

### Solve the following problems:

1.  $e^{x_1 x_2} \rightarrow \max, x_1^3 + x_2^3 = 1$
2.  $x_1^2 + 12x_1 x_2 + 2x_2^2 \rightarrow \text{extr}, 4x_1^2 + x_2^2 = 25$
3.  $x_1 x_2^2 x_3^3 \rightarrow \text{extr}, x_1^2 + x_2^2 + x_3^2 = 1$

### Puzzle: Re-distribute your Blu-ray collections

You and your friend decide to share your Blu-ray collections for the upcoming christmas holidays. You own 22 crime series Blu-rays, and your friend owns 13 comedy series Blu-rays. By redistributing these over the two of you, each of you will end up with an interesting set of series to watch. Your aim is to redistribute the 35 Blu-rays over the two of you in such a way that the social welfare is maximal. The social welfare here is defined as the sum of the utilities for the both of you. Your utility is  $4\ln(x) + \ln(y)$ , and your friends utility is  $5\ln(x) + 20\ln(y)$ , where  $x$  denotes the number of crime series Blu-rays this person has, and  $y$  denotes the number of comedy series Blu-rays this person has.

1. Which distribution of the Blue-rays gives maximal social welfare?
2. Comment on the fairness of the outcome obtained in 1.
3. Estimate how much this maximal social welfare would increase if you would suddenly obtain another crime series Blu-ray. *We will actually discuss this in the next lecture, but you could try yourself now.*