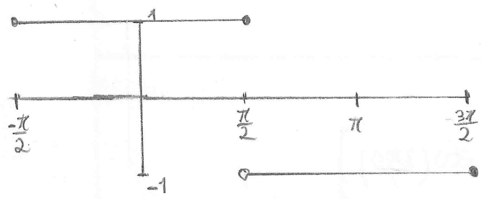
14/05/2019 - 9t2p

$$f(x) = \begin{cases} 1 & ; \ -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ -1 & ; \ \frac{\pi}{2} < x \le \frac{3\pi}{2} \end{cases}$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega x) + b_n \operatorname{sen}(n\omega x) \right)$$

$$a_0 = \frac{1}{T/2} \int_T f(x)\,dx \qquad a_n = \frac{1}{T/2} \int_T f(x) \cos(n\omega x)\,dx$$

$$b_n = \frac{1}{T/2} \int_T f(x) \operatorname{sen}(n\omega x)\,dx$$

ω = Frecuencia Fundamental

$\omega = \frac{2\pi}{T}$

T = Periodo

$T = 2\pi$

$\omega = \frac{2\pi}{T} = 1$

$$a_0 = \frac{1}{2\pi/2} \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x)\,dx = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} 1\,dx + \frac{1}{\pi} \int_{\pi/2}^{\frac{3\pi}{2}} (-1)\,dx$$

$$a_0 = \frac{1}{\pi} \left[x \Big|_{-\pi/2}^{\pi/2} - x \Big|_{\pi/2}^{3\pi/2} \right] = \frac{1}{\pi} \left[\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right) - \left(\frac{3\pi}{2} - \frac{\pi}{2} \right) \right] = \frac{1}{\pi} \left[\frac{\pi}{2} + \frac{\pi}{2} - \frac{3\pi}{2} + \frac{\pi}{2} \right] = \frac{1}{\pi} \left[\frac{3\pi}{2} - \frac{3\pi}{2} \right] = 0$$

$$a_n = \frac{1}{2\pi/2} \int_{-\pi/2}^{3\pi/2} f(x) \cos(n(1)x)\,dx = \frac{1}{\pi} \left[\int_{-\pi/2}^{\pi/2} (1) \cos(nx)\,dx + \int_{\pi/2}^{3\pi/2} (-1) \cos(nx)\,dx \right]$$

Integración por sustitución

$$\int \cos(nx)\,dx = \int \frac{\cos(u)\,du}{n} \ ; \quad u = nx$$

$$= \frac{1}{n} \int \cos(u)\,dx = \frac{1}{n} \operatorname{sen}(u) = \frac{1}{n} \operatorname{sen}(nx) + C$$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \cos(nx)\,dx = \frac{1}{n} \left[\operatorname{sen}\left(\frac{n\pi}{2}\right) - \operatorname{sen}\left(n\left(-\frac{\pi}{2}\right)\right) \right]$$

Puesto que sen (x) es una función par:
$\operatorname{sen}(-x) = -\operatorname{sen}(x)$

$$= \frac{1}{n} \left[\operatorname{sen}\left(\frac{n\pi}{2}\right) - \left(-\operatorname{sen}\left(\frac{n\pi}{2}\right)\right) \right] = \frac{2}{n} \operatorname{sen}\left(\frac{n\pi}{2}\right)$$

$$\int_{\pi/2}^{\frac{3\pi}{2}} \cos(nx)\,dx = \frac{1}{n} \left[\operatorname{sen}\left(n\left[\frac{3\pi}{2}\right]\right) - \operatorname{sen}\left(n\frac{\pi}{2}\right) \right] = \frac{1}{n} \left[\operatorname{sen}\left(\frac{3\pi n}{2}\right) - \operatorname{sen}\left(\frac{\pi n}{2}\right) \right]$$

$$\int_{-\pi/2}^{\pi/2} \cos(nx)\,dx - \int_{\pi/2}^{3\pi/2} \cos(nx)\,dx = \frac{2}{n} \operatorname{sen}\left(\frac{n\pi}{2}\right) - \frac{1}{n} \left[\operatorname{sen}\left(\frac{3\pi n}{2}\right) - \operatorname{sen}\left(\frac{\pi n}{2}\right) \right]$$

$$= \frac{1}{n} \left[2\operatorname{sen}\left(\frac{n\pi}{2}\right) - \operatorname{sen}\left(\frac{3\pi n}{2}\right) + \operatorname{sen}\left(\frac{\pi n}{2}\right) \right]$$

$$= \frac{1}{n} \left[3\operatorname{sen}\left(\frac{n\pi}{2}\right) - \operatorname{sen}\left(\frac{3\pi n}{2}\right) \right]$$

$$a_n = \frac{1}{n\pi} \left[3 \cdot \operatorname{sen}\left(\frac{n\pi}{2}\right) - \operatorname{sen}\left(\frac{3\pi n}{2}\right) \right]$$

$$b_n = \frac{1}{2\pi/2} \int_{-\pi/2}^{3\pi/2} f(x) \cos(n(1)x)\,dx = \frac{1}{\pi} \left[\int_{-\pi/2}^{\pi/2} (1) \operatorname{sen}(nx)\,dx + \int_{\pi/2}^{3\pi/2} (-1) \operatorname{sen}(nx)\,dx \right]$$

Integración por sustitución

$$\int \operatorname{sen}(nx)\,dx = \int \frac{\operatorname{sen}(u)}{n}\,du \qquad u = dx$$

$$\frac{1}{n} \int \operatorname{sen}(u)\,du = \frac{1}{n} \left(-\cos(u) \right) = -\frac{1}{n} \cos(u) = -\frac{1}{n} \cos(nx) + C$$

$$\int_{-\pi/2}^{\pi/2} \text{sen}(nx)\,dx = \frac{1}{n}\left[-\cos\left(\frac{n\pi}{2}\right) - \left(-\cos\left(n\left(-\frac{\pi}{2}\right)\right)\right)\right]$$

// Puesto que $\cos(x)$ es una función par: $\cos(-x) = \cos(x)$

$$= \frac{1}{n}\cdot\left[-\cos\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{2}\right)\right] = 0$$

$$\int_{\pi/2}^{3\pi/2} \text{sen}(nx)\,dx = \frac{1}{n}\left[-\cos\left(\frac{3\pi n}{2}\right) - \left(-\cos\left(\frac{\pi n}{2}\right)\right)\right] = \frac{1}{n}\left[\cos\left(\frac{\pi n}{2}\right) - \cos\left(\frac{3\pi n}{2}\right)\right]$$

$$b_n = \frac{1}{\pi n}\left[\cos\left(\frac{\pi n}{2}\right) - \cos\left(\frac{3\pi n}{2}\right)\right]$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega x) + b_n \,\text{sen}(n\omega x)\right)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \,\text{sen}(nx)\right)$$

$$f(x) = \frac{0}{2} + \sum_{n=1}^{\infty}\left(\frac{1}{n\pi}\left[3\cdot\text{sen}\left(\frac{\pi n}{2}\right) - \text{sen}\left(\frac{3\pi n}{2}\right)\right]\cos(nx) + \frac{1}{n}\left[\cos\left(\frac{\pi n}{2}\right) - \cos\left(\frac{3\pi n}{2}\right)\right]\text{sen}(nx)\right)$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{1}{n\pi}\left[3\cdot\text{sen}\left(\frac{\pi n}{2}\right) - \text{sen}\left(\frac{3\pi n}{2}\right)\right]\cos(nx) + \frac{1}{n}\left[\cos\left(\frac{\pi n}{2}\right) - \cos\left(\frac{3\pi n}{2}\right)\right]\text{sen}(nx)\right)$$