Project 2

Static and Dynamic Optimization

2019-11-03 18.06

The Chain. The objective in this assignment is primely to apply dynamic optimization on a specific problem rather than determining the shape of a suspended chain or cable. (This can be found in just about any text book in mechanics). In the report, it is important to give the results and an interpretation of those, but certainly also to describe the chosen method, its background and assumptions. Notice, question 4 in part 1 can be postponed and solved on a later stage (when the topic has been lectured).

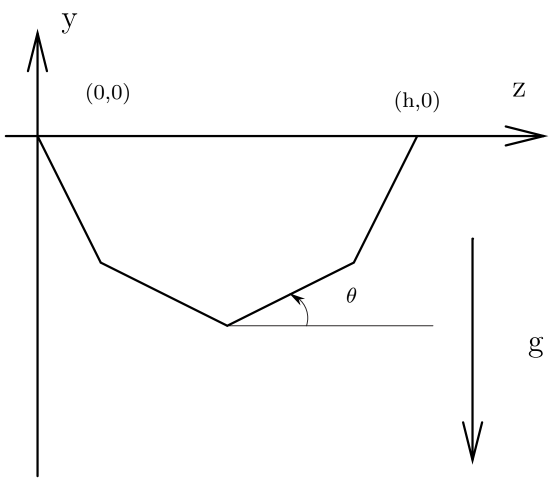

Figure 1. A (very) simple suspended chain with 4 elements

A chain, which is suspended in two points $(0, 0)$, $(h, 0)$ have a total length L, have the mass M and consists of N elements. For a start let $h = 6, L = 10, M = 14$ and $N = 6$. (Let these quantities be variable in your codes. Then you are able to change them easily.) Furthermore, let the gravity be $g = 9.81$. The elements have equal length and mass. Let

$$l = \frac{L}{N} \qquad m = \frac{M}{N}$$

be the length and mass of each elements, respectively. The shape of the chain (in the equilibrium) is characterized by the position of the end point (z_i, y_i) of each elements. If i is the element number and u_i and v_i is the difference between start and end point of the chain element in the z- and $y-$direction, respectively, then

$$\begin{bmatrix} z \\ y \end{bmatrix}_{i+1} = \begin{bmatrix} z \\ y \end{bmatrix}_i + \begin{bmatrix} u \\ v \end{bmatrix}_i \qquad i = 0,\ 1,\ ...\ N-1$$

Due to the length of each element we have the constraints:

$$u_i^2 + v_i^2 = l^2 \tag{1}$$

and since the end point are fixed:

$$\begin{bmatrix} z \\ y \end{bmatrix}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} z \\ y \end{bmatrix}_N = \begin{bmatrix} h \\ 0 \end{bmatrix}$$

In steady state the potential energy is minimal. The (steady state) potential energy can be written as:

$$J = \sum_{i=0}^{N-1} \frac{1}{2} mg(y_i + y_{i+1}) \tag{2}$$

Notice, this expression do not follow the standard formulation of a performance index, but can easily be rephrased to be in the standard form.

Question 1 Rewrite the potential energy from (2) in a form which match the standard form. □

The constraints in (1) can be respected by introducing the angle between the z-axis and the chain elements and using the fact

$$\begin{bmatrix} u \\ v \end{bmatrix}_i = l \begin{bmatrix} cos(\theta_i) \\ sin(\theta_i) \end{bmatrix} \tag{3}$$

meet the constraints (1). In this case the problem (for each elements) has been reduced from being a two dimensional constrained problem to an one dimensional unconstrained problem.

Question 2 Solve the problem by using the relation in (3) and plot the shape of the chain (for $N = 6$). Determine the value of the costate vector at the beginning of the chain. Increase the number of elements in the chain to e.g. $N = 100$ and plot the chain again. Also determine the value of the costate vector in the beginning of the chain. □

Question 3 Determine the vertical force in the origin ($i = 0$). Compare this with the costate at the origin. Discuss your observations. Give a qualified guess on the sign of horizontal force in the origin. □

Question 4 Solve the problem e.g. by using Pontryagins principle, i.e. by using the constraint in (1) directly (and not (3)). □

In the next question you can use (3) for satisfying the constraints in (1).

Question 5 Now consider the chain as two symmetric half chains. Utilize the symmetry in the problem to reduce the investigation to a problem just involving the one half of the chain. For simplicity assume N is even (i.e. $N = 2n$). *Notice that some of the boundary conditions might have changed.* □

Now, the chain is substituted by a wire and the problem becomes a continuous problem. Let $\rho = M/L$ and s the distance along the wire. The positions along the wire obey

$$\frac{d}{ds}\begin{bmatrix} z_s \\ y_s \end{bmatrix} = \begin{bmatrix} cos(\theta_s) \\ sin(\theta_s) \end{bmatrix}$$

with

$$\begin{bmatrix} z \\ y \end{bmatrix}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} z \\ y \end{bmatrix}_L = \begin{bmatrix} h \\ 0 \end{bmatrix}$$

The potential energy (in equilibrium) is

$$J = \int_0^L \rho g y_s ds$$

Question 6 Formulate the problem as a continuous problem and solve it (e.g. analytically or numerically). Plot the shape of the wire and discuss your observations. Determine the value of the costate vector in origin. Investigate the variation of the Hamiltonian function (i.e. the variation of the Hamiltonian as function of s). Plot the function as function of s and explain what you see - and why. □