The model

The following model has been implemented in Julia (see the file `network_planning.jl`):

$$\text{minimize} \quad \sum_{i,p,t}\left(\sum_{s} c_s y_{ispt} + c^{BR} m^{BR}_{ipt} + c^{BO} m^{BO}_{ipt} + c^{PR} m^{PR}_{ipt} + c^{PO} m^{PO}_{ipt} + h_p I_{ipt} + \Pi \delta_{ipt}\right)$$

$$\begin{aligned}
\text{subject to} \quad y_{ispt} &= \pi_{ps} x_{ipt} && \forall i,p,t,s \\
m^{BR}_{ipt} + m^{B0}_{ipt} &= \mu^{B}_{p} x_{ipt} && \forall i,p,t \\
m^{PR}_{ipt} + m^{PO}_{ipt} &= \mu^{P}_{p} x_{ipt} && \forall i,p,t \\
\sum_{p} m^{BR}_{ipt} &\leq 432000 && \forall i,t \\
\sum_{p} m^{PR}_{ipt} &\leq 432000 && \forall i,t \\
\sum_{p} m^{B0}_{ipt} &\leq 86400 && \forall i,t \\
\sum_{p} m^{P0}_{ipt} &\leq 86400 && \forall i,t \\
x_{ipt} + I_{ipt-1} &= I_{ipt} + D_{ipt} - \delta_{ipt} && \forall i,p,t \\
I_{ipt} &\geq ss_{ipt} && \forall i,p,t
\end{aligned}$$

Model sets	
Symbol	Meaning
L	Number of regions
P	Number of products
T	Number of weeks
S	Number of materials
Model variables	
Symbol	Meaning
x_{ipt}	Production quantity at plant i of product p in week t
y_{ispt}	Source quantity at plant i of (preblend/packaging) material s for product p in week t
m^{BR}_{ipt}	Regular time of the blend machine reserved at plant i for product p in week t
m^{PR}_{ipt}	Regular time of the packaging machine reserved at plant i for product p in week t
m^{B0}_{ipt}	Overtime of the blend machine reserved at plant i for product p in week t
m^{P0}_{ipt}	Overtime of the packaging machine reserved at plant i for product p in week t
I_{ipt}	Inventory level at warehouse i of product p at the end of week t
δ_{ipt}	Amount of demand at warehouse i of product p in week t that could not be satisfied
Model parameters	
Symbol	Meaning
π_{ps}	Amount of (preblend/packaging) material s required to produce one unit of product p
μ^{B}_{p}	Seconds of blending machine time needed to produce one unit of product p
μ^{P}_{p}	Seconds of packaging machine time needed to produce one unit of product p
c_s	Cost per unit of material s sourced
c^{BR}	Blending machine regular time cost (per second)
c^{PR}	Packaging machine regular time cost (per second)
c^{BO}	Blending machine overtime cost (per second)
c^{PO}	Packaging machine overtime cost (per second)
h_p	Cost of carrying one unit of inventory of product p over from one week to the next
I_{ip0}	Inventory level at plant i of product p in the beginning of week 1
D_{ipt}	Forecasted demand at warehouse i of product p in week t
ss_{ipt}	Safety stock required at warehouse i of product p at the end of week t
Π	Penalty per unit of demand not satisfied

KPIs

In addition to total cost, we will also consider the following KPIs concerning the production plan:

- *Utilization:* The % of available regular time a machine is producing in regular time
- *Overtime:* The % of available overtime a machine is in producing in overtime
- *Inventory:* The average inventory carried over from one week to the next
- *Fill rate:* The % of forecasted demand that is satisfied by the production plan

The KPIs should be calculated separately for each region. In addition, the last two should be calculated separately for each product. In other words, all KPIs should be averaged over time only.

Task 1: Initial analysis

Use the model implementation in Julia to produce a production plan for the next four weeks, and answer the following questions:

1. What are the KPI values for this plan?
2. What will the buffer production be in the next four weeks? The *buffer production* refers to units produced for the sole purpose of serving as safety stock.

Task 2: Promotion planning

Marketing would like to run some promotions for Product 3 in all three regions in the upcoming weeks, but needs to first check if there is enough capacity for the expected increase in demand. They would like to run the promotion in one of the first two weeks of the planning horizon, and not any later, and they suggest that the production plan should produce 30% more than the demand forecast to cover units needed for the promotion and the expected increase in sales in that week. The promotions do not need to run in the same week in different regions.

Adapt the model and the Julia implementation to suggest the best time (week 1 or week 2) to run the promotion in each of the three regions. Note that if the promotion runs in week t, then the demand for that week should be increased by 30%. In other words, the new demand forecast should be $1.3 \times D_{i3t}$. The demand forecasts for other weeks stay the same.

Use the model to create a new production plan with promotions, and answer the following questions:

1. When should the promotions be run? Why do you think the model chose these particular weeks?
2. Recalculate all the KPIs for this plan. How are they different from the KPI values from Task 1? Explain the differences.

Task 3: Promotion and capacity planning

It is clear from the previous task that, given the forecast of a 30% increase in demand, there is currently not enough capacity to run the promotion. You have been put to the task to investigate three strategies for increasing production: adding new production lines, outsourcing production, or using transshipment between the DCs.

Strategy I: New production lines

Since the current capacity is not enough, it is natural to try to increase capacity. This can be done in two ways: firstly by allowing overtime on Sunday's as well (currently it is only on Saturdays), and secondly by upgrading the machines. Upgrading the machines will increase their speed and therefore increase capacity. However, it is not clear at this point what kind of upgrades are possible and how much it will cost. Part of your analysis will be to see what speedups are needed and what the potential savings will be.

1. Assume the company is only allowed to use six hours of overtime on Sundays. Adapt the model from Task 2 (including the promotions) to include this possibility, create a new production plan, and answer the following questions:

   (a) In which weeks are overtime used on Sunday, and how much?

   (b) Recalculate all the KPIs for this plan. How are they different from the KPI values from Task 2? Explain the differences.

2. Investigate what the minimum speedup of the machines are that allows all demand to be satisfied. Adapt the model from Task 2 (including the promotions) to include the speedup and create a new production plan using this new speed. Recalculate all the KPIs for this plan. How are they different from the KPI values from Task 2? Explain the differences.

Strategy II: Outsourcing

The second option is to use a contract manufacturer to produce the amount that the current capacity cannot satisfy. There is a contract manufacturer that offers the following per unit prices for the three products (which includes delivery to the DCs):

Product 1	Product 2	Product 3
250	300	300

In addition, the contract manufacturer has a minimum production quantity — quantities lower than this would not be worthwhile for them to produce. Even though you are free to choose the week and region for which they should produce, they demand that at least 10 000 units be bought from them (per product) in whichever week you choose to outsource production to them.

Adapt the model from Task 2 (including the promotions) to also include the possibility of outsourcing. Use the model to create a new production plan, and answer the following questions:

1. In which weeks should the company outsource, and how much?

2. Recalculate all the KPIs for this plan. How are they different from the KPI values from Task 2? Explain the differences.

Strategy III: Transshipment

The final alternative is to use transshipments between DCs. This means that products can be shipped (within a single week) from one DC to another. So production in the plant of one region can satisfy the demand at a DC in another region, perhaps to help out when there are capacity problems in the other region's plant.

You may assume that transshipments can be done in a single day, so that transportation time is negligible on the level of weekly planning. The cost per unit transported (of any product) quoted by the carrier is as follows:

	DC1	DC2	DC3
DC1		0.5	1
DC2	0.5		1.2
DC3	1	1.2	

Adapt the model from Task 2 (including the promotions) to also include the possibility of transshipment between warehouses in each week. Use the model to create a new production plan, and answer the following questions:

1. Is transshipment used in the new production plan? Where and when?
2. Recalculate all the KPIs for this plan. How are they different from the KPI values from Task 2? Explain the differences.

Task 4: Final recommendation

Discuss the pros and cons of each of the three strategies from Task 3 and make a final recommendation.