Week 3
The Concept of Filtering

A DJ has to be familiar with signal processing！

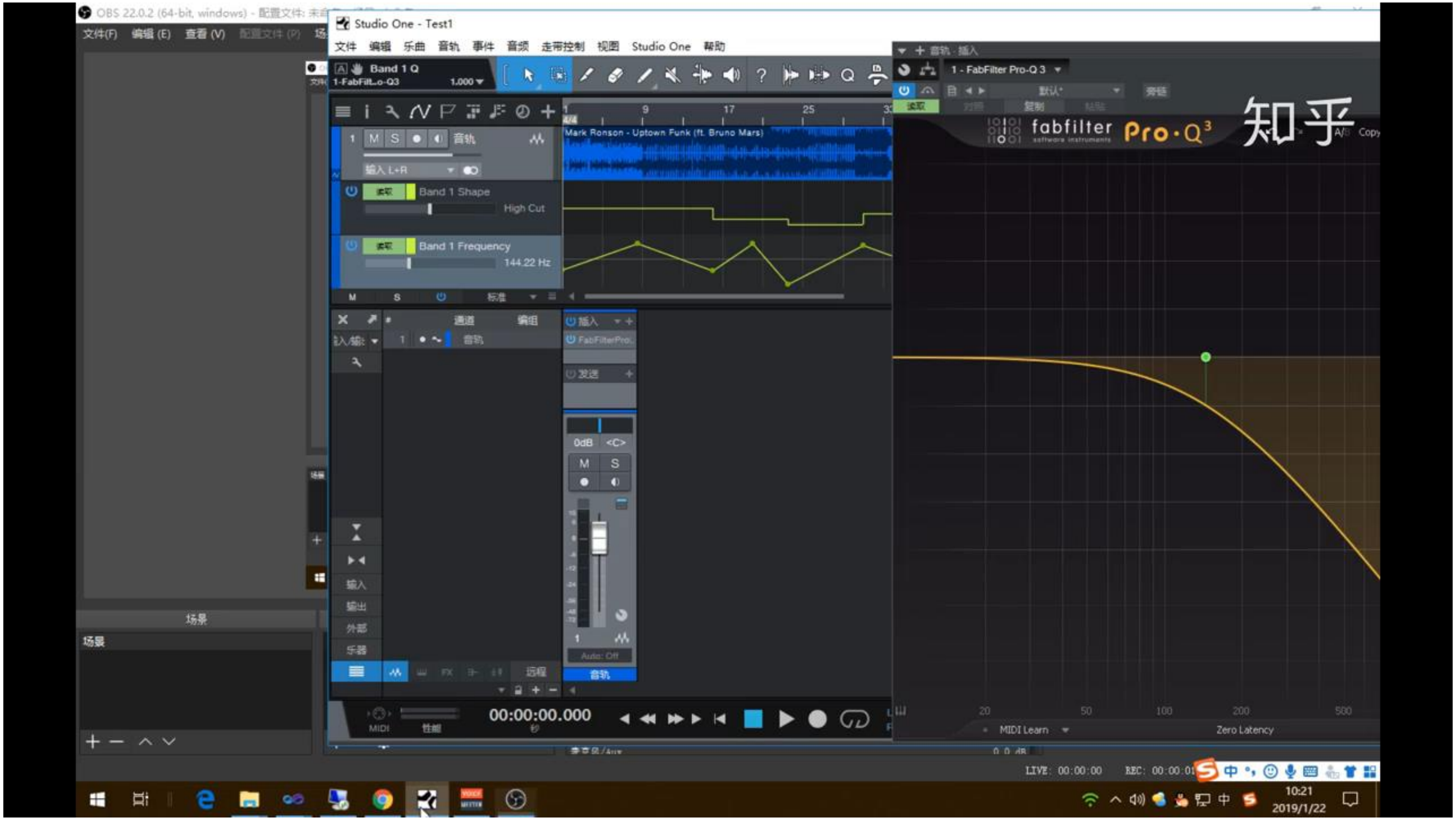
Equalizer

To adjust the balance of frequency components.

上海科技大学
ShanghaiTech University

How Equalizer Works?

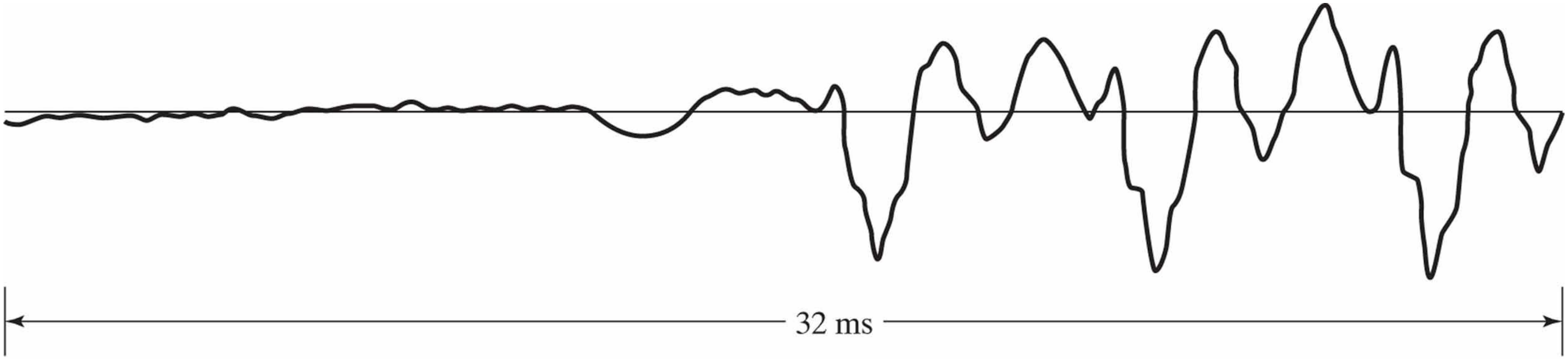
- Based on filters or filter banks



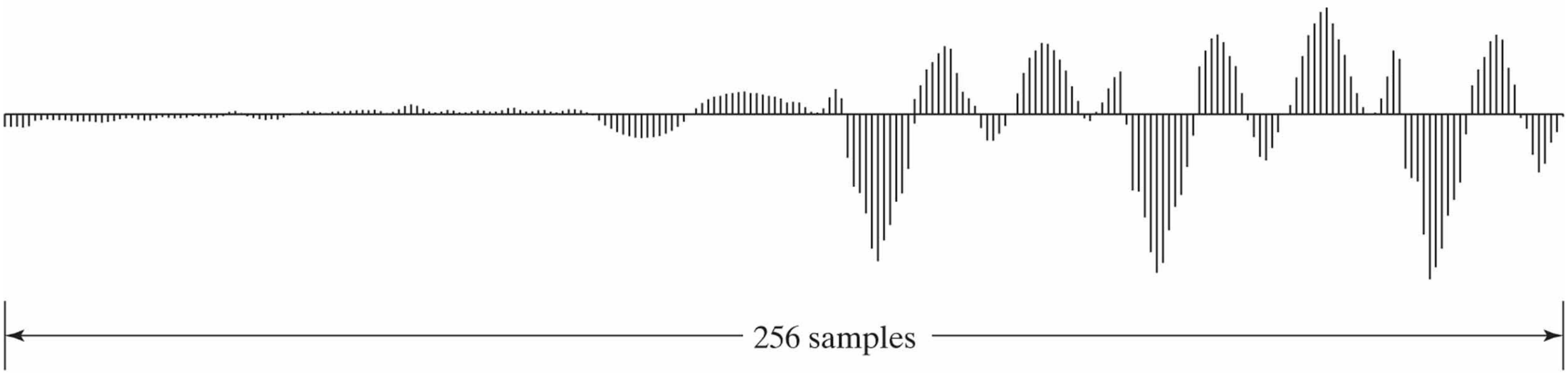
https://zhuanlan.zhihu.com/p/55543887

Some basic definitions

Discrete Time (DT) Signal



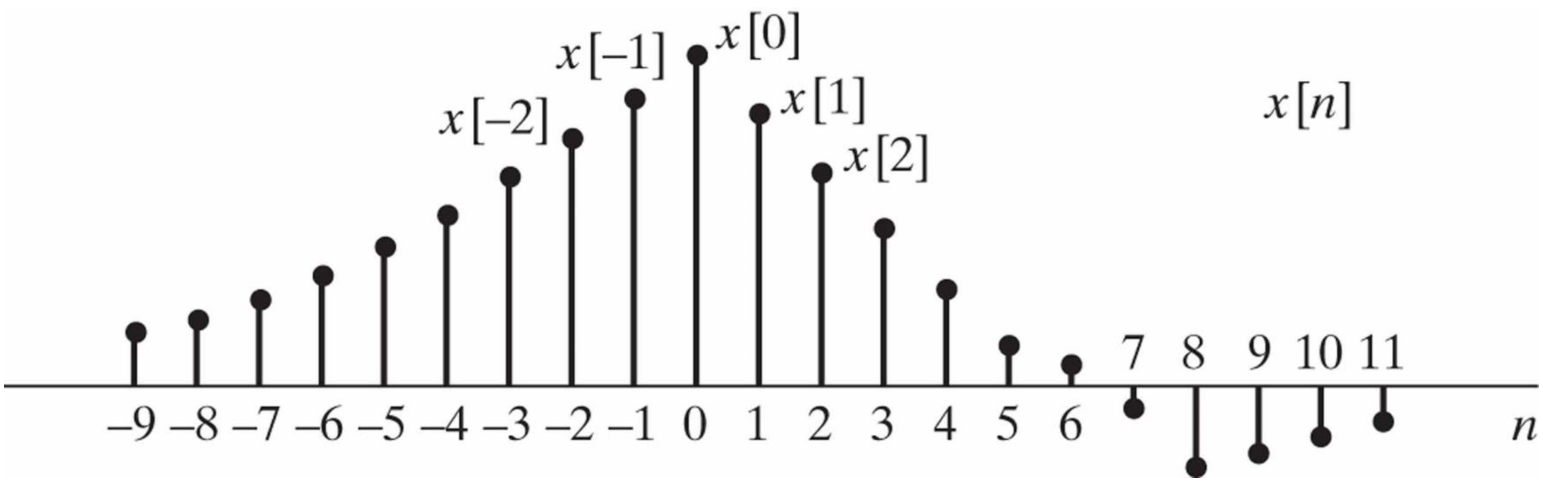
(a)



(b)

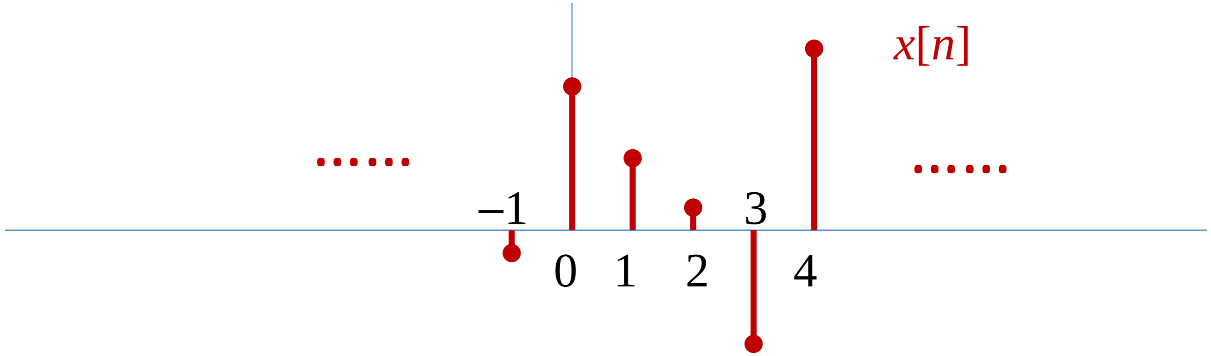
DT Signal

- Graphical representation of a discrete-time signal with real-valued samples



DT Signal

- Signals represented as sequences of numbers, called samples
- Sample value of a typical signal is denoted by $x[n]$ with n being an integer
- $x[n]$ is called the n^{th} sample of the sequence

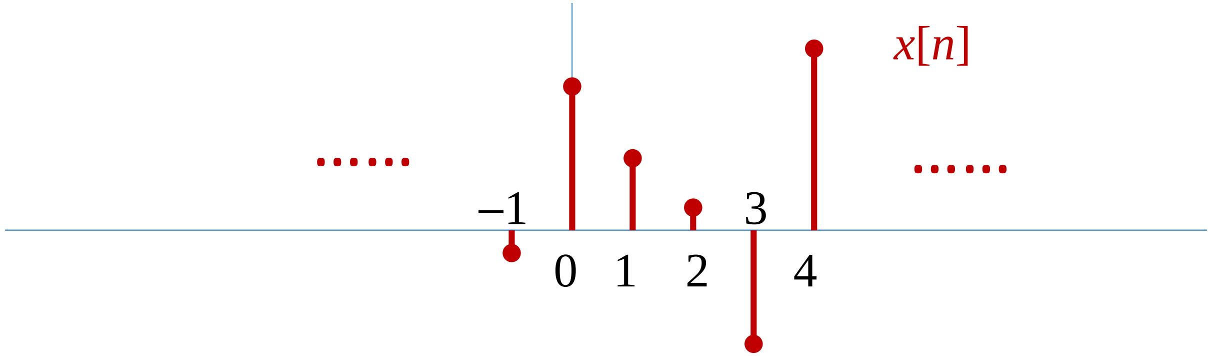


上海科技大学 ShanghaiTech University

DT Signal

- DT signals are defined only for integer values of n and undefined for non-integer values of n
- DT signals may also be written as a sequence of numbers inside braces

$$\{x[n]\} = \{\ldots, -0.2, 2.2, 1.1, 0.2, -1.9, 2.9, \ldots\}$$

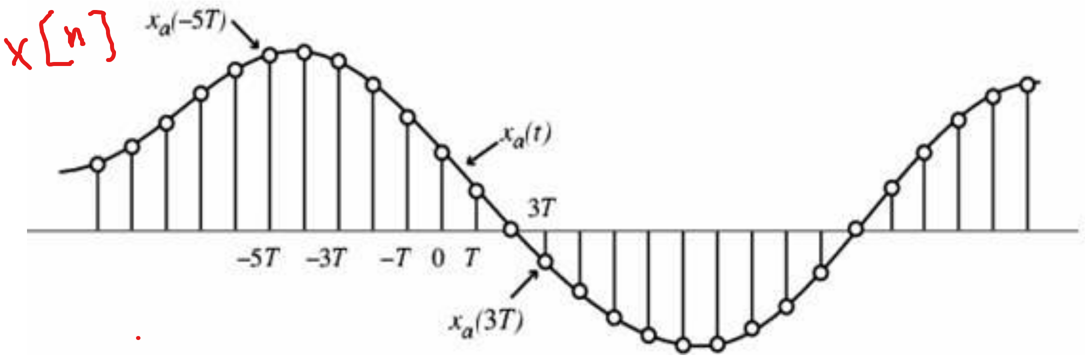


DT Signal

- Samples of a continuous-time signal

$$x[n] = x_a(nT),\ n = \ldots, -1, 0, 1, 2, \ldots$$

一段信号记作 x[n]

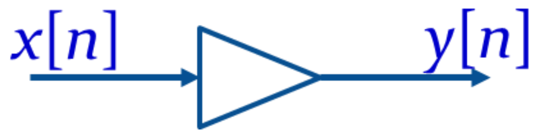


- The spacing T between two consecutive samples is called the sampling interval or sampling period
- Reciprocal of sampling interval T_s, denoted as f_s , is called the sampling frequency:

$$f_s = 1/T_s$$

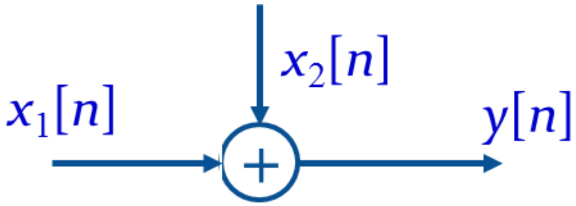
Elementary Operations

- Multiplication operation:
  - Multiplier



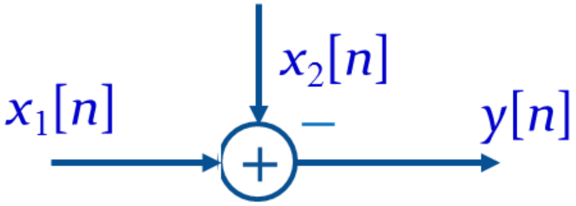
$$y[n] = \alpha x[n]$$

- Addition operation:
  - Adder



$$y[n] = x_1[n] + x_2[n]$$

- Subtraction operation:
  - Subtractor



$$y[n] = x_1[n] - x_2[n]$$

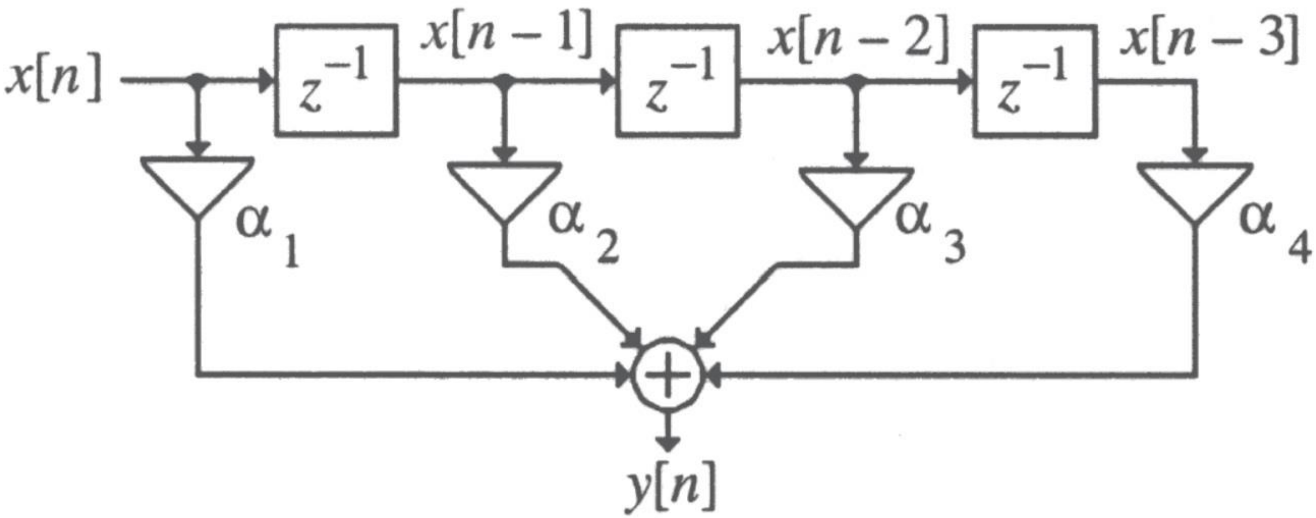
Elementary Operations

- Time-shifting operation: $y[n] = x[n - n_0]$, where n_0 is an integer
- If $n_0 > 0$, it is delaying operation
  - Unit delay

$y[n] = x[n - 1]$

Combinations of Basic Operations

- Example



$$y[n] = \alpha_1 x[n] + \alpha_2 x[n-1] + \alpha_3 x[n-2] + \alpha_4 x[n-3]$$

Relationship Between Frequencies

- The frequency we familiar with f, *in Hertz or Hz*
- For a signal with period T, we have

$$f = 1/T$$

- Angular frequency

$$\Omega = 2\pi f$$

- Digital frequency

$$\omega = 2\pi f / f_s$$

Fs is the sampling frequency

Relationship Between Frequencies

- Sampling frequency or f_s is the bridge between analog frequency and digital frequency

$$\omega = 2\pi f / f_s$$

$$f_s \rightarrow 2\pi$$

A Quick Example

- If $f_s = 44.1\text{K}$ $\omega = 2\pi f / f_s$

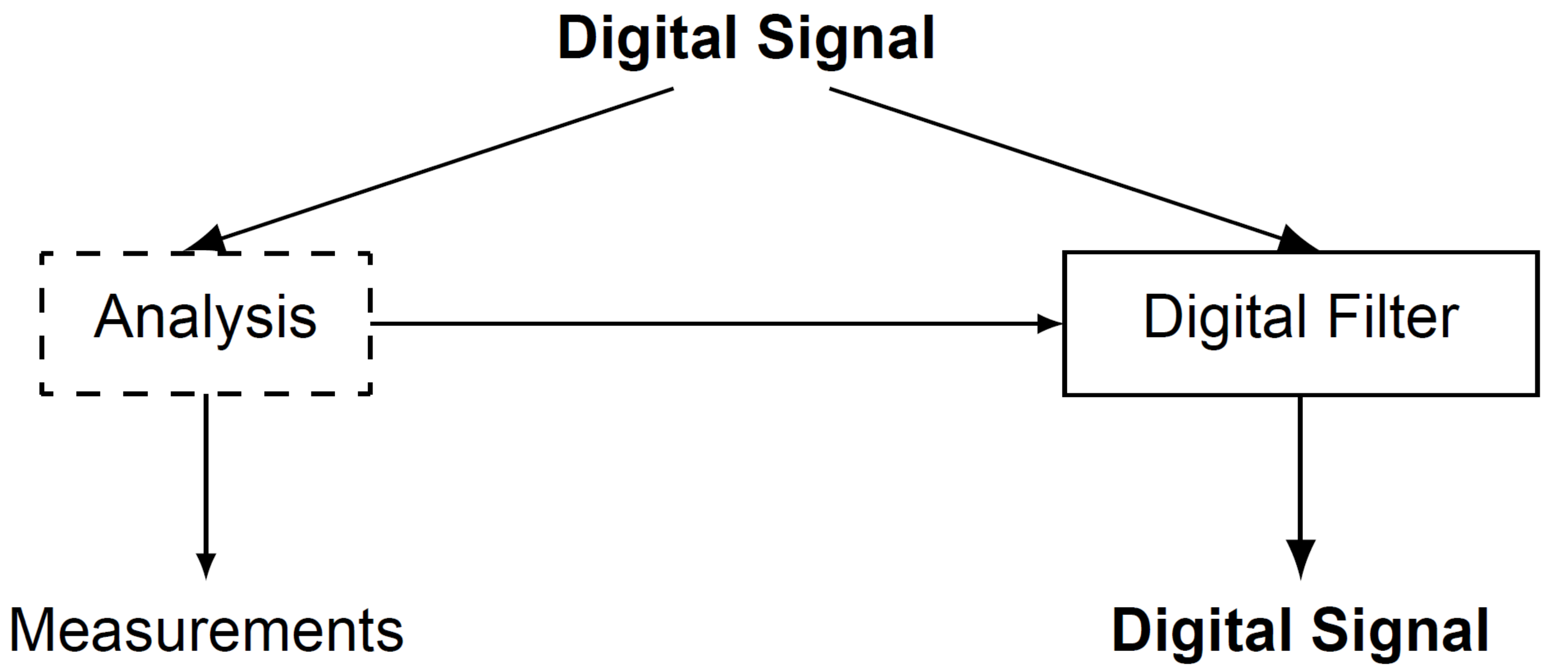
Frequency (Hz)	1209	1336	1477
697	1	2	3
770	2	5	6
852	7	8	9
941	*	0	#

The two digital frequencies of 3 are 0.0316π and $0.0670\,\pi$

The Concept of Filtering

The Objective of Signal Processing

- The objective of signal processing

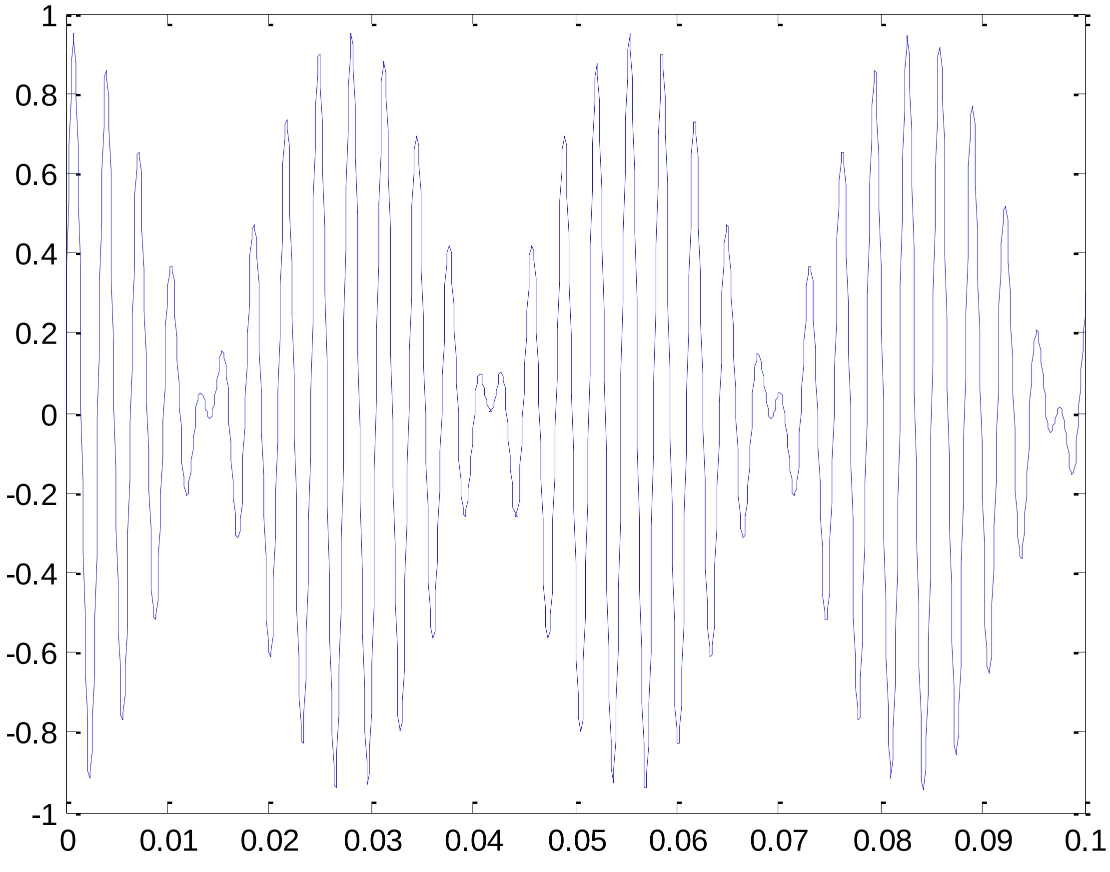


The Concept of Filtering

- To pass certain frequency components in an input signal without any distortion (is possible) and to block other frequency components

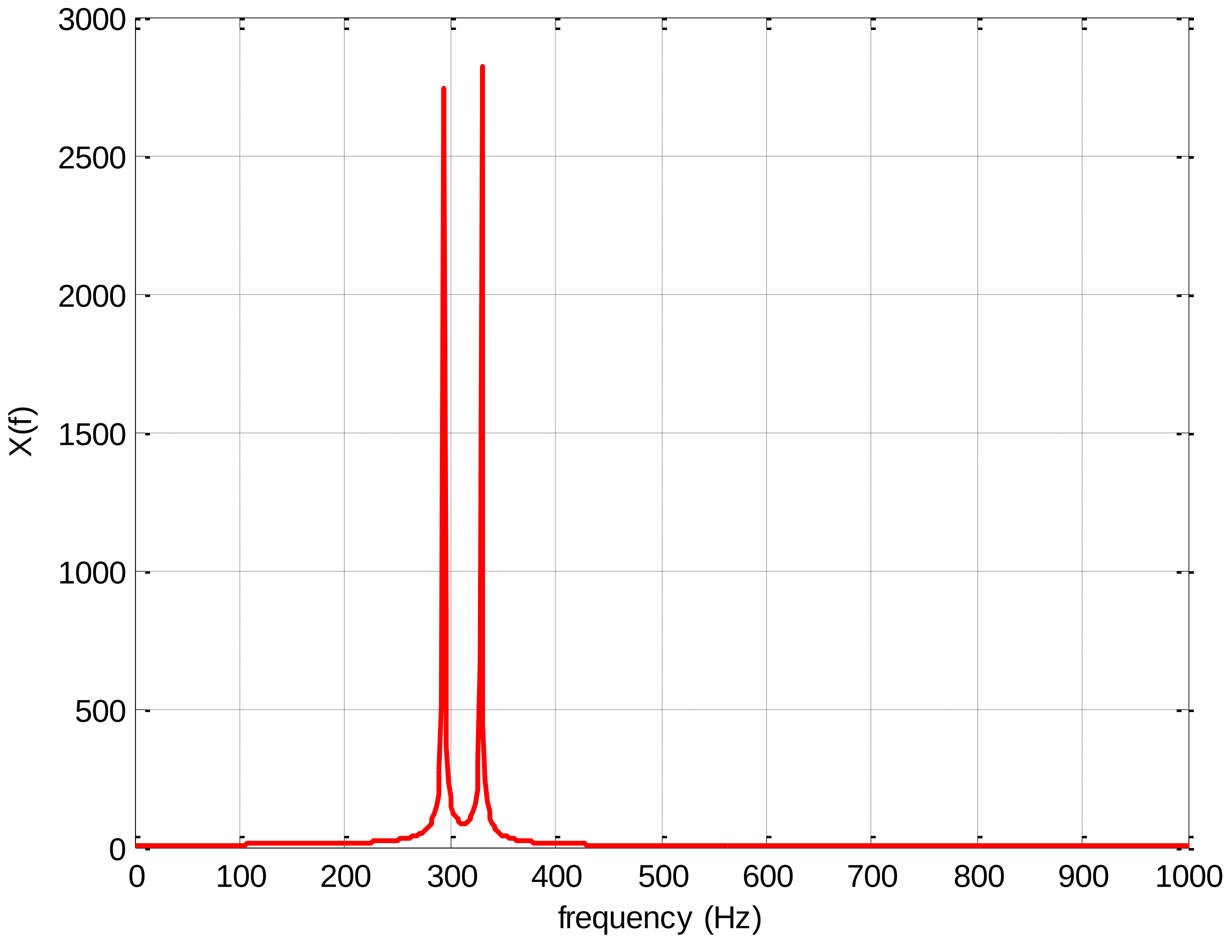
Back to Where We Begin

- Time domain



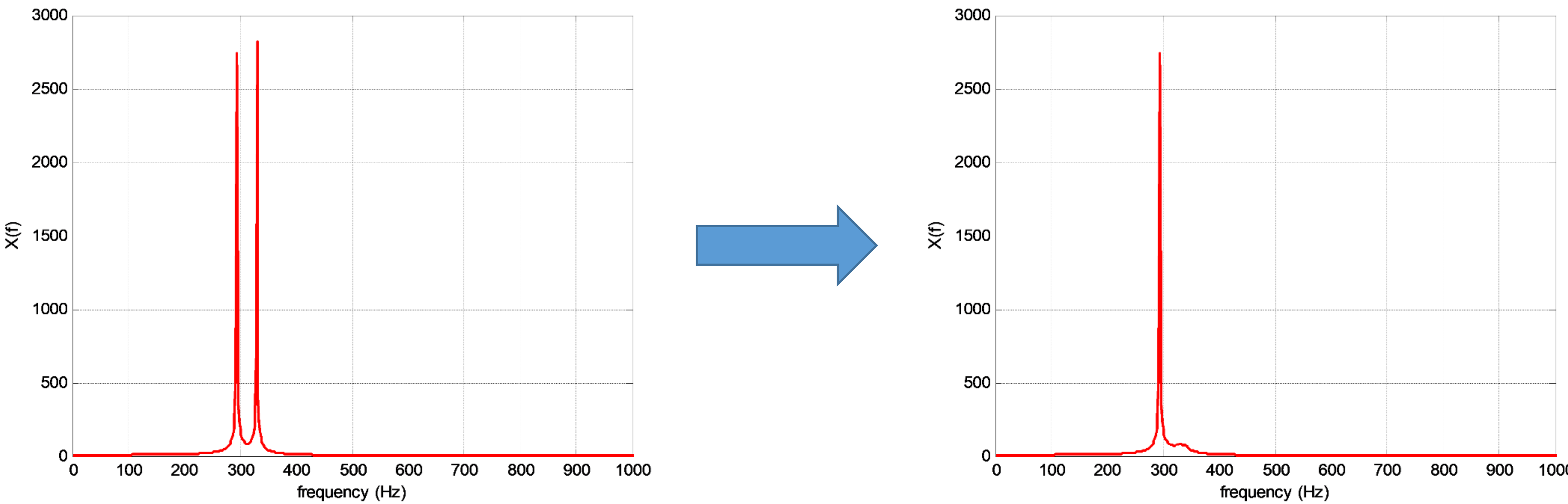
Back to Where We Begin

- Frequency domain



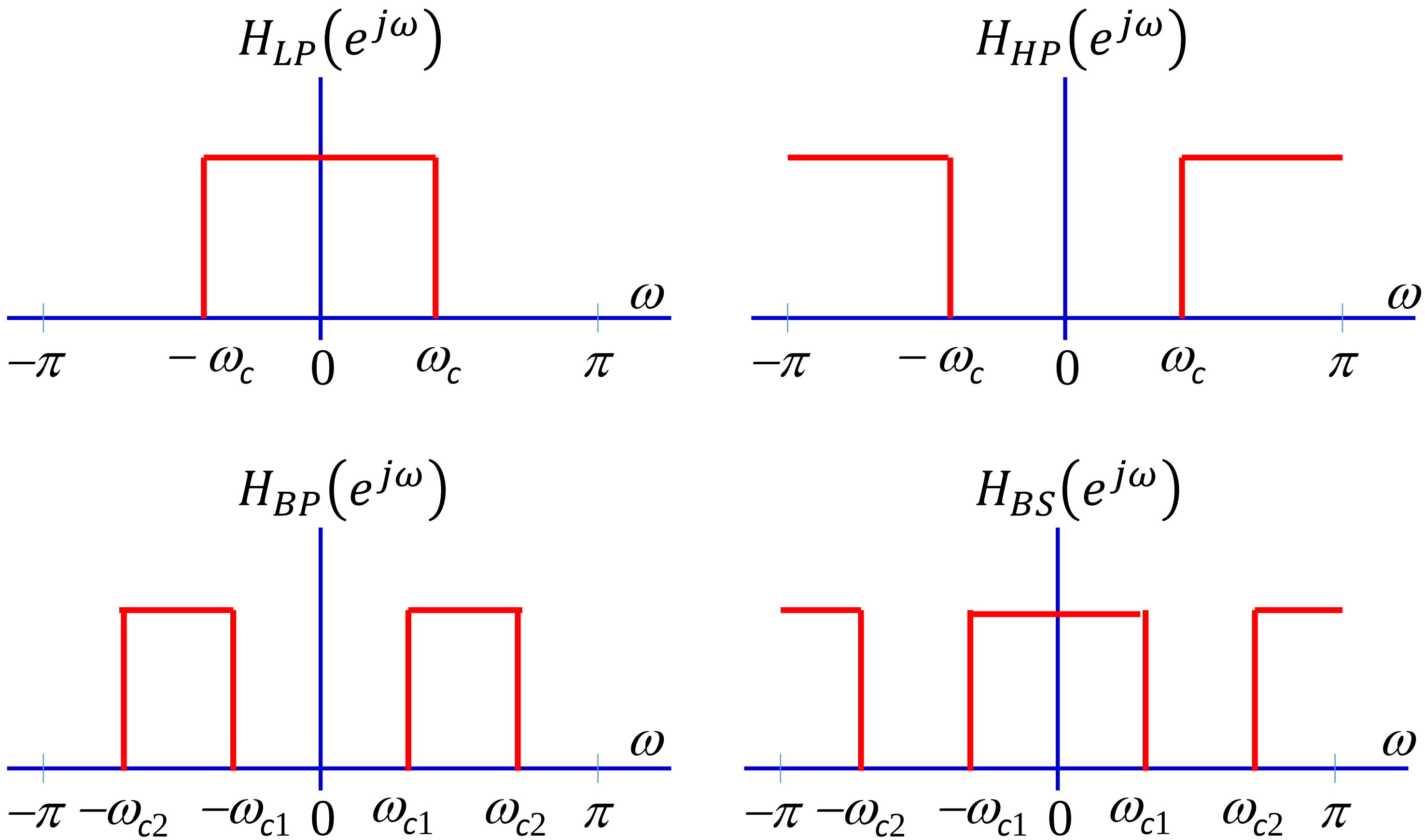
Back to Where We Begin

- Frequency domain



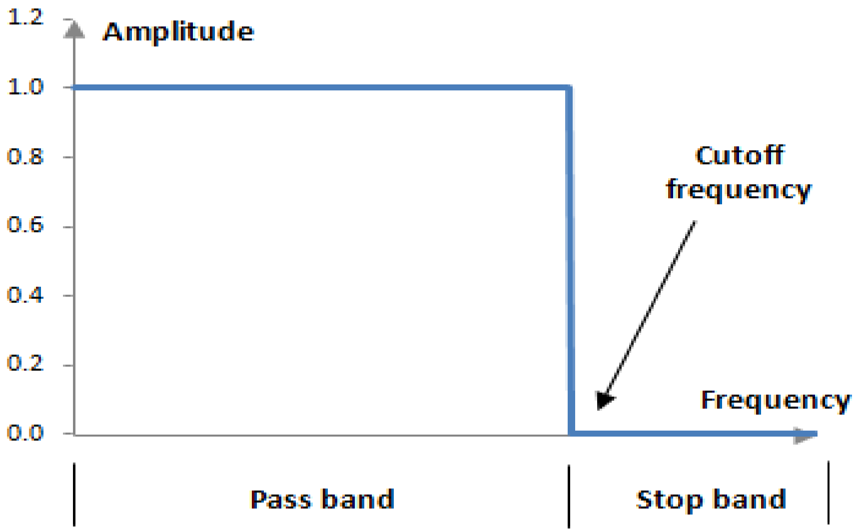
Magnitude Characteristics

- Digital Filter with Ideal Magnitude Responses

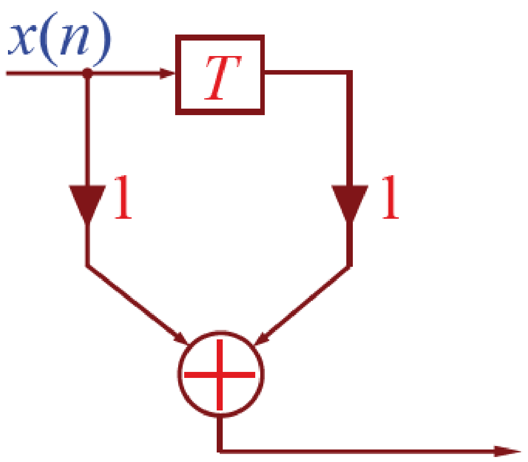


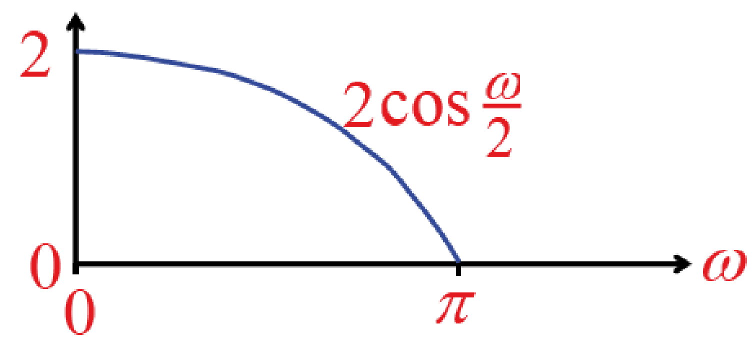
Passband and Stopband

- ❑ Passband
  - ➢The range of frequencies that is allowed to pass through the filter
- ❑ Stopband
  - ➢the range of frequencies that is blocked by the filter



Simple Examples

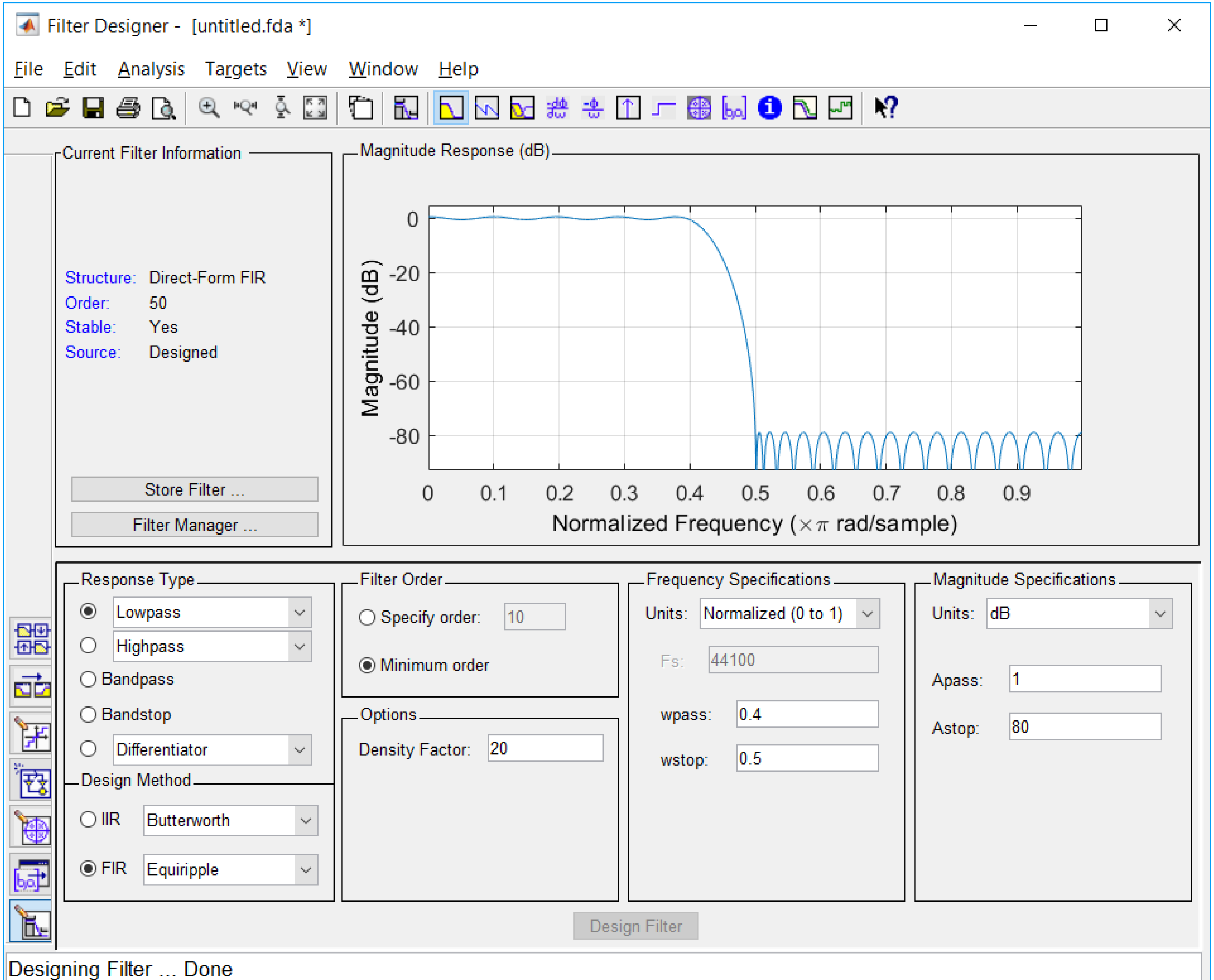




Filter Design Tool

- A tool to play with
  - The *filterDesigner* (*fdatool* for old versions) in Matlab

- How to use?
  - Just type *filterDesigner* in the Command Window



Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 50
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
0
-20
-40
-60
-80
Magnitude (dB)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Density Factor: 20
Frequency Specifications
Units: Normalized (0 to 1)
Fs: 44100
wpass: 0.4
wstop: 0.5
Magnitude Specifications
Units: dB
Apass: 1
Astop: 80
Design Filter
Designing Filter ... Done

Output of Filter Design

- A filter design process is to determine the filter coefficients

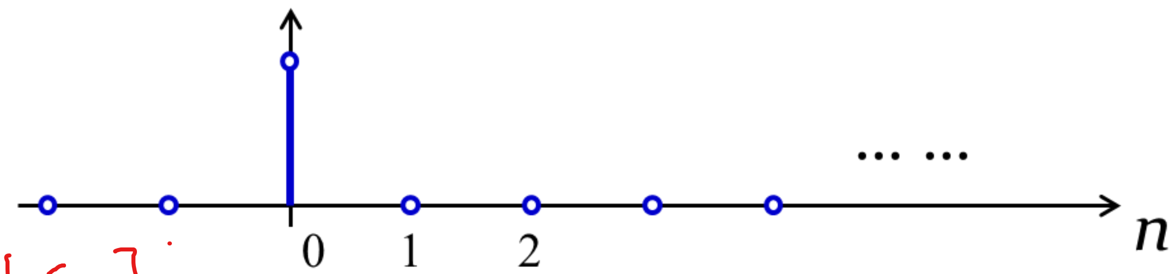
```
Num =

  Columns 1 through 14

   -0.0009   -0.0027   -0.0025    0.0037    0.0137    0.0174    0.0077   -0.0066   -0.0077    0.0061    0.0139    0.0004   -0.0169   -0.0089

  Columns 15 through 28

    0.0174    0.0207   -0.0123   -0.0342   -0.0010    0.0478    0.0274   -0.0594   -0.0823    0.0672    0.3100    0.4300    0.3100    0.0672

  Columns 29 through 42

   -0.0823   -0.0594    0.0274    0.0478   -0.0010   -0.0342   -0.0123    0.0207    0.0174   -0.0089   -0.0169    0.0004    0.0139    0.0061

  Columns 43 through 51

   -0.0077   -0.0066    0.0077    0.0174    0.0137    0.0037   -0.0025   -0.0027   -0.0009
```

Filter coefficients are also called the impulse response of a filter

The Unit Impulse and Impulse Response

- Unit impulse 单位冲击$\delta[n]$

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



对单位冲击的响应$h[n]$

- Impulse response: the response of a system to a unit impulse sequence

类似一种变换?

Why Impulse Response Matters

- It is the “DNA” of Linear Time-invariant systems

Linearity 线性

- Linearity:

If $y_1[n] = T\{x_1[n]\}$, and $y_2[n] = T\{x_2[n]\}$

  - Superposition:

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$$

  - Homogeneity:

$$T\{ax_1[n]\} = aT\{x_1[n]\} = ay_1[n]$$

Overall: $T\{a_1x_1[n] + a_2x_2[n]\} = a_1y_1[n] + a_2y_2[n]$

Time Invariance 时不变

- Time invariance:

  If: $y[n] = T\{x[n]\}$

  Then: $y[n-n_0] = T\{x[n-n_0]\}$ for all integer n_0

- For a specified input, the output is independent of the time the input is being applied

Output of LTI Systems

线性时不变系统

- Since the system is time-invariant, we have

① 系统是线性时不变的
② 给定单位冲击响应
⇒ 则能得到确定输出

Input		**Output**
$\delta[n+2]$	$\rightarrow$	$h[$
$\delta[n-1]$	$\rightarrow$	$h[$
$\delta[n-2]$	$\rightarrow$	$h[$
$\delta[n-5]$	$\rightarrow$	$h[$

Output of LTI Systems

- Since the system is linear, we have

Input		Output
$0.5\delta[n+2]$	$\rightarrow$	$0.5h[n+2]$
$1.5\delta[n-1]$	$\rightarrow$	
$\delta[n-2]$	$\rightarrow$	
$0.75\delta[n-5]$	$\rightarrow$	

- According to the superposition property, we get

$$y[n] = 0.5h[n+2] + 1.5h[n-1] - h[n-2] + 0.75h[n-5]$$

Output of LTI Systems

- The impulse response $h[n]$ completely characterizes an LTI system

"DNA of LTI"

$\otimes$ Convolution

$x[n] \otimes h[n]$

即 $x[n]$ 对 $h[n]$ 做卷积

$$\delta[n] \rightarrow \boxed{\text{LTI}} \rightarrow h[n]$$

$$\delta[n-k] \rightarrow \boxed{\text{LTI}} \rightarrow h[n-k]$$

$$x[k]\delta[n-k] \rightarrow \boxed{\text{LTI}} \rightarrow x[k]h[n-k]$$

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Output of LTI Systems

- Compute the output of an LTI system using $h[n]$ for the input:

$$x[n] = 0.5\delta[n+2] + 1.5\delta[n-1] - \delta[n-2] - 0.75\delta[n-5]$$