[INFO-F409] Learning Dynamics

Assignment 1: Game theory

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<u>The Hawk-Dove game (3 pts):</u>

1. Find all the (mixed strategy) Nash equilibria of this game. How do the results change when the order of the parameters V, D and T is changed (V>D, D>T, etc.)?

		Player 2	
		Escalate (Hawk)	Display (Dove)
Player 1	Escalate (Hawk)	(V-D)/2 (V-D)/2	0 V
	Display (Dove)	V 0	V/2-T V/2-T

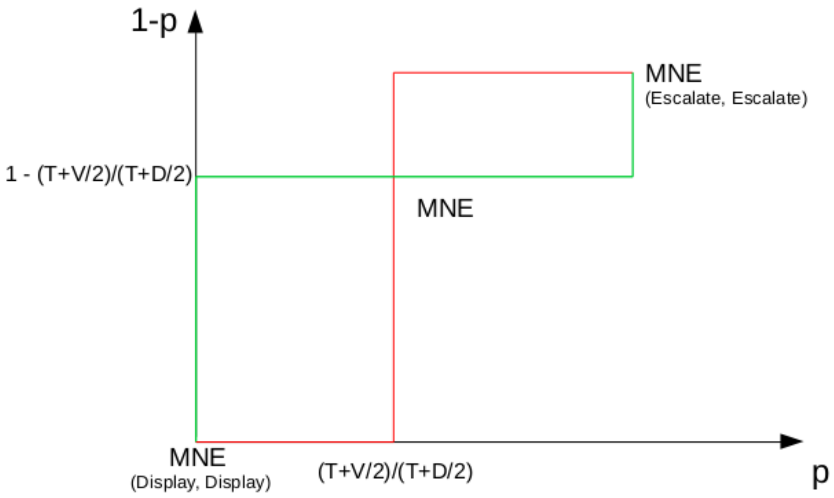
We know that:
- D represents fitness costs of injury, it can be a positive or negative number
- $V \geq 0$ because V represents the winning resources
- $T \geq 0$ because it's a wasting time

<u>If V > D:</u>
If player 2 escalates (Hawk), the best response for player 1 is to escalate because $(V-D)/2 > 0$.
If player 2 displays (Dove), the best response for player 1 is to escalate because $V \geq V/2-T$.
This game is symmetric, it means that it's the same responses when we switch the players.
To conclude, if V > D, «Escalate» strictly dominates «Display», (Escalate, Escalate) is the strict/pure Nash Equilibrium, it corresponds to {(1,0);(1,0)}.

<u>If V < D:</u>
If player 2 escalates (Hawk), the best response for player 1 is to display because $0 > (V-D)/2$.
If player 2 displays (Dove), the best response for player 1 is to escalate because $V \geq V/2-T$.
This game is symmetric, it means that it's the same responses when we switch the players.
To conclude, if V < D, we have two strict/pure Nash Equilibria: (Escalate, Display) and (Display, Escalate), it corresponds to {(1,0);(0,1)} and {(0,1);(1,0)}.
When (Escalate,Escalate) is not a Pure Nash equilibrium (in this case when V< D) and when both actions have the same utility, the player can use a mixed strategy, so we have a Mixed Nash Equilibrium corresponding to {(p, 1-p); (1-p, p)} .We must now determine the probability of escalate and the probability of display for a mixed strategy Nash Equilibrium. So, we have U(H) = U(D) with U(H) the utility of Escalate and U(D) the utility of Display.
The probability of escalate is p and 1-p is the probability to display.
$p(V - D)/2 + (1 - p)V = p * 0 + (1 - p)(V/2 - T)$
$p = (T+V/2)/(T+D/2)$
It's the same for both players because the payoffs matrix is symmetric.
If the value of T is high, p will tend towards 1 and if the value of T is low, p will tend towards V/D.

2. Under which conditions does displaying become more beneficial than escalating? Draw the set of all mixed strategies.

When T increase, it increase the chance to escalate, we can determine a thresholds with the inequality of the payoff:
$(V-D)/2 < V/2-T$
$T < D/2$
When $T < D/2$, it's more beneficial for both players to display.



3. Validate your results using NashPy. You may use the example provided in the CGT-Exercise.ipynb. Indicate here the Nash equilibria found for V=2, D=3 and T = 1.

For $V < D$:

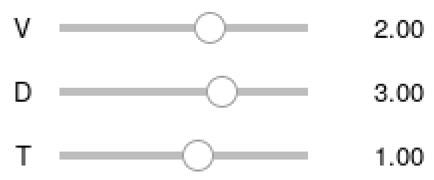
		Player 2	
		Escalate (Hawk)	Display (Dove)
Player 1	Escalate (Hawk)	-1/2 -1/2	0 2
	Display (Dove)	2 0	0 0

Here, we have a Mixed Nash Equilibrium with:
$p = (T+V/2)/(T+D/2) = 4/5 = 0{,}8$ and
$1 - p = 1 - (T+V/2)/(T+D/2) = 1 - 4/5 = 1/5 = 0{,}2$

and we have two strict/pure Nash Equilibria:
- (Hawk, Dove) = {(1,0);(0,1)}
- (Dove, Hawk) = {(0,1);(1,0)}

That's what we obtain with Nashpy:

```python
import nashpy as nash
import numpy as np
from ipywidgets import interact, FloatSlider

def hawk_dove_game_equilibria(V, D, T):
    """
    This function returns the Nash equilibria of
    a Hawk-Dove game with its payoff matrix being a
    function of V, D and T.
    """
    # payoff matrix for the row player
    hg_matrix_row = np.array([
        [ (V-D)/2, V],
        [ 0      , (V/2) - T],
    ])
    # payoff matrix for the column player
    hg_matrix_col = hg_matrix_row.T

    # Create game
    game = nash.Game(hg_matrix_row, hg_matrix_col)

    # Find all equilibria
    return list(game.support_enumeration())

@interact(V=FloatSlider(min=-10, max=10, step=0.1, value=2),
          D=FloatSlider(min=-10, max=10, step=0.1, value=3),
          T=FloatSlider(min=-10, max=10, step=0.1, value=1))
def show_articles_more_than(V=2, D=3, T=1):
    return hawk_dove_game_equilibria(V, D, T)



#hawk_dove_game_equilibria(2,3,1)
```




```
[(array([1., 0.]), array([0., 1.])),
 (array([0., 1.]), array([1., 0.])),
 (array([0.8, 0.2]), array([0.8, 0.2]))]
```

<u>Which social dilemma? (3 pts)</u>

Player A knows he’s confronted with one of three social dilemma’s; a prisoner’s dilemma, a snowdrift game or stag-hunt game (see above). In each game he needs to decide whether to cooperate (C) or defect (D), yet he is not sure in which he actually is. He’s sure that each game is equally likely. The other player, player B, knows in which game he’s playing. Determine the pure Nash equilibria using the Bayesian game analysis discussed in the course.

Prisonners dilemma

	C	D
C	2,2	0,5
D	5,0	1,1

Stag-Hunt game

	C	D
C	5,5	0,2
D	2,0	1,1

Snowdrift game

	C	D
C	2,2	1,5
D	5,1	0,0

Payoff matrix for the 3 games

The two best responses for each game are:

Prisonners dilemma

	C	D
C	2	5
D	0	1

Stag-Hunt game

	C	D
C	5	2
D	0	1

Snowdrift game

	C	D
C	2	5
D	1	0

We compute the player A matrix for all games:

C[C,C,C] = 1/3*2+1/3*5+1/3*2 =3
C[D,D,D] = 1/3*0+1/3*0+1/3*1 = 1/3
C[C,C,D] = 1/3*2+1/3*5+1/3*1 = 8/3
C[D,D,C] = 1/3*0+1/3*0+1/3*2 = 2/3
C[C,D,D] = 1/3*2+1/3*0+1/3*1 = 1
C[D,C,C] = 1/3*0+1/3*5+1/3*2 = 7/3
C[C,D,C] = 1/3*2+1/3*0+1/3*2 = 4/3
C[D,C,D] = 1/3*0+1/3*5+1/3*1 = 2
D[C,C,C] = 1/3*5+1/3*2+1/3*5 = 4
D[D,D,D] = 1/3*1+1/3*1+1/3*0 = 2/3
D[C,C,D] = 1/3*5+1/3*2+1/3*0 = 7/3
D[D,D,C] = 1/3*1+1/3*1+1/3*5 = 7/3
D[C,D,D] = 1/3*5+1/3*1+1/3*0 = 2
D[D,C,C] = 1/3*1+1/3*2+1/3*5 = 8/3
D[C,D,C] = 1/3*5+1/3*1+1/3*5 = 11/3
D[D,C,D] = 1/3*1+1/3*2+1/3*0 = 1

	CCC	DDD	CCD	DDC	CDD	DCC	CDC	DCD
C	3	1/3	8/3	2/3	1	7/3	4/3	2
D	4	2/3	7/3	7/3	2	8/3	11/3	1

Player A matrix for all games

We can now find Nash equilibria by matching the best responses between the two matrices.
Finally, we have C[D,C,D] and D[D,D,C] which are the Nash Equilibria.

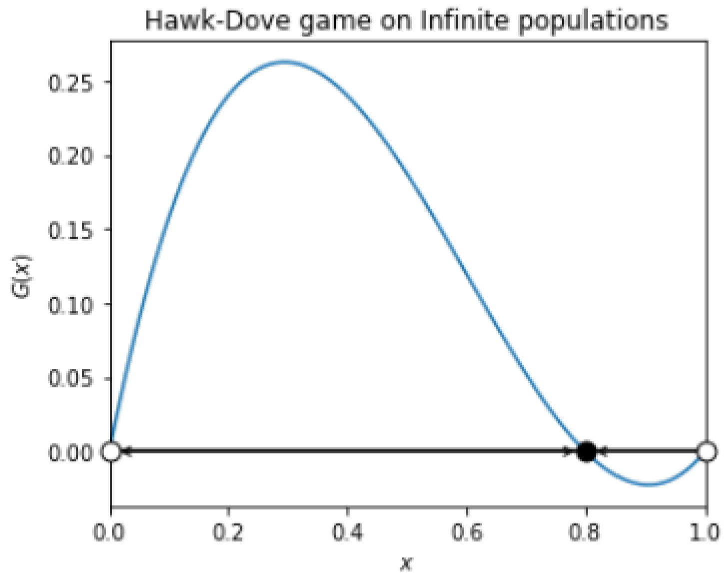
Evolutionary Dynamics in the Hawk-Dove game (4 pts)

For this exercise, you will need to use the code provided to you, and follow the CGT-Exercise.ipynb.Follow the instructions indicated in the notebook, and provide answers to the questions stated here. Youare going to study the evolutionary dynamics of the Hawk-Dove game both in infinite and finite populations.

1. Look at the plot of the gradient of selection for infinite populations, explain which saddle points are stable and which aren't, and why. Do the results here agree with those found in Exercise 1? Do you expect any changes if the population is finite?

```
import numpy as np
import matplotlib.pyplot as plt
from egttools.analytical import replicator_equation
from egttools.utils import find_saddle_type_and_gradient_direction
from egttools.plotting import plot_gradient


nb_points = 101
strategy_i = np.linspace(0, 1, num=nb_points, dtype=np.float64)
strategy_j = 1 - strategy_i
states = np.array((strategy_i,strategy_j)).T

# Payoff matrix
V = 2; D = 3; T = 1
A = np.array([
        [ (V-D)/2, V],
        [ 0      , (V/2) - T],
    ])



# Calculate gradient
G = np.array([replicator_equation(states[i], A)[0] for i in range(len(states))])



# Find saddle points (where the gradient is 0)
epsilon = 1e-7
saddle_points_idx = np.where((G <= epsilon) & (G >= -epsilon))[0]
saddle_points = saddle_points_idx / (nb_points - 1)

# Now let's find which saddle points are absorbing/stable and which aren't
# we also annotate the gradient's direction among saddle poinst
saddle_type, gradient_direction = find_saddle_type_and_gradient_direction(G,
 saddle_points_idx)
```

```



ax = plot_gradient(strategy_i,
                   G,
                   saddle_points,
                   saddle_type,
                   gradient_direction,
                   'Hawk-Dove game on Infinite populations',
                   xlabel='$x$')
plt.show()
```




```
print('saddle_points:', saddle_points)
```

```
saddle_points: [0.  0.8 1. ]
```

	HAWK	DOVE
HAWK	A	B
DOVE	C	D

This Saddle point 0 is unstable. It corresponds to the pure dove strategy for the whole population.

B >D , a Hawk player can invade a population of Dove players.

This Saddle 1 point is unstable. It is the point corresponding to the pure Hawk strategy for the whole population. C>A , a Dove player can invade a population of Hawk players.

This Saddle point 0,8 is stable. This point corresponds to 4/5 of the population being Hawk players and 1/5 being Dove players.

The results agree with those found in Exercise 1. Each of the two unstable saddle point corresponds to a Nash Equilibrium and the stable saddle point corresponds to the mixed strategy Nash equilibrium.
If the population is finite, we expect changes.

2. For finite populations, how are the dynamics affected when changing:
a. Population size (Z)
The greater the population size, the smaller random drift. The distribution will be narrower.

b. Intensity of selection (β)
The greater the β, the greater the selection pressure. The distribution will be narrower.

c. Probability of mutation (μ)
A greater μ will shift the distribution towards the middle.

3. Find the stationary distribution of the Hawk-Dove game for finite populations numerically. Assume that =10and =10!". For this, you will need to

a. Implement the Moran process with pairwise-comparison explained in CGT-Exercise.ipynb (and in the course).