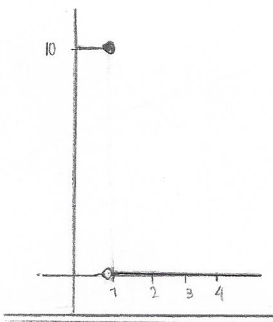
$$V(t)=\begin{cases}10 & ; 0 \le x \le 1\\ 0 & ; 1 < x \le 4\end{cases}$$



$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos(n\omega x)+b_n \operatorname{sen}(n\omega x)\right)$$

ω = Frecuencia Fundamental

$\omega = 2\pi/T$

T = Periodo

$$a_0=\frac{1}{T/2}\int_T f(x)\,dx \qquad a_n=\frac{1}{T/2}\int_T f(x)\cos(n\omega x)\,dx$$

$$b_n=\frac{1}{T/2}\int_T f(x)\operatorname{sen}(n\omega)\,dx$$

$$T=4$$
$$\omega=\frac{2\pi}{T}=\frac{2\pi}{4}=\frac{\pi}{2}$$

$$a_0=\frac{1}{2}\int_0^4 f(x)\,dx=\frac{1}{2}\left[\int_0^1 10\,dx+\cancel{\int_1^{10} 0\,dx}^{\,0}\right]$$

$$a_0=\frac{1}{2}\left[10x\right]_0^1=5x\Big|_0^1=5(1-0)=5$$

$$a_n=\frac{1}{2}\int_0^{10} f(x)\cos\left(\frac{\pi n x}{2}\right)dx=\frac{1}{2}\left[\int_0^1 10\cdot\cos\left(\frac{\pi n x}{2}\right)dx+\cancel{\int_1^{10} 0\cdot\cos\left(\frac{\pi n x}{2}\right)dx}^{\,0}\right]=5\int_0^1\cos\left(\frac{\pi n x}{2}\right)dx$$

Integración por sustitución

$$\int\cos\left(\frac{\pi n x}{2}\right)dx=\int\frac{2\cos(u)}{n\pi}\,du \qquad u=\frac{\pi n x}{2}$$

$$=\frac{2}{n}\int\cos(u)\,du=\frac{2}{n\pi}\operatorname{sen}(u)=\frac{2}{\pi n}\operatorname{sen}\left(\frac{n\pi x}{2}\right)$$

$$\int\cos\left(\frac{\pi n x}{2}\right)dx=\frac{2}{\pi n}\operatorname{sen}\left(\frac{n\pi x}{2}\right)+C$$

$$a_n=5\left(\frac{2}{n\pi}\operatorname{sen}\left(\frac{n\pi(x)}{2}\right)\right)\Big|_0^1=\frac{10}{n\pi}\left[\operatorname{sen}\left(\frac{n\pi}{2}\right)-\cancel{\operatorname{sen}(0)}^{\,0}\right]=\frac{10}{n\pi}\operatorname{sen}\left(\frac{n\pi}{2}\right)$$

$$b_n=\frac{1}{2}\int_0^{10} f(x)\operatorname{sen}\left(\frac{n\pi x}{2}\right)dx=\frac{1}{2}\left[\int_0^1 10\cdot\operatorname{sen}\left(\frac{n\pi x}{2}\right)dx+\cancel{\int_1^{10} 0\cdot\operatorname{sen}\left(\frac{n\pi x}{2}\right)dx}^{\,0}\right]=5\int_0^1\operatorname{sen}\left(\frac{\pi n x}{2}\right)dx$$

Integración por sustitución

$$\int\operatorname{sen}\left(\frac{\pi n x}{2}\right)dx=\int\frac{2\operatorname{sen}(u)}{n\pi}\,du \qquad u=\frac{\pi n x}{2}$$

$$=\frac{2}{n\pi}\int\operatorname{sen}(u)\,du=\frac{2}{n\pi}(-\cos(u))=-\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)$$

$$\int\operatorname{sen}\left(\frac{\pi n x}{2}\right)dx=-\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)+C$$

$$b_n=5\left[-\frac{2}{\pi n}\cos\left(\frac{n\pi(x)}{2}\right)\right]_0^1=\frac{10}{n\pi}\left[-\cos\left(\frac{n\pi}{2}\right)-(-\cos(0))\right]=\frac{10}{n\pi}\left(1-\cos\left(\frac{n\pi}{2}\right)\right)$$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos(n\omega x)+b_n\operatorname{sen}(n\omega x)\right)$$

$$f(x)=\frac{5}{2}+\sum_{n=1}^{\infty}\left(\frac{10}{n\pi}\operatorname{sen}\left(\frac{n\pi}{2}\right)\cdot\cos\left(\frac{n\pi x}{2}\right)+\frac{10}{n\pi}\left(1-\cos\left(\frac{n\pi}{2}\right)\right)\cdot\operatorname{sen}\left(\frac{n\pi x}{2}\right)\right)$$

$$=\frac{5}{2}\sum_{n=1}^{\infty}\left[\frac{10}{n\pi}\left(\operatorname{sen}\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{2}\right)+\left(1-\cos\left(\frac{n\pi}{2}\right)\right)\cdot\operatorname{sen}\left(\frac{n\pi x}{2}\right)\right)\right.$$

★ Calcular la serie de fourier correspondiente a V(t) hasta n=3

$$f(x) = 5 + \sum_{n=1}^{3} \left[\frac{10}{n\pi} \left(sen\left(\frac{n\pi}{2}\right) \cos\left(\frac{n\pi x}{2}\right) + \left(1 - \cos\left(\frac{n\pi}{2}\right)\right) sen\left(\frac{n\pi x}{2}\right) \right) \right]$$

$$f(x) = 5 + \frac{10}{\pi}\left(\cancelto{1}{sen\left(\frac{\pi}{2}\right)} \cos\left(\frac{\pi x}{2}\right) + \left(1 - \cancelto{0}{\cos\left(\frac{\pi}{2}\right)}\right) sen\left(\frac{\pi x}{2}\right)\right)$$

$$+ \frac{10}{2\pi}\left(\cancelto{0}{sen\left(\frac{2\pi}{2}\right)} \cos\left(\frac{2\pi x}{2}\right) + \left(1 - \cancelto{-1}{\cos\left(\frac{2\pi}{2}\right)}\right) sen\left(\frac{2\pi x}{2}\right)\right)$$

$$+ \frac{10}{3\pi}\left(\cancelto{-1}{sen\left(\frac{3\pi}{2}\right)} \cos\left(\frac{3\pi x}{2}\right) + \left(1 - \cancelto{0}{\cos\left(\frac{3\pi}{2}\right)}\right) sen\left(\frac{3\pi x}{2}\right)\right)$$

$$f(x) = 5 + \frac{10}{\pi}\left(\cos\left(\frac{\pi x}{2}\right) + sen\left(\frac{\pi x}{2}\right)\right) + \frac{5}{\pi}\left(1 - (-1)\right) sen(\pi x) + \frac{10}{3\pi}\left(- \cos\left(\frac{3\pi x}{2}\right) + sen\left(\frac{3\pi x}{2}\right)\right)$$

$$= 5 + \frac{10}{\pi}\left(\cos\left(\frac{\pi x}{2}\right) + sen\left(\frac{\pi x}{2}\right)\right) + \frac{5}{\pi} \cdot 2 sen(\pi x) + \frac{10}{3\pi}\left(sen\left(\frac{3\pi x}{2}\right) - \cos\left(\frac{3\pi x}{2}\right)\right)$$