Exercise 4 : The News Boy problem (single-stage) solved using Benders

The News Boy problem is the classic stochastic programming problem, which is also part of the course "Mathematical Programming Modelling" (42112). Here we will only implement Benders for the single-stage problem. Below follows the data etc. for the basic single-stage problem.

The newsboy buys news-papers in the morning and sells them during the day. He earns money by charging a higher price p than the purchase price c. He can return the non-sold newspapers as scrap at a lower price h, see below:

The primary data of the problem are as follows:

$c = 20$ purchase price of a newspaper (cost for the newsboy),

$p = 70$ selling price of a newspaper (revenue for the newsboy),

$h = 10$ scrap value of a leftover (surplus) newspaper that is sold for recycling.

The complication of the problem stems from the fact that the demand for newspapers is *uncertain* (i.e., it is not known with certainty by the newsboy at the time he has to decide the number of newspapers that he should order in the morning). Hence, his decision of how many newspapers to buy y in the morning, is taken in the face of uncertainty about the demand level that will actually materialize during the day. Consequently, the total revenue (and thus the total net profit) is not deterministically known to the decision maker.

From his past experience, the newsboy has estimated that the uncertain demand is properly described by the following discrete probability distribution (scenario set).

Scenario (s)	1	2	3	4	5	6	7	8	9	10
Demand (d_s)	12	14	16	18	20	22	24	26	28	30
Probability (π_s)	0.05	0.10	0.10	0.10	0.15	0.15	0.10	0.10	0.10	0.05

Table 1: 10 scenarios for the number of daily customers

Hence, the demand can take one of ten possible values d_s from the discrete scenario set $S = \{s \ : \ s = 1, \ldots, N\}; \ N = 10$, with associated probabilities

$\pi_s > 0$; $\sum_{s=1}^{N} \pi_s = 1$. The mean value (expected value) of the demand is $\bar{d} = \sum_{s=1}^{N} \pi_s d_s = 21$. You can download the data in Julia format from campusnet, the file "DataNewsBoy.jl".

Todays exercise:

First: Create the basic MIP model, with two variables $y \in Z^+$ (the number of bought variables) and $x_s \in R+$ the number of sold newspapers in scenario s. Notice, that eventhough the newsboy sells an integer number of newspapers, formulating the problem with a continuous variable x_s automatically leads to integer values. It is **VERY** important to find the smallest possible model here, since a larger correct model may lead to a lot of extra work. Hence, you should make a model with just two constraints.

Second: Create the dual of the sub problem (the original problem, with fixed y variables). Get the dual variables by using getdual(¡constraint name¿)
Third: Formulate the Benders sub-problem, with fixed $\overline{y}$ variables. Notice, you can now check that the dualization is correct by comparing the duals with the model from the second question (for similar ybar variables)

Fourth: Using the Benders template, create the full Benders program. **NOTICE:** This time it is a maximization problem, the Benders_Template.jl works for minimization problems.

Fifth: Formulate the sub-problem *s*cenario dependent (i.e. one LP model for each scenario). Make a new version of the Benders algorithm, where the full sub-problem is solved sequentially.

Sixth: (optional): When you look at the scenario dependent sub-problem, can you replace the LP sub-problem with an algorithmic approach ?