42111:Static & Dynamic Optimization
Project One

This assignment is to be handed in via CampusNet no later than **Sunday October** $13^{\mathbf{th}}$**, 2019**. Needs to be completed individually. If you have worked together, note clearly in assignment. All reports need to be written individually.

Part A

You receive 315 euros in unexpected tax returns. You decide to spend the money on going out for dinner, and going out for drinks. A dinner will typacilly set you back 35 euros, a drink 5 euros. Your utility from consuming x dinners and y drinks can be described as:

$$U(x, y) = 8lnx + lny$$

The logarithmic function represents the diminishing utility of additional dinners and drinks to you. You want to maximize your utility. How many dinners and drinks will you plan to buy?
Choose one of the optimization methods from the course, and motivate your selection.
Follow the 4 step method to solve the problem.

Part B

There is a festival coming up and you decide to open a stall and sell home-made beer. Assume that you have 3 weeks of work at 40 hours a week available for preparation. You decide to allocate this time between making IPA, and making Lager. Your output of X bottles of IPA and Y bottles of Lager depends only on the hours of labor that you spend, in the following way:

$$X = 3\sqrt{L_x}, Y = 4\sqrt{L_y} \tag{1}$$

The root function is chosen because when doing the same job for a long time, people generally get bored and slow down.

1. Imagine you are certain to sell all your production for fixed prices $P_X = 5$ and $P_Y = 4$. How would you allocate your time between brewing IPA and brewing Lager to maximize your profits?
2. Instead, consider spending 3 weeks making the production just for your own consumption. Your utility can be described as:

$$U = 6 \ln X + 4 \ln Y \tag{2}$$

   How should you divide your time between making IPA and Lager to maximize your own utility?
3. Reflect on the meaning of the lagrangian multipliers in 1. and 2.

One of your friends, Carl, has heard about the market, and offered to help you out making Lager (only) in return for a cut of the total profit. Carl has never made beer before in his life. You would need to learn him, and supervise him. You need to invest 5 hours of your own time in training him, and next spend 10% of your own time supervising him for every hour he works. Carl would produce Lager at:

$$Y = 2\sqrt{L_c} \tag{3}$$

with L_c the hours you ask Carl to help out. If you indeed ask Carl to help out, you will loose $5 + 0.1L_c$ hours to invest in your own production. Carl cannot work more than 20 hours per week.

4. How many hours would you ask Carl to help you out to maximize total revenue? How would you divide your own time in producing Lager and IPA in this case? What is the maximum amount you can offer Carl for his services, where you would obtain the same revenue for yourself as without his help (part 1), ignoring production costs?

Part C

Someone wants to divide their savings in three separate funds that have expected returns of 2%, 2% and 5%. The goal is to return at least 3%, while minimizing risk. The risk function for an investment in this combination of funds is:

$$100x_1^2 + 25x_2^2 + 150x_1x_2 + 599x_3^2 + 200x_2x_3$$

where x_i is the proportion of the savings in fund i. Determine the proportions that should be invested in each fund. Would it help if you are allowed to go short, that is, if the x_i are allowed to be negative?
Choose one of the optimization methods from the course, and motivate your selection.

Part D

Your family owns a private forest of a total of 30 acres, and managing it is a hobby that you, your uncle, and your cousin share. You want to maximize the profit you generate together, but you also have your personal preferences in which tasks to do.

The forest contains 10 acres of christmas trees, 15 acres available for hunting, and 5 acres on which there exists a cabin in the woods that can be rented out for events. An acre consists of maximally 400 christmas trees; hunting is allowed 10 weeks a year with a maximum of 3 trips per week, and you can rent out the cabin every weekend in high season, about 20 weekends a year, where a weekend takes 2 workdays.

You, your uncle and your cousin share the tasks of managing the forests and can respectively spend 40, 50, and 60 days a year in managing the forest. You enjoy the christmastree management most, and want to spend at least half of your time working for the christmas tree management. Your uncle cannot do any christmastree work because of back issues, and loves to hunt. He is really an expert in hunting and to remain having that expertise he wants to have at least 20 hunting trips per year, and spend not more than a third of his time in managing the cabin. Your cousin enjoys the cabin management most, and wants to spend at least three times as much time in managing the cabin events as you and your uncle do together.

A chirstmastree will return 20 dollars, a hunting trip will return 750 dollars, and renting out the cabin for a weekend event (weddings, parties) will return 2000 dollar. You can harvest 50 christmastrees per day of work spend on managing the christmastrees.

1. Together you want to maximize total revenue. Write down the Linear Programming Model for the above problem.
2. Describe an iterative method that can solve this problem to optimality, and write down the initial step and the following first iteration.
3. Formulate the dual of this problem.
4. Verify whether the below solution is optimal, and explain your approach.

	Number of days		
Who	Trees	Hunting	Cabin
You	40	0	0
Uncle	0	20	5
Cousin	20	10	15

5. At a big family meeting about the forest management, there is a heated discussion about how to increase profits. One family member argues that the current division of the forest is sub-optimal, and more revenue could be generated by re-distributing the acres over christmastrees, hunting grounds, and cabin area, where an additional allocation of acres would increase the possibilities of exploiting that activity further.

   Another family member argues that not the assignment of acres is the problem, but rather more people are needed to invest time in forest management. For example, you have an aunt interested to join the team in organizing hunting trips as well.

   Investigate both claims assuming the solution in (x) is the optimal solution, and present both your approach and your conclusion, specifically:

   - Could redistribution of ground lead to a higher profit, without increasing labour? If yes, which activities should receive more acres, and which less?
   - Would additional labour lead to a higher profit, without redistribution of acres? If yes, are there any restrictions on the preferences of the person offering the labour? E.g. will your aunt joining the forest management team increase profits as much as someone joining willing to do any job?

Part E

Consider:

$$\max 2x_1 + 3x_2 \tag{4}$$

s.t.

$$\begin{aligned} 2x_1 + x_2 &\leq 16 && (5)\\ x_1 - x_2 &\leq 2 && (6)\\ -x_1 + 2x_2 &\geq 4 && (7)\\ x_2 &\leq 6 && (8)\\ x_1, x_2 &\geq 0 && (9) \end{aligned}$$

1. Display the feasible region
2. Solve the above optimization problem graphically

3. Formulate a new constraint that changes the feasibility region in such a way that the optimal solution also changes. Motivate how you came to this constraint, and display the result graphically (including an indication of the new optimum)
4. Explain why it would be generally harder to solve the above problem in case the objective function would be non-linear. How could you still solve the above problem if the solution was non-linear? Only describe the approach, you do not need to resolve the problem.

1 Part F

$$\max x_1^2 + 6x_2 + 4x_3^2 \tag{10}$$

s.t.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &\leq 10 \qquad (11) \\ x_1, x_3 &\geq 0 \qquad (12) \\ x_2 \in (0, 1)& \qquad (13) \end{aligned}$$

1. Apply the Lagrangian Dual method to solve the above optimization problem
2. Are there other method(s) discussed in the course that you could use to solve the above problem? Motivate your answer by describing at least one concrete example of a method that can (if yes) or cannot (if no) be used. You do not need to resolve the problem.

Part G

Starting from the trial solution $(1, 1)$, apply two iterations of the gradient search procedure to obtain an approximate solution for the following problem:

$$\text{Maximize: } f(\boldsymbol{x}) = 4x + 2xy - y - 4x^2 - y^2,$$

then solve $\nabla f(\boldsymbol{x}) = \mathbf{0}$ to obtain the exact solution.