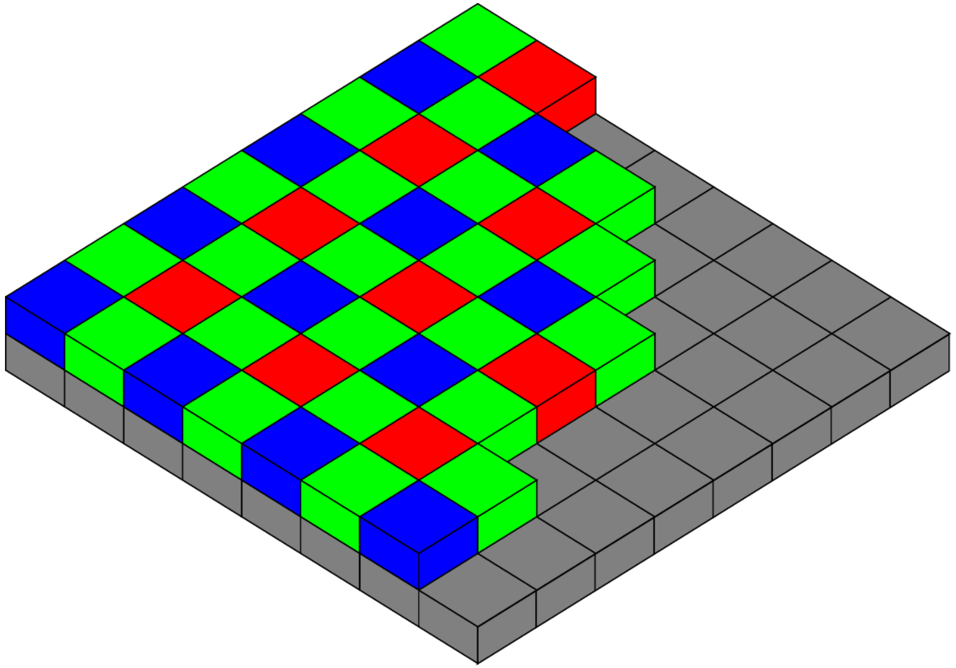
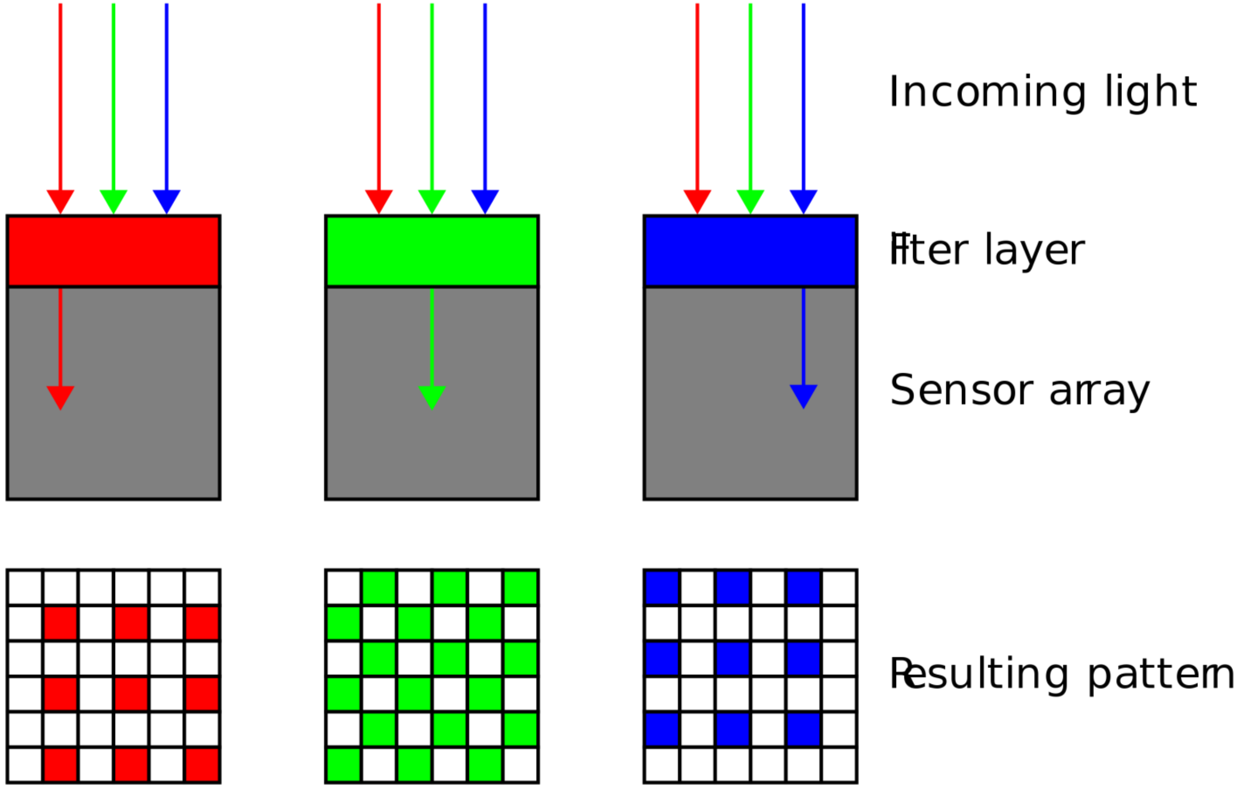
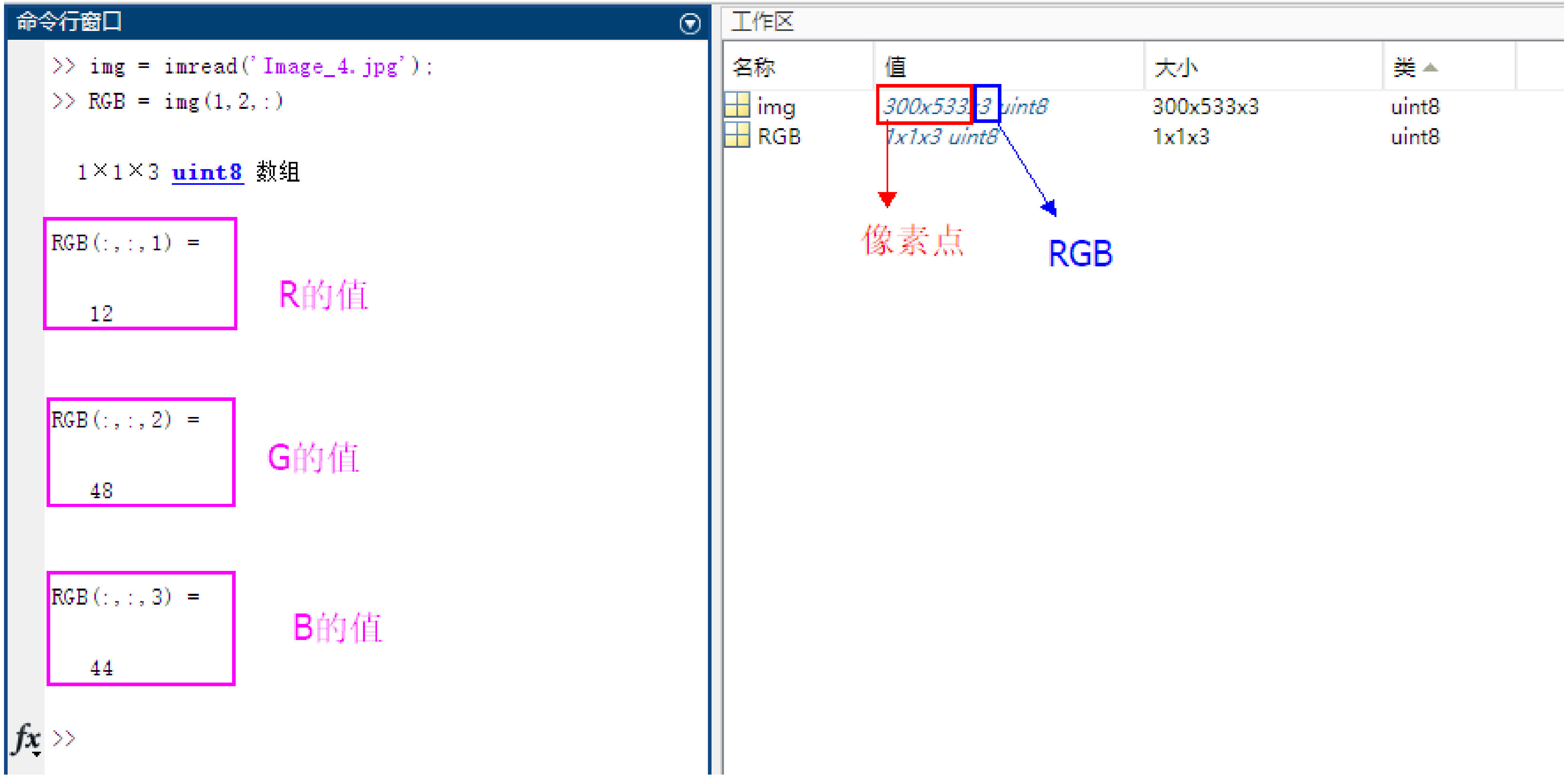
Bayer filter





$$\cancel{\mathrm{f}(x, y)} \longrightarrow \mathrm{f}(x, y, c)$$

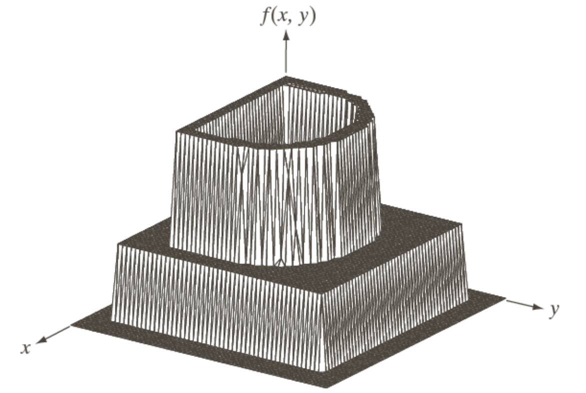
MATLAB


命令行窗口
>> img = imread('Image_4.jpg');
>> RGB = img(1,2,:)
1×1×3 uint8 数组
RGB(:,:,1) =
12
R的值
RGB(:,:,2) =
48
G的值
RGB(:,:,3) =
44
B的值
>>
工作区
名称
值
大小
类
img
300x533x3 uint8
300x533x3
uint8
RGB
1x1x3 uint8
1x1x3
uint8
像素点
RGB

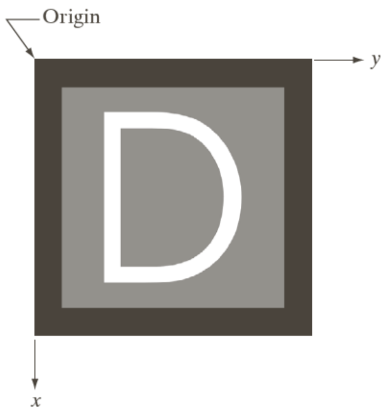
Matrix Representation

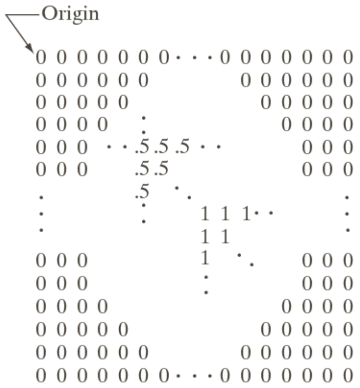
Three basic ways to represent *f*(*x, y*)

- Plot of function: *difficult to view and interpret*
- Visual intensity array: *for view*
- numerical array: *for processing and algorithm development*



$$[f(x,y)] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \ddots & \cdots & \vdots \\ f(M-1,0) & f(M-1,1)\cdots & \cdots & f(M-1,N-1) \end{bmatrix}$$





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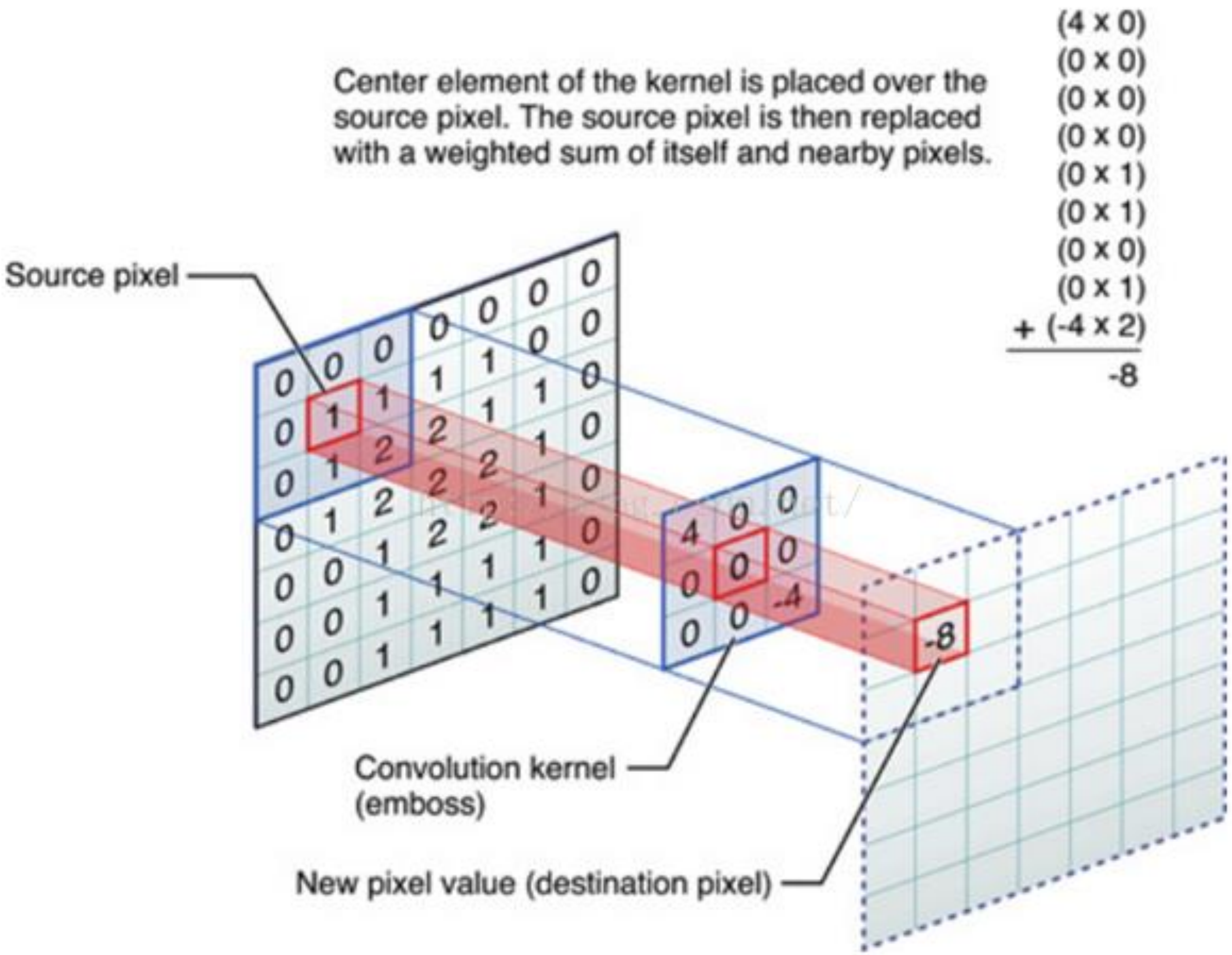
- Histogram Equalization

Convolution operation



Example

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

Example

$$* \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \Rightarrow$$

Example

$$* \begin{bmatrix} 0 & 0.2 & 0 \\ 0.2 & 0 & 0.2 \\ 0 & 0.2 & 0 \end{bmatrix} \Rightarrow$$

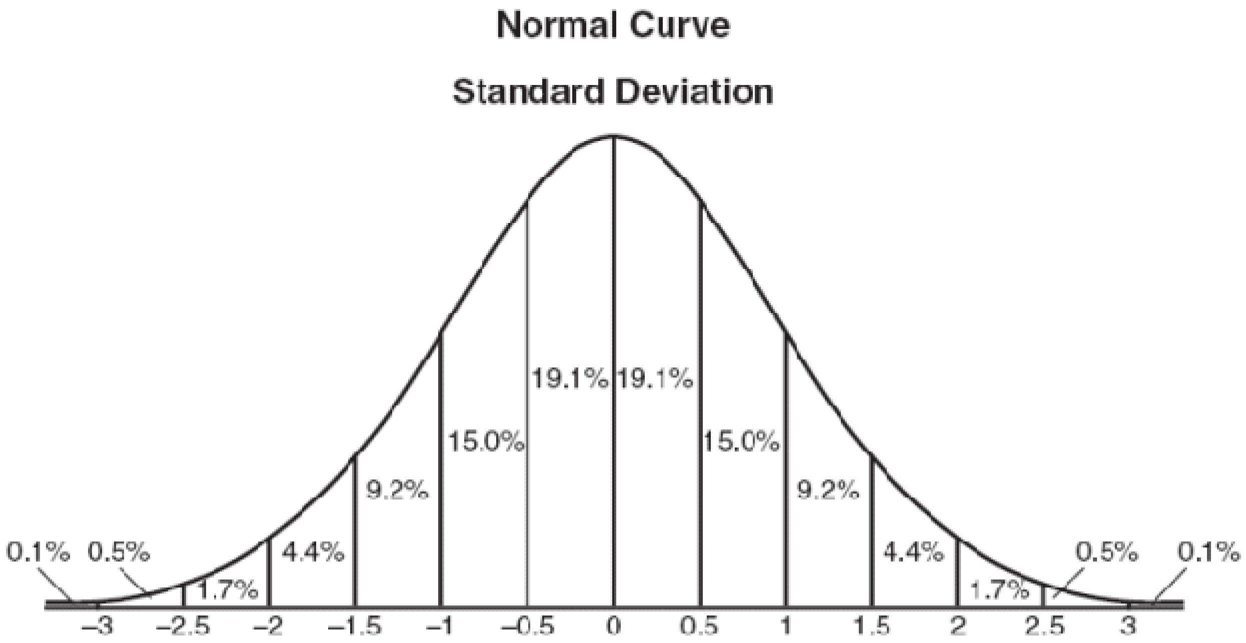
Example

$$* \begin{bmatrix} 0 & 0.25 & 0 \\ 0.25 & 0 & 0.25 \\ 0 & 0.25 & 0 \end{bmatrix} \Rightarrow$$

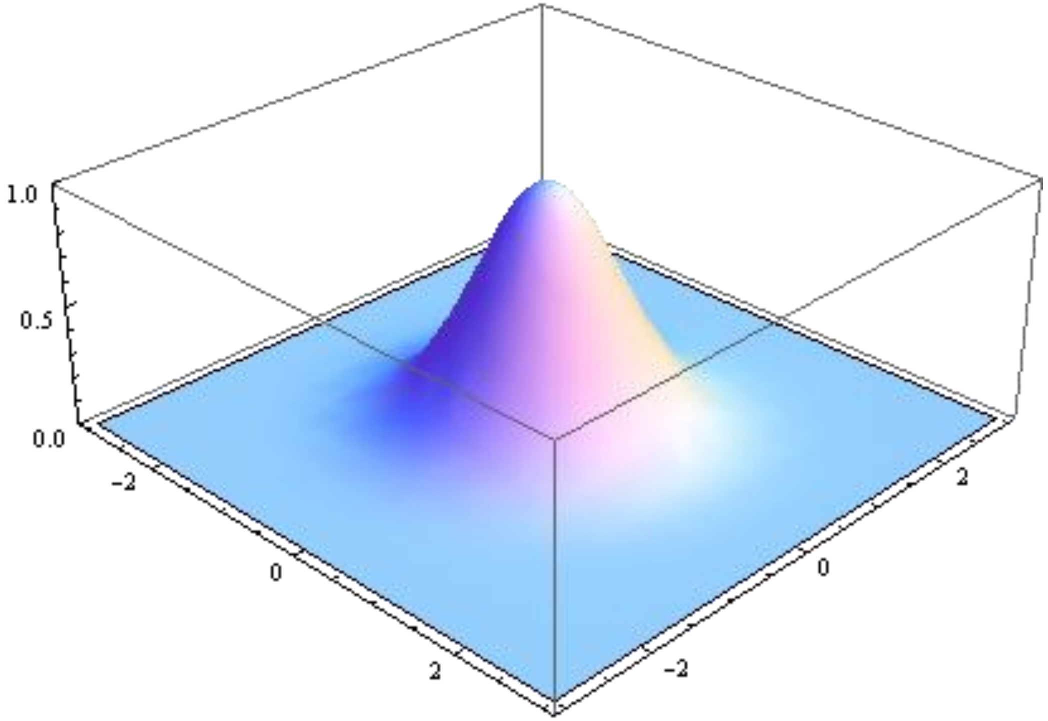
Example

1	1	1
1	1	1
1	1	1

Normal distribution



Normal distribution/Gaussian distribution



Two-dimensional normal Gaussian distribution

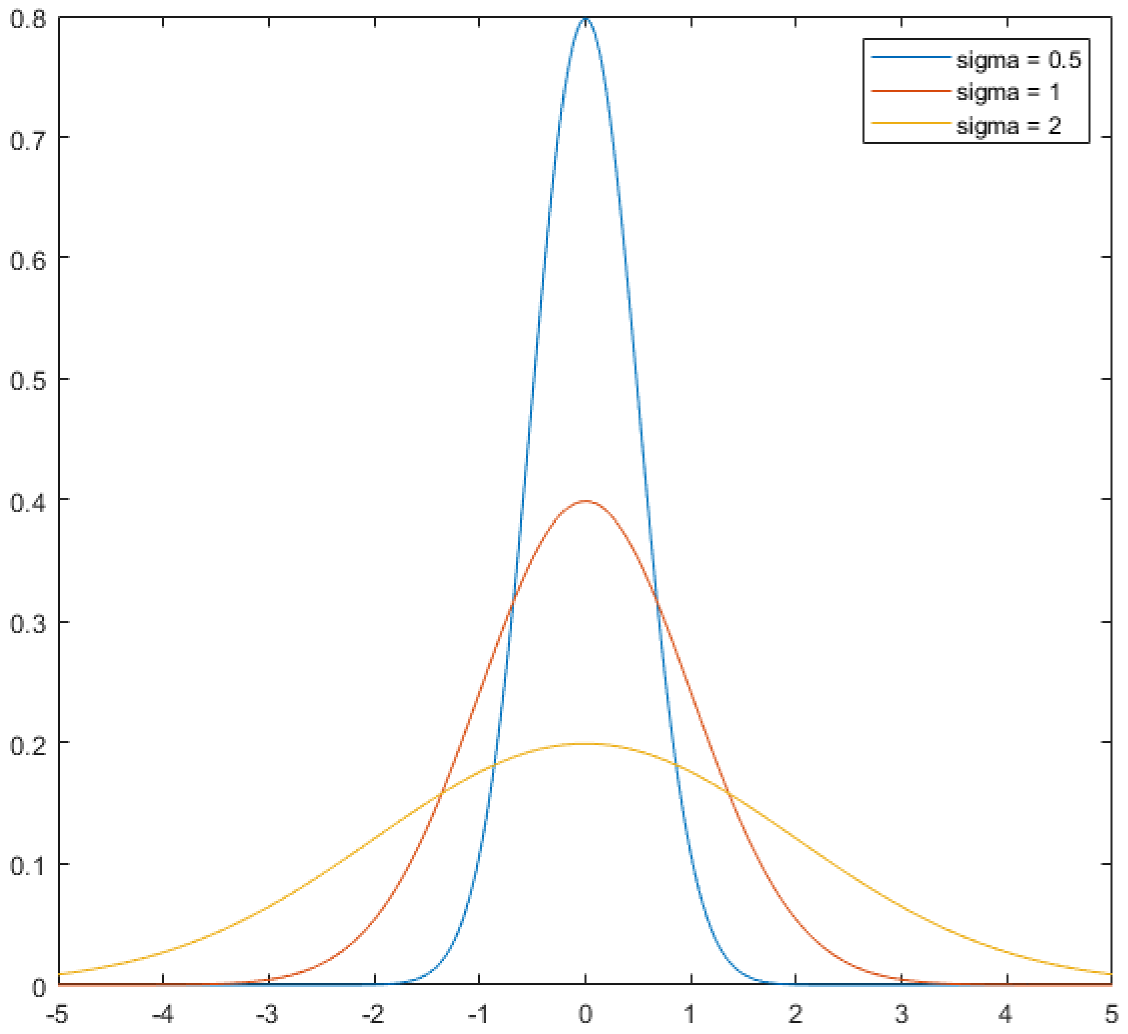
Normal distribution

- One-dimensional function

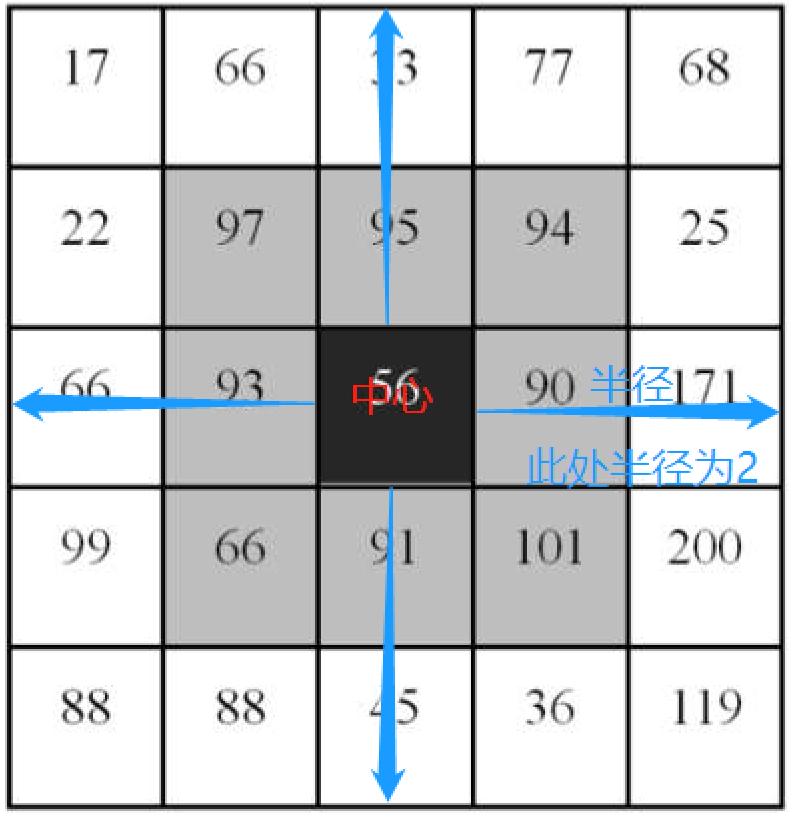
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \mu = 0$$

- Two-dimensional function

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$



Radius



Outline

Histogram Equalization

- **Histogram （直方图）**
  - Definition
  - Property
- **Histogram Processing**
  - Histogram Equalization

Definition

$$h(r_k) = n_k$$

Where r_k : the kth intensity value in the level range of [0, L-1]

n_k : the number of pixels in the image with intensity r_k

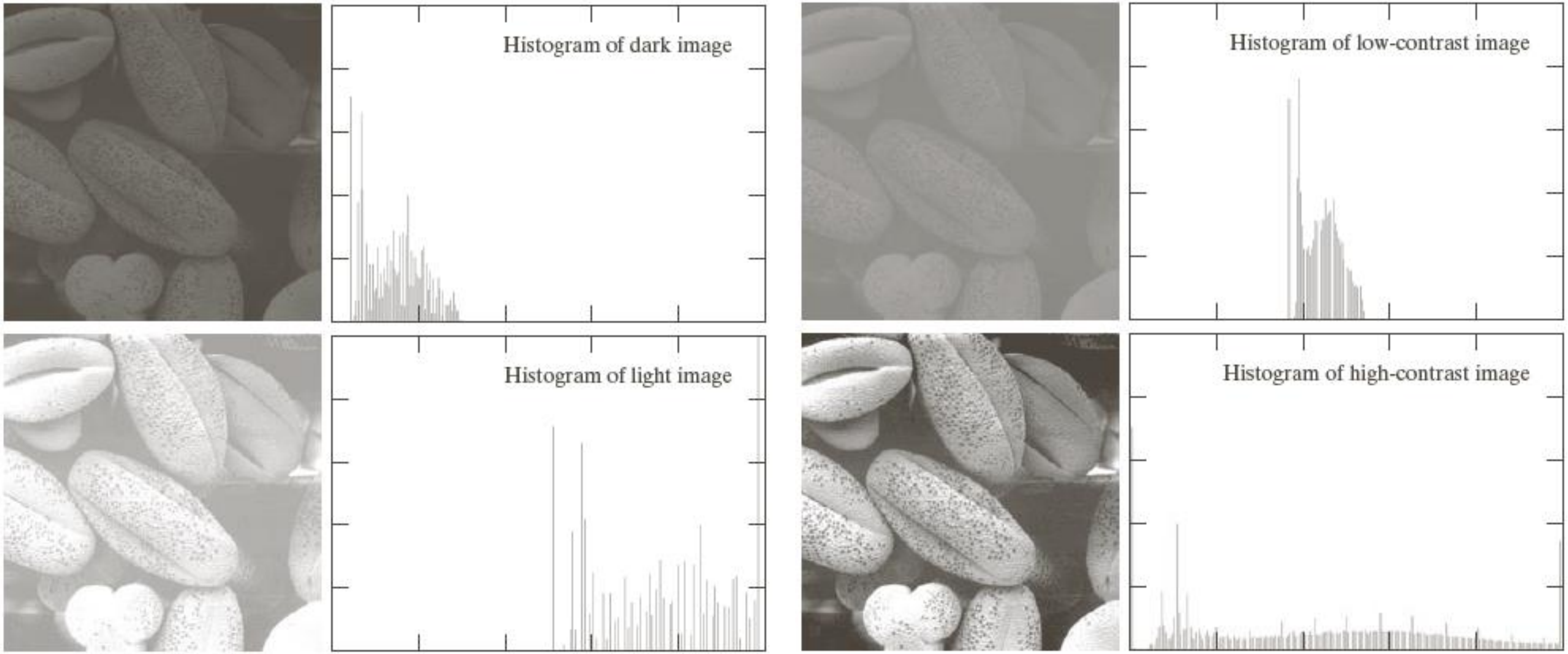
Normalized Histogram（归一化直方图）

$$p(r_k) = \frac{n_k}{MN}$$

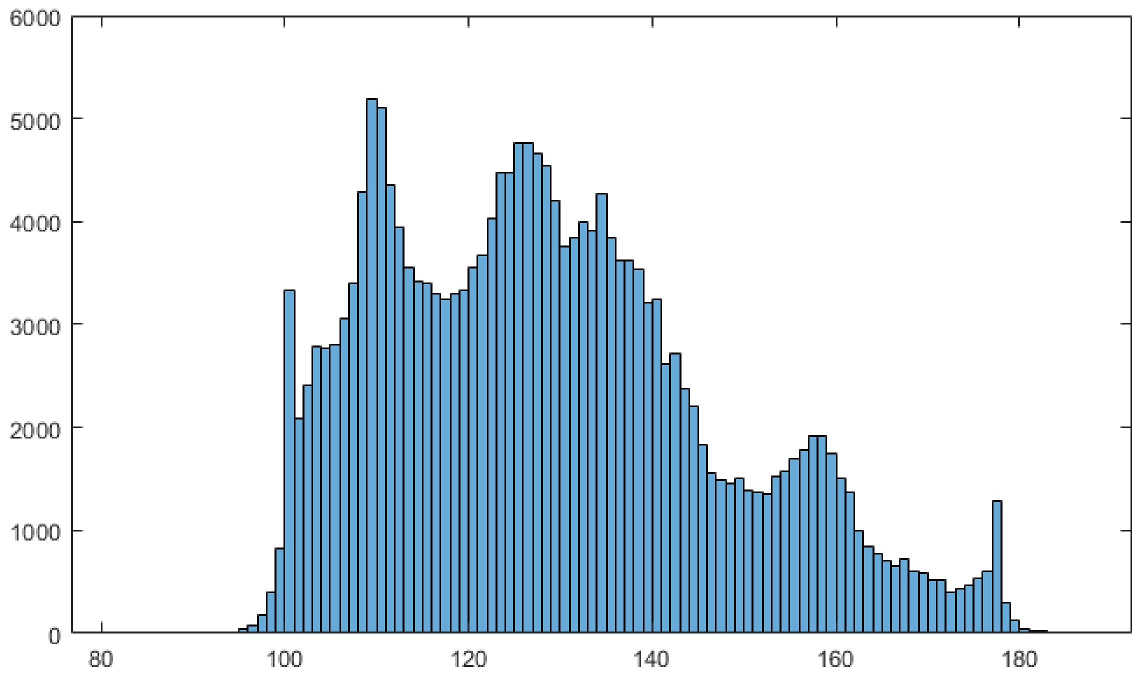
Where $p(r_k)$: the probability of occurrence of intensity r_k in an image

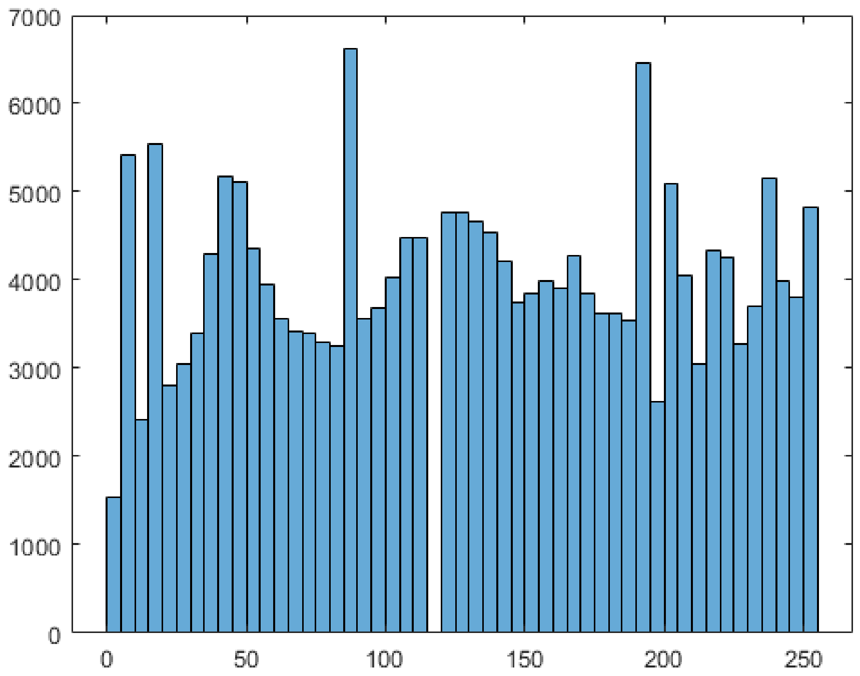
M, N: the row and column dimensions of the image

Basic Image Type



Histogram Equalization



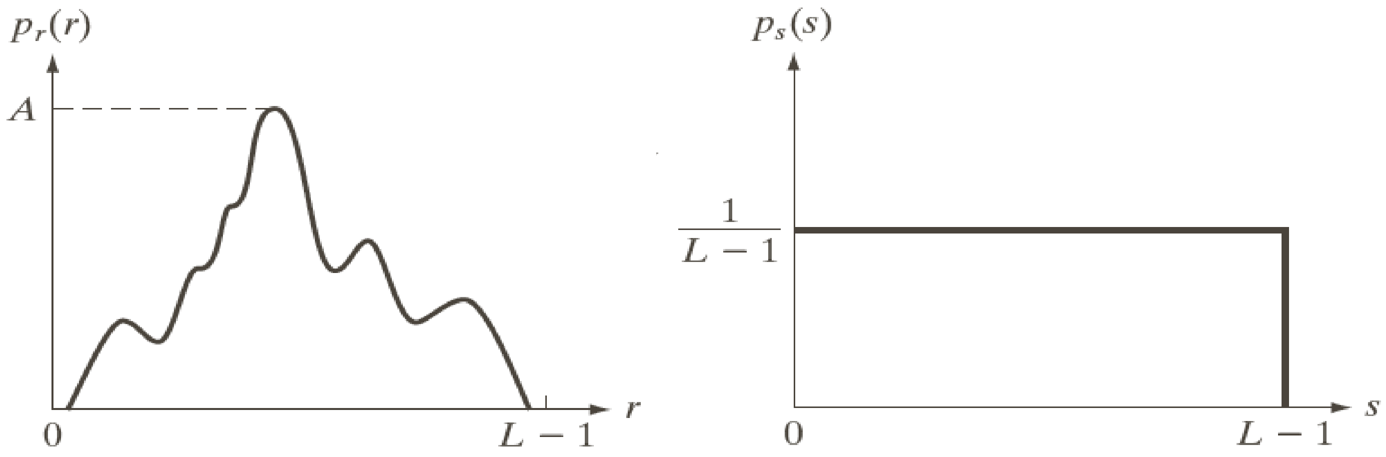


Histogram Equalization

- **Uniform Probability density function :** $p_s(s) = \frac{1}{L-1}$
- **The probability density function (PDF) of s is**

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds} \Longrightarrow p_r(r) \cdot \frac{dr}{ds} = \frac{1}{L-1} \Longrightarrow (L-1)p_r(r) \cdot dr = ds$$

- **Transformation function :** $s = T(r) = (L-1)\int_0^r p_r(w)dw$



Complementary prove

$$\boldsymbol{p_s(s) = p_r(r) \cdot \frac{dr}{ds}}$$

- Since $S = T(r)$ is strictly monotonically increasing function

$\Longrightarrow$ We have $s = T(r), \mathrm{v} = T(\mathrm{w})$, if $\mathrm{v} < \mathrm{s}$ then we have $\mathrm{v} < \mathrm{s} \Leftrightarrow \mathrm{w} < \mathrm{r}$

$\Longrightarrow P(\mathrm{v} < \mathrm{s}) = P(\mathrm{w} < \mathrm{r}\ )$

$\Longrightarrow (\int_{-\infty}^{s} P_s(v)dv)' = (\int_{-\infty}^{r} P_r(w)dw)'$

$\Longrightarrow P_s(s)ds = P_r(r)dr$

$\Longrightarrow \boldsymbol{p_s(s) = p_r(r) \cdot \frac{dr}{ds}}$

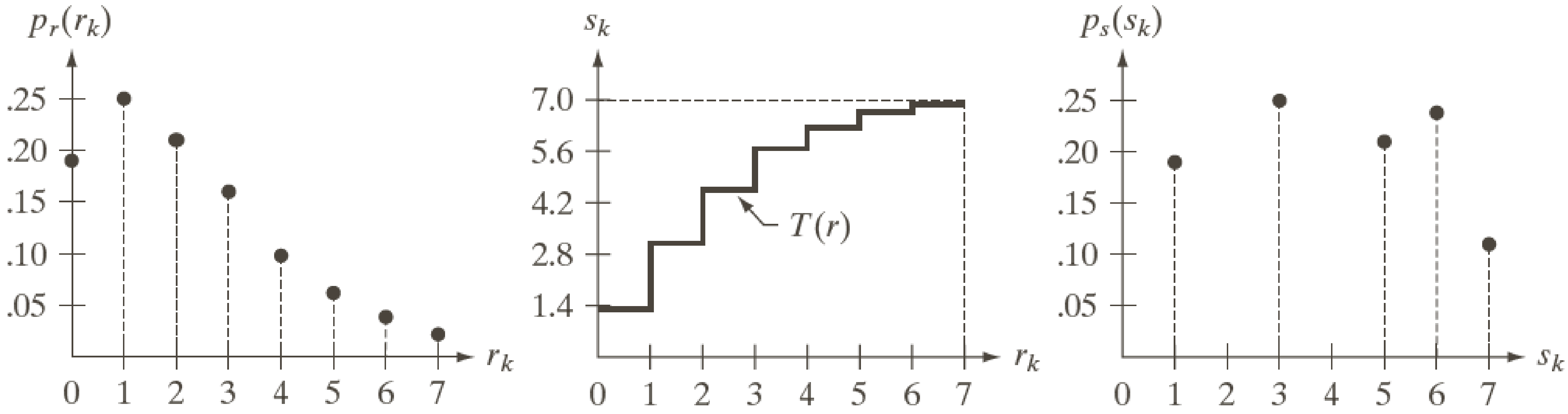
1) If $f(x)$ is continuous on [a,b], then $F(x) = \int_a^x f(t)dt$ is differentiable, and $F'(x) = f(x)$.
2) If $f(x)$ is continuous on [a,b], and $\varphi(x)$ is differentiable, then $(\int_a^{\varphi(x)} f(t)dt)' = f[\varphi(x)]\varphi'(x)$.

Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \,\mathrm{k}$$

r_k	n_k	$p_r(r_k)$	s_k		s_k	$p_s(s_k)$
0	790	0.19	1.33	1	0	0
1	1023	0.25	3.08	3	1	0.19
2	850	0.21	4.55	5	2	0
3	656	0.16	5.67	6	3	0.25
4	329	0.08	6.23	6	4	0
5	245	0.06	6.65	7	5	0.21
6	122	0.03	6.86	7	6	0.24
7	81	0.02	7.00	7	7	0.11

Example



$p_r(r_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
r_k
s_k
7.0
5.6
4.2
2.8
1.4
$T(r)$
0 1 2 3 4 5 6 7
r_k
$p_s(s_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
s_k

RBG to YUV

Y′ stands for the luma component (the brightness) and U and V are the chrominance (color) components; luminance is denoted by Y and luma by Y′ – the prime symbols (') denote gamma correction, with "luminance" meaning physical linear-space brightness, while "luma" is (nonlinear) perceptual brightness.

RBG to YUV

$$Y = 0.299R + 0.587G + 0.114B$$

$$U = -0.147R - 0.289G + 0.436B$$

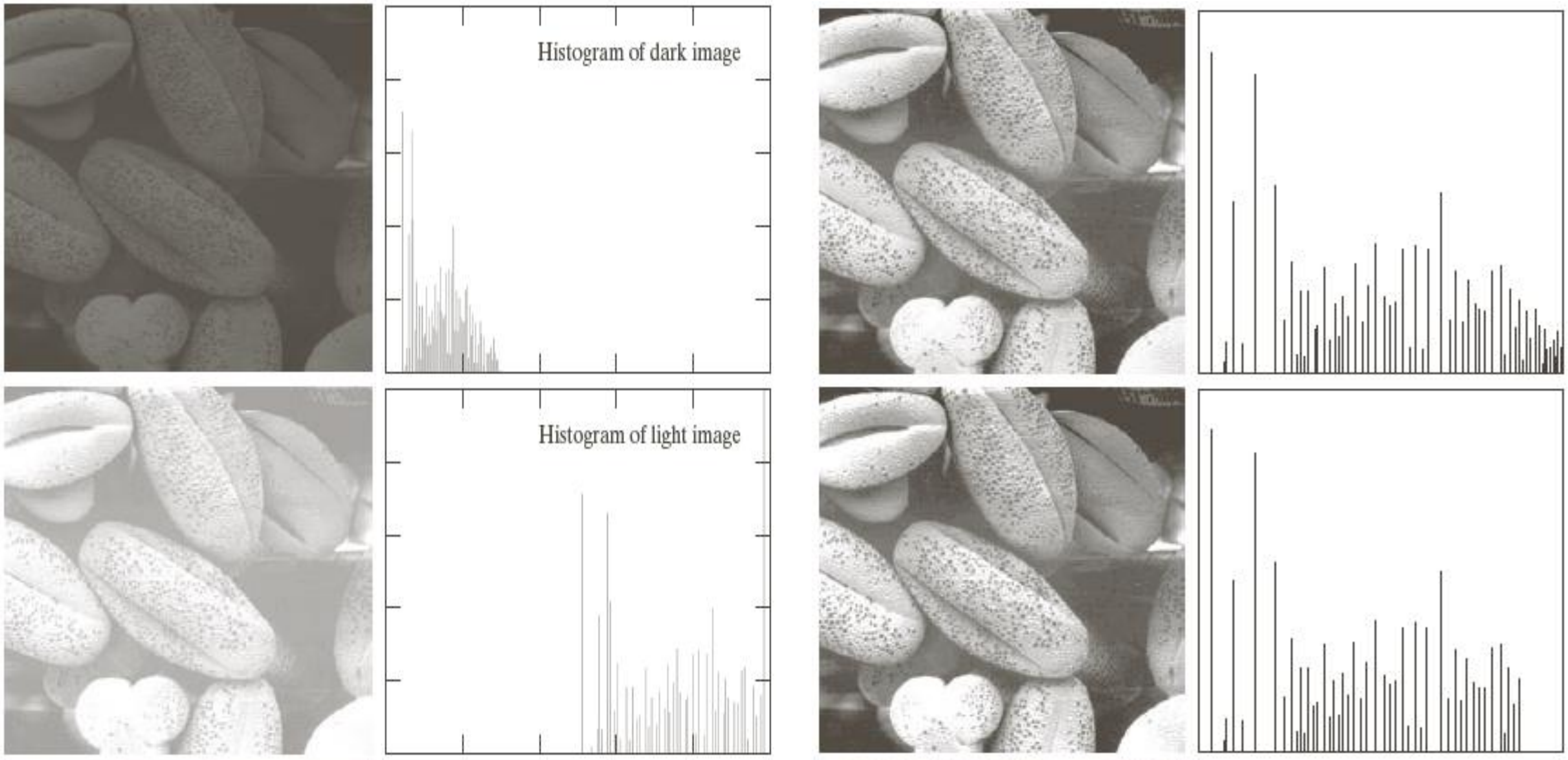
$$V = 0.615R - 0.515G - 0.100B$$

$$R = Y + 1.14V$$

$$G = Y - 0.39U - 0.58V$$

$$B = Y + 2.03U$$

Example



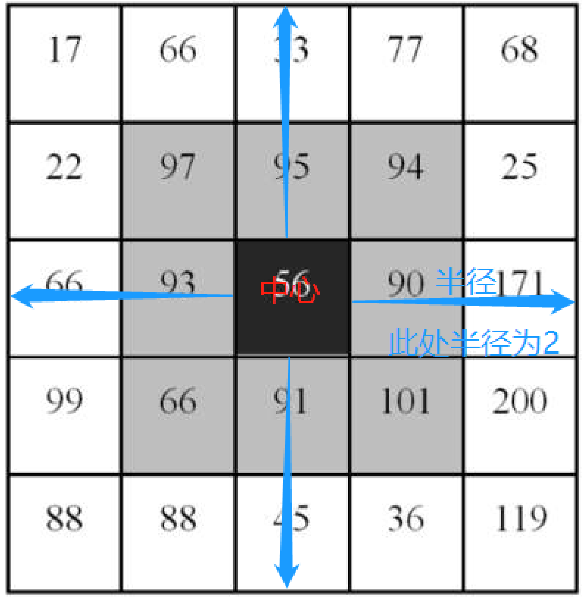
Homework

1、The requirements for implementing blur on image are as follows:

a. Draw a flowchart(Take a photo or post a screenshot to the report)

b. The user can specify the convolution kernel radius, variance (Input interface： input value by user)

c. Please implement the blur kernel by yourself according to the following formula, note that you cannot use the wrapper function in matlab



$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Homework

2、The requirements for implementing Histogram equalization on image are as follows:

a. Realize the histogram equalization function, note that you cannot use the wrapper function in matlab

b. Display pictures before and after equalization in a 1*2 subplot

c. Draw a histogram of brightness before and after equalization in a 1*2 subplot

Homework

3、Observe the code(unknown_funtion.m), find out 3 errors and correct them.

Note:
(1) After decolorizing Image3.jpeg, the relationship between the grayscale image J and the original image RGB is shown in the figure below.
(2)The recovery part of the code restores the grayscale image J to the RGB image T.

I_R	I_G	I_R	I_G	I_R	I_G
I_G	I_B	I_G	I_B	I_G	I_B
I_R	I_G	I_R	I_G	I_R	I_G
I_G	I_B	I_G	I_B	I_G	I_B
I_R	I_G	I_R	I_G	I_R	I_G
I_G	I_B	I_G	I_B	I_G	I_B

Note

- Generate **PDF** file and upload it to gradescope.
- Need to display both code and results.
- Discussions are encouraged and plagiarism is strictly prohibited, source code should not be shared in any form.
- File name: 学号+姓名

Office hour：

10.26(Monday)	18:30-21:30	SIST 1B-107
10.28(Wednesday)	18:30-21:30	SIST 1B-107

Submission: Before 10.30 23:59

If you are late to hand in your homework within one day, your score will discount 50 %. If it is more than one day, you will lose all points for this homework.

函数

- imread()，imshow()
- size
- Input
- imfilter
- ones
- zeros